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Energy-Based Control Design of an Underactuated 2-Dimensional TORA System

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***Abstract*— The translational oscillation with a rotational actuator (TORA) system has been used as a benchmark for motivating the study of nonlinear control techniques. In this paper, modeling and control of a novel 2-dimensional TORA (2DTORA) are presented. The 2DTORA is an underactuated mechanical system which has one actuated rotor and two unactuated translational carts. The dynamics of the 2DTORA system is derived based on Lagrange equations. The total energy of the system is employed to show the passivity property of 2DTORA, and then a simple state feedback control algorithm is developed based on a proper Lyapunov function including energy item. Finally, simulation results are demonstrated.**

I. INTRODUCTION

The nonlinear benchmark mechanical system TORA or RTAC (Rotational-Translational ACtuator) was originally studied as a simplified model of a dual-spin spacecraft to investigate the resonance capture phenomenon [1][2]. TORA system has two configuration states, translational position of the cart and rotating angle of the eccentric mass, to be controlled; however, only the rotating angle is actuated. Thus TORA system is an underactuated system. Consequently, the dynamic model of the TORA can't be globally feedback linearized and the direct application of well-known nonlinear control schemes such as feedback linearization can't guarantee its global stabilization.

The problem of controlling TORA was brought to attention by Bernstein [1] and has been studied extensively by several researchers. Global stabilization of the TORA system using state feedback and backstepping procedure has been introduced by Wan *et al*. [2] and global output tracking for the TORA system is addressed in [3]. Flexible backstepping control design using adequate Lyapunov functions is discussed in [4]. Jankovic *et al*. [5] explore solutions to this control problem based on the cascade and passivity paradigms. By solving the Hamilton-Jacobi-Isaacs equation, Panagiotis *et al*. [6] present a state-feedback nonlinear controller; a similar control design technique is also employed in [7]. A measurement-scheduled control for the TORA system is obtained in [8] using linear fractional representations. Based on linear parameter-varying gain-scheduling approach, three state-feedback controllers are developed for TORA system in [9]. A decoupled self-tuning signed-distance fuzzy sliding mode controller is applied to TORA system in [10]. In [11], a high pass filter is proposed to approximately differentiate the input position signal as an approximate velocity signal for use in the input control; thus, the TORA system can be controlled without the need for velocity measurements. Moreover, approaches in [12][13][14][15] have been validated through experimental results.

Passivity-based (dissipative, energy-based) control design methods have been widely and successfully applied to mechanical systems [16][17]. Researches have developed controllers for TORA based on its passivity [5][19]. And passivity property of 2DTORA is also presented and employed to develop the its controller in this paper. The energy-based controller design for the 2DTORA proposed here has been inspired by the work in [20] [21].

The rest of the paper is divided into four sections. In Section II, the dynamics of the 2DTORA is developed based on Lagrange equations and some characteristics of this underactuated system are analyzed. In Section III, passivity property of the 2DTORA is presented and then a simple state feedback controller without measuring unactuated system states is proposed. Simulations are performed for the proposed controller in Section IV. Finally, conclusions are given in Section V.

II. DYNAMICAL MODELING

The system shown in Fig. 1 represents a 2 dimensional translational oscillator with an eccentric rotational proof-mass actuator. The oscillator consists of a outer cart of mass M_y connected to a fixed wall by a linear spring of stiffness k_y and an inner cart of mass M_x connected onto a wall of outer cart by a linear spring with stiffness k_x. The outer cart and inner cart are constrained to have one-dimensional linear motion with y and x denoting the travel distance respectively. And also the two translational motions of the carts are perpendicular to each other. The proof-mass actuator attached to the inner cart has mass m and moment of inertia I about its center of mass, which is located a distance r from the point about which the proof-mass rotates. The motion occurs in a horizontal plane; therefore, no gravitational forces need to be considered. In Fig. 1, τ denotes the control torque applied to the proof-mass. F_x and F_y are the translational disturbance forces applied to the moving carts. Let x, $\dot{x}$ and y, $\dot{y}$ denote the translational position and velocity of the inner cart and outer cart respectively, and let θ and $\dot{\theta}$ denote the angular position and velocity of the rotational proof-mass.

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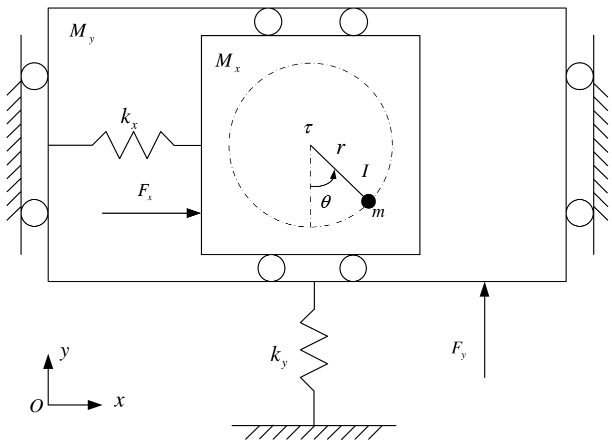


Fig. 1. 2DTORA system configuration

The total energy of the system is a sum of the kinetic energy K and the potential energy P. The total kinetic energy K is a sum of kinetic energy K_x corresponding to the equivalent mass of M_x, kinetic energy K_y corresponding to the equivalent mass of M_y and kinetic energy of the ball K_m.

$$\begin{aligned} K &= K_x + K_y + K_m \quad (1)\\ &= \frac{1}{2}(M_x+m)\dot{x}^2 + \frac{1}{2}(M_y+m)\dot{y}^2 \\ &\quad + mr\dot{\theta}(\dot{x}\cos\theta + \dot{y}\sin\theta) + \frac{1}{2}(mr^2+I)\dot{\theta}^2 \end{aligned}$$

The potential energy is given by

$$P = \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 \quad (2)$$

Remark 1: We consider the motion of the ball of 2DTORA system in the horizontal plane without gravity effects, otherwise there exists another potential item $-mgr\cos\theta$ in (2) if we consider the center point of inner cart as zero gravity potential.

Therefore, the Lagrangian of the system is given by

$$\begin{aligned} L &= K - P \quad (3)\\ &= \frac{1}{2}(M_x+m)\dot{x}^2 + \frac{1}{2}(M_y+m)\dot{y}^2 + mr\dot{\theta}(\dot{x}\cos\theta + \dot{y}\sin\theta) \\ &\quad + \frac{1}{2}(mr^2+I)\dot{\theta}^2 - \frac{1}{2}k_x x^2 - \frac{1}{2}k_y y^2 \end{aligned}$$

The Euler-Lagrange equations of motion for the 2DTORA system can be expressed as

$$\begin{aligned} \frac{d}{dt}(\frac{\partial L}{\partial \dot{x}}) - \frac{\partial L}{\partial x} &= -F_x \quad (4)\\ \frac{d}{dt}(\frac{\partial L}{\partial \dot{y}}) - \frac{\partial L}{\partial y} &= -F_y \quad (5)\\ \frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}}) - \frac{\partial L}{\partial \theta} &= \tau \quad (6) \end{aligned}$$

and thus after some calculating the dynamics of 2DTORA is given by

$$\begin{aligned} (M_x+m)\ddot{x} + mr\cos\theta\ddot{\theta} - mr\sin\theta\dot{\theta}^2 + k_x x &= -F_x \quad (7)\\ (M_y+m)\ddot{y} + mr\sin\theta\ddot{\theta} - mr\cos\theta\dot{\theta}^2 + k_y y &= -F_y \quad (8)\\ mr\cos\theta\ddot{x} + mr\sin\theta\ddot{y} + (mr^2+I)\ddot{\theta} &= \tau \quad (9) \end{aligned}$$

If we remove one of the translational motion from the above dynamics, it is easy to get the dynamics of TORA. As we remove the motion in y axis, the dynamics becomes

$$\begin{aligned} (M_x+m)\ddot{x} + mr\cos\theta\ddot{\theta} - mr\sin\theta\dot{\theta}^2 + k_x x &= -F_x (10)\\ mr\cos\theta\ddot{x} + (mr^2+I)\ddot{\theta} &= \tau \quad (11) \end{aligned}$$

which is the TORA dynamics used in [1].

In a compact form, the dynamics of 2DTORA system can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F = U \quad (12)$$

where

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

$$M(q) = \begin{bmatrix} M_x+m & 0 & mr\cos\theta \\ 0 & M_y+m & mr\sin\theta \\ mr\cos\theta & mr\sin\theta & mr^2+I \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & -mr\sin\theta\dot{\theta} \\ 0 & 0 & mr\cos\theta\dot{\theta} \\ 0 & 0 & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} k_x x \\ k_y y \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} F_x \\ F_y \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 \\ 0 \\ \tau \end{bmatrix}$$

From (12), it is clear that q is the configuration variable vector of the system with $q_2 = \theta$ the actuated variable vector and $q_1 = (x,y)^{\mathrm{T}}$ the unactuated variable vector of the system. Since there are three configuration variables to be controlled with only one actuated configuration variable, the 2DTORA system is an underactuated system. $M(q)$, $C(q,\dot{q})$, $G(q)$, F, and U are the inertia matrix, Coriolis and centrifugal force matrix, potential energy matrix, disturbance force vector, and control input vector respectively.

There are some general properties of these matrices.

Note that $M(q)$ is symmetric, and

$$\begin{aligned} \det(M(q)) &= (M_x+m)[(M_y+m)(mr^2+I) - (mr\sin\theta)^2] \\ &\quad -(mr\cos\theta)^2(M_y+m) \\ &= M_xM_ymr^2 + M_xm^2r^2\cos^2\theta + M_ym^2r^2\sin^2\theta \\ &\quad +(M_xm + M_ym + M_xM_y + m^2)I > 0 \quad (13) \end{aligned}$$

Therefore, $M(q)$ is positive definite defined for all q.

Calculating $\dot{M}-2C$ with (12), it follows that

$$\dot{M}(q)-2C(q,\dot{q})=\begin{bmatrix} 0 & 0 & mr\sin\theta\dot{\theta} \\ 0 & 0 & -mr\cos\theta\dot{\theta} \\ -mr\sin\theta\dot{\theta} & mr\cos\theta\dot{\theta} & 0 \end{bmatrix}$$

It is a skew-symmetric matrix which has an important property

$$z^T(\dot{M}(q)-2C(q,\dot{q}))z=0 \quad \forall z \tag{14}$$

This property will be used in establishing the passivity of the 2DTORA system.

The potential energy of the system P is related to $G(q)$ as

$$G(q)=\frac{\partial P}{\partial q}=\begin{bmatrix} k_x x \\ k_y y \\ 0 \end{bmatrix} \tag{15}$$

Note that the actuated variable in an underactuated system can be controlled directly by the input torque; however, the unactuated degree of freedom must be controlled through system coupling. In other words, the nonlinear coupling between the rotational angle of the proof-mass and translational motion of the carts provides the basis for control of 2DTORA. As for a TORA system (10)(11), the destination angle of the rotor can't be set to angles aligned to the translation direction, otherwise the translation position of the cart is uncontrollable. Similarly, the angular position θ of the rotor in 2DTORA can't be set to the angles aligned to the two translation directions. The reason is that if the destination angle of the rotor is aligned to the translation motion of the car, the nonlinear coupling will disappear, and the control input has no effect on translational motion of the cart any more. As we can see from (9), the coupling coefficient between rotation angle θ and translation position y is $mr\sin\theta$. If $\theta=k\pi$, where k is an integer, then $\sin\theta=0$, which means the nonlinear coupling between rotational proof-mass and translational motion of the cart in y direction has disappeared. As a result, the cart position in y direction can not be controlled by the input control torque τ after the θ is brought to $k\pi$ by the control input.

III. ENERGY-BASED CONTROL DESIGN

Control design for underactuated systems is challenging and attracts many researchers. The well known and extensively studied underactuated systems has one actuated variable and one unactuated variable such as the following benchmarks: Cart-Pole system (single stage inverted pendulum system), TORA, Pendubot, Acrobot. The number of actuated DOFs is no less than the number unactuated DOFs in most underactuated systems we deal with. There, however, exists system with more unactuated DOFs than actuated DOFs. A typical example is the double stage inverted pendulum [22]. Although 2DTORA also has one actuated variable and two unactuated variables as in (12), it is distinctly different from the double stage inverted pendulum systems. The matrices $M(q)$ and $C(q,\dot{q})$ of 2DTORA are based on the actuated variable while the matrices of the double stage inverted pendulum are based on the unactuated variables. In other words, the 2DTORA has actuated shape variable, while double stage inverted pendulum systems has unactuated shape variables [23].

A. Passivity of 2DTORA

Passivity concept can be defined by introducing the notions of storage function $S(x)$ and supply rate $\omega(u,y)$, where x is the system state, u is the input, and y is the output. A system is passive if it has a positive semi-definite storage function $S(x)$ and a bilinear supply rate $\omega(u,y)=u^{\mathrm{T}}y$, satisfying the inequality $S(x(T))-S(x(0))\leq\int_0^T\omega(u(t),y(t))dt$ for all u and $T\geq 0$. Passivity, therefore, is the property that the increase in storage S is not larger than the integral amount supplied [24]. Various properties of passive system make it "easy to control"[16], which means passivity property of a system can be employed to facilitate its control task.

To find out the passivity of the 2DTORA system, we consider the total energy of the system

$$\begin{aligned} E &= K+P \\ &= \frac{1}{2}\dot{q}^T M(q)\dot{q}+\frac{1}{2}k_x x^2+\frac{1}{2}k_y y^2 \end{aligned} \tag{16}$$

Neglecting the disturbance force matrix F in the dynamics of the 2DTORA and differentiating E, based on (12) and (14) we obtain

$$\begin{aligned} \dot{E} &= \dot{q}^T M(q)\ddot{q}+\frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q}+\dot{q}^T G(q) \\ &= \dot{q}^T(-C(q,\dot{q})\dot{q}-G(q)+U+\frac{1}{2}\dot{M}(q)\dot{q})+\dot{q}^T G(q) \\ &= \dot{q}^T U=\dot{\theta}\tau \end{aligned} \tag{17}$$

Integrating both sides of the above equation we get

$$\int_0^t \dot{\theta}(t)\tau dt=E(t)-E(0)\geq -E(0) \tag{18}$$

Therefore, the system having τ as input and $\dot{\theta}$ as output is passive.

Remark 2: The deduction process of the passivity of the 2DTORA shows that the system is always passive taking τ as input and $\dot{\theta}$ as output regardless of the motion of the ball affected by its gravity or not. When the motion of the ball occurs in a vertical plane, additional item for gravity potential should be added to the right of the inequality (18). However, when the motion of the rotor occurs in a vertical plane, the dynamics of the 2DTORA has two equilibriums with $\theta=0$ and $\theta=\pi$ for all $\tau=0$ and $\theta\in(-\pi,\pi]$. At the equilibriums, the coupling item to control the position of the outer cart y as shown in (9) disappears. As a result, when the rotor motion occurs in a vertical plane, the equilibriums of configuration variables of the 2DTORA system can't be achieved dynamically.

B. Controller design

The passivity property of the system suggests that the total energy E should be considered in the controller design. Since we wish to bring to zero E and angle θ to desired angle θ_f, we propose the Lyapunov function candidate as

$$V(q,\dot{q}) = k_1 E + \frac{k_2}{2}(\theta - \theta_f)^2 \qquad \theta_f \neq k\pi/2 \tag{19}$$

where k_1, k_2 are positive constants and k is an integer. Note that the Lyapunov function $V(q,\dot{q})$ is positive for any θ.

Differentiating V we get

$$\begin{aligned} \dot{V}(q,\dot{q}) &= k_1\dot{E} + k_2(\theta - \theta_f)\dot{\theta} \\ &= k_1\dot{\theta}\tau + k_2(\theta - \theta_f)\dot{\theta} \\ &= \dot{\theta}(k_1\tau + k_2(\theta - \theta_f)) \end{aligned} \tag{20}$$

Define

$$k_1\tau + k_2(\theta - \theta_f) = -k_3\dot{\theta} \tag{21}$$

where $k_3 > 0$, we obtain

$$\dot{V} = -k_3\dot{\theta}^2 \tag{22}$$

which leads to the control law

$$\tau = -\frac{1}{k_1}(k_2(\theta - \theta_f) + k_3\dot{\theta}) \tag{23}$$

The control input (23) can guarantee the deferential of the candidate Lyapunov function is a negative semi-definite function. Therefore, the closed-loop control system is stable with the stability criteria of Lyapunov.

Remark 3: The control law (23) suggests that the stability control of the system can be realized without measuring unactuated configurable variables and their speed items, i.e., the positions x, y of the carts and their speed $\dot{x}$, $\dot{y}$.

Remark 4: The above controller design can also be applied to the TORA system. And the controller derived here is the same as the controller developed for TORA in [5][6]. Compared to [5][6], the Lyapunov function including of the total energy E proposed in this paper is simpler and the controller is easier to be achieved.

IV. Simulation Results

In order to verify the analysis on the target angle control of the system and to observe the performance of the proposed control scheme, we performed the control system simulations on MATLAB using SIMULINK.

Referencing the parameters of TORA in [1], simulation parameters of 2DTORA system are chosen as in Table I; and the gains of the controller (23) are chosen as k_1=30, k_2=2, k_3=0.12.

TABLE I

SIMULATION PARAMETERS

Parameter	Value	Units	Description
M_x	1.3608	kg	Cart mass of axis x
M_y	2.7216	kg	Cart mass of axis y
m	0.096	kg	Ball mass
k_x	186.3	N/m	Spring stiffness of axis x
k_y	279.45	N/m	Spring stiffness of axis y
r	0.0592	m	Arm length
I	0.0592	$\mathrm{kg}\cdot\mathrm{m}^2$	Ball Inertia

The first column in Fig. 2 shows the simulation results of the control system with the initial condition $(x,\dot{x},y,\dot{y},\theta,\dot{\theta}) = (0.01,0,0.01,0,0,0)$ and target rotor angle $\theta_f = \pi/3$. As we can see, positions of the carts are stabilized to zero from their initial positions after 65 seconds; the rotor angle is bring to the set value $\pi/3$, i.e. 60°, and the total energy of the system decreases to zero. Therefore, the linear control law can stabilize the system effectively. At the same time, the carts position and the continuous control torque can meet the constrains of $x \leq 0.025$m and $\tau \leq 0.100$N·m [1].

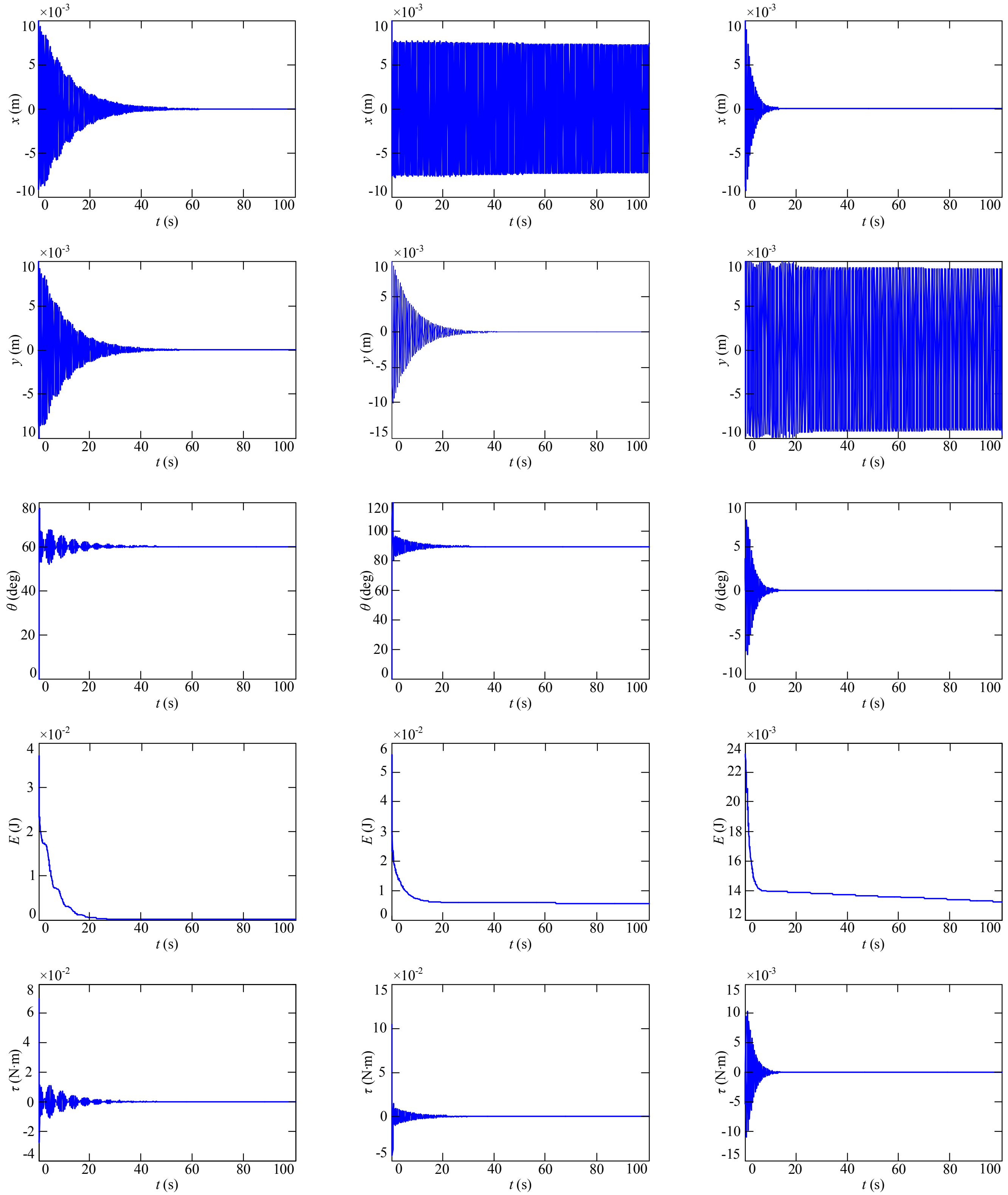
To illustrate that the target rotor angle can't be set to the $k\pi/2$ where k is an integer, the simulation results with $\theta_f = \pi/2$ and $\theta_f = 0$ are shown as the last two columns in Fig. 2. In the middle column of Fig. 2 we can see that if the target angle was set to $\theta_f = \pi/2$ which is aligned to the x axis, the position of the inner cart x can't be stabilized anymore while the position y and rotor angle θ can be stabilized a little faster with the same initial condition. As a result of the unstabilized cart position x, the total energy of the system doesn't decrease to zero. The analysis on the results with $\theta_f = 0$ is similar. The simulation results are in accordance with the previous analysis on the target rotational angle in section II.

We have neglected the disturbance forces of translational motions during our controller design. Suppose the disturbance forces be friction forces defined as $F_x = k_{fx}\dot{x}$, $F_y = k_{fy}\dot{y}$ where k_{fx} and k_{fy} are friction coefficients. It is anticipated that the damping time of the control system will be shortened with the same conditions once these disturbance forces are applied to the system. The simulation results under disturbance forces with the same initial condition $(0.01,0,0.01,0,0,0)$ and the same target rotor angle $\theta_f = \pi/3$ are shown in Fig. 3, and the friction coefficients are both set as 0.05. Comparing the results in Fig. 3 and the column (a) in Fig. 2, the system responses are consistent, while the stabilizing time for system with disturbance forces reduces to 50 seconds.

V. Conclusion

TORA is an underactuated mechanical system consisting of one passive translational cart and one actuated rotor, and its dynamics is taken as a benchmark nonlinear system to test control design techniques. Adding an unactuated DOF cart motion to the TORA leads to our proposed 2DTORA system. Based on the derived dynamics, the 2DTORA system has an actuated variable (rotor angle) can be controlled by the input torque directly and two unactuated variables (cart positions) must be controlled through system coupling. In order to keep the coupling items in the dynamics, the target rotor angle can't be set as $k\pi/2$ where k is an integer.

The total energy of the system is employed to show the passivity property of 2DTORA, and then a proper Lyapunov function including energy item is designed. Based on Lyapunov theory, a simple state feedback controller only with rotor angle and its deviation item is achieved for the underactuated 2DTORA system. Simulation results verified the analysis and controller design of the system.



(a) (b) (c)

Fig. 2. Simulation results: (a) initial condition (0.01, 0, 0.01, 0, 0, 0) and $\theta_f = \pi/3$; (b) initial condition (0.01, 0, 0.01, 0, 0, 0) and $\theta_f = \pi/2$; (c) initial condition (0.01, 0, 0.01, 0, 0, 0) and $\theta_f = 0$.

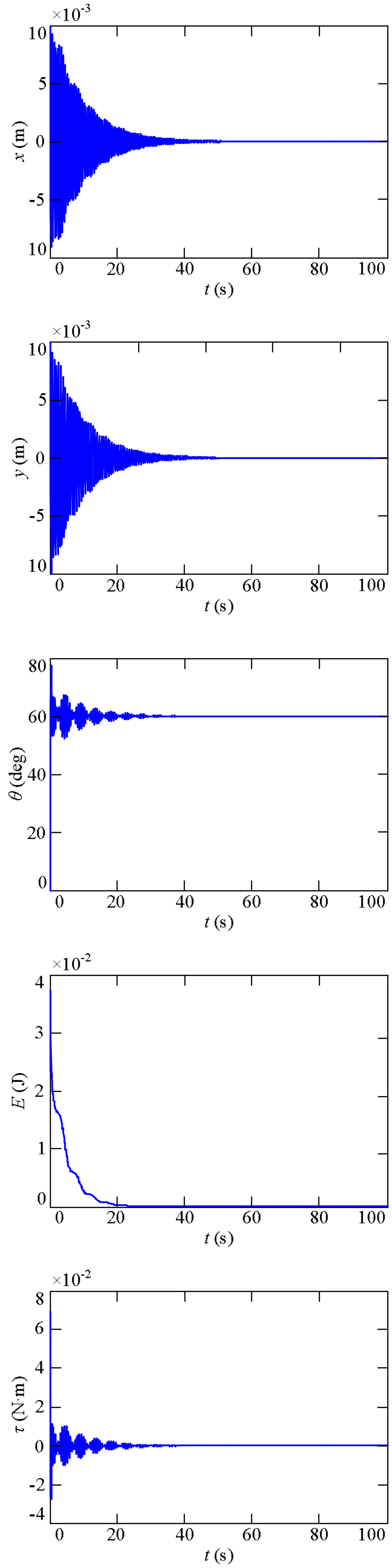


Fig. 3. Simulation results with initial condition (0.01, 0, 0.01, 0, 0, 0), $\theta_f = \pi/3$, and $F_x = 0.05\dot{x}$, $F_y = 0.05\dot{y}$.

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