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Motion Planning of Multirobot Formation

Shuang Liu, Dong Sun, and Changan Zhu

Abstract—**This paper presents a motion planning approach to coordinating multiple mobile robots in moving along specified paths. The robots are required to fulfill formation requirements while meeting velocity/acceleration constraints and avoiding collisions. Coordination is achieved by planning robot velocities along the paths through a velocity optimization process. An objective function for minimizing formation errors is established and solved by a linear interactive and general optimizer. Motion planning can be further adjusted online to address emergent demands such as avoiding suddenly-appearing obstacles. Simulations and experiments are performed on a group of mobile robots to demonstrate the effectiveness of the proposed coordinated motion planning in multirobot formations.**

I. Introduction

Multirobots have been widely used in industrial plants and warehouses. In many multirobot applications, robots are required to form formations to accomplish complex tasks such as transportation of large awkward objects [1][2], localization and mapping, search, and rescue [3][4]. Amongst these applications, optimal motion planning becomes increasingly important especially when the task is executed repeatedly or resources must be conserved [5][6]. This paper aims to address optimal motion planning of multirobots when moving along the desired paths to fulfill formation requirement.

In many multirobot applications, the robots move along fixed paths and coordinate the motions with each other through proper motion planning [7][8][9]. In the recent work [10], the robots were required to follow their designed paths while forming desired formations via a synchronization control approach. An integrated design of trajectory planning for multiple micro air vehicles was reported in [11].

This paper solves the optimal motion planning problem when multirobots move along specific paths while maintaining desired formations. The collision avoidance is also considered in the motion planning in a dynamic environment. A relevant work was reported in [12], where the coordination problem was formulated as a mixed integer nonlinear programming problem with optimal completion time, and then solved by mathematical tools such as optimizer package of AMPL [13] and CPLEX [14]. Recently, the coordination was extended by considering the communication constraint [15] to minimize the completion time. These approaches did not consider formation requirement.

In this paper, we model the formation relationship to be velocity dependent, and then formulate the motion coordination problem as a velocity optimization problem. An objective function is established to measure the formation performance of the generated velocity profiles. Through the use of linear interactive and general optimizer (Lingo) [16] to minimize the objective function, the velocity optimization problem can be solved. Further, the developed motion plan can be adjusted online to deal with emergent cases such as avoiding suddenly appeared moving obstacle that is not known when planning the motions with Lingo.

The remainder of this paper is organized as follows. Section II presents the coordination problem and the robot model. In Section III, the problem is modeled as a velocity optimization problem, which is achieved by lingo. A strategy is further developed to adjust the motion plan online for avoiding collision with suddenly appeared moving obstacle. In Section IV, simulations and experiments are performed to demonstrate the effectiveness of the proposed approach. Finally, conclusions of this work are given in Section V.

I. Problem Formulation

Consider that a group of robots move along the designed paths in the environment that contains moving obstacle, as shown in Fig. 1. The robots are required to form and maintain different formations along the paths, represented by a, b, c and d, respectively. The problem to be investigated is to design optimal velocity profiles for the robots to meet the formation requirements along the paths with collision avoidance.

Let x_i, y_i and θ_i be the coordinates of robot i, where $i = 1,2,...,n$. Denote the pose of the ith robot as $q_i = [x_i \ y_i \ \theta_i]^T$. Similar to [8][10], the kinematics of robot i can be represented by a unicycle model as follows

$$\dot{x}_i = v_i \cos\theta_i, \quad \dot{y}_i = v_i \sin\theta_i, \quad \dot{\theta}_i = w_i \tag{1}$$

where v_i and w_i denote the linear and angular velocities of robot i, respectively.

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S. Liu is with Control and Mechatronics Group, Suzhou Research Center of City University of Hong Kong and the University of Science and Technology of China, Suzhou, P. R. China (phone: 86-0512-87161379; fax: 86-0512-87161381; e-mail: shuliu@cityu.edu.hk).

D. Sun is with Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Hong Kong, P. R. China (e-mail: medsun@cityu.edu.hk).

C. Zhu is with Department of Precision Machinery and Instrumentations, University of Science and Technology of China, Hefei, P. R. China (e-mail: changan@ustc.edu.cn).

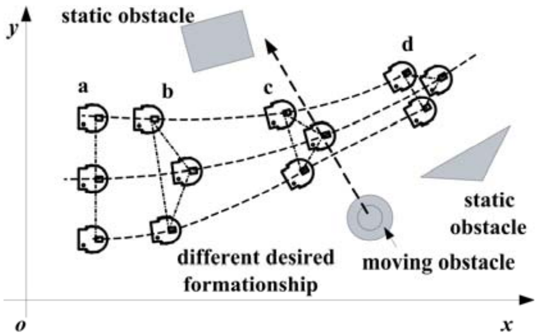


Fig. 1 Motion planning of multiple robots moving along the designed paths with formation requirement

Note that each robot is subject to the constraint of velocity and acceleration bounds, given as follows

$$0 \le v_i \le V_{\max} \tag{2}$$

$$|\dot{v}_i| \le a_{\max} \tag{3}$$

where $V_{\max}$ and $a_{\max}$ are the maximum bounds of the linear velocity and acceleration, respectively. The angular velocity w_i can be derived by the linear velocity v_i as follows

$$w_i(s_i) = k_i(s_i)v_i(s_i) \tag{4}$$

where the parameter s_i denotes the travel distance along the path, and $k_i(s_i)$ denotes the curvature of the path at position s_i. It is generally assumed that the curvature of the designed path is small enough such that the angular speed corresponding to the optimal speed is always achievable [15].

II. Coordinated Motion Planning with Formation Requirement

The formation can be achieved by coordinating the robots' velocities along the paths. Thus, the motion planning is posed as a velocity optimization problem as detailed below.

A. *Formation constraint*

The robots are required to maintain the formation relationship, which is represented by the desired relative positions of the robot pairs in the group. Fig. 2 illustrates an example of a group of robots in formation, where the arrows show that the two neighboring robots (namely robot pair) need to meet the desired relationship in the formation.

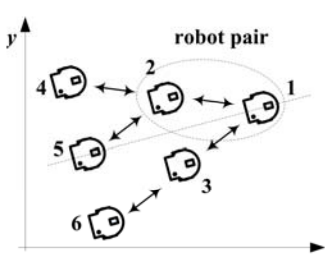


Fig. 2 Formation relationship in the robot network

In a particular robot pair i and j, the formation relationship is determined by the relative distance between the two robots in the global reference frame, denoted as

$$\begin{cases} \Delta x_{i,j} = x_j - x_i \\ \Delta y_{i,j} = y_j - y_i \end{cases} \tag{5}$$

Fig. 3 illustrates the relative distance between robot i and j, where $\Delta x'_{i,j}$ and $\Delta y'_{i,j}$ denote the relative distances in the reference frame of robot i. Note that $\Delta x_{i,j}$, $\Delta y_{i,j}$, $\Delta x'_{i,j}$ and $\Delta y'_{i,j}$ represent the predicted "actual" relative distances, obtained by the proposed motion planning.

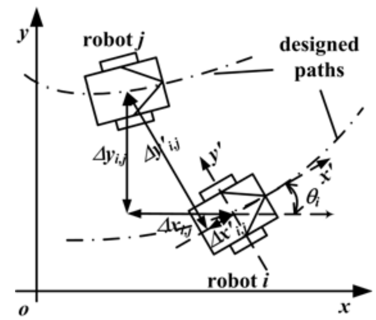


Fig. 3 Formation relationship of a robot pair

Denote $\Delta x'^{d}_{i,j}$ and $\Delta y'^{d}_{i,j}$ as the desired relative distances in the reference frame of robot i. Since the relative distances are determined by the locations of the robots, they can be represented as the functions of the travel distance s_i along the paths, expressed by $\Delta x'^{d}_{i,j}(s_i)$ and $\Delta y'^{d}_{i,j}(s_i)$. Then, the desired relative distances in the global reference frame, denoted by $\Delta x^{d}_{i,j}(s_i)$ and $\Delta y^{d}_{i,j}(s_i)$, are determined as

$$\begin{bmatrix} \Delta x^{d}_{i,j}(s_i) \\ \Delta y^{d}_{i,j}(s_i) \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \end{bmatrix} \times \begin{bmatrix} \Delta x'^{d}_{i,j}(s_i) \\ \Delta y'^{d}_{i,j}(s_i) \end{bmatrix} \tag{6}$$

where θ_i denotes the orientation of robot i. The first term in the right hand side in (6) is the rotation matrix that rotates the coordinate frame by the angle of θ_i.

Divide time t into m intervals, and each interval is denoted by Δt, i.e., $t = m\Delta t$, $\forall m = 0,1,...,M$, where $M\Delta t$ is the maximum time for any robot to complete the motion. Denote the velocity of robot i at each time interval as $v_i(t)$. The travel distance $s_i(t)$ can then be determined by $v_i(t)$ and $v_i(t+\Delta t)$ in each time interval as follows

$$s_i(t+\Delta t) = s_i(t) + \frac{1}{2}\Delta t\big(v_i(t) + v_i(t+\Delta t)\big) \tag{7}$$

Using the travel distance $s_i(t)$, the position of robot i at the particular time $t = m\Delta t$ can be parameterized as a function of $f_i(s_i(t))$, expressed as

$$q_i(t) = \left[x_i(s_i(t))\ y_i(s_i(t))\ \theta_i(s_i(t))\right]^T = f_i(s_i(t)) \tag{8}$$

Based on (7) and (8), the desired relative distances can be calculated by (6). Denote $q^{e}_{i,j}(t)$ as the variation of the relative distance between the predicted actual and the desired relative distances of the robot pair i and j, calculated by

$$q_{i,j}^{e}(t)=\begin{bmatrix}\Delta x_{i,j}^{e}(t)\\ \Delta y_{i,j}^{e}(t)\end{bmatrix}=\begin{bmatrix}\Delta x_{i,j}(t)-\Delta x_{i,j}^{d}(s_i(t))\\ \Delta y_{i,j}(t)-\Delta y_{i,j}^{d}(s_i(t))\end{bmatrix} \tag{9}$$

We then introduce an objective function Q^e, which can be used to guide the motion planning to meet the formation requirement, as follows,

$$Q^e=\sum_{t=0}^{M\Delta t}\sum_{i,j}^{n}\left\|q_{i,j}^{e}(t)\right\|^2 \tag{10}$$

where the term $\sum_{i,j}^{n}\left\|q_{i,j}^{e}(t)\right\|^2$ integrates the variations of the relative distances of all the robot pairs in each time interval, and Q^e integrates them during the whole motion. When the function Q^e equals zero, the robots move with the desired formation perfectly.

B. Velocity constraints

To meet the constraint (2) and (3), $v_i(t)$ is subject to the following boundary conditions

$$v_i(t)-a_{\max}\Delta t\le v_i(t+\Delta t)\le v_i(t)+a_{\max}\Delta t \tag{11}$$

$$0\le v_i(t)\le V_{\max} \tag{12}$$

Without loss of generality, we assume that the velocities of all the robots are zero at the initial and final times. Thus, the boundary condition of the velocity of robot *i* at the initial time $t=0$ and the final time $T_i\le M\Delta t$ are given by

$$v_i(0)=0\,,\quad v_i(T_i)=0 \tag{13}$$

Accordingly, to arrive in the goal positions within the prescribed time limit, the boundary conditions of the travel distances of robot *i* should be given by

$$s_i(0)=0\,,\quad s_i(T_i)=L_i \tag{14}$$

where L_i is the total arc length of the path of robot i.

Although the paths are designed for the robots to avoid collisions with static obstacles, robots may still collide with each other. Hence, the velocities of the robots are coordinated to ensure that the distance between any two robots i and j is larger than a safe distance denoted by D_{safe}, represented as

$$\left\|(q_i(s_i(t))-q_j(s_j(t))\right\|\ge D_{safe} \tag{15}$$

The safe distance D_{safe} is dependent on the factors such as the robot's size, fault tolerance of robot localization [17], etc.

C. Motion planning with velocity optimization

Since all the formations with constraints of velocity/acceleration bounds and collision avoidance are modeled to be dependent on the velocities of the robots, the motion coordination problem is formulated as a velocity optimization problem. The objective function (10) should be minimized while various velocity constraints should be met. This can be achieved by solving the following nonlinear optimization problem with a mathematical programming technique, linear interactive and general optimizer (Lingo) [16].

Minimize: $$Q^e=\sum_{t=0}^{M\Delta t}\sum_{i,j}^{n}\left\|q_{i,j}^{e}(s_i(t))\right\|^2$$

Subject to:

$t=0,\Delta t,2\Delta t,\ldots,m\Delta t$;

$0\le v_i(t)\le V_{\max}$, $v_i(0)=0$, $v_i(m\Delta t)=0$;

$s_i(0)=0$, $s_i(m\Delta t)=L_i$;

$s_i(t+\Delta t)=s_i(t)+\frac{1}{2}\Delta t\left[v_i(t)+v_i(t+\Delta t)\right]$;

$\left[x_i(s_i(t))\ y_i(s_i(t))\ \theta_i(s_i(t))\right]^T=f_i(s_i(t))$;

$\forall i,j$, $\left\|(q_i(s_i(t))-q_j(s_j(t))\right\|\ge D_{safe}$.

D. Online modification of motion planning for collision avoidance

The motion planning as described above is developed offline. In real application, however, it may need to be adjusted online to deal with some emergent cases such as avoiding some moving obstacle that is not known when planning the motion.

When the moving obstacle is observed by robot group, the position of the moving obstacle, represent by $q_{obs}(t)$, is recorded. If the obstacle moves close enough to the robot group, the robots must take action to avoid possible collision.

After collecting the obstacle's positions at different times, the moving trajectory of the obstacle can be estimated approximately[18]. A series of the future positions of the obstacle after time intervals $h\Delta t$, for $h=1,2,...,H$, can be predicted, where H is a maximum integer and $H\Delta t$ denotes the maximum time used for collision estimation. Denote these future positions of the obstacle as $q_{obs}(t_{obs}+h\Delta t)$, where t_{obs} is the time when the obstacle is observed. The distance between the obstacle and robot i can be estimated as $\left\|(q_i(t_{obs}+h\Delta t)-q_{obs}(t_{obs}+h\Delta t)\right\|$. Define D_{obs} as the safe observation distance between the moving obstacle and robot *i*. If the following condition is satisfied

$$\left\|(q_i(t_{obs}+h\Delta t)-q_{obs}(t_{obs}+h\Delta t)\right\|\ge D_{obs} \tag{16}$$

then the robot group is collision-free with the moving obstacle within the time $H\Delta t$.

If (16) is not satisfied, the robots may collide with the obstacle. Calculate the distances between robot *i* at the time t_{obs} and a series of future positions of the obstacle at the time $t_{obs}+h\Delta t$, which is denoted by $\left\|(q_i(t_{obs})-q_{obs}(t_{obs}+h\Delta t)\right\|$. If the following condition is satisfied

$$\left\|(q_i(t_{obs})-q_{obs}(t_{obs}+h\Delta t)\right\|\ge D_{obs} \tag{17}$$

the robots are collision-free with the moving obstacle within the time $H\Delta t$, if they stop the motions and stay at the current positions $q_i(t_{obs})$. Thus, the original plan is modified as

$$q_i'(t)=\begin{cases}q_i(t) & 0<t\le t_{obs}\\ q_i(t_{obs}) & t_{obs}<t\le t_{obs}+\Delta t_d\\ q_i(t-\Delta t_d) & t>t_{obs}+\Delta t_d\end{cases} \tag{18}$$

where Δt_d denotes the time delay that the robots stop to wait till the obstacle moves away. All the robots will resume their original motion plans after the delay time Δt_d.

If the condition (17) is not satisfied, the robots will collide with the moving obstacle even though they stop the motion. In this case, the robots must move back to let the obstacle pass first. Hence, the robot group reverses the original motion plan at the time t_{obs} as follows

$$q_i'(t) = \begin{cases} q_i(t) & 0 < t \le t_{obs} \\ q_i(t_{obs} - (t - t_{obs})) & t_{obs} < t \le t_{obs} + \Delta t_r \\ q_i(t_{obs} - \Delta t_r)) & t_{obs} + \Delta t_r < t \le t_{obs} + \Delta t_r + \Delta t_d' \\ q_i(t - 2\Delta t_r - \Delta t_d') & t > t_{obs} + \Delta t_r + \Delta t_d' \end{cases} \tag{19}$$

where Δt_r is the time of countermarch, and $\Delta t_d'$ is the waiting time that may be needed by the robot to let the obstacle pass. In the time period $(t_{obs}, t_{obs} + \Delta t_r + \Delta t_d']$, the robots reverse the original motion plan to move back to their previous positions at the time $t_{obs} - \Delta t_r$, and may further wait for a time period of $\Delta t_d'$ till the moving paths are clear. After the avoidance, the robot will resume the motion from the state $q_i(t_{obs} - \Delta t_r)$.

III. Simulations and Experiments

To demonstrate the effectiveness of the proposed approach, simulations and experiments were performed on a group of mobile robots.

A. Simulations

The designed paths were generated by using Matlab function Spline(), and parameterized by arc length such that the path function $f_i(s_i(t))$ in eq. (8) can be obtained. The optimization problem was modeled and solved in Lingo 9.0 in windows XP system with 2GB of main memory and a 2.2GHz clock speed.

We first performed the simulation on six mobile robots to form and maintain formations along the designed paths, as shown in Fig. 4. There were four different formations at sequential locations along the paths, denoted by a, b, c, and d. The arrow between the two neighboring robots represents the formation constraint of the robot pair in the robot network.

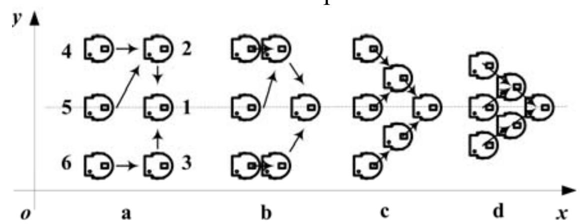


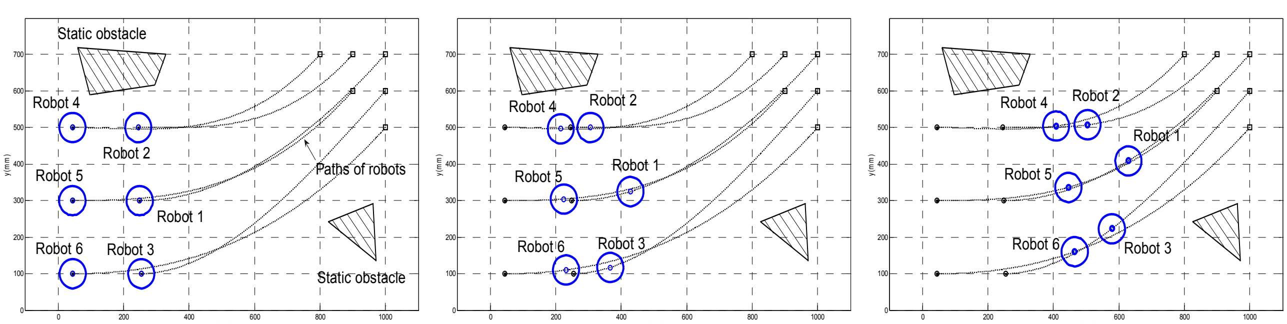
Fig. 4 Formation relationship in the robot network

The desired offsets $\Delta x_{i,j}^d$ and $\Delta y_{i,j}^d$ were parameterized as functions of the travel distance $s_i(t)$ based on (6). The desired offsets of the four different formations are listed in Table 1. The parameters in the simulations were $M = 15$, $\Delta t = 1s$, $V_{\max} =$ 120mm/s, $a_{\max} = 120$ mm/s^2, and $D_{safe} = 90$ mm.

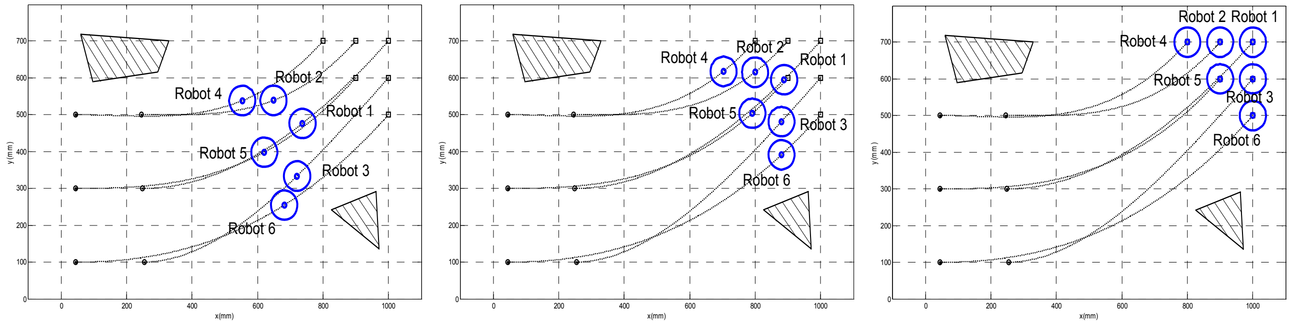
Fig. 5 shows the simulation results at six different times. The six robots moved from the initial position in the left hand side, marked by the circles, toward their goal positions in the right hand side. As shown in Fig. 5, the robots changed the formation along the designed paths. It is seen that the desired formations were well formed during the motions。

Table 1 Desired formation relationship

	Different positions (mm)			
Desired offsets	a	b	c	d
$\Delta x_{1,2}^d$	0	100	100	70.7
$\Delta y_{1,2}^d$	-200	-200	-100	-70.7
$\Delta x_{1,3}^d$	0	100	100	70.7
$\Delta y_{1,3}^d$	200	200	100	70.7
$\Delta x_{2,4}^d$	200	100	100	70.7
$\Delta y_{2,4}^d$	0	0	-100	-70.7
$\Delta x_{2,5}^d$	200	100	100	70.7
$\Delta y_{2,5}^d$	200	200	100	70.7
$\Delta x_{3,6}^d$	200	100	100	70.7
$\Delta y_{3,6}^d$	0	0	100	70.7



0 second (formation a)   2 seconds (formation a to b)   4 seconds (formation b)



6 seconds (formation c) 8 seconds (formation c to d) 10 seconds (formation d)

Fig. 5 Simulation results with formation constraint

We then compared our method to the generalized velocity optimization without formation constraint, and a well-known leader-follower approach. The leader-follower configuration has been shown in Fig. 4. We compared the variations $q^e_{1,2}(t)$, $q^e_{1,3}(t)$, $q^e_{2,4}(t)$, $q^e_{2,5}(t)$ and $q^e_{3,6}(t)$ under the three methods, as shown in Figs. 6. It is seen that the proposed velocity optimization with formation requirement exhibited the best formation performance since the variation $q^e_{i,j}(t)$ had the lowest values. The leader-follower method exhibited worse formation performance than our method, but was better than that without considering formation constraint. Using the velocity optimization only without formation constraint, the robot formation could not be maintained.

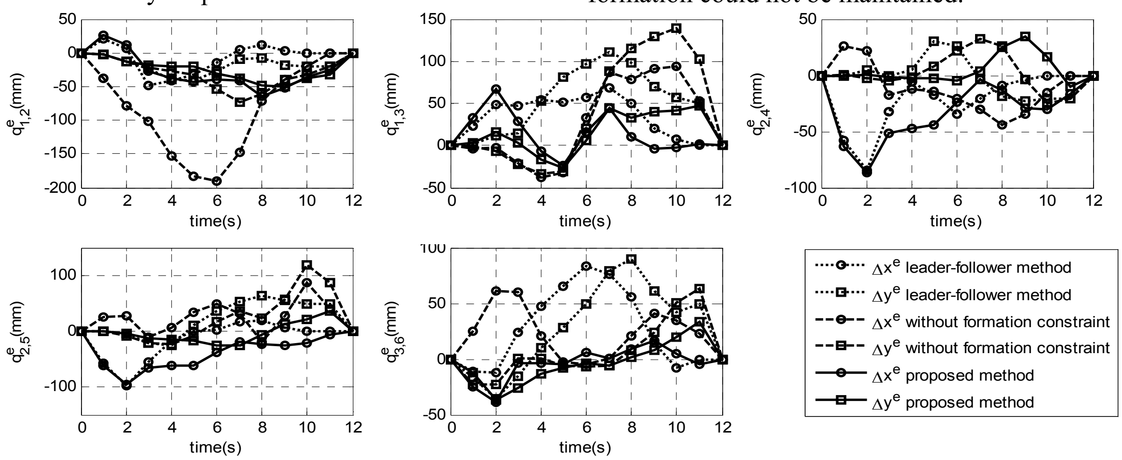


Fig. 6 Formation errors of robots

B Experiments

Experiments were performed on three PIONEER 3DX mobile robots in office environment [17]. The computer on each PIONEER 3DX has a Pentium III 800MHz CPU and 256M memory. The generated path of each robot was as a two-piecewise cubic spline function and parameterized by the arc length s_i , where $i = 1,2$.

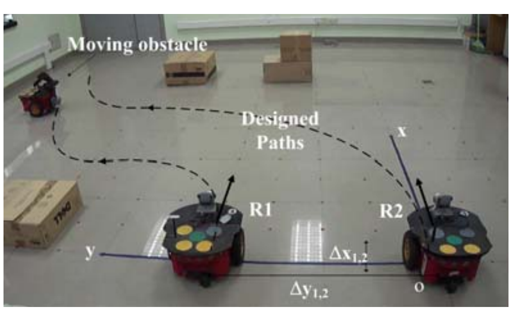


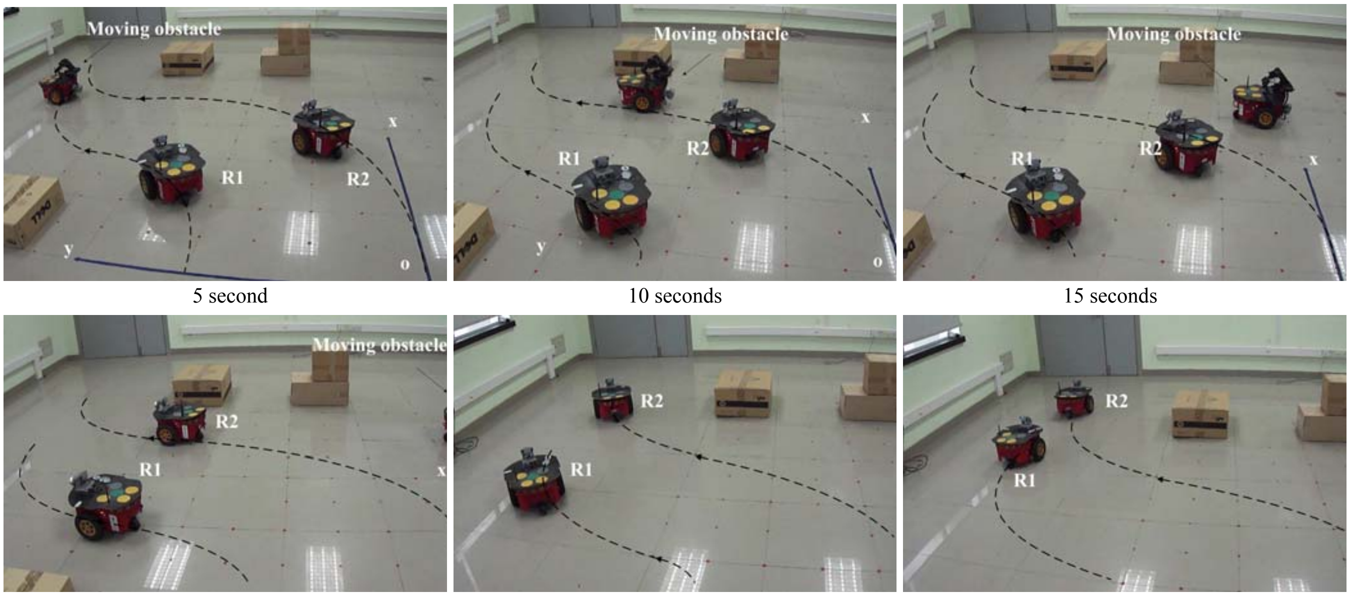
Fig. 7 Experiment scenario

In the experiments, two robots, labeled as R1 and R2, were required to maintain and change formations when moving along their desired paths, and another robot was considered as a moving obstacle, as shown in Fig. 7. The straight lines on the floor denote the x and y coordinate axes, and the distances $\Delta x_{1,2}$ and $\Delta y_{1,2}$ are labeled in the figures. The robot pair (1, 2) has two different formation requirements in steps a and b respectively, as shown in Table 2, where $\Delta x'^d_{1,2}$ and $\Delta y'^d_{1,2}$ represent the relative desired distances between robots 1 and 2 in the reference frame of robot 1.

Table 2 Formation requirements in experiment

	$\Delta x'^d_{1,2}$	$\Delta y'^d_{1,2}$
Step a	0	1200mm
Step b	-1200mm	0

Fig. 8 illustrates the robots moving in the experiment, and Fig. 9 illustrates the velocity profiles with the proposed motion planning method. At 5 seconds, a moving obstacle (another robot) appeared, and collision would occur. As a result, at 8 seconds, robot 1 stopped and retreated along the path to avoid the collision as shown in the velocity profile in Fig. 9. The robot 2 also retreated along its path to maintain the formation relationship. They resumed the motions at 15 seconds, and changed to formation step b at 20 seconds. At the end, the two robots arrived in their goals and form a line formation.



5 second 10 seconds 15 seconds

20 seconds 25 seconds 30 seconds

Fig. 8 Robots in experiment

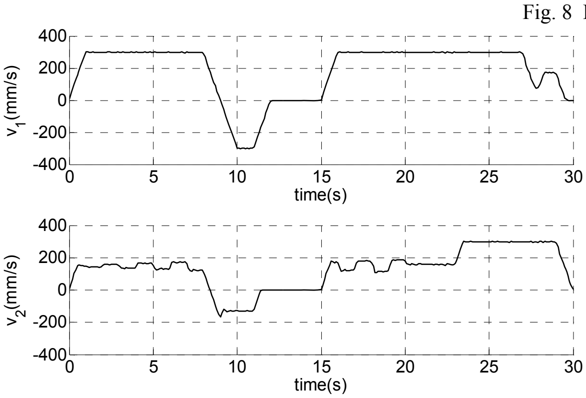


Fig. 9 Velocity profiles of the robots in the experiment

IV. CONCLUSIONS

In this paper, coordinated motion planning of multiple mobile robots is studied. The problem is formulated as a velocity optimization problem and a global optimal solution is obtained. A strategy is proposed to adjust the motion planning online to avoid moving obstacles. Simulations and experiments are performed on a group of mobile robots to demonstrate the effectiveness of the proposed approach. The future work will focus on reducing the computational complex of the problem and strategies will be studied to deal with more complex environment.

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