Accurate and stable mobile robot path tracking: An integrated solution for off-road and high speed context

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Abstract—This paper is focused on the problem of accurate and reliable path tracking control of a 4-wheels car-like mobile robot moving off-road at high speed. Dynamic and extended kinematic models that take into account the effects of wheel skidding are presented. Based on the extended kinematic model, an adaptive and predictive controller for path tracking is derived. This control law is combined to a stabilization algorithm of yaw motion, based on the dynamic model and the modulation of driven wheel forces. The overall control architecture is experimentally evaluated on a slipping terrain. Results demonstrate enhanced performances as the robot succeed in following the path at high speed, accurately and without loss of control.

I. INTRODUCTION

As the autonomous navigation in off-road conditions appears as a promising solution [5] with respect to social needs in many areas (such as surveillance, rescue or agriculture [10], etc.), the research in off-road mobile robotics has to propose devices fitting users expectations. In particular, in order to be actually usable, the proposed robots have to be accurate, reliable and move at relatively important speeds. This still constitutes an open issue since natural grounds are irregular and offer low grip conditions, moreover variable (see [1]). When using basic mobile robot control laws, such as proposed in [3], these specificities indeed generate at least important perturbations (decreasing accuracy) up to a total loss of stability (spun around). Furthermore, such a phenomena is emphasized at high speed because of the unavoidable settling time and delays of actuators.

With this aim, some approaches have been developed to address instabilities or the lack of accuracy due to low grip conditions. A first approach lies in the definition of a stability domain (velocity/steering angle) considering known grip conditions (such as proposed in [11]). Mainly dedicated to path planning, such algorithms do not account for online grip condition variation in motion control. In order to compensate skidding effects in real time, an alternative consists in considering sliding as a perturbation to be rejected by robust control (see for instance [15] or [4]). If it permits to obtain a satisfactory accuracy at low speed or in a structured environment context, the settling time and delays of low level do not permit an efficient off-road control at high speed, where sliding variables can reach important values. Another way to address sliding is to control robot dynamics to make it tend to the theoretical behavior under the rolling without sliding conditions. Some work based on the wheel velocity repartition is then proposed, such as in [9], permitting to limit the effect of skidding. Nevertheless, used solely, it does not permit to obtain a high accurate path tracking, as sliding is not totally compensated.

Then, solutions based on grip conditions estimation rise at promising approaches. In [8], the authors propose an online estimation procedure of the wheel-ground slippages, based on Terra-mechanics models. The slippage conditions are included in a trajectory controller in order to improve mobility over difficult terrains. Nevertheless, this approach needs an accurate estimation of vehicle motion to be reported into tire model, which is not always practicable. Simplified models, including sideslip angles as additive variables of a kinematic representation, have been proposed and classified in [13]. Adaptive and predictive algorithms (see [6]), based on such a modeling and coupled with an on-line estimation of sliding have then shown significant results from the path tracking accuracy point of view. They indeed permit to estimate and compensate for perturbations whatever the changing conditions and the geometry of terrain at relatively limited speed. If the last results demonstrate the capability of an accurate control at high speed (compatible with low level delay), such approaches assume that sliding effects are low enough to preserve the system controllability. As a result, if sliding is very large (likely to occur at important speed levels), robot can spin around.

As a result, it appears interesting to merge solutions allowing to first reduce sliding effects on robot behavior, and then estimate and compensate remaining sliding into motion control. This paper then proposes to gather on one hand a stabilization law and, on the other hand, an adaptive and predictive control, in order to ensure high accurate path tracking for off-road mobile robots acting at high speed. Based on previous developments, the algorithm presented in this paper first takes part of velocity repartition on each wheel to stabilize the robot dynamics (avoiding swing around situation) and limit sliding influence. Then, an advanced path tracking control law is derived for steering angle to compensate for residual effects of sliding and anticipate low level delays. It then results a stable and accurate positioning of the mobile robot with respect to a desired trajectory at high
speed, whatever the grip conditions and terrain irregularities.

The paper is organized as follows: in a first part the modeling of a mobile robot (including the reconstruction of unmeasured variables) for both parts of the control scheme is defined. Based on these models, a control part describing the path tracking and stabilization control acting in parallel, is presented. Finally, the complementarity of developments and the relevance of the overall approach are investigated through full scale experiments at high speed (up to 8 m/s) on natural and irregular ground.

II. OFF-ROAD MOBILE ROBOT MODELING
A. Four wheels mobile robot model

First of all, a complete dynamical model of a mobile robot can be considered, such as depicted in Figure 1. It allows to access to the relationship between forces and accelerations.

![Fig. 1. System dynamics](image)

In particular the four longitudinal and lateral tire forces, denoted respectively by $F_{x+s}$ and $F_{y+s}$ (with $f$ and $r$ for front and rear, and $l$ and $r$ for left and right) can be related to the yaw acceleration ($\ddot{\theta}$) thanks to the yaw torque (denoted in the following $T_{\theta}$),

$$\ddot{\theta} = f(F_{x+s}, F_{y+s}, \delta_l, \delta_r) \quad (1)$$

where $\delta_l$ and $\delta_r$ denote respectively left and right steering angle, related to the equivalent steering angle $\delta_F$ via the robot wheelbase $L$ and the half width $W$.

The equation (1), detailed in [9], permits to analyze the effect of each force on yaw acceleration. If the longitudinal forces $F_{x+s}$ can be controlled directly by the wheel velocity actuators, lateral forces, acting also on the yaw rate, rely mainly on the steering angle, the robot velocity and grip conditions. As a result, this dynamical approach appears to be suitable for yaw rate control via wheel velocity in order to reduce effects of sliding, while the steering angle actuation is investigate thanks to another level of modeling.

B. Extended bicycle kinematic model

1) Model Description: A path tracking control based on a complete model such as depicted in Figure 1 would require the knowledge of numerous parameters (hardly measurable and variable off-road). As a consequence, the design of control laws for robot motion has to be based on a lighter model. Nevertheless, the kinematic model classically used in path tracking applications basically relies on the rolling without sliding assumption, which is not applicable off-road. The direct use of such control laws indeed leads to large tracking errors, due to neglected dynamics (mainly low grip conditions, actuator delays).

Consequently, an alternative model (so called "extended kinematic model") is considered in this paper, preserving a kinematic representation. As detailed in [6], it consists in adding a limited number of variables representative of low grip conditions into a pure kinematic model. As depicted in Figure 2, the two sideslip angles $\beta_F$ and $\beta_R$ (denoting the difference between the tire direction and the actual speed vector orientation) have been introduced into a bicycle representation of the mobile robot as in [12]. Notations, depicted in Figure 2, are listed below.

- $O$ is the center of the rear axle and constitutes the point to be controlled.
- $\theta = \theta - \theta_G$ is the vehicle angular deviation with respect to $\Gamma$.
- $v$ is the vehicle linear velocity at point $O$, assumed to be strictly positive.
- $\beta_F$ and $\beta_R$ are the front and rear side slip angles.
- $M$ is the point on the path $\Gamma$ to be followed, which is the closest to $O$. $M$ is assumed to be unique.
- $c(s)$ is the curvature of the path $\Gamma$ at point $M$.
- $y$ is the vehicle lateral deviation at point $O$ with respect to $\Gamma$.

Except the two sideslip angles $\beta_F$ and $\beta_R$, all these variables are supposed to be measured or known by a preliminary calibration. Thanks to this representation framework, the evolution of the vehicle state with respect to the path $\Gamma$ to be followed can be described by the set of equations (2) (see [6] for more details).

$$\begin{align*}
\dot{s} &= v \cos(\theta + \beta_R) \\
\dot{y} &= v \sin(\theta + \beta_R) \\
\dot{\theta} &= v [\cos(\beta_R) \lambda_1 - \lambda_2]
\end{align*} \quad (2)$$

![Fig. 2. Path tracking parameters](image)
with: $\lambda_1 = \frac{\tan(\delta_F + \beta_F) - \tan(\beta_R)}{\frac{L}{1 - c(s) y}}$,
$\lambda_2 = \frac{c(s) \cos(\theta + \beta_R)}{1 - c(s) y} \\
\frac{1}{c(s)} y$.

The existence of this model is guaranteed since $y \neq \frac{1}{c(s)}$ (i.e. the point $O$ is supposed to be never at the center of the reference path curvature). It can be checked that the proposed structure is then consistent with classical point of view. As a consequence, as soon as sideslip angles are correctly estimated, the properties of such kinematic structures can be exploited.

2) Real-time sideslip angle estimation: In order to build from model (2) a path tracking control accounting for sliding, the knowledge of the variables $\beta_F$ and $\beta_R$ is mandatory. As it does not exist any simple sensor to proceed a direct measurement, their indirect estimation must be achieved. If observers relying solely on the proposed model can be designed (as achieved in [7]), they appear to be not reactive enough at considered speeds. As a result, a new observer scheme is proposed based on [6], mixing the extended kinematic model and a dynamic representation (summarized in Figure 3).

In a first step, a preliminary observer (depicted in red/dashed box) permits to extract a first estimation of sideslip angles. These angles are computed as a control law, imposing the convergence of observed lateral and angular deviations to the measured one thanks to the model (2). As they appear to be quite accurate but low reactive, a second step consists in using these variables, coupled with the measured yaw rate, to estimate slow varying parameters (the cornering stiffnesses) of a dynamical model. Knowing these cornering stiffnesses, a classical observer can be designed, allowing a relevant and reactive estimation of sideslip angles. As a result, the model (2) is entirely known and updated in real time.

III. RELIABLE PATH TRACKING CONTROL ALGORITHM

A. Adaptive and predictive control

The extended kinematic model (2), coupled with the observer described in Figure 3, allows an accurate description of mobile robots in the considered conditions, with a kinematic structure. As a result, a control law based on chained system form linearization can be derived, such as proposed in [6]. It consists in two steps: (i) an adaptive control law ensuring the convergence of the tracking error to zero and (ii) a predictive curvature servoing, which compensates for steering actuator delays. The adaptive layer is based on the exact conversion of model (2) (on line updated with sideslip angles estimation) into a chained form. Then a classical PD control is proposed for the auxiliary inputs in order to ensure the convergence of the actual lateral deviation to zero. The reverse transformation provides finally the non-linear expression (3) for the steering control law.

$$\delta_F = \arctan\left(\frac{\tan(\beta_R) + \frac{L}{\cos(\beta_R)} \left(c(s) \cos(\theta + \beta_R) + A \cos^2(\theta)\right)}{\lambda_1}ight) - \beta_F$$

with:

$$\begin{cases}
\lambda_2 = \frac{\alpha}{1 - c(s) y} \\
\alpha = 1 - c(s) y \\
A = -K_p y - K_d \alpha \tan \hat{\theta}_1 + c(s) \alpha \tan^2 \hat{\theta}_1
\end{cases}$$

In addition to this non-linear control expression, Model Predictive Control is applied to address specifically curvature servoing in expression (3). The steering control law can indeed be split into two additive terms:

$$\delta_F = \delta_{Traj} + \delta_{Deviation}$$

where $\delta_{Deviation}$ is a term mainly concerned with errors and sliding compensation, while $\delta_{Traj}$ deals with the reference path shape: it imposes that path and robot curvatures are equal. As the future curvature of the path to be followed can be known, as well as steering actuator features, a model predictive algorithm can be derived: the value of $\delta_{Traj}$ (called $\delta_{Traj}^{Pred}$ in the sequel) to be applied at the current time, to reach "at best" the future curvature on a fixed horizon of prediction, is then computed. This optimal term is then substituted to term $\delta_{Traj}$, so that the adaptive and predictive control law is finally:

$$\delta_F = \delta_{Traj}^{Pred} + \delta_{Deviation}$$

B. Modulation of wheel velocity for yaw rate regulation

Due to the presence of slippage at the wheel-ground contact, especially at high speed, estimated sideslip angles can reach high values. Such sliding levels may not be compensated properly by the path tracking algorithm and lead to swing around. In such a case, the actual yaw rate of the robot $\dot{\theta}$ is widely different from the theoretical yaw rate in rolling without sliding condition $\dot{\theta}^t$ defined as:

$$\dot{\theta}^t = \frac{v \tan \delta_F}{L}$$

PENDING on the ground slippage conditions and the robot configuration, over-steer ($\dot{\theta} > \dot{\theta}^t$) or under-steer ($\dot{\theta} < \dot{\theta}^t$) appears during turning maneuvers. In order to reduce such differences and consequently decreasing sideslip angles (improving efficiency of the control (6), an additive regulation of the yaw rate is designed. Based on the partial dynamic model given in equation (1), the modulation of longitudinal forces produced by the wheels is considered. First, for a given state, the influence of propulsion forces $F_{x,\ast}$ on yaw motion is analyzed. Let us define the propulsion forces as
a sum of the force resulting from the extended kinematic controller $F^k_{x**}$ and a force stabilizing the yaw motion $F^s_{x**}$:  
$$F_{x**} = F^k_{x**} + F^s_{x**}$$

Now, let us consider the error between the theoretical yaw rate (without sliding) and the measured one, defined as $\varepsilon = \dot{\theta}^t - \dot{\theta}$. The goal of the stabilization algorithm is then to determine a set of propulsion forces that compensates this error $\varepsilon$ (as detailed in [9]).

$$\begin{cases} F^s_{x**} = \Phi(\delta_F, \varepsilon) \\
\varepsilon = \dot{\theta}^t - \dot{\theta} \end{cases}$$

In the case where the system control inputs are the wheel velocities instead of the wheel torques, one can use wheel angular acceleration which is homogeneous to wheel torque (and consequently to longitudinal force), as it is claimed, for example, in [14]. Thus, the stabilization control law becomes:

$$\begin{cases} \frac{d\omega^s_k}{dt} = K_\omega \Phi(\delta_F, \varepsilon) \\
\varepsilon = \dot{\theta}^t - \dot{\theta} \end{cases}$$

and, then, the wheel velocity to be applied on one of the wheel $\omega^k$ at instant $k$ can be computed as follows:

$$\omega^k = \omega^{k-1} + \frac{d\omega^s_k}{dt} T_e$$

where $K_\omega$ is the conversion constant between propulsion force and wheel acceleration, $T_e$ is the sampling period and $\omega^k$ is the wheel velocity computed from the extended kinematic controller. The function $\Phi$ is detailed in [9] and can be summarized as follows:

$$\begin{cases} F^s = -K_\varepsilon \\
\text{If } (\delta_F < 0) \& (\varepsilon < -\varepsilon_1) \text{ then } F^s_{xrr} = F^s \\
\text{If } (\delta_F < 0) \& (\varepsilon > \varepsilon_1) \text{ then } F^s_{xll} = F^s \\
\text{If } (\delta_F > 0) \& (\varepsilon < -\varepsilon_1) \text{ then } F^s_{xrr} = F^s \\
\text{If } (\delta_F > 0) \& (\varepsilon > \varepsilon_1) \text{ then } F^s_{xll} = F^s \end{cases}$$

The limit $\varepsilon_1$ defines the threshold of activation of this wheel velocity control (WVC) and $K$ is a strictly positive constant.

### C. Overall algorithm

The two proposed control approaches act at different levels in the steering control of the car-like mobile robot. While the adaptive and predictive law (6) is devoted to steer the front angle for path tracking, the regulation of yaw rate defined by (12) is applied on one wheel velocity. This latter regulation then permits to impose a robot behavior closer than theoretical motion in rolling without sliding, consequently reducing the level of sliding to be accounted in the steering control law. This reduction in sliding level then improves the efficiency of the path tracking, preventing the robot from swing over situations (due to too large sideslip angles). As the incidence of WVC on sideslip angles can be accounted on-line thanks to the observer defined in Figure 3, both of control laws can be applied in parallel such as on the scheme depicted on the figure 4.

### IV. EXPERIMENTAL RESULTS

#### A. Experimental mobile platform

The experimental platform is shown in Figure 5. It consists of an electric off-road vehicle, whose maximum reachable speed is $8 m.s^{-1}$. Designed for all-terrain mobility, it can climb slopes up to $45^\circ$ and has the following properties:

| Total mass | $m = 350 kg$ |
| Yaw inertia | $I = 270 kg.m^2$ |
| Wheelbase | $L = 1.2 m$ |
| Rear half-wheelbase | $b = 0.58 m$ |

**TABLE I**

**EXPERIMENTAL ROBOT DYNAMIC PARAMETERS**

The main exteroceptive sensor on board is a Magellan ProFlex 500 RTK-GPS receiver, which can supply an absolute position accurate to within $2 cm$, at a $20 Hz$ sampling frequency. The GPS antenna is located vertically above the center of the rear axle, so that the absolute position of point $O$ (i.e. the point to be controlled, see Figure 2) is straightforwardly obtained from the sensor. In addition, a gyrometer supplying a yaw rate measurement accurate to within $0.1^\circ .s^{-1}$ is fixed on the chassis, to feed both the observer algorithm, and the distributed wheel velocity control (WVC).

#### B. Stabilization results

The first contribution of the proposed approach lies in the trajectory tracking stabilization of mobile robots acting at high speed on natural ground. In order to point out the effect of the distributed wheel velocity control, an half-turn has first
been manually recorded on a wet grass ground at a speed of 1m.s\(^{-1}\). This trajectory (depicted in black plain line in Figure 6) constitutes the reference path.

![Fig. 6. Comparison of trajectories with and without stabilization](image)

This path has first been followed autonomously, at a velocity of 8m.s\(^{-1}\) using only the front steering angle control law (6), without the differential wheel velocity control. The obtained trajectory is reported in red dashed line in Figure 6. An instability can be observed, since the mobile robot swings around during the curve. Then, the same control law enhanced with the wheel velocity control WVC has been used at the same speed. As it can be seen (the trajectory resulting from this test is reported in blue dashed-dotted line in the same figure), the mobile robot succeeds in following the reference path with a limited tracking error (below 1.5m at 8m.s\(^{-1}\))\(^{1}\).

As the stabilization algorithm attempts to make the robot yaw rate converge to the ideal one (computed in non sliding case), it reduces sideslip angles, by limiting one wheel velocity. This can be seen on the figure 7, where one wheel velocity decreases of about 30 percent (the rear right wheel) to prevent oversteering for a negative yaw rate error. Then, when the vehicle is turning to the left in the positive \(\theta\) direction, it understeers, so the yaw rate error becomes positive (\(\varepsilon > 0\)) and a velocity decrease of almost 60 percent is applied to the rear left wheel. At the end of the curve, a velocity decrease of more than 20 percent and then about 35 percent are successively applied to the front right and the rear right wheels to prevent an oversteering in the curve exit.

![Fig. 7. Percentage of wheel braking](image)

\(^{1}\)Comparison of behaviors can be visually checked thanks to the video available at ftp://ftp.clermont.cemagref.fr/pub/Tscf/Lenain/Videoiros2010/

C. Path tracking accuracy results

This stabilization permits to reduce the influence of sliding. Nevertheless, skidding effects still remains. On the previ-ous trajectory, using WVC without integrating sideslip angles into adaptive control (4) and (6) indeed leads to large errors, especially at 8m.s\(^{-1}\). The lateral error then reaches more than 5m during the curve (mobile robot is stopped for security reasons). In order to point out benefits of adaptive control gathered with WVC, tracking errors obtained at lower speed (6m.s\(^{-1}\)), with different configurations of the control scheme for the path depicted in Figure 6, are proposed in Figure 8.

![Fig. 8. Comparison of tracking errors at 6m/s](image)

In this figure, the tracking error with a classical path tracking control law (sliding is neglected), and without differential wheel control velocity, is reported in black plain line. It can be noticed that during the curve, a large deviation (close to 4m) is recorded. As previously pointed out, the differential control of wheels allows to reduce the effect of sliding. Consequently, the deviation reported in blue dashed line, when only WVC is active (sliding is neglected in steering control), is slightly reduced, but still important (up to 2.5m) during the curve. The same remark can be achieved when using only the steering control (with sliding accounted, but WVC inactive), the error of which is reported in green dotted line. The error is considerably reduced during the curve, but large deviations are recorded at the end of the curve, because of the huge variation of sliding during the transition curve/straight line. Finally, when combining both algorithms (WVC active and sliding accounted), it can be noticed that tracking error (reported in magenta dashed dotted line) is significantly reduced all the path long. The largest error is indeed limited to 1m (transient overshoot at abscissa 40m), while the behavior is much more stable (in terms of oscillations).

Benefits of the complete algorithm can be tested further on more complex trajectories, even non admissible at high speed. For instance, a "double S" trajectory has been recorded manually at a quite limited speed of 1m.s\(^{-1}\), still on a wet grass ground. This reference path, depicted in black line in Figure 9, is quite difficult to follow properly at high speed because of low level delays and high speed transition of sideslip angles. It finally becomes non admissible (it is physically not achievable) at the maximal speed of 8m.s\(^{-1}\).

Nevertheless, the proposed algorithm permits to ensure the stability of the path tracking with a limited deviation, even during the harsh conditions, as it can be noticed by considering the trajectories obtained with the entire
algorithm at 4, 6 and 8 m.s\(^{-1}\). If the first curve is similarly followed at each of the considered speed, the fast modification of curvature sign generates unavoidable but limited overshoots at high speed, so does in the second S. Nevertheless, the stability stays ensured and the mobile robot does not swing around.

To go further and point out the contributions brought by all parts of the proposed approach, the comparison of several lateral errors resulting from different configurations of the controller at a speed of 4 m.s\(^{-1}\) (as the reference path is still admissible at this speed) is reported in Figure 10.

As it can be noticed, a control law based on classical model (sliding neglected) appears to be quite inaccurate during the curve. As the influence of grip conditions are not accounted, important deviations (more than 2m) are recorded during the curves (between curvilinear abscissas 25-38m and 55-65m). As expected, the WVC activation (tracking error is reported in blue dashed line) permits to limit the effects of slip, since it attempts to make the robot behavior close to a non-sliding one. As a result, the error is slightly reduced during both S curves with a maximal deviation limited to 1.2m. Finally, the benefit of merging WVC and adaptive and predictive algorithm (enclosing an indirect estimation of sliding), clearly appears: the error obtained with the complete algorithm (in magenta dashed-dotted line) stays very small during all the path tracking. Despite the harsh transition (linked to the reference path geometry), the bad grip conditions (wet grass soil and low tire width) and the relatively high speed, the tracking error does not exceed 0.45m. It finally shows the complementarity of the advanced path tracking control and WVC for an accurate and stable path tracking in the considered conditions.

V. CONCLUSION AND FUTURE WORK

A robust complete controller for a generic 4-wheel-steering mobile robots has been presented. This controller is suitable for high speed path tracking on uneven winding terrains. It is able to handle sliding soils to preserve accuracy and stability of path tracking control. The results obtained with the implementation of complementary algorithms have shown their efficiency in such conditions. Further, the extension of this work to a stabilization algorithm acting simultaneously on the four wheels of the robot is currently being investigated, as well as the integration of the stability with respect to a rollover risk.

REFERENCES