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Sliding mode observer to estimate both the attitude and the gyro-bias by using low-cost sensors

A. El Hadri and A. Benallegue

Abstract— **This paper presents a nonlinear observer algorithm for attitude estimation that improves the quality of measures obtained by using low-cost inertial measurements (IMU). It is based on sliding mode observer that provides both the estimates of the gyro-bias and the actual attitude of the rigid body. The algorithm was developed in order to address the well-known problem of the weak dynamics of the tilt sensors and magnetometers, which can be modeled by low pass filters, and of the measurement bias of the gyros. In its design the observer uses the real measurements given by the low-cost attitude sensors (inclinometers and magnetometers) and the gyros, the filters modeling the sensors and the kinematics equation of the rigid body. The asymptotic convergence of the estimation of the attitude and bias-gyros was proven using Lyapunov stability method. The effectiveness of the algorithm has been shown from experimental tests using a rotary platform equipped with several sensors with axes of rotation coincide with orientation of the rigid body. Also, tests for comparison with a linear complementary filter are given.**

I. INTRODUCTION

The attitude control problem of rigid bodies (Walking Robots, Unmanned Aerial Vehicles, Autonomous Vehicles, ...) has been widely studied in literature (control, aerospace and robotics), and several control strategies have been proposed [4], [8], [14], [17]. The effectiveness of these controls depends on the availability and reliability of measurements. In most applications in this field, these measurements are derived from sensors such as rate gyros, inclinometers, accelerometers and magnetometers. These sensors are used to perform the attitude estimation. If these sensors are of very high quality, then on the one hand the use of information from accelerometer or inclinometer and magnetometer can provide very accurate estimation of attitude that is valid only on low bandwidth. In the other hand, the rate gyros can be used to derive attitude by integrating the kinematic equations of the rigid body. Such high quality-sensors are very expensive and not suitable for commercial applications.

Nowadays, the progress in micro electro-mechanical system (MEMS) and technology of the anisotropic magneto-resistive has enabled the development of low-cost inertial measurement units (IMU). However, these low-cost sensors (gyroscopes, accelerometers and magnetometers) are usually noisy and provide a biased measurement. The multiplication of the applications using low-quality sensors has lead to a strong interest in attitude estimation algorithms in order to improve the performance. Several authors in the literature proposed estimation algorithms providing an estimation of the bias assuming that the attitude is well known [10], [16]. In case of small angles variation, a linear complementary filtering technique can be used to provide relatively accurate attitude estimation obtained through the fusion process [2], [14]. A nonlinear complementary filtering approach with gyro-bias estimation has been proposed in [10]. A Survey of nonlinear attitude estimation methods is proposed in [5]. A high gain observer based on a low-pass sensors model and Euler equations of a rigid body has been studied for roll and pitch angles estimation by combining sensors data from gyros and inclinometers [11]. In [12], the authors show an experimental evaluation of this observer compared with a standard extended Kalman filter. In [1], the authors formulated the rigid body attitude control with state estimation using Rodrigues parameters and assuming measurements from gyros and low-pass inclinometers. In [15], a model with quaternion parameterization using a first-order low-pass filter on a ”virtual” angular velocity is used to design an observer combined with complementary filter for providing estimates for the gyro-bias and the actual attitude. Several other authors in literature have used the Kalman filter or extended Kalman filter to estimate the attitude of the rigid body with low-cost sensors (see for example [9], [18]).

In this paper, we consider the problem of rigid body attitude estimation with gyro-bias compensation based on low-cost sensors. In fact, in order to improve the quality of measurements, we combine the measurements derived from inclinometers and magnetometers and rate-gyros with an estimation algorithm based on a nonlinear observer for providing estimates of the gyro-bias and the real attitude of the rigid body. In practice the orientation of the rigid body is obtained using low-pass sensors such as inclinometers (based on accelerometers) and magnetometers. These sensors with generally very close bandwidth provide relatively accurate attitude measurements at low frequencies. On other hand, the gyros have often large bandwidth but the angular velocity measurement is biased.

To develop the algorithm for estimating the attitude that covers a wide frequency range, we consider that the sensors measuring the attitude at low frequencies can be modeled as a low pass filter like as proposed in [11] and then using the kinematics equation of a rigid body we propose a nonlinear observer based on sliding mode technique (see [13]) to reconstruct the true attitude and provide an estimate of the gyro-bias. By using Lyapunov analysis stability of the observer, we show the asymptotic convergence of the estimation of the attitude and gyro-bias. This one is considered constant

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but varies at each use. To evaluate the effectiveness of the algorithm we have performed some experimental tests using a rotary platform equipped with several sensors and whose axes of rotation coincide with orientation of the rigid body. Also, tests for comparison with a linear complementary filter have been performed to show the effectiveness of the proposed algorithm in case of large angle variations.

II. Rigid-body attitude description

The attitude control problems of rigid bodies such as the stabilization and navigation require the transformation of measured and computed quantities between various frames of references. The position and the attitude of a rigid-body is based on measurements from sensors attached to a rigid-body. Indeed, inertial sensors (accelerometer, gyro,...) are attached to the body-platform and provide inertial measurements expressed relative to the instrument axes. In most systems, the instrument axes are nominally aligned with the body-platform axes. Since the measurements are performed in the body frame we describe in Fig. 1 the orientation of the body-fixed frame $B(x_m, y_m, z_m)$ with respect to the inertial reference frame $R_I(x_a, y_a, z_a)$. Various mathematical representation can be used to define the attitude of the rigid-body with respect to coordinate inertial reference frame. In this paper, we consider the Euler angles representation in which a transformation from one coordinate frame to another is defined by three successive rotations about different axes taken in turn. The Euler rotation angles used here corresponds to the following rotation sequence: yaw(ψ)-pitch(θ)-roll (ϕ).

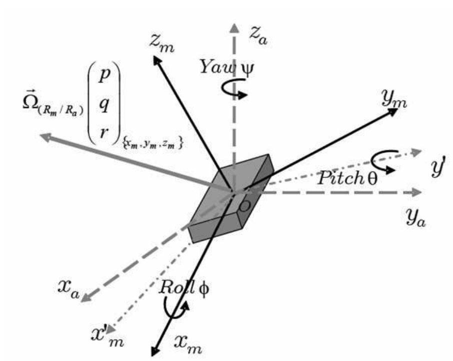

Fig. 1. Coordinate system of a rigid body

In this case, the coordinate transformation relating body frame B to the inertial reference frame R_I is given by the rotation matrix R and expressed as a function of the attitude:

$$R = \begin{bmatrix} c\theta c\psi & -c\phi s\psi + s\theta c\psi s\phi & s\phi s\psi + c\phi s\theta c\psi \\ c\theta s\psi & c\phi c\psi + s\theta s\phi s\psi & -c\psi s\phi + c\phi s\theta s\psi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \quad (1)$$

where $c(\cdot)$ and $s(\cdot)$ denote functions $cos(\cdot)$ and $sin(\cdot)$, respectively.

The rotation matrix $R \in SO(3)$ satisfies the following rigid body kinematic differential equation:

$$\dot{R} = S(\Omega)R \quad (2)$$

where $S(\Omega)$ is a skew-symmetric matrix such that $S(\Omega)V = \Omega \times V$ for any vector $V \in \mathbb{R}^3$, where $\times$ is the vector cross product. Ω is the angular velocity vector of the body expressed in the body-fixed frame B. The roll, pitch and yaw angular rates (p, q, r) measured by gyros are the components of the angular velocity vector Ω.

From the matrix equation (2) we can derive expression which relates the Euler angle rates $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ to the equivalent angular velocity (p, q, r) as follows:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \quad (3)$$

III. Problem statement

In this paper, our objective is to design a high-quality inertial measurement unit (IMU) based on low-cost sensors and using an algorithm estimation that is based on the kinematics equation of the rigid body. In fact, in order to improve the performance of measures provided by the IMU, we take into account in the estimation algorithm the sensors dynamics combined with the kinematics of the rigid body (see Fig. 2).

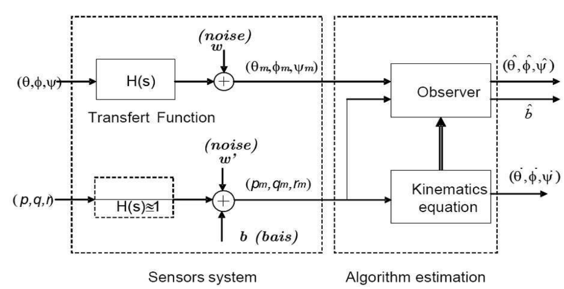

Fig. 2. Scheme of the estimation algorithm

The low-cost sensors used to measure the orientation of the rigid body are generally characterized by close bandwidth and can provide a relatively accurate measure of attitude only at low frequencies. This measure can also be corrupted by noise. In this case, we assume that the attitude measure $(\phi_m, \theta_m, \psi_m)$ is related to the actual attitude (ϕ, θ, ψ) through the following first low order filters in matrix form:

$$\begin{pmatrix} \dot{\phi}_m \\ \dot{\theta}_m \\ \dot{\psi}_m \end{pmatrix} = \begin{pmatrix} \frac{1}{\tau_\phi} & 0 & 0 \\ 0 & \frac{1}{\tau_\theta} & 0 \\ 0 & 0 & \frac{1}{\tau_\psi} \end{pmatrix} \begin{pmatrix} \phi - \phi_m \\ \theta - \theta_m \\ \psi - \psi_m \end{pmatrix} \quad (4)$$

These equations define the dynamics of the low-cost sensors used in IMU. The positive constants τ_i describe the time constants of the sensors.

The gyros used to obtain the angular velocities are often large bandwidth but the measurements are biased. Then, we consider the real angular velocity vector Ω written as:

$$\Omega = \Omega_m - b \tag{5}$$

where Ω_m is a measurement provided by the gyros, b is the unknown gyro-bias.

Now we consider that the state vectors $\Theta = (\phi, \theta, \psi)^T$ and $\Theta_m = (\phi_m, \theta_m, \psi_m)^T$ respectively represent the real and the measured attitude of the rigid-body. The equation (4) can be rewritten as:

$$\dot{\Theta}_m = \Pi(\Theta - \Theta_m) \tag{6}$$

where matrix $\Pi = diag(\frac{1}{\tau_\phi}, \frac{1}{\tau_\theta}, \frac{1}{\tau_\psi})$.

By using kinematics equation (3) and relationship (5) we can write:

$$\dot{\Theta} = M(\Theta)(\Omega_m - b) \tag{7}$$

where :

$$M(\Theta) = \begin{pmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{pmatrix} \tag{8}$$

Then given (6) and (7), we can propose a nonlinear observer that provide estimates of both the real attitude Θ and the gyro-bias b.

IV. Design of the nonlinear observer for estimating attitude and gyro-bias

In this section, we aim to design a nonlinear observer scheme for the attitude and gyro-bias estimation. We will show how the measurement data provided by IMU sensors can be used in the observer design and how to find stability conditions of the observer to have an asymptotic convergence of the estimate errors to zero.

As described previously, we can write the following system:

$$\left\{ \begin{array}{l} \dot{\Theta}_m = \Pi(\Theta - \Theta_m) \\ \dot{\Theta} = M(\Theta)(\Omega_m - b) \end{array} \right. \tag{9}$$

The sliding mode observer can be designed by using system (9) with choice of a sliding manifold based on the error observation measurement. Then, we propose the following observer:

$$\left\{ \begin{array}{l} \dot{\hat{\Theta}}_m = \Pi(\hat{\Theta} - \Theta_m) + L_1 sign(\tilde{\Theta}_m) \\ \dot{\hat{\Theta}} = M(\hat{\Theta})(\Omega_m - \hat{b}) + L_2 sign(\tilde{\Theta}_m) \\ \dot{\hat{b}} = -M^T(\hat{\Theta})\Pi^{-1} L_1 sign(\tilde{\Theta}_m) \end{array} \right. \tag{10}$$

where $\tilde{\Theta}_m = \Theta_m - \hat{\Theta}_m$, L_i $(i = 1, 2)$ are diagonal positive definite matrix gains and $sign$ represents the usual function $sign(\cdot)$, understanding the components of the vector $((\phi_m - \hat{\phi}_m), (\theta_m - \hat{\theta}_m), (\psi_m - \hat{\psi}_m))^T$

Let define $\tilde{\Theta}_m = \Theta_m - \hat{\Theta}_m$, $\tilde{\Theta} = \Theta - \hat{\Theta}$ and $\tilde{b} = b - \hat{b}$ to be the observation errors. Then, the error dynamics of the observer can be obtained by subtracting (9) and (10), it is given by:

$$\dot{\tilde{\Theta}}_m = \Pi\tilde{\Theta} - L_1 sign(\tilde{\Theta}_m) \tag{11}$$

$$\dot{\tilde{\Theta}} = \Delta f - M(\hat{\Theta})\tilde{b} - L_2 sign(\tilde{\Theta}_m) \tag{12}$$

$$\dot{\tilde{b}} = M^T(\hat{\Theta})\Pi^{-1} L_1 sign(\tilde{\Theta}_m) \tag{13}$$

where $\Delta f = f(\Theta) - f(\hat{\Theta})$ is obtained by defining the function $f : D \subset \mathbb{R}^3 \to \mathbb{R}^3$ as $f(\Theta) = M(\Theta)\Omega$.

In order to analyze the stability of the observer we first consider the following assumptions.

A1 : Assume that the real angular velocity Ω and its first derivative $\dot{\Omega}$ are bounded.

A2 : The pitch angle θ is assumed bounded as $|\theta| \leqslant \frac{\pi}{2}(1 - \delta)$, with $0 < \delta < 1$.

A3 : The nonlinear function f verify the inequality condition as $\|\Delta f\| \leqslant K \left\|\tilde{\Theta}\right\|$ for all $\Theta, \hat{\Theta} \in D \subset \mathbb{R}^3$ and constant $K > 0$

The assumption A2 can be justified by the fact that in many applications such as walking robot, autonomous vehicle, stabilized aero-vehicle, the control algorithms would keep the pitch angle in the range very less then $\pm\frac{\pi}{2}$.

The assumption A3 can be verified by showing that f is a Lipschitz function. So, we deal with vector function defined by f of a variable $\Theta = (\phi, \theta, \psi)$ witch is continuously differentiable on a bounded set $D \subset \mathbb{R}^3$.

$$f = \begin{pmatrix} p + r\tan\theta\cos\phi + q\tan\theta\sin\phi \\ q\cos\phi - r\sin\phi \\ \frac{r}{\cos\theta}\cos\phi + \frac{q}{\cos\theta}\sin\phi \end{pmatrix} \tag{14}$$

By the mean-value theorem, we can have:

$$\left\|f(\Theta) - f(\hat{\Theta})\right\| \leqslant \sup_{0<\alpha<1} \left\|J_f(\Theta + \alpha(\Theta - \hat{\Theta}))\right\| \left\|\Theta - \hat{\Theta}\right\| \tag{15}$$

where $J_f(\cdot)$ is the Jacobian matrix.

By use of the vector norm $\|\cdot\|_\infty$, we can write:

$$\|\Delta f\|_\infty \leqslant (\check{q} + \check{r}) \left(\left|\tan(\theta + \alpha\tilde{\theta})\right| + \frac{1}{\cos^2(\theta + \alpha\tilde{\theta})} \right) \left\|\tilde{\Theta}\right\|_\infty \tag{16}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ and $\check{q}$ and $\check{r}$ correspond to the maximum of the angular velocity Ω.

Then if $\left|\tilde{\theta}\right|$ is bounded such that $\left|\theta + \alpha\tilde{\theta}\right| < \frac{\pi}{2} - \varepsilon$ with ε is a positive constant $< \frac{\pi}{2}$, thus we can obtain:

$$\|\Delta f\|_\infty \leqslant (\check{q} + \check{r}) \left(\left|\tan(\frac{\pi}{2} - \varepsilon)\right| + \frac{1}{\cos^2(\frac{\pi}{2} - \varepsilon)} \right) \left\|\tilde{\Theta}\right\|_\infty \tag{17}$$

and so we can take:

$$K = (\check{q} + \check{r}) \left(\left|\tan(\frac{\pi}{2} - \varepsilon)\right| + \frac{1}{\cos^2(\frac{\pi}{2} - \varepsilon)} \right) < \infty \tag{18}$$

Theorem 1: Under assumptions A2 and A3 and by appropriate choice of the observer gains L_1 and L_2, the observer (10) for the system (9) ensure, in finite time, the convergence to zero of the estimation errors $\tilde{\Theta}_m$ and thereafter the asymptotic convergence to zero of the actual attitude estimation errors $\tilde{\Theta}$ and the bias errors $\tilde{b}$.

Proof: The stability analysis of the observer can be made in two steps. In the first step, we show the convergence to zero in finite time of $\tilde{\Theta}_m$, and then after we prove the asymptotic convergence to zero of $\tilde{\Theta}$ and $\tilde{b}$.

For the first step, the following candidate Lyapunov function is chosen as:

$$V_1 = \frac{1}{2}\tilde{\Theta}_m^T\tilde{\Theta}_m \tag{19}$$

Using (11) in the expression of the time derivative of the Lyapunov function (19) we obtain:

$$\dot{V}_1 = \tilde{\Theta}_m^T\dot{\tilde{\Theta}}_m = \tilde{\Theta}_m^T(\Pi\tilde{\Theta} - L_1 sign(\tilde{\Theta}_m)) \tag{20}$$

If we choose the observer gains of the matrix L_1 sufficiently large, then when we have $\left\|\tilde{\Theta}\right\| < \frac{\lambda_{\min}(L_1)}{\|\Pi\|}$ the sliding occurs in finite time along the sliding manifold $\tilde{\Theta}_m = 0$, see [13] for more details. As shown in [7] and according to the equivalent control method, in sliding mode ($\tilde{\Theta}_m = 0$ and $\dot{\tilde{\Theta}}_m = 0$) the system behaves as if the discontinuous term $L_1 sign(\tilde{\Theta}_m)$ is replaced by its equivalent value which can be deduced from (11) as:

$$(L_1 sign(\tilde{\Theta}_m))_{eq} = \Pi\tilde{\Theta} \tag{21}$$

Now, substituting (21) into (12) and (13), we obtain the reduced system:

$$\begin{cases} \dot{\tilde{\Theta}} = \Delta f - M(\hat{\Theta})\tilde{b} - L_2 L_1^{-1}\Pi\tilde{\Theta} \\ \dot{\tilde{b}} = M^T(\hat{\Theta})\tilde{\Theta} \end{cases} \tag{22}$$

In this step, the following candidate Lyapunov function is chosen as:

$$V_2 = \frac{1}{2}\tilde{\Theta}_m^T\tilde{\Theta}_m + \frac{1}{2}\tilde{\Theta}^T\tilde{\Theta} + \frac{1}{2}\tilde{b}^T\tilde{b} \tag{23}$$

If the condition obtained in the first step holds for all $t > t_1$, then we have $\tilde{\Theta}_m = 0$ and the time derivative of the Lyapunov function (23) gives:

$$\begin{aligned} \dot{V}_2 &= \tilde{\Theta}^T\dot{\tilde{\Theta}} + \tilde{b}^T\dot{\tilde{b}} \\ &= \tilde{\Theta}^T(\Delta f - M(\hat{\Theta})\tilde{b} - L_2 L_1^{-1}\Pi\tilde{\Theta}) + \tilde{b}^T M^T(\Theta)\tilde{\Theta} \\ &= \tilde{\Theta}^T(\Delta f - L_2 L_1^{-1}\Pi\tilde{\Theta}) - \tilde{b}^T M^T(\Theta)\tilde{\Theta} + \tilde{b}^T M^T(\Theta)\tilde{\Theta} \end{aligned}$$

The time derivative of the Lyapunov function V_2 becomes:

$$\dot{V}_2 = \tilde{\Theta}^T(\Delta f - L_2 L_1^{-1}\Pi\tilde{\Theta}) \tag{24}$$

However based on assumption A2, the function f is bounded Lipschitz if $\left|\tilde{\theta}\right| < \frac{\pi}{2}\delta$. Then if the gains matrix L_1 is such $\lambda_{\max}(L_1) < \frac{\pi}{2}\delta\left\|\Pi\right\|$, the condition of the assumption is guaranteed. So, the time derivative of the Lyapunov function V_2 given in (24) can be upper bounded as follows:

$$\dot{V}_2 \leqslant -(\frac{\lambda_{\min}(L_2)\left\|\Pi\right\|}{\lambda_{\max}(L_1)} - K)\left\|\tilde{\Theta}\right\|^2 \tag{25}$$

Thus, by appropriate choice of the matrix gain L_2 such as $\lambda_{\min}(L_2) > \frac{\lambda_{\max}(L_1)K}{\|\Pi\|}$, we obtain the asymptotic convergence of the attitude observation $\tilde{\Theta}$ to zero and $\tilde{b}$ is bounded. However by using Barbalat's Lemma we can show according to first equation of the system (22) and assumption A1, $\lim\limits_{t\to\infty}\dot{\tilde{\Theta}}(t) = 0$, so we can conclude to $\lim\limits_{t\to\infty}\tilde{b}(t) = 0$. ■

The result shows that it is possible to improve the quality of measurements derived from low-cost sensors using the nonlinear observer described in (10). In the next section, we present the experimental validation of this observer.

V. Experimental setup

For the experimental validation of the proposed observer, we have designed a rotational platform (see Fig. 3) for generating a known motion in order to have the possibility to compare the attitude estimation to the true attitude of the platform. Yaw, pitch and roll angles are the three degrees of freedom of this platform. Each degree of freedom coincides with a rotational axis of the platform and all rotational joints are equipped with potentiometer sensors. Then to evaluate the estimation algorithm, we have mounted onto the platform a low-cost gyros IDG300 Gyro and ADXL330 Accelerometer ICs. All sensors of the platform are connected to a PC Pentium, equipped with a dSpace DS1103 PPC real-time controller board using *Matlab* and *Simulink software*. The sampling frequency has been fixed to 1 kHz.

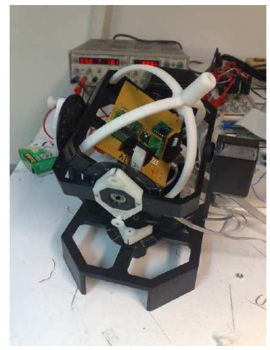

Fig. 3. Experimental platform: three rotational axis

VI. Experimental results

To implement the estimation algorithm, first we performed some tests to identify the static and dynamic characteristics of sensors. The data analysis shows that these senores can be modeled by low pass filters with a transfer function given by:

$$H(s) = \frac{k}{1+\tau s} \tag{26}$$

where k is the gain and τ is the time constant. The identified parameters are given in the following table I

TABLE I

PARAMETERS OF THE LOW-PASS SENSORS

Rotational axis	k	τ
Roll angle ϕ	0.995	0.04
Pitch angle θ	1.01	0.045
Yaw angles ψ	0.99	0.05

Also, the gyros-bias are identified in order to be able to compare them with those estimated by the algorithm. The table II gives the identified gyros-bias:

To implement the estimation algorithm defined by the observer (10), we have used the parameters of transfer functions identified previously (see table I) to write the matrix Π and we have take $\delta = 0.5$ in assumption A2. To

TABLE II
GYROS-BIAS

$Gyro - bias$	b
$Roll\ rate\ (p)$	-0.59
$Pitch\ rate\ (q)$	1.91
$Yaw\ rate\ (r)$	-0.62

ensure the condition of the bounded Lipschitz of f, the initial condition of the pitch angle is $\theta(0)$ and by taking $\varepsilon = 0.3$, then the matrix gains $L_1 = diag(10, 10, 10)$ and $L_2 = diag(100, 100, 100)$ satisfy the stability conditions of the observer. To perform the test, we manually operated platform, by combining the movements of roll, pitch and yaw for coupling between the different dynamics. We have also made slow and fast movements combining small and large angle variations.

The obtained results are shown in the following figures. In Fig.4, we compare the real attitude Θ given by potentiometers and that provided by tilt sensors Θ_m. We can observe a phase delay of the tilt sensors responses. This explain the use of the model (6).

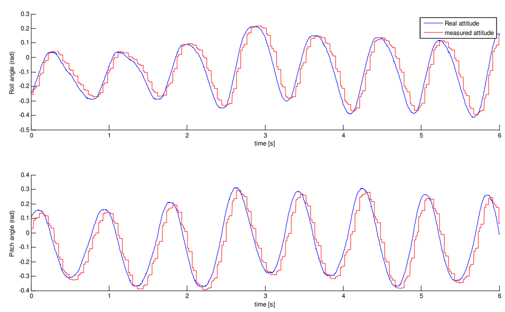

Fig. 4. Real attitude Θ given by potentiometers and its measure Θ_m provided by tilt sensors.

Fig. 5 presents the result of observation of the measurement of attitude and shows the establishment of the sliding mode and the convergence of the observation error to zero

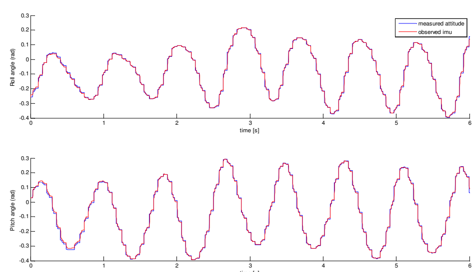

Fig. 5. Measured attitude and its observation

In Fig. 6, we present the result of estimation of the real attitude. In this figure, we show that the attitude estimation $\hat{\Theta}$ obtained from observer (10) converges to the real attitude Θ based on the Θ_m data measure derived from IMU sensors.

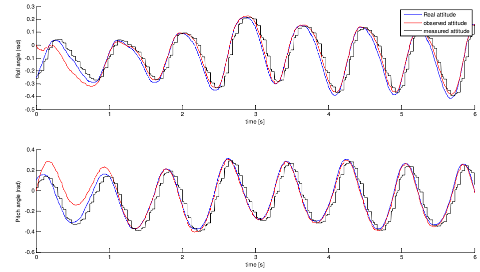

Fig. 6. The estimate of the attitude obtained from observer

In the Fig. 7, we present the estimation of the gyro-bias. These results show that the gyro-bias estimates converge to the values given in the table (II).

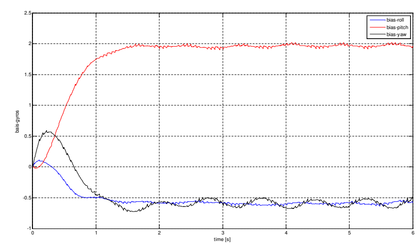

Fig. 7. Estimation of the gyro-bias

These results show the effectiveness of the observer to improve the quality of measurements provided by low-cost sensors.

In the following we will compare the performance of the proposed observer with estimation obtained from known linear complementary filter [2]. The complementary filter is designed as shown in Fig. 8.

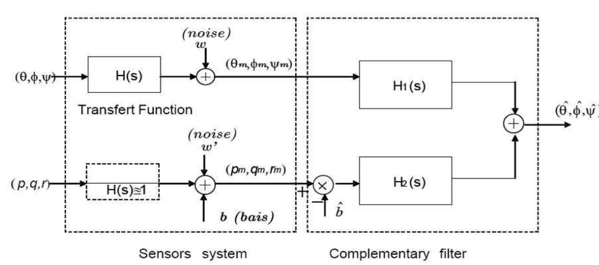

Fig. 8. Linear complementary filter.

The transfer functions are written as:

$$H_1(s) = \frac{b_{11}s + 1}{a_{21}s^2 + a_{11}s + 1}$$

$$H_2(p) = \frac{s(b_{12}s + b_{22})}{a_{32}s^3 + a_{22}s^2 + a_{12}s + 1}$$

TABLE III

PARAMETERS OF THE COMPLEMENTARY FILTER

$parameters$	$Roll$	$Pitch$	Yaw
$b11$	0.842	0.966	1.05
$b12$	0.0067	0.0097	0.0125
$b22$	0.1936	0.2539	0.3
$a11$	0.8	0.92	1
$a21$	0.16	0.21	0.25
$a12$	0.842	0.966	1.05
$a22$	0.1936	0.2539	0.3
$a32$	0.0067	0.0097	0.0125

The parameters a_{ij} and b_{ij} are given in the table III of each rotational dynamics.

In Fig. 9 and Fig. 10, we present the result of this comparison. For the complementary filter the gyro-bias has been compensated manually. These results show that the complementary filter is not suitable for estimating the attitude in the case of combined movement with large variations in angles. As against, the proposed observer is very effective for the reconstruction of the real attitude in this case.

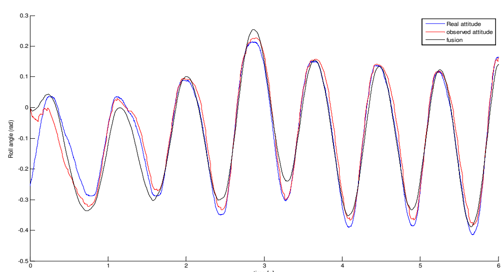

Fig. 9. Estimation of the roll angle: observer vs complementary filter

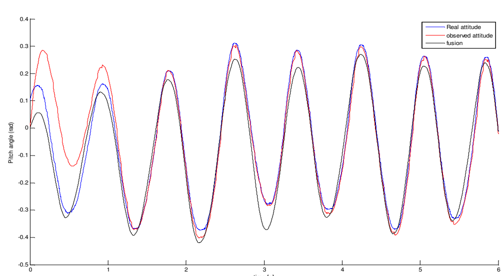

Fig. 10. Estimation of the pitch angle: observer vs complementary filter

VII. CONCLUSIONS

In this paper we have presented a nonlinear observer algorithm to estimate both the attitude and the gyro-bias in order to improve the quality of measures obtained by using low-cost inertial measurements (IMU). The algorithm was developed by assuming that the sensors measuring the attitude at low frequencies can be modeled as a low pass filter and using the kinematics equation of a rigid body. The observer is designed based on the sliding-mode approach known to be robust with respect to parametric uncertainties and modeling errors

The asymptotic convergence of the estimation of the attitude and gyro-bias was proven using Lyapunov stability method. The effectiveness of the algorithm has been shown from experimental tests using a rotary platform equipped with several sensors with axes of rotation coincide with orientation of the rigid body. Also, we have compared the results of the proposed observer with a linear complementary filter.

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