

Controlling Balancer and Steering for Bicycle Stabilization

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Abstract—In this paper, we propose a control algorithm for stabilizing of an unmanned bicycle at zero speed by using a nonlinear control based on an output-zeroing controller. The simplified model of the bicycle with the balancer is derived from Lagrangian and nonholonomic constraints with respect to translation and rotation relative to the ground plane. We derived a controller using a steering torque and a balancer torque to stabilize the bicycle at zero linear velocity. The output-zeroing controller is designed by using the angular momentum of two-link system and by adding the effect of steering angle to the derivative of the angular momentum. Numerical simulation and experimental results are shown to verify the effectiveness of the proposed control strategy.

Index Terms—Balancing Control, Output-Zeroing, Autonomous Bicycle, Nonlinear System.

I. INTRODUCTION

Research on the stabilization of bicycles has gained momentum over the last decade in a number of robotic laboratories around the world. Modeling and control of bicycles became a popular topic for researchers in the latter half of the last century. The bicycle literature is comprehensively reviewed from a control theory perspective in [1], which also describes interesting bicycle-related experiments. But almost all of those papers are focusing on modeling and stabilizing the bicycle with the steering handlebar and the rear wheel. Getz [2] studied feedback control law for nonlinear, nonholonomic, nonminimum phase model of a two-wheeled bicycle with non-zero rear-wheel velocity. Defoort [3] proposed an innovative robust control strategy based on second order sliding mode control (SMC) for the stabilization of an autonomous bicycle. Yi [4] presented a trajectory tracking and balancing control for an autonomous motorcycle using only steering handlebar. Dynamic models of the motorcycle were developed from an existing modeling approach [2], modified by adding the bicycle caster angle, and the model can capture the steering effect on the vehicle tracking and balancing. In [5], the simplified dynamic model of a bicycle with a balancer was modeled by using Lagrange dynamic equations. Simulation study has been carried out to show the effectiveness of the proposed model. Yamakita [8], [9] utilized an input-output linearization method to design a trajectory tracking controller for an automatic bicycle. The control methods are designed independently for trajectory tracking and balancing. The proposed algorithm for that problem used an output function which is defined by an angular momentum and the new state function is controlled

to zero. In [7], an autonomous bicycle designed and balanced based on output-zeroing controller by using only a balancer to stabilize the bicycle. The performance of this controller was not so good when the mass of the bicycle is increased. Recently, Hwang [6] proposed a controller namely variable structure under-actuated control to balance an electrical bicycle. The balancing control was designed based on a steering handlebar and a balancer and it cannot track a given trajectory.

In this paper, we extended our balancing control from [7] by using both a steering handlebar and a balancer to stabilize the bicycle and we can enlarge the initial position of the bicycle. In order to see the effectiveness of the proposed control, we conduct some simulations and experiments by comparing the bicycle stabilization with only balancer, with only steering handlebar and with both. This paper is composed of six sections. In section II, we present a simplified dynamic model of the bicycle with the balancer. In section III, we discuss control system design for stabilizing the bicycle. Numerical simulation and experimental results are presented in section IV and V respectively. The conclusions are summarized in section VI.

II. BICYCLE DYNAMICS

In this paper, we use the simplified bicycle models that was developed in [5] and [7]. A detailed model of a bicycle is complex since the system has many degrees of freedom and the motions have constraints. The coordinate system used to analyze the bicycle is defined in Fig. 1. The details of the bicycle assumptions are presented in [5]. The bicycle and the balancer parameters were identified from an experimental setup and it is shown in Table I. The key parameters are:

- β Balancer angle
- ψ Steering shaft angle
- α Roll angle
- θ Yaw angle
- ϕ Front wheel direction angle
- v_r Rear wheel longitudinal velocity
- v_{\perp} Lateral velocity

In order to simplify calculation the dynamic model of the bicycle, we used the trajectory curvature σ as the virtual steering angle.

$$\sigma = \frac{\tan \phi}{b}.$$

We consider the generalized velocities \dot{x} , \dot{y} , $\dot{\theta}$, $\dot{\alpha}$, $\dot{\beta}$, and $\dot{\sigma}$ such that it will be convenient when we introduce the constraints. The generalized velocities of the bicycle with the balancer are partitioned as $\dot{r} = [\dot{\alpha}, v_r, \dot{\beta}, \dot{\sigma}]^T$

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TABLE I
THE BICYCLE AND CONTROL PARAMETERS

Parameter descriptions	Parameters	Value
Bicycle mass	m	45 kg
Height of the bicycle center of mass	h	0.45 m
Distance between ground and balancer	l_1	0.81 m
Bicycle wheelbase	b	1.06 m
Distance between rear wheel and COG	c	0.5 m
Bicycle head angle	η	65°
Bicycle trail	Δ	0.1 m
Moment inertia of steering mechanism	J_s	0.35 kgm ²
Balancer mass	m_b	13.2 kg
Height of the balancer center of mass	h_b	0.29 m
Moment inertia of balancer	I_b	0.22 kgm ²

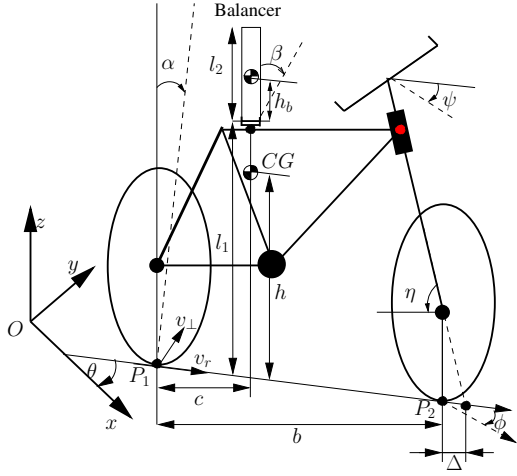


Fig. 1. Coordinate system of the bicycle with the balancer.

and $\dot{s} = [\dot{\theta}, v_{\perp}]^T$. In these velocity coordinates, the nonholonomic constraints associated with the front and the rear wheels which are assumed to roll without slipping, are expressed simply by $\dot{s} + A(r, s)\dot{r} = 0$ or

$$\begin{bmatrix} \dot{\theta} \\ v_{\perp} \end{bmatrix} + \begin{bmatrix} 0 & -\sigma & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\alpha} & v_r & \dot{\beta} & \dot{\sigma} \end{bmatrix}^T = 0. \quad (1)$$

From [5], we can obtain the bicycle dynamics with nonholonomic constraints as

$$M\ddot{q} = K + Bu, \quad \text{or} \quad \begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 \\ M_{21} & M_{22} & 0 & 0 \\ M_{31} & 0 & M_{33} & 0 \\ 0 & 0 & 0 & M_{44} \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \dot{v}_r \\ \ddot{\beta} \\ \ddot{\sigma} \end{bmatrix} = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} \quad (2)$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_r \\ \tau_b \\ \tau_{\sigma} \end{bmatrix},$$

where

F_r is the reaction force at the rear wheel, τ_b is the balancer torque input,

τ_{σ} is the steering torque input, and

$$\begin{aligned} M_{11} &= I_b + mh^2 + m_b(h_b^2 + l_1^2) + 2h_b l_1 m_b \cos(\beta), \\ M_{12} &= M_{21} = -chm\sigma \cos \alpha, \\ M_{13} &= M_{31} = I_b + h_b m_b (h_b + l_1 \cos \beta), \\ M_{22} &= m(c^2 \sigma^2 + (1 + h\sigma \sin \alpha)^2) + \\ &\quad m_b(1 + l_1 \sigma \sin \alpha + h_b \sigma \sin(\alpha + \beta))^2, \\ M_{33} &= I_b + h_b^2 m_b, \\ M_{44} &= \frac{J_s b^2}{\sin^2 \eta (1 + b^2 \sigma^2)^2}, \\ K_1 &= cmg\Delta \sigma \sin \eta \cos \alpha + hm(g \sin \alpha + v_r(\sigma v_r(1 + \\ &\quad h\sigma \sin \alpha) + c\dot{\sigma}) \cos \alpha) + m_b(g(l_1 \sin \alpha + \\ &\quad h_b \sin(\alpha + \beta)) + \sigma v_r^2(l_1(1 + l_1 \sigma \sin \alpha) \cos \alpha + \\ &\quad h_b(1 + h_b \sigma \sin(\alpha + \beta)) \cos(\alpha + \beta) + \\ &\quad h_b l_1 \sigma \sin(2\alpha + \beta)) + h_b l_1 \dot{\beta}(2\dot{\alpha} + \dot{\beta}) \sin \beta), \\ K_2 &= -m(h(c\sigma \dot{\alpha}^2 + v_r \dot{\sigma}(1 + h\sigma \sin \alpha)) \sin \alpha + \\ &\quad \sigma v_r(2h\dot{\alpha}(1 + h\sigma \sin \alpha) \cos \alpha + c^2 \dot{\sigma}) \\ &\quad - 2ch\dot{\alpha} \dot{\sigma} \cos \alpha) - m_b(2h_b \sigma v_r((\dot{\beta} + \dot{\alpha})(1 \\ &\quad + h_b \sigma \sin(\alpha + \beta)) + l_1 \dot{\sigma} \sin \alpha) \cos(\alpha + \beta) \\ &\quad + \sigma v_r \dot{\sigma}((l_1 \sin \alpha + h_b \sin(\alpha + \beta))^2) \\ &\quad + h_b v_r \dot{\sigma} \sin(\alpha + \beta) + l_1 v_r \dot{\sigma} \sin \alpha + 2l_1 \sigma v_r \dot{\alpha}((1 \\ &\quad + l_1 \sigma \sin \alpha) \cos \alpha + h_b \sigma \sin(\alpha + \beta))), \\ K_3 &= h_b m_b(-l_1 \dot{\alpha}^2 \sin \beta + g \sin(\alpha + \beta) + \\ &\quad v_r^2 \sigma \cos(\alpha + \beta)(1 + \sigma(l_1 \sin \alpha + h_b \sin(\alpha + \beta))), \\ K_4 &= c\Delta gm \sin \eta \sin \alpha + \frac{2b^4 J_s \sigma \dot{\sigma}^3}{\sin^2 \eta (1 + b^2 \sigma^2)^3}. \end{aligned}$$

In order to fix the balancer joint or handle joint, we introduce a nonholonomic constraint for the joints and it is represented as

$$J\ddot{q} = 0. \quad (3)$$

where J is a constraint Jacobi matrix which is switched according to the joint situations. The equation of motion become

$$\begin{bmatrix} M & J^T \\ J & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} Bu + K \\ 0 \end{bmatrix}. \quad (4)$$

where λ is the corresponding Lagrange multiplier.

III. CONTROL ALGORITHM

The basic idea is that an output function is defined so that the relative degree from input to the output becomes 3 and the zero dynamics becomes stable. Then, the output-zeroing controller is designed. In this case, a new state is defined then the new output function is easily determined since the angular momentum is integrable for 2 D.O.F. system.

A. Model of two-link system

By projecting the motion of the balancer on $X-Z$ plane, the system can be considered as a two-link system. In the two-link model, the bicycle body and steering handlebar consist of the first link and the balancer is considered as

the second link. The control torque for the system is only applied to the second joint of the balancer. We can find the angular momentum L from the first row of equation (2) as

$$\begin{aligned} L &= M_{11}\dot{\alpha} + M_{13}\dot{\beta} \\ &= (d_1 + d_3 + 2d_2 \cos \beta) \dot{\alpha} + (d_3 + d_2 \cos \beta) \dot{\beta}, \end{aligned} \quad (5)$$

where

$$\begin{aligned} d_1 &= mh^2 + m_b l_1^2, \quad d_2 = m_b l_1 h_b, \\ d_3 &= m_b h_b^2 + I_b. \end{aligned}$$

and it can be easily shown that the time derivative of L just contains a gravity term and it is calculated as

$$\dot{L} = e_1 \sigma \cos \alpha + e_2 \sin \alpha + e_3 \sin(\alpha + \beta), \quad (6)$$

where

$$\begin{aligned} e_1 &= cmg\Delta \sin \eta, \quad e_2 = g(mh + m_b l_1), \\ e_3 &= gm_b h_b. \end{aligned}$$

In (6), we can see that it contains a term σ which means that the steering actions affect to the momentum. Using the angular momentum expressed in (5), a new function p is defined to satisfy the following:

$$L = (d_1 + d_3 + 2d_2 \cos \beta) \dot{p}. \quad (7)$$

From the equation above, p can be determined as

$$\begin{aligned} p &= \alpha + \int_{\beta_0}^{\beta} \frac{d_3 + d_2 \cos \beta}{d_1 + d_3 + 2d_2 \cos \beta} d\beta - C \\ &= \alpha + w(\beta), \end{aligned} \quad (8)$$

where C is an integral constant and is determined as $p = 0$ when the system is at the upright position. Using L and p , a new coordinate function $q = (p, L, \beta, \dot{\beta}, \sigma, \dot{\sigma})$ can be represented as

$$\begin{bmatrix} \dot{p} \\ \dot{L} \\ \dot{\beta} \\ \ddot{\beta} \\ \dot{\sigma} \\ \ddot{\sigma} \end{bmatrix} = \begin{bmatrix} L/H(\beta) \\ G(p, \beta, \sigma) \\ \dot{\beta} \\ 0 \\ \dot{\sigma} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_b \\ u_\sigma \end{bmatrix}, \quad (9)$$

where

$$\begin{aligned} H(\beta) &:= d_1 + d_3 + 2d_2 \cos \beta, \\ G(p, \beta, \sigma) &:= e_1 \sigma \cos(p - w(\beta)) + e_2 \sin(p - w(\beta)) \\ &\quad + e_3 \sin(p - w(\beta) - \beta). \end{aligned}$$

and u_b, u_σ are new inputs defined as $u_b := \ddot{\beta}$, $u_\sigma := \ddot{\sigma}$ respectively.

B. Output-zeroing controller

For the system (9), an output function y is defined as

$$y = L + a_1 p, \quad (10)$$

where $a_1 > 0$ is a design parameter and it is controlled zero since

$$y = 0 \rightarrow \dot{p} = (-a_1/H)p \quad \text{and} \quad \dot{L} = a_1 \dot{p}.$$

this zero dynamics is stable. Since L and p have relative degree 3 to the control input, we can easily determine a control input which attains the dynamics of the output function. By taking a derivative of L , we have

$$y^{(3)} = L^{(3)} + a_1 p^{(3)}, \quad (11)$$

$$\ddot{L} = \frac{dG}{dt} = \frac{\partial G}{\partial p} \frac{L}{H} + \frac{\partial G}{\partial \beta} \dot{\beta} + \frac{\partial G}{\partial \sigma} \dot{\sigma}, \quad (12)$$

$$\begin{aligned} L^{(3)} &= \frac{d}{dt} \left(\frac{\partial G}{\partial p} \frac{L}{H} \right) + \frac{d}{dt} \left(\frac{\partial G}{\partial \beta} \right) \dot{\beta} + \frac{\partial G}{\partial \beta} u_b \\ &\quad + \frac{d}{dt} \left(\frac{\partial G}{\partial \sigma} \right) \dot{\sigma} + \frac{\partial G}{\partial \sigma} u_\sigma, \end{aligned} \quad (13)$$

$$\ddot{p} = \frac{G}{H} - \frac{(\partial H / \partial \beta) L}{H^2} \dot{\beta}, \quad (14)$$

$$\begin{aligned} p^{(3)} &= \frac{d}{dt} \left(\frac{G}{H} \right) - \frac{d}{dt} \left(\frac{(\partial H / \partial \beta) L}{H^2} \right) \dot{\beta} \\ &\quad - \frac{(\partial H / \partial \beta) L}{H^2} u_b. \end{aligned} \quad (15)$$

We can determine control inputs which attains the dynamics of the output function as

$$y^{(3)} + a_2 \ddot{y} + a_3 \dot{y} + a_4 y = 0, \quad (16)$$

and y converges to zero asymptotically if $a_2 > 0$, $a_3 > 0$, $a_4 > 0$ are determined appropriately (other robust stabilizing controls of y can be also used).

By rearranging the equations from (11) to (16) and we can put it in the form

$$A_1 u_b + A_2 u_\sigma = B, \quad (17)$$

where

$$A_1 = \frac{\partial G}{\partial \beta} - a_1 \frac{(\partial H / \partial \beta) L}{H^2},$$

$$A_2 = \frac{\partial G}{\partial \sigma},$$

$$\begin{aligned} B &= -\frac{d}{dt} \left(\frac{\partial G}{\partial p} \frac{L}{H} \right) - \frac{d}{dt} \left(\frac{\partial G}{\partial \beta} \right) \dot{\beta} - \frac{d}{dt} \left(\frac{\partial G}{\partial \sigma} \right) \dot{\sigma} \\ &\quad - a_1 \frac{d}{dt} \left(\frac{G}{H} \right) + a_1 \frac{d}{dt} \left(\frac{(\partial H / \partial \beta) L}{H^2} \right) \dot{\beta} - a_2 \ddot{y} \\ &\quad - a_3 \dot{y} - a_4 y. \end{aligned}$$

From (17), we can get

$$\begin{bmatrix} u_b \\ u_\sigma \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \frac{B}{A_1^2 + A_2^2}. \quad (18)$$

To simplify the control input, we linearize the parameters A_1 and A_2 by assuming α and β are small. We can get the parameters \tilde{A}_1 and \tilde{A}_2 as

$$\tilde{A}_1 = A_1|_{\alpha \approx 0, \beta \approx 0} = -e_1 \left(\frac{d_3 + d_2}{d_1 + 2d_2 + d_3} \right) + e_2 \left(\frac{d_1 + d_2}{d_1 + 2d_2 + d_3} \right), \quad (19)$$

$$\tilde{A}_2 = A_2|_{\alpha \approx 0, \beta \approx 0} = e_3. \quad (20)$$

Thus, \tilde{A}_1 and \tilde{A}_2 are constants then the final control law are

$$\begin{bmatrix} u_b \\ u_\sigma \end{bmatrix} = \begin{bmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{bmatrix} \frac{B}{\tilde{A}_1^2 + \tilde{A}_2^2}. \quad (21)$$

We can find the relationship between u_b and u_σ as

$$\frac{u_b}{u_\sigma} = \frac{\tilde{A}_1}{\tilde{A}_2}. \quad (22)$$

To scale the two control inputs, we introduce γ to be weighting gain and we can get

$$\frac{u_b}{u_\sigma} = \gamma \frac{\tilde{A}_1}{\tilde{A}_2}. \quad (23)$$

where $\gamma > 0$. Of course, we can determine (u_b, u_σ) to consider an optimal control with the equation (17).

IV. NUMERICAL SIMULATION

The simulation is conducted on an Intel Core 2 Duo, 2.2GHz, 2GB RAM computer, and all simulations were performed in MATLAB using an adaptive step-size Runge-Kutta integrator, ode45. In order to explain the effectiveness of the proposed method, several numerical simulations are shown where the parameters of bicycle and control parameters are shown in Table I and Table II. The parameters of the bicycle were identified from an experimental setup. The limitation of control input for the balancer is set to 100[Nm] and for the steering is set to 40[Nm]. To show the validity of the

TABLE II
THE CONTROL PARAMETERS

Parameters	Values	Parameters	Values
a_1	120	a_2	50
a_3	80	a_4	80

proposed control for bicycle stabilization at zero velocity, we perform three types of simulation to compare the response of the bicycle stability.

A. Stabilizing the bicycle with only a balancer

In this simulation, we fixed the steering shaft and back-wheel from the dynamic equation (4) by using the constraint Jacobi matrix J and we set $J = [0 \ 1 \ 0 \ 1]$. The control input for the balancer is required and we set the scale gain to $\gamma = 2$. The initial angle of the bicycle and balancer are set to $\alpha_0 = 5^\circ$ and $\beta_0 = 0^\circ$. Fig.2 shows the angles of α , β and σ versus t. Fig.3 shows the control input of τ_b and τ_σ versus t. The maximum torque for the balancer is 71[Nm]. From this simulation result, we can see that the bicycle can be stabilized with only a balancer by using the proposed method.

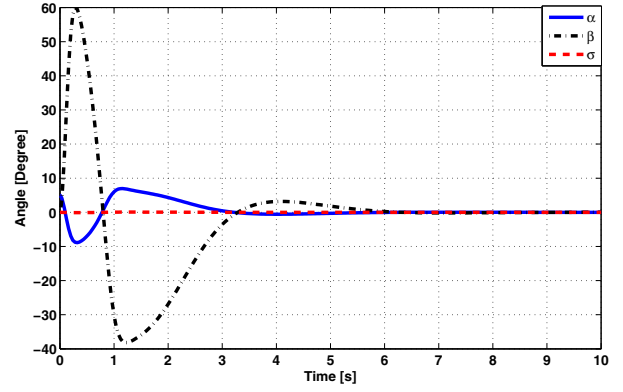


Fig. 2. Roll angle α , Balancer angle β and Steering angle σ with $\gamma = 2$ and $\alpha_0 = 5^\circ$.

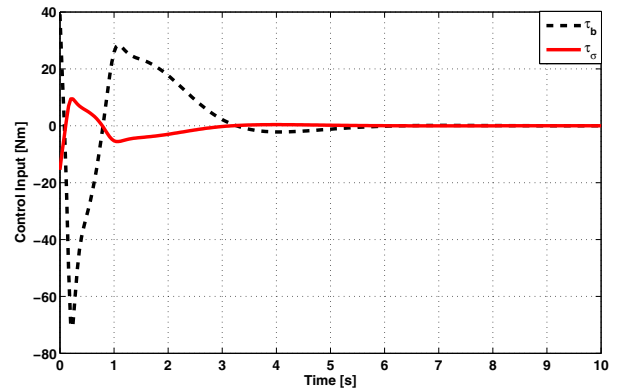


Fig. 3. The control input for balancer τ_b and steering τ_σ .

B. Stabilizing the bicycle with only steering handlebar

In this simulation, we fixed the balancer from the dynamic equation (4) and we set $J = [0 \ 0 \ 1 \ 0]$. In this case, the control input for the steering is needed and we set the scale gain to $\gamma = 0.1$. The initial angle of the bicycle and balancer are set to $\alpha_0 = 5^\circ$ and $\beta_0 = 0^\circ$. Fig.4 shows the angles of α , β and σ versus t. Fig.5 shows the control input of τ_b and τ_σ versus t. The maximum torque for the steering torque is 34[Nm] that is required for stabilization the bicycle. From these figures, we can see that the bicycle can be stabilized with only a steering by using the proposed method.

C. Stabilizing the bicycle with balancer and steering handlebar

From the two previous simulation results, The proposed method can stabilize the bike system by using only balancer or steering, but the performance of the balancing systems were not so good and the maximum of initial bicycle roll angle is $\alpha_{max} = 7^\circ$. In order to improve the performance of the bike system and to enlarge the initial roll angle, we used the balancer and steering handlebar to stabilize the bike system. The dynamic equation (2) is used to perform this simulation. The both control input for the balancer and

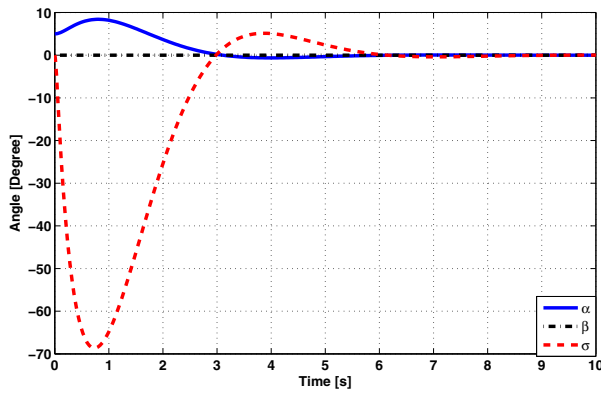


Fig. 4. Roll angle α , Balancer angle β and Steering angle σ with $\gamma = 0.1$ and $\alpha_0 = 5^\circ$.

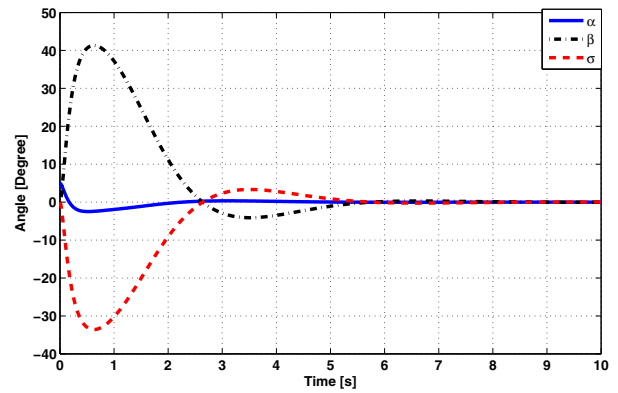


Fig. 6. Roll angle α , Balancer angle β and Steering angle σ with $\gamma = 2$ and $\alpha_0 = 5^\circ$.

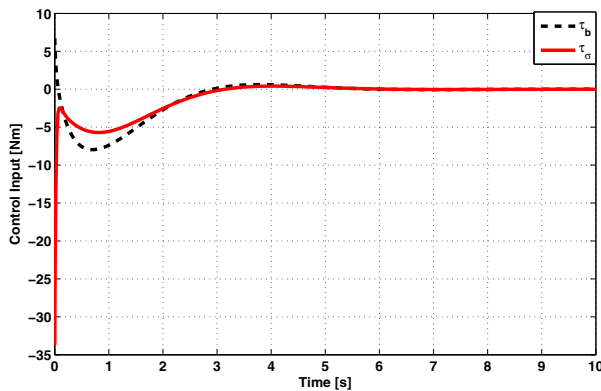


Fig. 5. The control input for balancer τ_b and steering τ_σ .

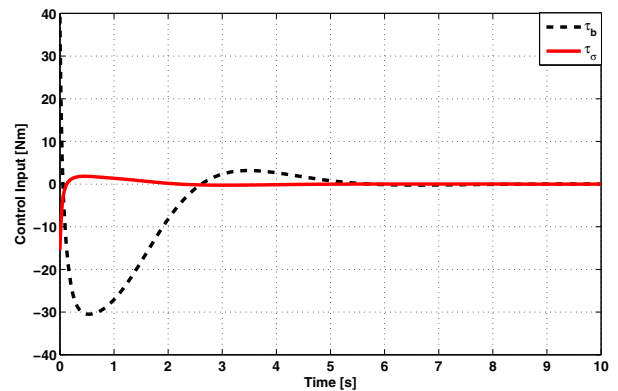


Fig. 7. The control input for balancer τ_b and steering τ_σ .

steering are required and we set the scale gain to $\gamma = 2$. The initial angle of the bicycle and balancer are set to $\alpha_0 = 5^\circ$ and $\beta_0 = 0^\circ$. In Fig.6 and 7, it can be seen that the state converges to the upright position. From this simulation results, we clearly saw that the cooperation between the balancer and the steering gave a better performance. The maximum of the bicycle roll angle can enlarge up to $\alpha_{max} = 15^\circ$.

V. EXPERIMENTAL RESULTS

In order to see the validity of the proposed method, some experiments using a real system was conducted. Fig. 8 shows the real system of the bicycle and Fig. 9 shows the hardware system of the bicycle. The base system is a commercial available electric motor bicycle named ‘Passol’ produced by Yamaha co.ltd. We attached a balancer which can move in a lateral plane and can keep the balance of the bicycle system. One motor is attached to the top of the handle for steering control. An IMU sensor is also attached to the bicycle. This sensor detects roll angle α and roll angular velocity $\dot{\alpha}$. For a realization of real-time control, XPC Target is employed in the control system as an operating system. The sampling rate of the controller is 2[ms]. In order to see the

performance of the proposed control method, stability of the bike system with the output-zeroing controller were checked by keeping the system in upright position for more than 30[*min*]. Due to limitation of the memory for data logger, we can get experiments data for duration less than 5[*min*]. Figure 10 shows the experimental results of the balancing system with only the balancer. In this case, we set $\gamma = 2$ for controlling the bike system and the we fixed the steering handlebar by PI controller. With the real bicycle, the steering handlebar can not move over 40° and the bicycle trail is only 0.1*m*. Thus, we can not successfully stabilized the bicycle with only the steering handlebar. Figure 11 and 12 show the experimental results of the balancing system with both balancer and steering with different scale gain. If the scale gain is large, the effect of the control input for steering is small. From these figures, it can be seen that the proposed method can work very well.

VI. CONCLUSIONS

In this paper, we presented a new proposed control method for stabilizing a bicycle with both balancer and steering. From simulation results of balancing bicycle with a steering handlebar and a balancer, it was shown that a novel proposed

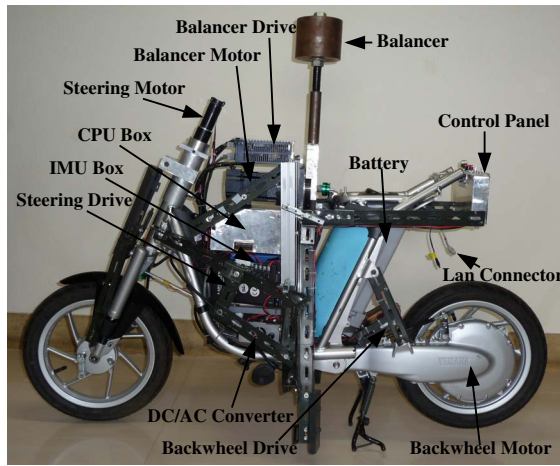


Fig. 8. Bicycle with a Balancer Hardware.

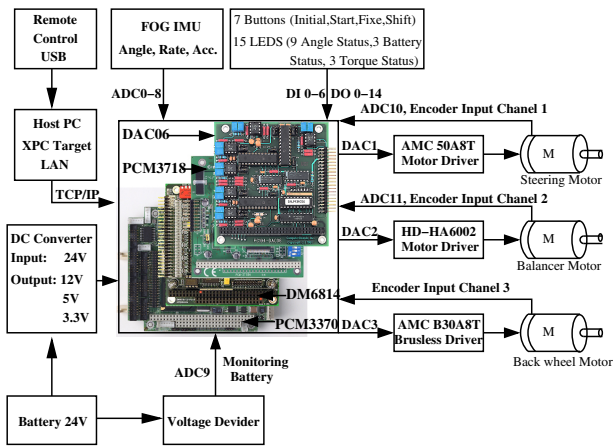


Fig. 9. Bicycle with a Balancer Hardware Scheme.

control has better performance than balancing control of a bicycle with only a balancer or only a steering. The proposed method also has been confirmed by the experimental results and it was shown that this controller can work very well.

REFERENCES

- [1] K. Astrom, R. Klein and A.Lennartsson , "Bicycle Dynamics and Control," *IEEE Control System Magazine*, vol. 25, no. 4, pp. 26-47, 2005.
- [2] N. Getz, "Dynamic Inversion of Nonlinear Maps with Applications to Nonlinear Control and Robotics," *Ph.D dissertation, Department of Electrical Engineering and Computer Sciences*, University of California at Berkeley, CA, 1995.
- [3] M. Defoort and T. Murakami, "Second order sliding mode control with Disturbance Observer for bicycle stabilization," *IEEE/RSJ International Conference on Intelligent Robots and Systems.*, pp. 2822-2827, 2008.
- [4] J. Yi, D. Song, A. Levandowski and S. Jayasuriya, "Trajectory Tracking and Balance Stabilization Control of Autonomous Motorcycle," *Proceeding IEEE Int. conf. on Robotics and Automation*, pp. 2583-2589, 2006.
- [5] L. Keo and M. Yamakita, "Dynamic Models of a Bicycle with a Balancer System," *The 26th annual conference of the Robotics Society of Japan.*, pp. 43-46, 2008.
- [6] C. Hwang, H. Wu and C. Shih, "Autonomous Dynamic Balance of an Electrical Bicycle Using Variable Structure Under-Actuated Control," *IEEE/RSJ International Conference on Intelligent Robots and Systems.*, pp. 3737-3743, 2008.

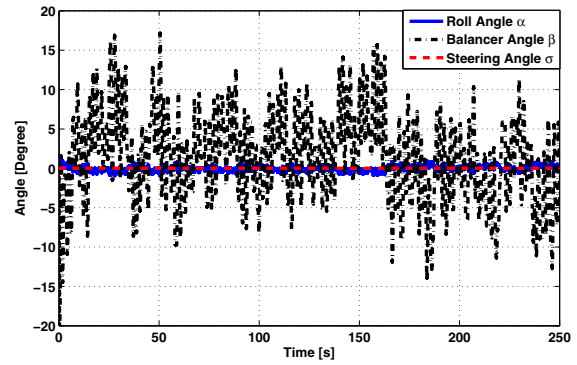


Fig. 10. Roll angle, Balancer angle and Steering angle with $\gamma = 2$.

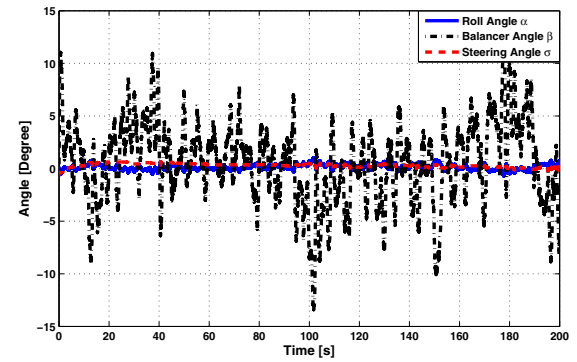


Fig. 11. Roll angle, Balancer angle and Steering angle with $\gamma = 2$.

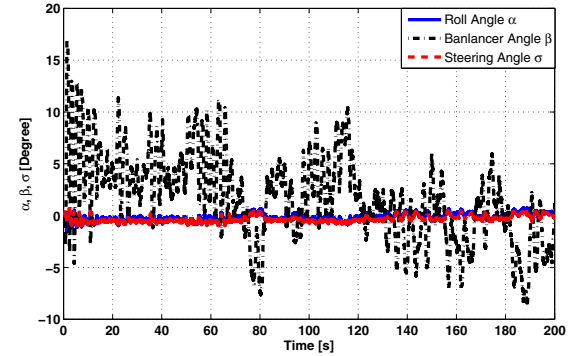


Fig. 12. Roll angle, Balancer angle and Steering angle with $\gamma = 1$.

- [7] L. Keo and M. Yamakita, "Trajectory Control for an Autonomous Bicycle with Balancer," *IEEE/ASME Int. Conf. on Ad. Intel. Mechatronics.*, pp. 676-681, 2008.
- [8] M. Yamakita and A. Utano, "Automatic Control of Bicycle with Balancer," *IEEE/ASME Int. Conf. on Ad. Intel. Mechatronics.*, pp. 1245-1249, 2005.
- [9] M. Yamakita, A. Utano and K. Sekiguchi, "Experimental Study of Automatic Control of Bicycle with Balancer," *IEEE/ASME Int. Conf. on Ad. Intel. Mechatronics.*, pp. 1245-1249, 2005.