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Adaptive Output Feedback Control for Robot Manipulators Using Lyapunov-based Switching

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Abstract— **In the face of large scale parametric uncertainties, the single model (SM)-based classical adaptive control approach demands high observer, controller and adaptation gains in order to achieve good tracking performance. The well known problem of having high-gain based design is that it amplifies the input and output disturbance as well as excites hidden unmodeled dynamics causing poor tracking performance. In this paper, a multi-model based adaptive design is proposed to reduce the level of parametric uncertainty in order to reduce the observer-controller gains. The key idea of this approach is to allow the parameter estimate of the SM-based classical adaptive control design to be reset into a model that best approximates the plant among a finite set of candidate models. For this purpose, we uniformly distribute the compact set of unknown parameters into a finite number of smaller compact subsets. Then we design a family of candidate controllers for each of these smaller compact subsets. The derivative of the Lyapunov function candidate is used as a resetting criterion to identify a candidate model that closely approximates the plant at each instant of time. The proposed method is evaluated on a 2-DOF robot manipulator to demonstrate the effectiveness of the theoretical development.**

Key words:Adaptive Control, Output Feedback, Robotics, Lyapunov-based Switching.

I. Introduction

Over the past decades, there have been tremendous effort in the development of high performance tracking controllers for robotic systems, (see for example [1-10, [16] and references therein], to name a few). Most of these studies, however, only provide asymptotic convergence property while transient tracking behavior do not consider in their stability analysis. Another important reason for showing poor transient tracking performance is that the *certainty equivalence* (CE)-based design is based on using well-known dynamical properties of the robotic systems [3], [7], [8], where nonlinear robot dynamics are required to be appear linearly with respect to uncertain parameters. Therefore, if we consider that the unknown parameters and initial conditions belong to a known, but relatively large compact set, then the existing single-model CE principle based adaptive control, either for state or output feedback, approach provides poor transient tracking performance. To improve transient tracking response, one may use high observer-controller gains to speed up the convergence rate of the state and parameter estimates. Specifically, state feedback (position-velocity) based classical adaptive design requires high learning gains to obtain fast convergence rate of the parameter estimator. In output feedback case, high observer-controller gains are essential to achieve robust reconstruction of true velocity signal and fast parameter learning to ensure good transient tracking performance. The requirement of high observer gain makes the CE-based adaptive output feedback approach even more complex to realize in real-time applications as high observer gain amplifies the input and output disturbance causing high-frequency control chattering activity. As a consequence, the classical adaptive design, either for the state or the output feedback design, might not be practically implementable or might be very expensive as, in practice, the control effort in most nonlinear control systems are limited. If applicable, then high controller-observer gain excites unmodeled high-frequency dynamics as well as amplifies input-output disturbance resulting poor tracking performance, see for example [11]-[13].

To deal with the problem associated with high observer-controller gain as well as the robustness of classical adaptive control (CAC) approach, we introduce multi-model control (MMC) technique for multi-input multi-output (MIMO) robotic systems. This method extends the existing CE-based CAC approach by allowing the parameter estimate to be changed into a family of candidate parameters model. First we propose a pre-routed switching-logic strategy, where an inequality for the derivative of the Lyapunov function is used as a resetting criterion. Results in this direction can be found in the literature [15] and references therein. We show that the pre-routed switching nature may cause an undesirable transient tracking errors and high-frequency control oscillation in the presence of large number of candidate controllers. This is due to the fact that when the number of the candidate controllers become large then the supervisor requires to scan through a large number of candidate controller before converging to the one that guarantees the resetting criterion. Moreover, this method is based on a strict assumption that the position-velocity signals are available for feedback design. To remove the transient tracking phenomenon from the pre-routed switching-logic as well as from CAC design, we allow the controller to be reset instantaneously so that the control system can improve overall tracking performance. In contrast with the pre-routed switching design, the proposed approach can be applied for both state and output feedback control technique.

The rest of the paper is organized as follows: In section II, we derive the CE-based CAC approach for robotic systems. Section III proposes multi-model based adaptive control design in order to improve transient tracking perfomance from CE-based CAC approach and the pre-routed switching-

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logic based multiple model/control design. Section IV provides implementation results to demonstrate the theoretical development of this paper. Finally, section IV concludes the paper.

II. Classical adaptive control for robotic manipulators

We first represent the robot model [1], [2], [6] in the following error state space form

$$\dot{e}_1 \quad = \quad e_2, \dot{e}_2 = \phi_1(e) + \phi_2(e_1)\tau - \ddot{q}_d \qquad (1)$$

where $e_1 = q - q_d$, $e_2 = \dot{q} - \dot{q}_d$, $\phi_1(e) = -M^{-1}(e_1 + q_d)[C(e_1+q_d, e_2+\dot{q}_d)(e_2+\dot{q}_d)+G(e_1+q_d)]$ and $\phi_2(e_1) = M^{-1}(e_1+q_d)$, $e_1 \in \Re^n$ is the joint position vector, $e_2 \in \Re^n$ is the joint velocity vector, $\dot{e}_2 \in \Re^n$ is the joint acceleration vector, $\tau \in \Re^n$ is the input torque, $M(e_1 + q_d) \in \Re^{n\times n}$ is a symmetric and uniformly positive definite inertia matrix, $C(e_1 + q_d, e_2 + \dot{q}_d)(e_2 + \dot{q}_d) \in \Re^n$ is the coriolis and centrifugal loading vector and $G(e_1 + q_d) \in \Re^n$ is the gravitational loading vector. The objective of this work is in two folds. In the first part of this paper, we develop single-model CE principle based CAC design for the tracking error model (1). Then, we introduce multi-model based adaptive control approach. Our main focus of this method is to extend CAC design in order to reduce the observer-controller gains through on-line estimation of the derivative of the Lyapunov-function inequality.

State feedback controller: Let us first design an adaptive controller as a state feedback such that the manipulator joint position $q(t)$ asymptotically tracks the desired joint position $q_d(t)$. To obtain this objective, we assume that $q_d(t)$, its first and second derivatives are bounded. Then define $Q_d = [q_d\ \dot{q}_d\ \ddot{q}_d]^T \in \Omega_d \subset \Re^{3n}$ with compact set Ω_d. To meet this control objective, we consider the following adaptive control law as

$$\tau(e, Q_d, \hat{\theta}) \quad = \quad Y(e, \dot{q}_d, \ddot{q}_d)\hat{\theta} - K_P e_1 - K_D e_2 \qquad (2)$$

with $\dot{\hat{\theta}} = -\Gamma Y^T(e, \dot{q}_d, \ddot{q}_d)S$, where $Y(e, \dot{q}_d, \ddot{q}_d)\hat{\theta} = \hat{M}(q)\ddot{q}_d + \hat{C}(q, \dot{q}_r)\dot{q}_d + \hat{G}(q)$ [1], [2], $K_P \in \Re^{n\times n}$, $K_D \in \Re^{n\times n}$, $S = e_2 + \lambda e_1$, $\dot{q}_r = (\dot{q}_2 - \lambda e_1)$, $\lambda = \frac{\lambda_0}{1+\|e_1\|}$, $\lambda_0 > 0$ and $\hat{M}$, $\hat{C}(.)$ and $\hat{G}(.)$ define the estimates of the $M(.)$, $C(.)$ and $G(.)$, respectively,. The adaptation mechanism is used to cope with structured parametric uncertainty. To alleviate the discontinuous property from $\hat{\theta}$, the learning estimates can be adjusted with the smooth parameter projection scheme [5] as, $\dot{\hat{\theta}}_i = [Proj(\hat{\theta}, \Phi)]_i$ for $\theta \in \Omega = \{\theta \mid a_i \leq \theta_i \leq b_i\}, 1 \leq i \leq p\}$, where Φ_i is the i the element of the column vector $-Y^T(e, \dot{q}_d, \ddot{q}_d)S$ and $\delta > 0$ is chosen such that $\Omega_\delta = \{\theta \mid a_i - \delta \leq \theta_i \leq b_i + \delta\}, 1 \leq i \leq p\}$. This adaptive controller design is based on using the following control Lyapunov function [8]

$$V(e, \tilde{\theta}) = \frac{1}{2}S^T MS + \frac{1}{2}e^T K_P e + \frac{1}{2}\tilde{\theta}\Gamma^{-1}\tilde{\theta} \qquad (3)$$

with $S = e_2 + \lambda e_1$ and $\tilde{\theta} = (\hat{\theta} - \theta)$. The time derivative, $\dot{V}(e, \tilde{\theta})$, along the closed loop trajectories, formulated by using the error system (1) and the control law (2) along with the parameter projection, satisfies the following asymptotic stability condition

$$\dot{V}(e, \tilde{\theta}) \leq -\lambda_{min}(\Pi)\|e\|^2 \qquad (4)$$

$\forall\hat{\theta}(0) \in \Omega$, $\forall\theta(0) \in \Omega$, $\forall e(0) \in \Omega_{co}$, $\forall\hat{\theta} \in \Omega_\delta$, $\forall e \in \Omega_c$ with $\Omega_c = \{e \mid e^T Q_{sm} e \leq c\}$, $c > 0$ and $Q_{sm} = \begin{bmatrix} 0.5M & 0.5M\lambda \\ 0.5M\lambda & 0.5\left(\lambda^2 + K_P\right) \end{bmatrix}$, $\Pi = \Theta^T \Delta \Theta$ with Δ and Θ defined as $\Delta = \begin{bmatrix} K_1 I & 0 \\ 0 & K_2 I \end{bmatrix}$ and $\Theta = \begin{bmatrix} \frac{\lambda I}{2} & I \\ \frac{\lambda I}{2} & 0 \end{bmatrix}$ where $K_1 = [\lambda_{min.}(K_D) - 3\lambda_0 M_M - 2\lambda_0 C_M]$ and $K_2 = \left[\frac{4\lambda_{min.}K_P}{\lambda_0} - \lambda_{max.}(K_D) - 2\lambda_0 M_M - 2\lambda_0 C_M\right]$. Due to space limitation, we remove the details proof.

Output feedback with linear observer: Let us now consider the practical situation where velocity sensors are not available, see for example [11], [12], [13]. This means that the state vector e in the control law (2) is not available for measurement and is required to be estimated by a suitable estimator $\hat{e}$. If e is replaced by an estimate $\hat{e}$ in $\tau(\hat{e}, Q_d, \hat{\theta})$, then one obtains $\dot{e}_2$ of (1) as, $\dot{e}_2 = \phi_1(e) + \phi_2(e_1)\tau(\hat{e}, Q_d, \hat{\theta}) - \ddot{q}_d$ with $\tau(\hat{e}, Q_d, \hat{\theta}) = Y(\hat{e}, \dot{q}_d, \ddot{q}_d)\hat{\theta} - K_P\hat{e}_1 - K_D\hat{e}_2$, $\hat{\dot{q}}_r = \hat{e}_2 + \dot{q}_d - \lambda\hat{e}_1$ and e is replaced by the following model-free linear estimator as

$$\dot{\hat{e}}_1 \quad = \quad \hat{e}_2 + \frac{H_1}{\epsilon}\tilde{e}_1, \dot{\hat{e}}_2 = \frac{H_2}{\epsilon^2}\tilde{e}_1 \qquad (5)$$

where $\tilde{e}_1 = e_1 - \hat{e}_1$, $\hat{e}_1$ is the estimate of e_1, $\hat{e}_2$ is the estimate of e_2, ϵ is a small constant design parameters needs to be specified, H_1 and H_2 are chosen such that the matrix $\begin{bmatrix} -H_1 & I \\ -H_2 & 0_{n\times n} \end{bmatrix}$ is Hurwitz. Then, the observer error can be written in the following standard singularly perturbed closed-loop observer error-model as

$$\epsilon\dot{\eta} \quad = \quad A_o\eta + B\epsilon[-\ddot{q}_d + \phi_1(e) + \phi_2(e_1)\tau(\hat{e}, Q_d, \hat{\theta})] \qquad (6)$$

where $\tilde{e}_1 = \epsilon\eta_1$, $\eta_2 = e_2 - \hat{e}_2 = \tilde{e}_2$, $\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$, $\epsilon\dot{\eta}_1 = \eta_2 - H_1\eta_1$, $\epsilon\dot{\eta}_2 = \epsilon[-\ddot{q}_d + \phi_1(e) + \phi_2(e_1)\tau(\hat{e}, Q_d, \hat{\theta})] - H_2\eta_1$, $\hat{e} = e - \zeta(\epsilon)\eta$, $\zeta(\epsilon) = \begin{bmatrix} \epsilon I_{n\times n} & 0_{n\times n} \\ 0 & I_{n\times n} \end{bmatrix}$, $A_0 = \begin{bmatrix} -H_1 & I \\ -H_2 & 0_{n\times n} \end{bmatrix}$, $H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$ and $B = \begin{bmatrix} 0_{n\times n} \\ I_{n\times n} \end{bmatrix}$. The resulting adaptive output feedback system has the following form

$$\dot{e} \quad = \quad B[\phi_1(e) + \phi_2(e_1)\tau(\hat{e}-, Q_d, \hat{\theta}) - \ddot{q}_d] + Ae \qquad (7)$$

with $\dot{\hat{\theta}} = Proj(\hat{\theta}, \phi(e - \zeta(\epsilon)\eta, Q_d, \hat{\theta}))$, where $A = \begin{bmatrix} 0 & I_{n\times n} \\ 0_{n\times n} & 0 \end{bmatrix}$. As the speed of the observer dynamics increase with the increase of the inertial parameter and initial condition of interest, then the large control input may enter into the system resulting poor transient tracking performance. To protect the plant from large control action due to high speed observer in the presence of large parametric uncertainty, one requires to saturate the control using known saturation level, τ_{max}, outside the region of interest Ω_c of the adaptive state feedback control law $\tau(e, Q_d, \hat{\theta})$ [14]. We now present the main results for the SM-based classical adaptive output feedback (CAOFB) design.

Theorem 1 : Consider the closed-loop system (6) and (7). Then, for any given $\forall e(0) \in \Omega_{co} \subseteq \Omega_c$, $\forall \hat{e}(0) \in \Omega_{co}$, $\forall \hat{\theta}(0) \in \Omega$ and $\forall \theta(0) \in \Omega$, there exists a small ϵ_1^* such that for all $0 < \epsilon < \epsilon_1^*$, all the state variables of the closed loop system are bounded and their bound can be made very small using small value of ϵ.

Proof: The proof of theorem 1 consists of two parts. In the first part, it is proven that there exists a short transient period $T_1(\epsilon) \in [0, T_2]$ during which the fast variable η approaches a function of the order $O(\epsilon)$, while the slow variables (e, θ) remain in a subset of the domain of attraction. In the second part, the boundedness of the signal $e(t)$ was given for all $t \in [T_1(\epsilon), T_3]$, where $T_1(\epsilon) \in (0, \frac{T_2}{2}]$ and $T_3 \geq T_2$ is the first time $(e(t), \theta(t))$ exists from the set Ω_c. In this part, T_3 is shown to tend to infinity, which implies that the state variables $(e(t), \theta(t))$ remain bounded for $t \geq 0$. This proof makes use of the fact that the fast variable η is of the order $O(\epsilon)$ and there exists a Lyapunov function for the fast model $W(\eta) = \eta^T P \eta$ such that the fast variables converge to the set $\Omega_\epsilon = \{\eta \mid W(\eta) \leq \epsilon^2 \beta\}$ where $\beta = 16\|P\|^2 k_1^2 \lambda_{max.}(P) = 16\|P\|^3 k_1^2$, $\|P\| = \lambda_{max.}(P)$ and P is the solution of the Lyapunov-equation as $PA_o + A_o^T P = -I$, for all $t \in [T_1(\epsilon), T_3]$. This implies that the state trajectory (e, θ) is trapped inside the set which can be made very small by using small value of observer design constant ϵ as

$$\dot{V} \leq -\lambda_{min.}(\Pi)\|e\|^2 + \chi\epsilon \tag{8}$$

with $\chi > 0$. The proof can be shown along the line of the output feedback design proposed in [9], [10] and [16]. So, we omitted the details proof for brevity and can be obtained from the authors. It is worth noting that, for the given set of initial conditions of interest, the designer can calculate the minimum bound on the value of ϵ *a priori* via using combined Lyapunov-function candidate as, $V_Q = (1-d)\left[\frac{1}{2}S^T M S + \frac{1}{2}e^T K_P e\right] + d\frac{1}{2}\eta^T P \eta$ with $d > 0$ [4]. However, such *a priori* calculated bound on ϵ may not be applied for the real-time applications as the value of ϵ depends on the sampling time, the output and the input disturbance noise [11], [12], [13].

III. Adaptive feedback using multi-model control approach

The main drawback of the CE-based single model CAC approach for state and output feedback design is its poor transient tracking response. Specifically, if the initial conditions and parameter errors become large then the transient tracking performance will also become unacceptably large values. This is mainly because of the assumption that the nonlinear functions are assumed to be appeared linearly with respect to uncertain parameters. The common technique is to use high values of controller-observer gains in order to achieve desired transient tracking performance. The main practical problem, however, is that the observer-controller gains require to increase with the increase of the parametric uncertainty resulting very large control efforts. In particular, when the level of uncertainty is large, two parameters ($\frac{1}{\epsilon}$ and Γ) of the observer-controller design are required to be very high to ensure good transient tracking performance. To increase the domain of interest (stability domain), the control saturation levels [9], [14] (maximum bound on the state feedback control input, τ_{max},) are required to increase causing unacceptable transient peaking phenomenon. In fact, the use of high-gains and large saturation levels are not a practical solution as they may increase the control chattering activity resulting poor tracking performance. In practice, such a large control effort based design may not be realizable as available control input in most system designs is restricted. To tackle the problem associated with high observer-controller gains, we propose to use multi-parameter models based adaptive control technique that allows to keep smaller value of Γ and higher value of ϵ. The main idea behind this approach is to reduce the level of uncertainty via resetting the parameter estimate of CAC design into a model which best approximates the plant among a finite set of candidate models at each instant of time. This implies to identify a control vector corresponding to a model θ that closely approximates the parameters of the manipulator and its payload that operating in the workspace. To identify best possible model from a family of candidates, we propose to use on-line estimation of the derivative of the Lyapunov-function candidate. The design steps can be described as follows. First, we consider that the unknown plant parameters, θ, belongs to a known but comparatively large compact set Ω. Then, we equally distribute the parameter set Ω into a finite number of smaller compact subsets such that $\theta_i \in \Omega_i$ with $\Omega = \bigcup_{i=1}^{N} \Omega_i$ and $\theta \in \Omega_i$. Then, for a given compact set of the initial condition of interest $e(0) \in \Omega_{co}$, we design a family of candidate controllers, bounded in e via saturating outside the region of interest Ω_c, correspond to each of these smaller parameter subsets as

$$\tau^i(e, Q_d, \theta_i) = Y(e, \dot{q}_d, \ddot{q}_d)\theta_i - K_P e_1 - K_D e_2 \tag{9}$$

with $(\theta, \theta_i) \in \Omega_i$, such that for every $\theta \in \Omega_i$ all the signals in the closed-loop system (1) and (9) started inside the sets Ω_{co} are bounded, and the output tracking error trajectories converge to zero, $e(t) \to 0$ as the time goes to infinity. The constant diagonal elements of the positive definite matrices K_P and K_D are chosen such that they ensures an acceptable transient and steady state tracking performance of the closed-loop system [4]. The control gains K_P and K_D are common to all the candidate controllers N. The regressors model $Y(e, \dot{q}_d, \ddot{q}_d)$ [1] is also common to all candidate controllers.

Remark 1: The model selection is based upon the known bound of the robot dynamics and its operating environments. If the manipulator parameters and the masses of the working loads are known to be within a specified range then the model sets can be distributed within the given specified range around with the nominal parameters value. To simplify the control design, the compact parameter sets, Ω, is partitioned into a finite number of smaller compact subsets, Ω_i,. We consider nominal parameter for each compact subsets, Ω_i,. The parameter sets can also be split into non-uniform and non-overlapping regions as long as it covers the entire parameter space. For non-overlapping and non-uniform case, one has to use strict assumption that there exists a controller corresponding to a model that guarantees asymptotic tracking property.

We now develop resetting-logic to identify a suitable model/controller , $\tau(e, Q_d, \theta)$, from a finite set of candidates, $\tau^i(e, Q_d, \theta_i)$,. More specifically, a logic needs to be selected in such a way that guarantees all the signals in the closed-loop systems are bounded, and the error trajectories converge to zero as time goes to zero. We consider that there exists a small time constant t_d such that the solution of the closed-loop system is well defined. This property holds as the manipulator parameters and the masses of the working loads belong to the compact subsets Ω_i The switching condition in Algorithm 1 below is based on using the inequality for the derivative of the Lyapunov-function candidate (4). Our approach can be applied for both state and output feedback control design.

Algorithm 1: *Suppose that the controller index $i \in \mathcal{M}$ is acting in the loop at time t. Then, we follow the following pre-routed switching-logic to identify a controller that satisfies the pre-specified Lyapunov inequality*
[A.] *Assuming that the initial time $t_o = 0$, controller index $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$ and a dwell time constant $t_d > t_o$.* **[B.]** *Then, we put the CAC algorithm, $\tau(e, Q_d, \hat{\theta})$ with $\hat{\theta}$ is provided by classical adaptation law in the loop and dwell it for a short period of time $t \in [t_o, t_o + t_d]$.* **[C.]** *For $t \geq t_o + t_d$, we check the pre-specified resetting inequality using with the derivative of the Lyapunov-function candidate $\dot{V}(t) \leq 0$. If the inequality satisfies then we keep the classical control in the loop. If not then we put the first candidate controller, $\tau^i(e, Q_d, \theta_i)$, with $i = 1$.* **[D.]** *We again dwell this controller for small time t_d and monitor the inequality for the derivative of the Lyapunov function to see whether or not the function decreasing sufficiently fast to switch to the next candidate controller. If the controller does not satisfy the inequality then we switch again to the next candidate controller, $\tau^i(e, Q_d, \theta_i)$, with $i = 2$. We repeat the search until we find a controller that satisfies the derivative of the Lyapunov inequality.*

Using our above analysis, let us state the main results in the following Theorem 2 by assuming that the position and velocity signals are available for candidate controllers (9) design.

Theorem 2 : Consider the closed loop system formulated by (1) and (9) under the switching-logic defined by Algorithm 1. Then, there exists a time such that the controller according to the logic stated in Algorithm 1 is tuned to the plant that ensures $\dot{V}(t) \leq 0$.

Remark 2: As the plant parameters θ belongs to one of the compact subsets, Ω_i, then there exists a finite number of search such that at least one of the candidate controller satisfies the Lyapunov inequality. Therefore, the number of search as well as the switching period is finite.

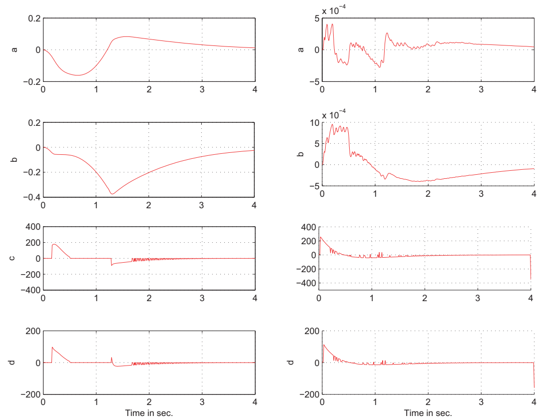

Fig. 1. The implementation results with Theorem 2 and Theorem 4 under $\tilde{\theta} = 8$. First column is for Theorem 2 and the second column is for Theorem 4, where a: output tracking errors (radians) for joint 1, b: output tracking errors (radians) for joint 2, c: control input for joint 1, d: control input for joint 2.

Let us consider the velocity signals e are unavailable in (9). To reproduce unknown velocity signals, we then replace e by the output of the linear estimator (5). Then, we can modify the algorithm (9) to formulate multi-model based adaptive output feedback (AOFB) as

$$\tau^i(\hat{e}, Q_d, \theta_i) = Y(\hat{e}, \dot{q}_d, \ddot{q}_d)\theta_i - K_P\hat{e}_1 - K_D\hat{e}_2 \qquad (10)$$

We also need to estimate the Lyapunov inequality (4) that modifies the resetting criterion for the multi-models based AOFB design. To do that, our first task is to ensure the robust reconstruction of unknown velocity state vectors. Notice from the Lyapunov inequality (8) that one cannot make state estimation error to zero as $\epsilon \neq 0$. Note that the existence of ϵ can be shown along the line of the idea introduced in [4]. This means that we have to find the bound on the non-vanishing estimation error term in the resetting inequality provided by the derivative of the Lyapunov-function candidate (8). For a given observer design constant ϵ there exists a short transient period such that the state estimates $\hat{e}$ decay exponential fast to a small compact set Ω_ϵ. The short transient peaking time $T_1(\epsilon)$ can be determined as, $T_1(\epsilon) = \frac{\epsilon}{\gamma}\ln\left(\frac{k_o}{\beta\epsilon^4}\right)$ where ϵ is known constant, $k_o = k^2\lambda_{max.}(P) = \frac{k^2}{2\gamma}$, $\gamma = \frac{1}{2\lambda_{max.}(P)}$ and $e(0) - \hat{e}(0) \leq k$ with $k \geq 0$. After this transient peaking time, the estimation error converge to a small value, namely $O(\epsilon)$,. To ensure that, the value of t_d requires to choose such that $T_1(\epsilon) < t_d$. Then, we propose to use $\dot{V} + \lambda_{min}(\Pi)\|e\|^2 \leq k_f$ as the modified resetting inequality for the derivative of the Lyapunov function for the multi-model AOFB design. Based on the above analysis, let us state our main results for the multi-model based AOFB design in the following Theorem 3.

Theorem 3 : Consider the closed loop control system designed by using (1), (10) and (5) under the switching-logic *defined in Algorithm 1 with the resetting inequality for the derivative of the Lyapunov-function $\dot{V}(t) \leq k_f$. Then, for the given $(e(0), \hat{e}(0)) \in \Omega_{co}$, $\theta \in \Omega_i$ and $\theta_i \in \Omega_i$ with $i \in \mathcal{M}$, there exists $\epsilon > 0$ and $t_d > T_1(\epsilon)$ such that the candidate controller, corresponding to an appropriate model, according to the Algorithm 1, is tuned to the plant which ensures that all the state variables of the closed-loop system are bounded.*

Proof: The proof of Theorem 2 and Theorem 3 can be shown along the logic defined in Algorithm 1. Due to space limitation, we remove the details proof and can be obtained from authors.

The problem of the pre-routed switching Algorithm 1 is that if the number of candidate controllers become large then the long switching search (number of search) may produce unacceptable transient tracking errors and high-frequency control oscillation. This is mainly because, in the presence of large number of candidate controllers, the switching has to scan through a large number of candidate controllers before converging to the one that satisfies the Lyapunov inequality. On the other hand, if the parameter changes after switching events (if any due to fault) then the logic stated in Algorithm 1 will be insensitive to the parameter change which may cause large transient tracking performance. To avoid unacceptable transient tracking and control oscillation from pre-routed switching-logic, we allow the parameter estimates to be reset instantaneously using with the following switching Algorithm 2. To design that switching logic, we first consider that a family of Lyapunov-function candidates corresponding to a family of candidate controllers (9) as, $\alpha_2^i\|e\|^2 \leq V_i(e, \tilde{\theta}_i) \leq \alpha_3^i\|e\|^2$ $\forall e \in \Omega_c^i = \{(e, \tilde{\theta}_i) \mid V_i(e, \tilde{\theta}_i) \leq c\}$ and $\forall(\theta, \theta_i) \in \Omega_i$, where $c > 0$, $\tilde{\theta}_i = (\theta_i - \theta)$ and α_2^i and α_3^i are bounded positive constant. Then we follow the following logic to identify a candidate controller corresponding to a model which best approximates the plant at each instant of time such that all the trajectories asymptotically converge to zero as the time goes to infinity.

Algorithm 2: *Suppose that the candidate controllers $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$ as well as candidate Lyapunov-functions, $V_i(e, \tilde{\theta}_i)$, are available at any time t. Then, we apply the following logic-based switching to identify a candidate model/controller which closely approximates the plant.* **[A.]** *Define the initial time $t_o = 0$, the switching index $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$ and a small positive dwell time constant $t_d > 0$.* **[B.]** *Put the classical control, $\tau(e, Q_d, \hat{\theta})$, with standard adaptation mechanism for a short period of time $t \in [t_o, t_o + t_d]$.* **[C.]** *For $t \geq t_o + t_d$, we continuously monitor the inequality for the multiple Lyapunov-function candidates to see which candidate generates guaranteed decrease in the value of $\triangle W_i(t) = V_i(t_s) - V_0(t) \leq 0$ where $t_s \geq t_o + t_d$ is the resetting time and $V_0(t)$ is the Lyapunov function (3).* **[D.]** *For $t \geq t_o + t_d$, we continuously monitor the inequality for the multiple Lyapunov-function candidates to see which candidate generates guaranteed decrease in the value of $\triangle W_i(t) = V_i(t_s) - V_0(t) \leq 0$. We keep the CAC law in the loop until the moment of time $t_i \geq t_o + t_d$ when the resetting inequality violated. If the classical controller does not satisfy the Lyapunov inequality then, at $t = t_i$,*

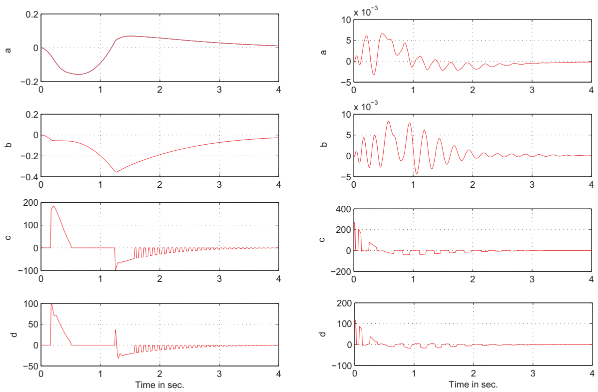

Fig. 2. The implementation results with Theorem 3 and Theorem 5 under $\tilde{\theta} = 8$. First column is for Theorem 3 and the second column is for Theorem 5, where a: output tracking errors (radians) for joint 1, b: output tracking errors (radians) for joint 2, c: control input for joint 1, d: control input for joint 2.

we reset to the candidate control law that generates largest guaranteed decrease in the value of $\triangle W_i(t) \leq 0$. **[E.]** *If the resetting inequality, $\triangle W_i(t) \leq 0$, never violated then there will not be any switching. This implies that the plant output tracks the desired trajectory, e.i., $q(t) \to q_d(t)$ as the time goes to infinity. If at some time, say t_i with $t_i \geq t_o + t_d$ and $t_i = t_o$, the controller that acting in the loop does not satisfy $\triangle W_i(t) \leq 0$ then another candidate will be put in the system as there always exists a controller that provides guaranteed minimum value of $\triangle W_i(t) \leq 0$ at that instant of time.*

We now summarize the results for the multi-model based adaptive control as a state feedback design in the following Theorem 4.

Theorem 4 : Consider the closed-loop system composed of (1) and (9) under the switching-logic defined in Algorithm 2. Then, there exists a time such that, according to Algorithm 2, the control law corresponding to the guaranteed decrease in the value of $\triangle W_i(t) \leq 0$ is tuned to the plant which ensures that all the signals in the closed-loop model are bounded and $e(t) \to 0$ when $t \to \infty$.

Theorem 4 can be applied when all the state vectors are available for feedback to construct multi-model based candidate controllers (9). We now replace the velocity signals in the control law (9) by the output of the linear estimator (5) to formulate multi-model based AOFB (10). Then, we present the main results for the multi-model based AOFB design in the following Theorem 5.

Theorem 5 : Consider the closed-loop system (1), (10) and (5) under the switching-logic Algorithm 2. Then, for the given $(e(0), \hat{e}(0)) \in \Omega_{co}$ and $\theta_i \in \Omega_i$ with $i \in \mathcal{M}$, there exists a small value of $\epsilon > 0$ and $t_d > T_1(\epsilon)$ such that the controller corresponding to a guaranteed decrease in the value of $\triangle W_i(t) \leq 0$ is tuned to the plant. Then, the AOFB control system ensures that all the state variables of the closed-loop system are bounded by a bound that can be made closed to zero.

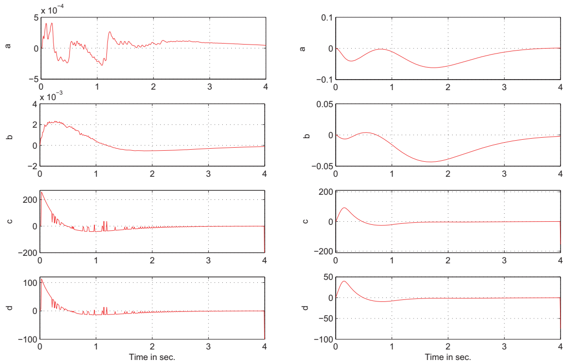

Fig. 3. The implementation results with Theorem 4 and SM-based CAC design under $\tilde{\theta} = 8$. First column is for Theorem 4 and the second column is for the single-model CAC, where a: output tracking errors (radians) for joint 1, b: output tracking errors (radians) for joint 2, c: control input (newton-meters) for joint 1, d: control input (newton-meters) for joint 2.

Proof: The proof of Theorem 4 and Theorem 5 can be shown along the line of the logic introduced in Algorithm 2. The main idea of this logic is to compare candidate controllers, $\tau^i(e, Q_d, \theta_i)$ with $i \in \mathcal{M}$, at each instant of time to see which candidate provides the highest decrease in the value of the Lyapunov inequality, i.e., $\triangle W_i(t) = V_i(t_s) - V_0(t)$, with $V_0(t) = \frac{1}{2}e^T(t)Q_{sm}e(t) + \frac{1}{2}\tilde{\theta}^T(t)\Gamma^{-1}\tilde{\theta}(t)$ and $V_i(t_s) = \frac{1}{2}e^T(t_s)Q_{sm}e(t_s) + \frac{1}{2}\tilde{\theta}_i^T(t_s)\Gamma^{-1}\tilde{\theta}_i(t_s)$ with $\tilde{\theta}(t) = \left(\hat{\theta}(t) - \theta\right)$, $\tilde{\theta}_i(t_s) = (\theta_i(t_s) - \theta)$ and $t_s > t_o + t_d$ is the time when the parameter estimate, $\hat{\theta}(t)$, provided by classical adaptation mechanism is reset into a model from a family of candidate model sets, $\theta_i(t_s)$, that best approximates the plant θ. This implies that the reset will occur if $\triangle W_i(t)$ is a non-increasing sequence with respect to i, that is, $\triangle W_i(t) \le 0$. Due to space limit, we omitted the remaining proof and can be obtained from authors.

IV. Design and Implementation Results

In this section, we show the design and implementation process of the multi-model/control based adaptive control strategy on robotic systems. To do that, we consider a 2-link robotic manipulator system [4], [9], [10]. The dynamic equations for this robot system can be defined as,

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

with $m_{11} = (\theta_1 + 2\theta_2 + 2\theta_2 \cos q_2)$, $m_{12} = (\theta_2 + \theta_2 \cos q_2)$, $m_{21} = (\theta_2 + \theta_2 \cos q_2)$, $m_{22} = \theta_2$, $c_{11} = -2\dot{q}_2\theta_2 \sin q_2$, $c_{12} = -\dot{q}_2\theta_2 \sin q_2$, $c_{21} = \dot{q}_1\theta_2 \sin q_2$, $c_{22} = 0$, $\theta_1 = m_1 l^2$, $\theta_2 = m_2 l^2$, $l = l_1 = l_2$ is the link lengths and m_1 and m_2 are the masses of links 1 and link 2, respectively. The robot operates in the horizontal plane so the gravitational force vector is $G = 0$. We now generate the reference trajectory, $q_d(t)$, for the given robot model to follow, a square wave with a period of 8 seconds and an amplitude of ± 1 radians is pre filtered with a critically damped 2nd-order linear filter using a bandwidth of $\omega_n = 2.0$ rad/sec. Specifically, our main target is to use a desired trajectory that usually uses in industrial robotic systems [11], [12], [13]. We first consider that the plant parameter $\theta \in \Re^2$ is assumed to be unknown but belong to a known compact set as $\Omega \in [-10, 10]$. We define the initial conditions of interest as $e(0) = 2$, $\hat{e}(0) = 2$ and $\hat{\theta}(0) = 0$. Then, we split the parameter set Ω equally into a finite number of smaller compact subsets as $\theta_i \in \Omega_i$ with $\Omega = \bigcup_{i=1}^{41}\{\Omega_i\}$, that is, $\Omega = \bigcup_{i=1}^{41}\{\theta_i\} = \{-10, -9.5, ., ., ., ., ., ., ., ., 9.5, 10\} \times \{-10, 10\}$. The control design parameters λ_0, K_P and K_D are common to all $i = 41$ candidate controllers. The learning gains Γ are chosen such that $(\theta, \theta_i) \in \Omega_i$. For our evaluation, the control design parameters are chosen as $\lambda_0 = 2$, $K_{P1} = 60$, $K_{P2} = 60$, $K_{D1} = 60$, $K_{D2} = 60$ and $\Gamma = 10I_{2\times 2}$.

A. Comparison between Theorem 2 and Theorem 4

We first compare the tracking performance of Theorem 2 (pre-routed resetting logic) and Theorem 4 (continuous resetting logic) on the given robotic system. For this purpose, we apply the above design constants to construct a family of candidate controllers as a state feedback as, $\tau^i(e, Q_d, \theta_i) = Sat\left[Y(e, \dot{q}_d, \ddot{q}_d)\theta_i - K_P e_1 - K_D e_2\right]$ with $i = 41$.We then define small value of $t_d = 0.03$. The implemented results are given in Figure 1 (state feedback case). Figure 1 depicts the conducted results under $\tilde{\theta} = 8$, that is, $i* = 37$. The first column of this Figure is for Theorem 2 and the second column is for Theorem 4. By comparing left and right column of this Figure, we can see that comparatively large transient tracking errors under pre-routed switching-logic of Algorithm 1 than Algorithm 2. We also notice from our results that the tracking error under Theorem 2 increase with the increase of the number i^* as the pre-routed search has to travel larger number of candidate controllers before converging to the one that satisfies the fixed resetting criterion.

B. Comparison between Theorem 3 and Theorem 5

Let us now compare the performance obtained under state feedback based design can be recovered by using output feedback design. To illustrate that, we construct multi-model AOFB as, $\tau^i(\hat{e}, Q_d, \theta_i) = Sat\left[Y(\hat{e}, \dot{q}_d, \ddot{q}_d)\theta_i - K_P\hat{e}_1 - K_D\hat{e}_2\right]$ where $i = 41$. Then, we define slower observer speed as $H_1 = 20I_{2\times 2}$, $H_2 = 20I_{2\times 2}$, $\epsilon = 0.1$. For fair comparison, we keep the same controller design parameters that used for the evaluation of the state feedback based design. The value of t_d is chosen as $t_d = 0.005$ to guarantee $t_d > T_1(\epsilon)$. Then, we choose the value of $k_{f1} = 0.05$ and $k_{f2} = 0.05$. With these set up, we follow the logic defined by Algorithm 1 (Theorem 3) and Algorithm 2 (Theorem 5) on the given system. The tested results are given in Figure 2 with the chosen parameter $\theta = 8$. The left column of this Figure shows the control performance under Theorem 3. The right column of Figure 2 depicts the tracking convergence with the Theorem 5. By comparing left and right column of Figure 2, one can notice that results under multi-model based output feedback approach recover the performance achieved under state feedback design. However, like Theorem 2, undesirable transient tracking under pre-routed switching-logic of Theorem 3 can be observed.

C. Comparison between Theorem 4 and CAC design

In this part of the paper, we compare the tracking convergence property of Theorem 4 with the SM-based CAC design. For this purpose, we implement Theorem 4 and the SM-based CAC algorithm (2) on the given robotic system. The control design parameters are kept similar to our previous evaluation of Theorem 4 except the learning gains under classical control law (2) are used ten times higher than the learning gains employed with multi-model based approach. The conducted results are depicted in Figure 3. Figure 3 is depicted the tracking convergence with the chosen parameter $\tilde{\theta} = 8$. The left colum of this Figure is for Theorem 4 and the right column of the Figure 3 is for CAC approach. In view of the left and right column of Figure 3, we can notice that the output tracking under multi-model based adaptive design converge to the desired one as tracking errors almost converge to zero. But, quite a large tracking errors under single model based classical design can be seen. Note that the learning speed under CAC design is used ten times faster than multi-model based adaptive control design.

D. Comparison between Theorem 5 and Theorem 1

Our aim is now to compare the tracking performance of Theorem 5 with the CAOFB design of Theorem 1. For comparison, we define slower observer design constants as, $H_1 = 20I_{2\times2}$, $H_2 = 20I_{2\times2}$ and $\epsilon = 0.1$. But, we keep the same controller design sets that used for our last evaluation. Note that the learning gains under classical AOFB design are used ten times higher than the learning gains applied for the multi-model based approach of Theorem 5. The tested results are given in Figure 4 with $i* = 37$. The left column of the Figure 4 is for Theorem 5 and the right column is for Theorem 1. In view of the left and right column of Figure 4, we can observe the superiority of the multi-model based AOFB design of Theorem 5 over CAC design of Theorem 1.

V. Conclusion and Future work

In this paper, we have shown that multi-model based adaptive control strategy can be employed to improve overall tracking performance for trajectory control problem of robotic systems. The specific interest in the proposed design is to reduce the control gain of classical adaptive control scheme by reducing the level of parametric uncertainty through on-line estimation of the Lyapunov-function inequality. The method increases the convergence speed of adaptation mechanism via resetting the parameter estimate of CAC technique into a family of candidate models which best approximates the plant at each instant of time. The evaluation on a 2-DOF robotic system has been used to demonstrate the theoretical development for the real-time applications. The implementation of the proposed design on an industrial robotic system will be focused on our future work.

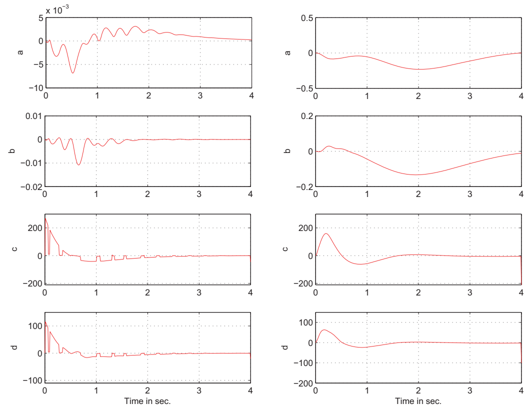

Fig. 4. The implementation results with Theorem 5 and SM-based design of Theorem 1 under $\tilde{\theta} = 8$ with $H_1 = 20I_{2\times2}$, $H_2 = 20I_{2\times2}$ and $\epsilon = 0.1$. First column is for Theorem 5 and the second column is for classical AOFB design, where a: output tracking errors (radians) for joint 1, b: output tracking errors (radians) for joint 2, c: control input (newton-meters) for joint 1, d: control input (newton-meters) for joint 2.

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