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Active Model Based Predictive Control for Unmanned Helicopter in Full Flight Envelope

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Abstract- **For the control of unmanned helicopters in full flight envelope, an active model based control scheme is developed in this paper. An adaptive set-membership filter (ASMF) is used to online estimate both the model error due to flight mode change and its boundary, taking advantage of ASMF, so that the model error can be assumed unknown but bounded (UBB). The proposed approach is practical because the model error depends on both helicopter dynamics and flight states, and may not be assumed as white noise. An active modeling based stationary increment predictive control (AMSIPC) is also proposed based on the estimated model error and its boundary to optimally compensate the model error, as well as the aerodynamics time delay. The proposed method has been implemented on the ServoHeli-20 unmanned helicopter platform and experimentally tested, and the results have demonstrated its effectiveness.**

I. INTRODUCTION

Unmanned helicopters are increasingly popular platforms for unmanned aerial vehicles (UAVs). With the abilities such as hovering, taking off and landing vertically, unmanned helicopters extend the potential applications of UAVs. However, due to the complex mechanism and complicated aero-flow during flight, it is almost impossible to accurately model the dynamics of an unmanned helicopter in full flight envelope, and the significant model uncertainties associated with a nominal model may degrade the performance and even stability of an onboard controller.

Due to the difficulty in obtaining a high fidelity full envelope model, the multi-mode modeling technique has been proposed for helicopters such as tilt-rotor aircraft XV-15 [1], helicopter BO-105 [2], UH-60 [3], R-50 [4] and X-Cell [5]. The mode-dependent model, which is identified and simplified according to a specific flight mode such as hovering, cruising, taking off and landing, can be used for control design for the corresponding flight mode. However, the mode-dependent control suffers from at least two problems: one is the difficulty in accommodating the mode transition dynamics [6], and the other is the compensation of the 'model drift' due to flight dynamics change within one particular mode. Up to now, for the purpose of practical implementation, the mode transition problem can be partially dealt with by limiting the mode switching conditions, e.g., mode change is made through hovering mode.

Robust and adaptive control techniques [7-8], on the other hand, have been used to deal with the 'model shift' within a flight mode. However, such control schemes normally need to know the boundary of internal and external uncertainties and relative noise distribution, which are difficult to identify accurately for a helicopter in full flight envelope.

In recent years, the encouraging achievement in sequential estimation makes it an important direction for online modeling and model-reference control [9]. Among stochastic estimations, the most popular one is the Kalman-type filters (KFs) [10, 11, 12]. Although widely used, the KFs suffer from sensitivity to bias and divergence in the estimates, relying on assumptions on statistic distribution such as white noise and known mean or covariance for optimal estimation. In many cases, it is more practical to assume that the noises or uncertainties are unknown but bounded (UBB). In view of this, the set-membership filter (SMF), which computes a compact feasible set in which the true state or parameter lies only under the UBB noise assumption, provides an attractive alternative [13-14].

On the control issue, model predictive control (MPC) can compensate for the aerodynamics delay and does not require a high accuracy reference nonlinear model [15]. Among these methods, linear generalized predictive control (GPC) has become one of the most popular MPC methods in industry and academia. However, the normal GPC is sensitive to process noise and model errors, which are unknown but bounded for helicopters in full flight envelope.

In this paper, for realizing the coupling control of unmanned helicopters in full flight envelope, an active modeling based controller is developed based on a modified generalized predictive control and adaptive set-membership filter estimation (ASMF). The time varying model error and its boundary are estimated by the adaptive set-member filter Based on this active estimation and the modified GPC controller is developed. Flight experiments have been conducted to test the performance of the proposed controller on our UAV platform, and experimental results have demonstrated the effectiveness of the proposed method.

II. ACTIVE MODEL BASED CONTROL SCHEME AND REFERENCE MODEL OF A HELICOPTER

Fig. 1 illustrates the active model based control scheme. The error between the reference model and the actual dynamics of the controlled plant is estimated by an on-line modeling strategy. The control, which is designed according

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to the reference model, should be able to compensate the estimated model error and it in real time. In the followings of this paper, we use the ASMF as the active modeling algorithm and the modified GPC as the control.

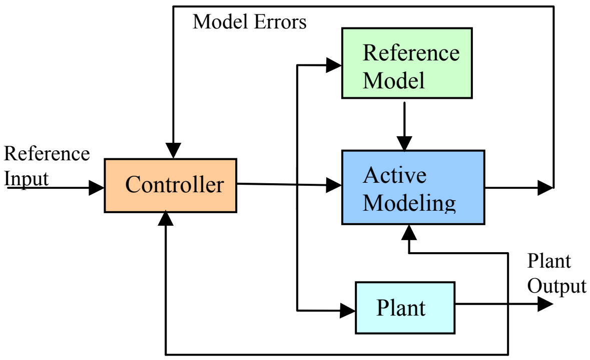

Figure 1: The scheme of active model based control

A reference model is typically obtained by linearizing the nonlinear dynamics of a helicopter at one flying mode. The model errors from linearization, external disturbance, simplification, and un-modeled dynamics can be considered as additional process noise [16]. Thus, a linearized state-space model for helicopter dynamics in full flight envelope can be formulated as

$$\begin{cases} \dot{X} = A_0 X + B_0 U + B_f f(X, \dot{X}, W) \\ Y = CX \end{cases} \tag{1}$$

where $X \in R^{13}$ is the state, including 3-axis velocity, pitch and roll angle, 3-axis gyro values, flapping angles of main rotor and stabilizer bar and the feedback of yaw gyro. $Y \in R^8$ is the output, including 3-axis velocity, pitch and roll angle and 3-axis gyro values, A_0 and B_0 contain parameters that can be identified in different flight modes, and we use them to describe the parameters in hovering mode. $U \in R^4$ is the control input vector. The detail of building the nominal model and physical meaning of parameters is explanted in Ref.[17].

III. ASMF Based Active Model Error Estimation

As illustrated in Fig.1, adopting the active modeling process to get the model error f and system state X is the basis for elimination of the model error. Controller can only work based on nominal model and feedback of state and model error from active modeling process. In this section, the active modeling process is built based on an adaptive set-membership filter (ASMF) [14] since the UBB process noise.

First, we must obtain the reference equation for estimation. Compared with the sampling frequency (often >50Hz for flight control) of the control system, the model error $f(X, \dot{X}, W)$ can be considered as a slow-varying vector, which means $f_{t+1} = f_t + h_t$, where f_t is the sampling value of $f(X, \dot{X}, W)$ at sampling time t, and h_t is the assumed unknown but bounded (UBB) process noise.

Let the extended sampling state $X_t^a = \begin{pmatrix} X_t^T & f_t^T \end{pmatrix}^T$, and then we can obtain the discrete equation from Eq. (1) as

$$\begin{cases} X_{t+1}^a = A_d^a X_t^a + B_d^a U_t + W_t^a \\ Y_t = C_d^a X_t^a + V_t \end{cases} \tag{2}$$

where $A_d^a = \begin{pmatrix} A_d & B_f \\ 0_{13\times13} & I_{13\times13} \end{pmatrix}$, $B_d^a = \begin{pmatrix} B_d \\ 0_{13\times4} \end{pmatrix}$, $C_d^a = \begin{pmatrix} C_d & 0_{8\times13} \end{pmatrix}$, $W_t^a = \begin{pmatrix} W_t^T & h_t^T \end{pmatrix}^T$, $B_f = I_{13\times13}$ and f_t is a 13×1 vector for model errors. Here, t is the sampling time, $I_{m\times m}$ is the $m\times m$ unit matrix and $0_{m\times n}$ is the $m\times n$ zero matrix. $\{A_d, B_d, C_d\}$ is the discrete expression of system $\{A_0, B_0, C\}$.

Both the model error and the process noise W^a are vehicle dynamics and flight states dependent, and do not necessarily have a normal distribution. Thus, the Kalman type filter cannot be applied, and adaptive set-membership filter, which is developed for UUB process noise and can get the uncertain boundaries of the states, is considered to estimate the states and model errors here.

In this section we only present the result of ASMF and please refer to [14] for the details about ASMF. With respect to Eq. (2), we can build the adaptive set-membership filter as Eq. (3),

$$\begin{cases} \rho_t = \dfrac{\sqrt{r_{mt}}}{\sqrt{r_{mt}} + \sqrt{p_{mt}}} \\ W_t = \dfrac{C_d^a P_{t|t-1} C_d^{aT}}{1-\rho_t} + \dfrac{R^a}{\rho_t} \\ K_t = \dfrac{P_{t|t-1} C_d^{aT} W_t^{-1}}{1-\rho_t} \\ \delta_t = 1 - (Y_t - C_d^a \hat{X}_{t|t-1}^a)^T W_t^{-1} (Y_t - C_d^a \hat{X}_{t|t-1}^a) \\ \hat{X}_{t|t}^a = \hat{X}_{t|t-1}^a + K_t (Y_t - C_d^a \hat{X}_{t|t-1}^a) \\ P_{t|t} = \delta_t \left(\dfrac{P_{t|t-1}}{1-\rho_t} - \dfrac{P_{t|t-1}}{1-\rho_t} C_d^{aT} W_t^{-1} C_d^a \dfrac{P_{t|t-1}}{1-\rho_t} \right) \\ \hat{X}_{t+1|t}^a = A_d^a \hat{X}_{t|t}^a + B_d^a U_t \\ \beta_t = \dfrac{\sqrt{Tr(Q^a)}}{\sqrt{Tr(Q^a)} + \sqrt{Tr(A_d^a P_{t|t} A_d^{aT})}} \\ P_{t+1|t} = \dfrac{A_d^a P_{t|t} A_d^{aT}}{1-\beta_t} + \dfrac{Q^a}{\beta_t} \end{cases} \tag{3}$$

where $Q^a \in R^{26\times26}$ and $R^a \in R^{8\times8}$ are the initial elliptical boundary of process noise W_t^a and measurement noise V_t, respectively, r_{mt} is the maximum eigenvalue of R^a at sampling time t, p_{mt} is the maximum eigenvalue of $C_d^a P_{t|t-1} C_d^{aT}$ at sampling time t, operational symbol $Tr(\bullet)$ is the trace of a matrix, δ_t and β_t are the adaptive parameters of the filter, $\hat{X}^a_{t+1|t}$ is the estimation of the extended state X^a_{t+1} at sampling time t, $\hat{X}^a_{t|t}$ is the estimation of $\hat{X}^a_t$ at sampling time t, $P_{t+1|t}$ is the estimation of elliptical boundary P_{t+1} at sampling time t, and $P_{t|t}$ is the estimation of elliptical boundary P_t at sampling time t . We can also obtain the boundary of the ith element X^a_{it} of extended state X^a_t as $\left(\hat{X}^a_{it}-\sqrt{P^{ii}_t}, \hat{X}^a_{it}+\sqrt{P^{ii}_t}\right)$ at sampling time t, where P^{ii}_t is the ith diagonal element of matrix $P_{t|t}$.

IV. Modified GPC For Unmanned Helicopters

We describe the normal GPC in Part A, and then, the modified scheme is proposed in Part B&C to eliminate the negative influence of model errors in real applications.

A. Preliminary work for Generalized Predictive Control

Generally, for a linear system with actuator time delay like,

$$\begin{cases} X_{t+1} = A_d X_t + B_d u_{t-k} + W_t \\ y_t = C_d X_t \end{cases} \tag{4}$$

where $X_t \in R^{n\times 1}$ is the system state vector at sampling time t, $y_t \in R^{l\times 1}$ is the output vector, $u_t \in R^{m\times 1}$ is the control input vector, k is the actuators' time-delay, and W_t is process noise; traditional Generalized Predictive Control (GPC) can be designed as [18].

However, with application to the unmanned helicopters, this kind of GPC algorithm has the following three disadvantages:

1) It cannot reject the influence of working mode changes, i.e., if

$$X_t = x_t - x_0 > \pi(x_0) \tag{5}$$

where x_0 is the current operation point, $\pi(x_0)$ is the valid range for model linearization and x_t is the absolute state at time t, the biased prediction will bring steady errors for velocity tracking.

2) Normal GPC is sensitive to mismatch of the nominal model, which means slow change in parameters (A_d, B_d) may result in prediction error and unstable control.

3) The transient model errors of the nominal model from external disturbance, estimated by ASMF, cannot be eliminated, and this will also result in the instability of the close loop.

B. Stationary Increment Predictive Control

To reject the influence of working mode change and sensitivity to nominal parameters change in real application, we assume that the process noise W_t's increment in Eq. (4) is a stationary random process, which means

$$W^0_t \triangleq \Delta W_t = W_t - W_{t-1} \tag{6}$$

is normal distribution. Where $\Delta = 1-q^{-1}$ is the difference operator; q^{-1} is one-step delay factor. Thus, Eq. (4) can be rewritten as follows,

$$\Delta X_{t+1} = A_d \Delta X_t + B_d \Delta u_{t-k} + W^0_t \tag{7}$$

To avoid the step signal reference tracking, which is dangerous for unmanned helicopter system, we use a low pass filter to calculate the set-point inputs of the output in the future i-th step, i=1, …, p.

Let $SP_t \in R^{l\times 1}$ be the set-point input at time t, then we have

$$r_{t+k+i} = SP_t + \alpha(r_{t+k+i-1} - SP_t) \quad ,1 \le i \le p \tag{8}$$

where α is the cut-off frequency of the filter, the initial value $r_{t+k} = \hat{y}_{t+k|t}$, r_{t+k+i} is the i-th set-point input, and $\hat{y}_{t+k|t}$ is the estimate of output at time t+k.

Thus, the set-point problem is solved and the output prediction can be implanted based on increment model (7) as follows:

Let $\Delta\hat{X}^1_{t+i|t} = \hat{X}^1_{t+i|t} - \hat{X}^1_{t+i-1|t}$, then

$$\begin{aligned} \hat{X}_{t+k+i|t} &= \hat{X}_{t+k+i-1|t} + A_d \Delta\hat{X}_{t+k+i-1|t} + B_d \Delta u_{t+i} \\ &= \hat{X}^1_{t+k+i-1|t} + A_d \Delta\hat{X}^1_{t+k+i-1|t} \\ &+ \sum_{m=0}^{i-1}\left\{\{\sum_{n=0}^{i-1-m} A^n_d B_d\}\Delta u_{t+m}\right\} \\ &= \hat{X}^1_{t+k+i|t} + \sum_{m=0}^{i-1}\left\{\{\sum_{n=0}^{i-1-m} A^n_d B_d\}\Delta u_{t+m}\right\}, 1 \le i \le p \end{aligned} \tag{9}$$

Hence, the above problem 1), which comes from working mode change, is solved because x_0 disappears in predictive equation (9).

We can obtain the following prediction matrix for the output from Eq. (9) :

$$\begin{aligned} \hat{Y}_t &= \left(\hat{y}_{t+k+1|t} \quad \hat{y}_{t+k+2|t} \quad \dots \quad \hat{y}_{t+k+p|t}\right)^T \\ &= \left(C_d\hat{X}^1_{t+k+1|t} \quad C_d\hat{X}^1_{t+k+2|t} \quad \dots \quad C_d\hat{X}^1_{t+k+p|t}\right)^T \\ &+ G\left(\Delta u_t^T \quad \Delta u_{t+1}^T \quad \dots \quad \Delta u_{t+p-1}^T\right)^T \\ &= Y^1_t + G\Delta U \end{aligned} \tag{10}$$

where Y^1_t is the known part of p steps' prediction, which cannot be influenced by current control input, and matrix G has the following form:

$$G = \begin{pmatrix} C_d B_d & 0 & \dots & 0 \\ C_d B_d + C_d A_d B_d & C_d B_d & \dots & 0 \\ \dots & \dots & \dots & \dots \\ C_d \sum_{i=0}^{p-1} A^i_d B_d & C_d \sum_{i=0}^{p-2} A^i_d B_d & \dots & C_d B_d \end{pmatrix} \tag{11}$$

Compared with the normal GPC, the prediction of SIPC has better characteristics that can be described by the following theorem, which solves the above problem 2) in Part A.

Theorem: for nominal model (7), the state prediction obtained by Eqs. (9) maintains unbiased even when the nominal model parameters (A_d, B_d) change online. The characteristic is also maintained in normal GPC conditions, where W_t is normal distribution.

In order to reduce the computational burden of Eq. (10), we propose here a 'step plan' technique,

$$\Delta u_{t+i+1} = \beta \Delta u_{t+i} \tag{12}$$

where β is a $m \times m$ diagonal matrix presenting the length of one step. Then, we can simplify Eq. (10) by only calculating the unknown control,

$$\begin{aligned}\hat{Y}_t &= Y_t^1 + G\left(I_{m\times m} \quad \beta \quad \ldots \quad \beta^{p-1}\right)^T \Delta u_t \\ &= Y_t^1 + G_2 \Delta u_t\end{aligned} \tag{13}$$

where $I_{m\times m}$ is a $m \times m$ unit matrix.

The cost function of the stationary increment predictive control is designed as:

$$J = (R_t - \hat{Y}_t)^T W (R_t - \hat{Y}_t) + \Delta u_t^T \lambda \Delta u_t \tag{14}$$

where $R_t = \left(r_{t+k+1}^T \quad r_{t+k+2}^T \quad \ldots \quad r_{t+k+p}^T\right)^T$, $W \in R^{lp\times lp}$ is the weight matrix for tracking error, and $\lambda \in R^{m\times m}$ is the weight matrix of the control increment.

By minimizing the cost function of Eq. (14), we can calculate the control vector as follows:

$$\begin{aligned}\Delta u_t &= (G_2^T W G_2 + \lambda)^{-1} G_2^T W (R_t - Y_t^1) \\ &= K_f (R_t - Y_t^1)\end{aligned} \tag{15}$$

where $K_f = (G_2^T W G_2 + \lambda)^{-1} G_2^T W$ can be completed offline.

Consequently, the stationary increment predictive controller (SIPC) can be designed as followings.

Step I: Make increment prediction

Based on the current and history measure value, use Eqs. (9-10) to obtain the prediction for future output $\hat{Y}_t$ and initial plan point $r_{t+k} = \hat{y}_{t+k|t}$.

Step II: Plan for the set-point input

Use Eq. (8) to plan the future set-points, and obtain

$$R_t = \left(r_{t+k+1}^T \quad r_{t+k+2}^T \quad \ldots \quad r_{t+k+p}^T\right)^T$$

Step III: Receding horizon optimization

Calculate the control increment Δu_t, based on Eq. (15).

Step IV: Control implementation

Current control input $u_t = u_{t-1} + \Delta u_t$, which is used as the control to the real plant. After that, go back to step I at the next time instant.

C. Optimal strategy for model error compensation

In order to compensate the model error in Eq. (1), the control vector has to match the following equation, which can be directly obtained from Eq. (1):

$$B_d U_t + B_f f_t = B_d U_t^0 \tag{16}$$

where U_t^0 is the control vector need to be calculated by the predictive controller in section IV.B, designed based on the original model (1) without the model error f.

Thus, we introduce the following cost function with quadratic form to solve the above problem 1).

$$\begin{aligned}&U_t^* = \arg\min_{U_t} J_t(U_t) \\ &J_t(U_t) \triangleq \left(B_d U_t + B_f f_t - B_d U_t^0\right)^T H \left(B_d U_t + B_f f_t - B_d U_t^0\right)\end{aligned} \tag{17}$$

where H is a weight matrix, which can be selected.

Actually, the convergence of ASMF algorithm is also influenced by the control action U_t. This is because the stability of the ASMF can be represented by the filter parameter δ_t, while δ_t in Eq. (3) can be rewritten as follows,

$$\begin{aligned}\delta_t &= 1 - (Y_t - C_d^a \hat{X}_{t|t-1}^a)^T W_t^{-1} (Y_t - C_d^a \hat{X}_{t|t-1}^a) \\ &= 1 - (Y_{t+1} - C_d^a (A_d^a \hat{X}_{t|t}^a + B_d^a U_t))^T \\ &\qquad W_t^{-1} (Y_{t+1} - C_d^a (A_d^a \hat{X}_{t|t}^a + B_d^a U_t))]\end{aligned} \tag{18}$$

In [14], it has been shown the stability of the ASMF can be represented by the filter parameter δ_t, i.e., the ASMF is stable when $\delta_t > 0$.

Firstly, define

$$\begin{aligned}J_t^{\delta}(U_t, Y_{t+1}) &\triangleq (Y_{t+1} - C_d^a (A_d^a \hat{X}_{t|t}^a + B_d^a U_t))^T \\ &\qquad W_t^{-1} (Y_{t+1} - C_d^a (A_d^a \hat{X}_{t|t}^a + B_d^a U_t))]\end{aligned} \tag{19}$$

Then,

$$\begin{aligned}J_t^{\delta*}(U_t, Y_{t+1}) &= \max_{\hat{X}_{t|t}^a} J_t^{\delta}(U_t, Y_{t+1}) \\ &= \max_{\hat{X}_{t|t}^a} \{\left[Y_{t+1} - C_d^a \left(A_d^a \hat{X}_{t|t}^a + B_d^a U_t\right)\right]^T W_t^{-1} \\ &\qquad \left[Y_{t+1} - C_d^a \left(A_d^a \hat{X}_{t|t}^a + B_d^a U_t\right)\right]\} \le 1\end{aligned} \tag{20}$$

We should select an U_t to make $J_t^{\delta*}(U_t, Y_{t+1})$ small as far as possible, that is,

$$J_t^{\delta*}(Y_{t+1}) = \min_{U_t} J_t^{\delta*}(U_t, Y_{t+1}) \tag{21}$$

We introduce the following cost function $J_t(U_t)$ with consideration of both (17) and (20) at the same time:

$$\begin{aligned}&{}^*U_t = \arg\min_{U_t} \bar{J}_t(U_t) \\ &\bar{J}_t(U_t) \triangleq J_t(U_t) + \alpha J_t^{\delta*}(U_t, Y_{t+1})\end{aligned} \tag{22}$$

where $\alpha = 1 - \delta_t \in R$ are the positive definite weight matrix. To minimize $J_t(U_t)$, considering $J_t(U_t) > 0$, the control can be obtained at $\partial J_t(U_t) / \partial U_t = 0$, i.e.,

$$\partial J_t(U_t) / \partial U_t = 2(MU_t + N) \tag{23}$$

where

$$M = B_d^T H B_d + \alpha B_d^{aT} C_d^{aT} W_t^{-1} C_d^a B_d^a$$
$$N = B_d^T H (B_f f_t - B_d U_t^0) - \alpha B_d^{aT} C_d^{aT} W_t^{-1} Y_{t+1}$$

Thus, we can obtain the optimal control that minimizes $J_t(U_t)$ as:

$$
\begin{aligned}
U_t(Y_{t+1}) &= -M^{-1}N \\
&= (B_d^T H B_d + \alpha B_d^{aT} C_d^{aT} W_t^{-1} C_d^a B_d^a)^{-1} \\
&\quad \left[\alpha B_d^{aT} C_d^{aT} W_t^{-1} Y_{t+1} - B_d^T H (B_f f_t - B_d U_t^0) \right]
\end{aligned} \tag{24}
$$

For the unknown measurement at time t+1 in Eq. (24), we consider that the control system is stable, so, $Y_{t+1} \in \Delta(Y_t)$. Here, $\Delta(Y_t)$ is the elliptical domain of Y_t. Thus, we first define array S_t^i to include the estimate of the i-th element's two boundary endpoints as

$$
S_t^i \triangleq \left\{ \hat{Y}_{t+1}^i \mid \{ Y_t^i + (-1)^h (\underset{j_l = \{\pm\sqrt{p_{ll}}, l=1,\dots,13\}}{Max} \left| C_d Col\{j\} \right|_i) \} \right\} \tag{25}
$$

where Y_t^i is the i-th element in the vector Y_t, $\hat{Y}_{t+1}^i$ is the corresponding output Y_{t+1}'s endpoints estimation. For set S_t^i, $i \in \{1, 2, ..., 8\}$ and h is 0 or 1 for every i, $|\bullet|_i$ is the absolute value of the i-th element in vector $\bullet$, and the function $Col\{j\}$ is defined as follows:

$$
Col\{j\} = \begin{pmatrix} j_1 & \dots & j_{13} \end{pmatrix}^T \tag{26}
$$

Then, we define a set S_t to describe all possible endpoint vector of the Y_{t+1} as

$$
S_t \triangleq \left\{ \hat{Y}_{t+1}^{EP} \mid \begin{pmatrix} S_t^1 & \dots & S_t^{13} \end{pmatrix} \right\} \tag{27}
$$

where $\hat{Y}_{t+1}^{EP}$ is the possible endpoint (EP) for output Y_{t+1} at next sampling time t+1.

Thus, the proposed active modeling based predictive controller can be implemented by using the following steps:

Step I: Make increment prediction

Based on the current estimated state $\hat{X}_{t|t}^a$, use the stationary increment predictive controller, as in section IV.B, to obtain the nominal control input U_t^0;

Step II: Model error estimation and elimination

Based on U_t^0, compute the optimal control input $^*U_t$:

a) Estimate the values and boundaries of state X_t and model error f_t, using ASMF in (3);

b) Calculate the corresponding $U_t(\hat{Y}_{t+1}^{EP})$ for every $\hat{Y}_{t+1}^{EP}$ in set S_t by Eq. (24);

c) For every $U_t(\hat{Y}_{t+1}^{EP})$ in step 1), use Eq. (19) to obtain the maximum of function $J_t^\delta(U_t(\hat{Y}_{t+1}^{EP}), \hat{Y}_{t+1}^{EP})$, and get the $^*\hat{Y}_{t+1}^{EP}$ to let $^*\hat{Y}_{t+1}^{EP} = \underset{\hat{Y}_{t+1}^{EP} \in S_t}{\arg Max} \left\{ J_t^\delta \{ U_t(\hat{Y}_{t+1}^{EP}), \hat{Y}_{t+1}^{EP} \} \right\}$;

d) The corresponding $U_t(^*\hat{Y}_{t+1}^{EP})$ is the optimal control $^*U_t$ at time t, i.e. $^*U_t = U_t(^*\hat{Y}_{t+1}^{EP})$.

Step III: Receding horizon strategy

Go back to **step *I*** at the next time instant.

V. FLIGHT EXPERIMENT

All flight tests are conducted on the Servoheli-20 setup, more details of this experimental platform can be found in [19].

Figure 2: SERVOHELI-20 small-size helicopter platform

Generalized predictive control (GPC), stationary increment predictive control (SIPC) and active model based stationary increment predictive control (AMSIPC) are all tested in the same flight conditions, and the comparison results are shown in Figs. 3-5. We use the identified parameters in Ref.[17] as nominal model.

It can be seen that, when the helicopter increases its longitudinal velocity and changes flight mode from hovering to cruising, GPC (brown line) has a steady velocity error (>20%) and increasing position error because of the model errors. SIPC (blue line) has a smaller velocity error because it uses increment model to reject the influence of the changing operation point and dynamics' slow change during the flight. However, the increment model may enlarge the model errors due to the uncertain parameters and sensor/process noises, resulting in the oscillations (>30%) in the constant velocity period (clearly seen in Fig. 3 and 4). While for AMSIPC (green line), because the model error has been online estimated by the ASMF, the proposed AMSIPC successfully reduces velocity oscillations (<5%) and tracking errors (<5%).

VI. CONCLUSIONS

An active model based predictive control scheme was proposed in this paper to compensate model error due to flight mode change and model uncertainties, and realize full flight envelope control without multi-mode models and mode-dependent controls. The ASMF was adopted as an active modeling technique to online estimate the error between reference model and real dynamics. The proposed control scheme was implemented on our developed ServoHeli-20 unmanned helicopter. Experimental results have demonstrated clear improvements over the normal GPC without active modeling enhancement.

It should be noted that, at present, we have only tested the control scheme with respect to the flight mode change from hovering to cruising, and vice versa. Further mode change conditions, including ground effect, coordinated turning and etc, will be flight-tested in near future.

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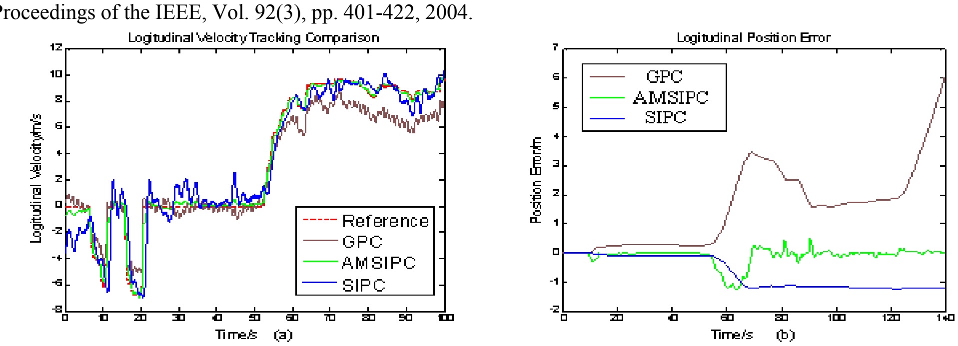

Figure 3: Longitudinal tracking results: (a) velocity; (b) position error (<50s hovering, >50s cruising)

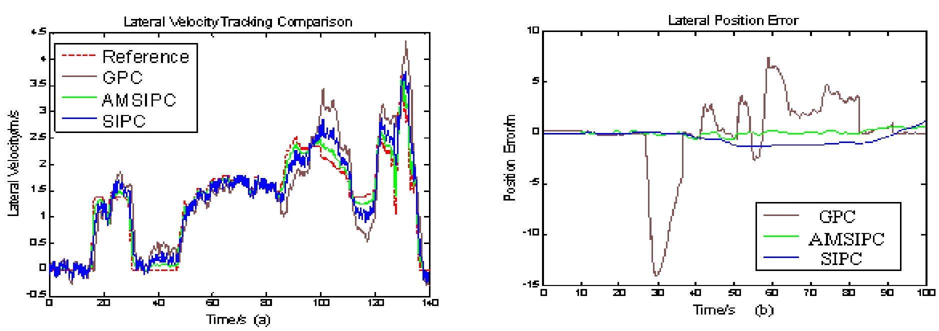

Figure 4: Lateral tracking results: (a) velocity; (b) position error (25s~80s cruising, others hovering)

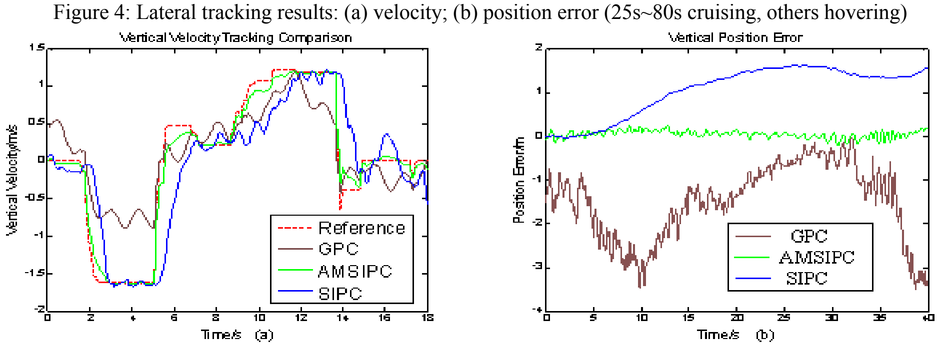

Figure 5: Vertical tracking results: (a) velocity; (b) position error (<5s hovering; >5s cruising)