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On Stability Region Analysis for a Class of Human Learning Controllers

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Abstract— **In this paper, we study the stability region for a set of intelligent controllers developed by learning human expert control skills using support vector machines (SVMs). Based on the discrete-time system Lyapunov theory, a Chebychev points based estimation approach is proposed to evaluate the stability region, a key property of this set of SVM-based human learning controllers. One of such learning controllers has been implemented in vertical balance control of a dynamically stable, statically unstable single wheel mobile robot – Gyrover. The experimental results validate the proposed scheme for estimation of the stability region.**

I. INTRODUCTION

Controller design for complex and dynamically stable systems becomes an increasingly important and challenging topic. These examples include unmanned aerial vehicles (UAVs), motorcycles, humanoid robots, etc. Many remarkable results in this area have been obtained owing to the advances in nonlinear control theory [6], [7]. But, practical applications of the approaches are limited in control of these systems because they rest on the exact knowledge of the system dynamics and accurate models of the plant nonlinearities, which are usually not trivial to obtain. In order to relax some of the exact model-matching restrictions, several adaptive schemes have been introduced to solve the problem of parametric uncertainties [2], [7]. For those classes of systems with structure uncertainties and fast-changing unmodeled dynamics, the schemes usually do not work well. In general, these systems exhibit dynamics that are highly coupled, nonlinear, and vary substantially depending on precise configuration of the systems; moreover, friction and other difficult-to-model physical properties often impact these dynamically stable systems violently. In the recent several years, intelligent controllers developed using artificial neural network, fuzzy logic, genetic algorithms, or a combination thereof are appealing to deal with such complicated systems [5], [8]. However, it is difficult to collect the needed training data to build a sufficiently accurate learning model for an intelligent controller, when the dynamically stable systems are intrinsically unstable and never work in the domain of interest without a well-worked feedback controller, whereas that is our goal. In such a case, modeling human expert control strategy is one of the promising solutions.

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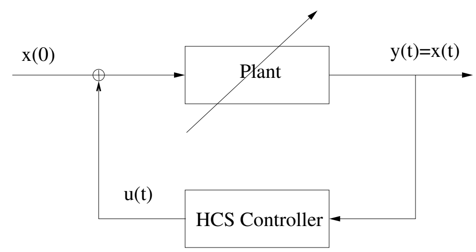

Fig. 1. A physical plant and a HCS controller combined into a feedback system.

Modeling human expert control strategy (HCS) with neural network based algorithms, is a fine solution for the control of dynamically stable systems with unstructured uncertainties and fast-changing un-modeled dynamics. Additionally, those algorithms are parallel and easily implemented with hardware circuits. Thus, the algorithms can be executed in a very high speed to satisfy the real-time requirement. There are many successful examples. Asade and Liu [1] have imparted human control skills to a deburring robot. Yang, Xu and Chen [18] have implemented a different state-based approach to open-loop skill learning using Hidden Markov Models (HMM) in the tele-operation control of a space robot system. In 1998, Montgomery and Bekey [10] proposed a model-free "teaching by showing" methodology through training a fuzzy-neural controller for an autonomous robot helicopter.

Despite their successful applications, HCS-based control methods are not widely accepted in the control community because this type of methods usually lack rigorous studies, even for their convergence analysis, such as asymptotically stable proofs. In addition, raw HCS controllers that have not been tested, usually diverge very quickly and only very few ones among them can work. Even for the limited HCS controllers that are stable, their performances are significantly different from each others. Therefore, it is desirable if we can estimate their stability regions before practical testing, because the larger their stability regions, the wider disturbances and model errors tolerable for these HCS controllers. Then, we can save time and efforts in only testing the few most promising ones. In this paper, an approach to estimate the stability region for this type of controllers is proposed. Stability region or domain of attraction around an asymptotically stable equilibrium is a critical performance index for locally stable feedback control laws, especially for this class of learning controllers. This issue has been studied by a number of researchers [12], [13].

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In the recent several years, support vector machines (SVMs) have gained popularity for two reasons. First, it is satisfying from a theoretical point of view: support vector learning is based on some simple ideas, and provides an insight into the relationship between inputs and outputs. Second, it demonstrates high performance in practical applications. Thus, SVMs have been extensively applied in many areas, such as pattern identification, function regression, and even nonlinear equalization modeling, etc., [14], [16].

The paper is organized as follows. Section II describes the class of nonlinear systems to be controlled and our problems in this paper. Theorem of Ehilich and Zeller and a Chebychev points based estimation approach to evaluate the stability region for the learning controllers are presented in Section III. In Section IV, the effectiveness of the proposed method is illustrated via a simulation study based on the data from learning human expert controlling of a single wheel robot. Finally, we close the paper by stating the conclusions in Section V.

II. Problem Statement

If we use the learning controller $\mathbf{u}$ to control the system, we have a closed-loop discrete time dynamic system and it is described by the difference equations of the form

$$\begin{cases} \mathbf{x}(k+1) &= f_x(\mathbf{x}(k),\mathbf{u}(k)) \\ \mathbf{u}(k+1) &= f_u(\mathbf{x}(k),\mathbf{u}(k)). \end{cases} \tag{1}$$

In addition, by letting $X=[\mathbf{x}^T\ \mathbf{u}^T]^T$ and $f=[f_x^T\ f_u^T]^T$, we obtain

$$X(k+1)=f(X(k)) \tag{2}$$

and let $\hat{X}=[\hat{\mathbf{x}}^T\ \mathbf{u}^T]^T$ and $\hat{f}=[\hat{f}_x^T\ f_u^T]^T$, and then an estimation for (2) is given as

$$\hat{X}(k+1)=\hat{f}(X(k)). \tag{3}$$

where $\hat{\mathbf{x}}=[\hat{x}_1,\hat{x}_2,...,\hat{x}_n]^T\in\mathbb{R}^n$ is an estimation for the state vector $\mathbf{x}$, $\hat{f}_x=[\hat{f}_1,\hat{f}_2,...,\hat{f}_n]^T : R^{n+m}\to R^n$ is an estimation for f_x, $\mathbf{u}=[u_1,u_2,...,u_m]^T\in\mathbb{R}^m$ is a learning controller, and $f_u=[f_{n+1},f_{n+2},...,f_{n+m}]^T : R^{n+m}\to R^m$ is an estimation for the next time human control.

In this paper, support vector machine (SVM) with polynomial kernels will be used as a neural network structure to learn the human expert control process. After an off-line training process, we obtain the support values (α and α^*) and the corresponding support vectors X_i. By inserting the simple polynomial kernel into SVM formula, we obtain $\hat{f}(X)=[\hat{f}_1,\hat{f}_2,...\hat{f}_{m+n}]^T$, which is a vector and its element $\hat{f}_i$ is a nonhomogeneous form of degree d in $X\in\mathbb{R}^{n+m}$ (containing all monomials of degree $\le d$)

$$\hat{f}_i=\sum_{0\le|\beta|\le d} c_j x_1^{\beta_1}x_2^{\beta_2}...x_n^{\beta_n}u_1^{\beta_{n+1}}...u_m^{\beta_{n+m}},\quad i=1,2,...,n+m, \tag{4}$$

where β is a vector $(\beta_1,\beta_2,...,\beta_{n+m})$, and its component β_l is a nonnegative integer, called a multiindex of order $|\beta|=\beta_1+\beta_2+...+\beta_{n+m}$; and $c_j\in\mathbb{R}$ are weight coefficients, $j=1,2,...,M$, where $M=\binom{n+m+d}{n+m}$. By defining

$$X=[x_1,x_2,...,x_n,u_1,...,u_m], \tag{5}$$

and rearranging the monomials with respect to their orders, we can write

$$\hat{f}(X)=C+AX+g(X), \tag{6}$$

where $C=[c_1,c_2,...c_{m+n}]^T$ is a constant vector, $A\in\mathbb{R}^{(n+m)\times(m+n)}$ is a constant coefficient matrix, and $g(X)=[g_1(X),g_2(X),...,g_{n+m}(X)]^T$ is a vector, and its elements $g_i(X)$ are polynomials where the degree of each term is greater than or equal to 2, and $i=1,2,...,n+m$.

Assume that $m+n$ multiple-input-single-output SVM models have been built through the data collected from a human expert controlling the physical system over a period of time. It has been proved by Vapnik [17] that any continuous mapping over a compact domain can be approximated as accurately as necessary by an SVM given the training data number is large enough, This implies that for any $\varepsilon>0$, if $\bar{n}$ is the number of sample data, there exists an $N>0$, such that if $\bar{n}>N$,

$$\| f(x)-\hat{f}(x) \| < \varepsilon,\ \forall x\in\mathscr{D},$$

where f is the function to be approximated, $\hat{f}$ is an SVM-based learning model and $\mathscr{D}$ is a compact domain of a finite dimensional normal vector space.

The following assumptions are held for the SVM-based regression models.

Assumption 1: For system (3), in the compact domain $\mathscr{D}$, the sample data number is sufficiently large and uniformly distributed.

Remark 1: The assumption 1 is usually required in control design with neural networks for function approximation [5]-[8]. Then from the above analysis, we have assumption 2.

Assumption 2: For system (3), in the region $\mathscr{D}$, the learning precision is high enough.

Let

$$e=f(X)-\hat{f}(X).$$

Then, we can write

$$X(k+1)=\hat{f}(X(k))+e. \tag{7}$$

According to assumption 2, the model (3) is sufficiently accurate, i.e., e is small enough and can be neglected here. Inserting equation (6) into (7) gives

$$X(k+1)\approx\hat{f}(X(k))=C+AX(k)+g(X(k)). \tag{8}$$

If the origin is an equilibrium point of the system, we have

$$C=[0,0,...0]^T,$$

and if not, by redefining the systems states as $(X(k)-C)$, we yet can drop term C in (8) to obtain,

$$X(k+1)=AX(k)+g(X(k)). \tag{9}$$

Note that as the SVM modeling error e and other noises always exist, even assumed sufficiently small, the left side of system (9) is approximately equal to the right side. A detailed description of the system can be found in our previous paper [11]. In [11], the stable conditions for the closed-loop system (9) are investigated, whereas we study the stability region of the closed-loop system (9), if it is stable.

III. Computation of Stability Region

With Assumptions 1 and 2 being satisfied in the region $\mathscr{D}$, we rewrite the system (3) as (9). If $\bar{A}$ is defined as A^TA-I and it is a Hurwitz' matrix, the equilibrium point $X=0$ is strongly stable under perturbations (SSUP). Thus, our task in this part is to address a scheme to estimate the stability region (SR) around the point of $X=0$ of (9) through using the Lyapunov function defined as

$$V(X(k)) = X^T(k)X(k). \tag{10}$$

The set

$$\Omega_r = \{X|\sqrt{X^TX} \le R\},\ R>0, \tag{11}$$

is contained in the unknown stability region if the inequality

$$\begin{aligned}\Delta V(X) &= X^T(k)(A^TA-I)X(k)+X^T(k)A^Tg(X(k))\\ &+ g^T(X(k))AX(k)+g^T(X(k))g(X(k))\\ &= X^T\bar{A}X+\bar{g}(X)\\ &< 0,\end{aligned} \tag{12}$$

holds for all

$$X\in\Omega_r,\ X\neq 0, \tag{13}$$

where

$$\begin{aligned}\bar{g}(X(k)) &= X^T(k)A^Tg(X(k))+g^T(X(k))AX(k)\\ &+ g^T(X(k))g(X(k)).\end{aligned} \tag{14}$$

The problem is to maximize R, which is actually an optimization problem, such that (12) - (13) are satisfied. Namely, the corresponding set Ω_r is the largest subset of the domain of attraction which can be guaranteed with the chosen Lyapunov function.

Let

$$s(X) = -\Delta V(X). \tag{15}$$

Apparently, $s(X)$ should be strictly positive in Ω_r, except for $X=0$. In addition, we reformulate states X from a Cartesian coordinate into a hyper-sphere coordinate with the following replacements.

$$\begin{aligned}X_1 &= r\cos\theta_1\,\cos\theta_2\,\cdots\,\cos\theta_{n+m-2}\,\cos\theta_{n+m-1}\\ X_2 &= r\cos\theta_1\,\cos\theta_2\,\cdots\,\cos\theta_{n+m-2}\,\sin\theta_{n+m-1}\\ X_3 &= r\cos\theta_1\,\cos\theta_2\,\cdots\,\cos\theta_{n+m-3}\,\sin\theta_{n+m-2}\\ &\vdots\\ X_i &= r\cos\theta_1\,\cos\theta_2\,\cdots\,\cos\theta_{n+m-i}\,\sin\theta_{n+m-i+1}\\ &\vdots\\ X_{n+m} &= r\sin\theta_1,\end{aligned} \tag{16}$$

where r is the radius and $\theta=[\theta_1,\theta_2,...,\theta_{n+m-1}]^T$ are the angles. In this case

$$V(X) = V(r,\theta) := V(y) = r^2, \tag{17}$$

where y is the vector $[r,\theta_1,\theta_2,...,\theta_{n+m-1}]^T$. The function $s(X)$ can be represented with the equation (18).

$$s(X) = s(r,\theta) = s(y) = a_kr^k + a_{k-1}r^{k-1}+...+a_2r^2, \tag{18}$$

where coefficients a_i are functions of the angles θ, $(i=2,...,k)$.

Next, inspired by [15], we address the Chebychev approximation scheme to compute Ω_r. In the following, $I=[a,b]$ denotes a nonempty real interval with $I\subset\mathbb{R}$. We define a set of Chebychev points in the interval I for a given integer $N>0$ by $x(N,I) := \{x_1,x_2,...,x_N\}$, where

$$x_i := \frac{a+b}{2}+\frac{b-a}{2}\cos(\frac{(2i-1)\pi}{2N}),\quad i=1,2,...,N. \tag{19}$$

Let F_m be the set of polynomials s in one variable with $deg(s)\le n$. Then, the following result is given by Ehilich and Zeller in [3].

$$\|s\|_\infty^I \le C\left(\frac{n}{N}\right)\|s\|_\infty^{x(N,I)}, \tag{20}$$

with $N>n$ is valid for all $s\in F_m$ and every interval I, where

$$\|s\|_\infty^I := \max_{x\in I}|s(x)|, \tag{21}$$

and

$$C(q) = [\cos(\frac{q}{2}\pi)]^{-1}\quad\text{for}\quad 0<q<1. \tag{22}$$

Using (20), the following inequalities

$$s^I_{min} \ge \frac{1}{2}\left\{\left(C\left(\frac{n}{N}\right)+1\right)s^{x(N,I)}_{min} - \left(C\left(\frac{n}{N}\right)-1\right)s^{x(N,I)}_{max}\right\}, \tag{23}$$

$$s^I_{max} \le \frac{1}{2}\left\{\left(C\left(\frac{n}{N}\right)+1\right)s^{x(N,I)}_{max} - \left(C\left(\frac{n}{N}\right)-1\right)s^{x(N,I)}_{min}\right\}. \tag{24}$$

are given by Gartel in [4], where

$$s^I_{min} := \min_{x\in I}s(x),\quad s^I_{max} := \max_{x\in I}s(x). \tag{25}$$

For trigonometric polynomials, the set of Chebychev points in I are defined with following equality

$$x_i = a+\frac{i-1}{N}(b-a),\quad i=1,2,...,N,$$

where a and b belong to the interval $[0,2\pi]$ and $N>2n$.

We can extend the inequalities (20), (23) and (24) for polynomials in one variable to several variables using the following replacements. The interval I is replaced by

$$\hat{I} = [a_1,b_1]\times[a_2,b_2]\times...\times[a_{n+m},b_{n+m}], \tag{26}$$

which represents a hyper-rectangle. For the degree of s with respect to the i-th variable x_i we introduce the abbreviation n_i and the set of Chebychev points in $\hat{I}$ is given by

$$\begin{aligned}x(\hat{N},\hat{I}) :=& x(N_1,[a_1,b_1])\times x(N_2,[a_2,b_2])\times...\\ &\times x(N_{n+m},[a_{n+m},b_{n+m}]),\end{aligned} \tag{27}$$

where N_i is the number of Chebychev points in the interval $[a_i,b_i]$. Then, the inequalities

$$s^{\hat{I}}_{min} \ge \frac{1}{2}\left\{(K+1)s^{x(\hat{N},\hat{I})}_{min} - (K-1)s^{x(\hat{N},\hat{I})}_{max}\right\}, \tag{28}$$

$$s^{\hat{I}}_{max} \le \frac{1}{2}\left\{(K+1)s^{x(\hat{N},\hat{I})}_{max} - (K-1)s^{x(\hat{N},\hat{I})}_{min}\right\}. \tag{29}$$

with

$$K = \prod_{i=1}^{n+m}C(\frac{m_i}{N_i}),\quad N_i>n_i,\quad i=1,2,...,m+n$$

are achieved.

Using the theorem of Ehilich and Zeller in [3], we can find out with following inequality whether the polynomial $s(y)$ is strictly positive in a hyper-sphere with radius r (without the origin),

$$(K+1)s_{min}^{y(\hat{N},\hat{I})} - (K-1)s_{max}^{y(\hat{N},\hat{I})} > 0, \quad (30)$$

where the angles θ vary in the interval $[0,2\pi]$ and the radius r varies in the interval $(0,r]$. If the inequality (30) holds, the following inequality are also valid,

$$(K+1)s(y[i]) - (K-1)s(y[j]) > 0, \quad i,j = 1,2,...,\hat{N} \quad (31)$$

with

$$s_{min}^{y(\hat{N},\hat{I})} \le s(y[i]) \le s_{max}^{y(\hat{N},\hat{I})}, \quad i = 1,2,...,j,...,\hat{N} \quad (32)$$

where $y[i], y[j] \in y(\hat{N},\hat{I})$ are two Chebychev points. For $\hat{N}$ Chebychev points we have $\hat{N}^2$ inequalities of type (31) which are equivalent to (30). According to (28), for $i \neq j$, the inequalities (31) provide us with the sufficient conditions for the strict positivity of polynomial $s(y)$.

In addition, for $i = j$, the inequalities (31) can be written as

$$s(y(i)) = s(r,\theta) > 0, \quad i = 1,2,...,\hat{N}, \quad (33)$$

which give us the necessary conditions for the strict positive of polynomial $s(y)$.

Let r^* be the maximum radius of the hyper-sphere, in which the polynomial $s(y)$ is strictly positive. The hyper-sphere will give us the largest subset of the stability region with the chosen Lyapunov function $V(X)$. If the inequalities (31) are numerically solved, an inner approximation r_{in}^* to the maximum radius r^* is determined. The solution of inequalities (33) gives us an outer approximation r_{out}^* to r^*. In this case

$$r_{in}^* \le r^* \le r_{out}^* \quad (34)$$

is valid. The maximum is equal to $R^* = (r^*)$ and gives the set Ω_{r^*}

$$\Omega_r^* = \{X | \sqrt{X^T X} \le R^*\}, \ R > 0, \quad (35)$$

where R^* lies

$$R_{in}^* \le R^* \le R_{out}^* \quad (36)$$

and Ω_{r^*} is the largest subset of the stability region with the given Lyapunov function.

The main tool in achieving this goal is the use of an appropriate Lyapunov function. In fact, there are almost infinity kinds of Lyapunov functions that can be applied for this aim. However, a large number of experiments show that quadratic Lyapunov functions usually can work out a desirable stability region if an SVM learning controller could have fine performance in practical control experiments. In the following we will address the type of quadratic Lyapunov functions.

$$V(X) = X^T P X, \quad (37)$$

where P is a positive definite symmetric $(n+m)\times(n+m)$ matrix. Assume that the matrix

$$Q = -(A^T P A - P), \quad (38)$$

is positive definite, $V(X)$ is a valid Lyapunov function for the system $X(k+1) = AX(k)$, i.e., the linear part of (9). The relationship among A, P and Q is given by the the following theorem [9].

Theorem 1: If these are positive definite matrices P and Q satisfying (38), then A is stable. Conversely, if A is stable, then given any Q, equation (38) has a unique solution; if Q is positive definite, P is positive definite, and if Q is symmetric, P is symmetric as well.

We need to ensure that the systems states and control inputs can vary in a large area, while the system (3) is still stable. In Section III, a scheme to estimate the SR based on the Lyapunov function as (10) has been proposed. To take the advantage of this, we will present an approach to transform the type of Lyapunov function into (37) to the formation of (10).

As P is symmetric, define

$$P = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,n+m} \\ p_{1,2} & p_{2,2} & \cdots & p_{2,n+m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1,n+m} & p_{2,n+m} & \cdots & p_{n+m,n+m} \end{pmatrix}. \quad (39)$$

By means of the Cholesky Factorization, as a positive definite symmetric matrix, P can be efficiently decomposed into a lower and upper triangular matrices with the following equation.

$$P = L^T L. \quad (40)$$

Here the matrix L is an upper triangular matrix in the form

$$L = \begin{pmatrix} l_{1,1} & l_{1,2} & l_{1,3} & \cdots & l_{1,n+m} \\ 0 & l_{2,2} & l_{2,3} & \cdots & l_{2,n+m} \\ 0 & 0 & l_{3,3} & \cdots & l_{3,n+m} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & l_{n+m,n+m}. \end{pmatrix} \quad (41)$$

The element of the matrix L are calculated as follows.

$$\begin{aligned} l_{1,1} &= \sqrt{p_{1,1}}, \quad l_{1,j} = \frac{p_{1,,j}}{l_{1,1}}, && j = 2,...,n+m \\ l_{i,i} &= \sqrt{(p_{i,i} - \textstyle\sum_{k=1}^{i-1} l_{k,i}^2)}, && i = 2,...,n+m \\ l_{i,j} &= \frac{1}{l_{i,i}}(p_{i,j} - \textstyle\sum_{k=1}^{i-1} l_{k,i} l_{k,j}), && i = 2,...,n+m-1, \\ & && j = i+1,...,n+m. \end{aligned} \quad (42)$$

By defining

$$\tilde{X} = LX, \quad (43)$$

we can rewrite the system (9) as

$$\begin{aligned} \tilde{X}(k+1) =& \tilde{A}\tilde{X}(k) + \tilde{g}(\tilde{X}(k)) \\ =& LAL^{-1}\tilde{X}(k) + Lg(L^{-1}\tilde{X}(k)), \end{aligned} \quad (44)$$

where $\tilde{X}$ is the new state vector. Apparently, $\tilde{X} = 0$ is still an equilibrium point of the new state space representation (44). We can present the Lyapunov function (37) with the new state vector $\tilde{X}$,

$$V(X) = X^T P X = X^T L^T L X = \tilde{X}^T \tilde{X} := \tilde{V}(\tilde{X}). \quad (45)$$

Therefore, we have a new state space equation

$$\tilde{X}(k+1)=\tilde{A}\tilde{X}+\tilde{g}(\tilde{X}(k)), \tag{46}$$

with the Lyapunov function

$$\tilde{V}(\tilde{X})=\tilde{X}^T\tilde{X}. \tag{47}$$

Assume that the new system (46) is SSUP. Following the scheme described in this Section, using the system (46) and its Lyapunov function (47), we compute a maximum radius $\tilde{R}^*$ of a new estimate $\Omega_{\tilde{r}}$ for the stability region,

$$\Omega_{\tilde{r}}=\{\tilde{X}|\sqrt{\tilde{X}^T\tilde{X}}\leq\tilde{R}^*\}. \tag{48}$$

where $\Omega_{\tilde{r}}$ is a $n+m$ dimensional sphere for the new states $\tilde{X}$, while for the original states X, the maximum radius $\tilde{R}^*$ defines a $n+m$ dimensional ellipsoid

$$\begin{aligned}\Omega_{\tilde{r}^*}=&\{X|\sqrt{(LX)^TLX}\leq\tilde{R}^*\}\\=&\{X|\sqrt{X^TPX}\leq\tilde{R}^*\}.\end{aligned} \tag{49}$$

The goal here is to design a symmetric positive definite P, such that $\tilde{R}^*$ is maximized. Since $P=L^TL$, it is more convenient to design L to achieve our goal. As L is a upper triangular $(n+m)\times(n+m)$ matrix, we have

$$\bar{N}=\frac{(n+m)(n+m+1)}{2} \tag{50}$$

independent variables that we can choose. In addition, two constraints should be satisfied in selecting the $\bar{N}$ variables: (1) Q is positive definite; (2) $|L|\approx 1$, where $|L|$ denotes the determine of matrix L. According to Theorem 1, if Q is positive definite, then P is positive definite. The aim of the second constraint $|L|\approx 1$ is to avoid that the increase of $\tilde{R}^*$ arises from the increase of $|L|$ or $|P|$.

It is possible that the estimation $\Omega_{\tilde{r}^*}$ (49) for the stability region for the system (9) is better than the estimation Ω_{r^*} (35), i.e., the volume of the set $\Omega_{\tilde{r}^*}$ is larger than the volume of the set Ω_{r^*}. However, this kind of approximation has its limitations, because it is influenced by the chosen Lyapunov function, namely the given P matrix.

In fact, the size of the stability region is the property of the systems (9) and should be independent on the selected P matrix. In addition, the shape of the stability region may be complicated and unlikely an ellipsoid. Therefore, we should vary the P matrix in $\mathbb{R}^{(n+m)\times(n+m)}$. Suppose that we have worked out all the $\Omega_{\tilde{r}^*}$ referring to all variants of the P matrix and combine them into a union M_{Ω_r}, this union M_{Ω_r} is still a subset of the stability region and is larger than any single estimation $\Omega_{\tilde{r}^*}$.

IV. Experimental Study

In this section, we will provide an experimental result to illustrate the theoretical analysis and the estimate scheme for the stability region.

Fig. 2. Gyrover: A single-wheel robot.

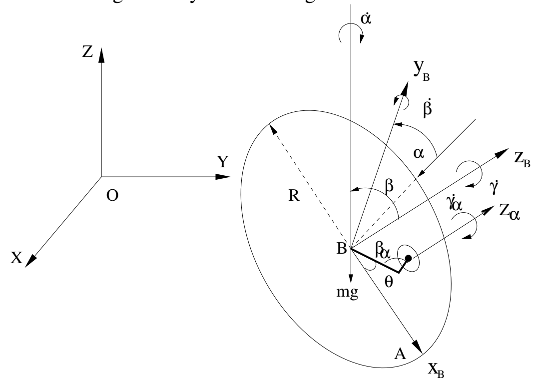

Fig. 3. Definition of the Gyrover's system parameters.

A. Experimental System – Gyrover

The single-wheel gyroscopically-stabilized robot, Gyrover, takes advantage of the dynamic stability of a single wheel. Fig. 2 shows a photograph of the third Gyrover prototype.

Gyrover is a sharp-edged wheel with an actuation mechanism fitted inside the rim. The actuation mechanism consists of three separate actuators: (1) a spin motor, which spins a suspended flywheel at a high rate and imparts dynamic stability to the robot; (2) a tilt motor, which steers the Gyrover; and (3) a drive motor, which causes forward and/or backward acceleration by driving the single wheel directly.

To represent the dynamics of the Gyrover, we need to define the coordinate frames: three for position (X,Y,Z), and three for the single-wheel orientation (α,β,γ). The Euler angles (α,β,γ) represent the precession, lean and rolling angles of the wheel respectively. (β_a,γ_a) represent the lean and rolling angles of the flywheel respectively. They are depicted in Fig. 3.

B. Task and Experimental Description

The aim of this experiment is to illustrate the proposed theorems and scheme through testing an SVM-based learning controller with Gyrover.

The control problem consists of controlling Gyrover in maintaining it vertically balanced, i.e., keeping it from falling down to the ground. We have built up an SVM-based learning controller through learning imparted from human expert's demonstrations.

There are mainly two control inputs available: U_0 controlling the rolling speed of the single wheel $\dot{\gamma}$, and U_1

controlling the angular position of the flywheel β_a. The spinning rate of the flywheel is given a particular value that is not treated as a control input. In the manual-model (i.e., controlled by a human), U_0 and U_1 can be controlled from a joystick, and in the auto-model, they are sent from a software based controller running in an on-board computer. In these experiments, we only use U_1, while U_0 is fixed to zero ($U_0 = 0$) all the time. The two lean angles β and β_a are used as the state variables and U_1 is the control input in the SVM-based learning model (1), i.e., $\mathbf{x} = [\beta\ \beta_a]^T$ and $\mathbf{u} = U_1$.

A human expert controls Gyrover to maintain a balance and then generates about 2400 training samples. Table I displays some raw sensor data from the human expert control procedure.

TABLE I
SAMPLE HUMAN CONTROL DATA.

states		controller
β	β_a	U_1
5.5034	2.4023	179.0000
5.7185	2.3657	176.0000
5.6012	2.1313	170.0000
5.1271	2.1460	170.0000
5.9433	1.0425	143.0000

After calibrating the data into the same scale $[-1,1]$, respect to individual variable, we divide the sample data into two groups: a training data set and a testing data set to train an SVM-based learning model, which is also the controller. In the learning model, Vapnik's polynomial kernel of order 2 is used. Three three-input-one-output SVM models are built for the three variables $\hat{\beta}_a$, $\hat{\beta}$, and U_1. In each of the SVM models the three current values are served as inputs. For $\hat{\beta}_a(k+1)$, $\hat{\beta}(k+1)$, and $u(k+1)$, we have 977, 989 and 905 support vectors generated, respectively. The SVM-based learning models are quite accurate. Fig. 4 shows the comparison of U_1 from the Human control and SVM-based learning model.

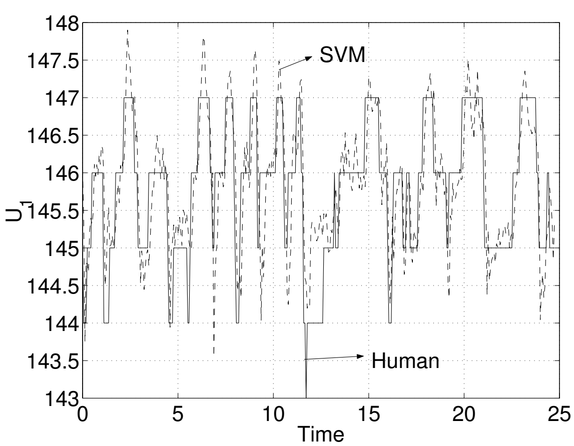

Fig. 4. U_1 comparison of the same Human control and SVM-based learner.

By expanding the three SVM models according to the Vapnik's polynomial kernel we obtain the following state space equation

$$X(k+1) = AX(k) + g(X(k)), \tag{51}$$

where $X = [\hat{\beta}_a,\ \hat{\beta},\ U_1]^T := [x_1, x_2, x_3]^T$,

$$A = \begin{bmatrix} 0.8818 & 0.0074 & -0.2339 \\ -0.1808 & 0.8615 & -0.2389 \\ 0.0154 & -0.0007 & 0.5167 \end{bmatrix} \tag{52}$$

and

$$g(X) = \begin{bmatrix} 0.0004x_1^2 - 0.0013x_1x_2 + 0.0028x_1x_3 - 0.0017x_2x_3 - 0.0006x_2^2 + 0.0027x_3^2 \\ 0.0004x_1^2 + 0.0002x_1x_2 + 0.0034x_1x_3 + 0.0017x_2x_3 + 0.0003x_2^2 + 0.0049x_3^2 \\ 0.0002x_1^2 + 0.0002x_1x_2 - 0.0002x_1x_3 - 0.0001x_2x_3 - 0.0001x_2^2 - 0.0007x_3^2 \end{bmatrix} \tag{53}$$

Then, as $r(A) = 0.8680$, A is stable. Thus, the system is SSUP.

Next, we need to estimate the stability region Ω. As

$$\bar{A} = A^TA - I = \begin{bmatrix} -0.1895 & -0.1492 & -0.1551 \\ -0.1492 & -0.2578 & -0.2079 \\ -0.1551 & -0.2079 & -0.6212 \end{bmatrix} \tag{54}$$

is negative definite, we will use the following positive definite function $V(X)$ to estimate the stability region for this learning controller.

$$V(X) = X^TX. \tag{55}$$

The first difference of the function is given by

$$\begin{aligned} \Delta V(X) = \ & (AX+g(X))^T(AX+g(X)) - X^TX \\ = \ & -0.19x_1^2 + 0.057x_1^3 + 0.0036x_1^4 - 0.0298x_1x_2 \\ - \ & 0.17x_1^2x_2 - 0.008x_1^3x_2 - 0.26x_2^2 - 0.085x_1x_2^2 \\ + \ & 0.015x_1^2x_2^2 + 0.051x_2^3 + 0.016x_1x_2^3 + 0.0046x_2^4 \\ - \ & 0.295x_1x_3 + 0.37x_1^2x_3 + 0.049x_1^3x_3 - 0.42x_2x_3 \\ + \ & 0.32x_1x_2x_3 - 0.060x_1^2x_2x_3 + 0.28x_2^2x_3 \\ + \ & 0.038x_1x_2^2x_3 + 0.031x_2^3x_3 + 0.15x_3^2 - 0.037x_1x_3^2 \\ + \ & 0.25x_1^2x_3^2 + 0.83x_2x_3^2 - 0.033x_1x_2x_3^2 + 0.056x_2^2x_3^2 \\ - \ & 0.50x_3^3 + 0.49x_1x_3^3 + 0.076x_2x_3^3 + 0.32x_3^4, \end{aligned} \tag{56}$$

and the polynomial $p(X)$ is $-\Delta V(X)$. We can now express the polynomial $p(X)$ in a polar coordinate by using the following transformation.

$$\begin{aligned} x_1 &= r\cos\theta_1\sin\theta_2, \\ x_2 &= r\cos\theta_1\cos\theta_2, \\ x_3 &= r\sin\theta_1. \end{aligned} \tag{57}$$

Then the interval I can be given as

$$\hat{I} = [0,1] \times [0,2\pi] \times [0,2\pi]. \tag{58}$$

We find out an inner and outer approximation to the maximum radius r^* by the inequalities (31) and (33) . With 192 Chebychev points the following approximation to r^* has been found by means of the scheme provided in the section III.

$$0.424 \le r^* \le 0.426.$$

Therefore,the lean angle β is in a region of $[-22,22]$ degree, which is large enough to allow Gyrover staying vertically for quite a while.

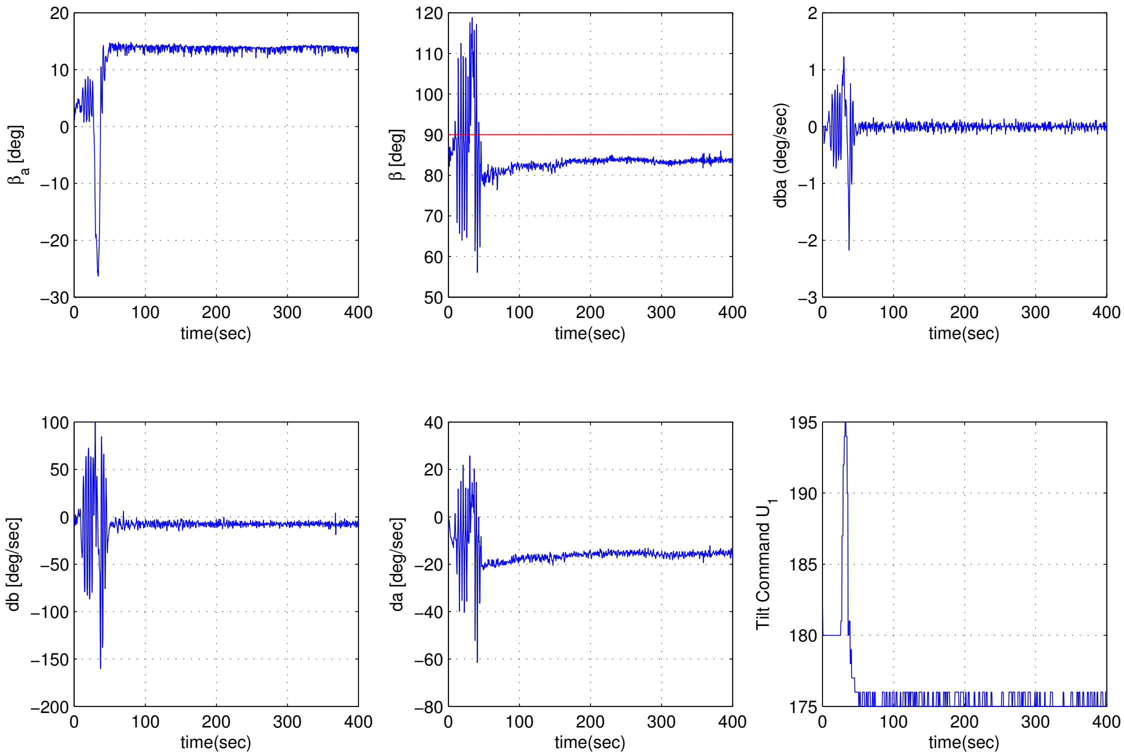

Fig. 5. Sensor data of the SVM-based learning control results.

C. Experimental Results

By using the SVM-based learning controller to execute the experiment of balance, the experiment is successful and Gyrover can independently maintain itself balanced for more than 5 minutes.

V. Conclusion

In this paper, convergence analysis for a class of intelligent controllers through learning human expert control skills using SVMs are investigated. One Chebychev approximation based scheme is proposed to estimate stability region, the key property for this class of intelligent controllers. An experimental study is given to validate the proposed estimation approach and the theoretical discussions therein. The further exploration for methods to enlarge the estimated stability region is a part of the future work.

References

[1] H. Asada and S. Liu. Transfer of human skills to neural net robot Ccontrollers. In *Proceedings of the IEEE Conference on Robotics and Automation*, pages 2442–2448, 1991.
[2] S. Behatsh. Robust output tracking for nonlinear systems. *International Journal of Control*, 51:1381–1407, 1995.
[3] H. Ehlich and K. Zeller. Schwankung von polynomen zwischen gitterpunkten. *Mathematische Zeitschrift*, 86:41–44, 1964.
[4] U. Gartel. Error estimation for second order verctor Boundary value functions, especially for problems in the chemical Reactions-Diffusions-Theory. *Ph.D. Thesis, University of Cologne*, 1987.
[5] S. S. Ge, C. C. Huang, and T. Zhang. control of nonlinear systems by state and output feedback. *IEEE Trans. on Systems, Man, and Cybernetics–Part B: Cybernetics*, 29(6):818–828, 1999.
[6] A. Isidor, editor. *Nonlinear control system*. Spring-Verlag, second edition.
[7] H. K. Khalil. daptive output feedback control of nonlinear system represented by input-output models. *IEEE Transactions on Automatica Control*, 41:177–188, 1993.
[8] Y. H. Kim and F. L. Lewis. Neural network output feedback control of robot manipulators. *IEEE Transactions on Robotics and Automation*, 15(2):301–309, 1999.
[9] J. P. LaSalle. *The Stability and Control of Discrete Processes*. Applied Mathematical Sciences. Springer-Verlag.
[10] J. F. Montgomery and G. A. Bekey. Learning helicopter control through “teaching by showing”. In *Proceedings of the IEEE Conference on Decision and Control*, volume 4, pages 3647 –3652, 1998.
[11] Y. Ou and Y. Xu. Convergence Analysis for a Class of Human Learning Controllers. pages 2653–2658, 2004.
[12] A. Pavlov, N. van de Wouw, and Henk Nijmeijer. The local output regulation problem: convergence region estimates. *IEEE Trans. Automat. Contr.*, 49:814–819, 2004.
[13] N. Peterfreund and Y. Baram. Convergence analysis of nonlinear dynamical systems by nested lyapunov functions. *IEEE Trans. Automat. Contr.*, 43:1179–1184, 1998.
[14] D. J. Sebald and J. A. Buckle. Support vector machine techniques for nonlinear equalization. *IEEE Transactions on Signal Processing*, 48:3217–3226, 2000.
[15] B. Tibken and K. F. Dilaver. Computation of subsets of the domain of attraction for polynomial systems. In *Proceedings of the IEEE Conference on Decision and Control*, pages 2651–2656, 2002.
[16] T. B. Trafalis and H. Ince. Support vector machine for regression and applications to financial forecasting. In *Proceedings of the IEEE-INNS-ENNS International Joint Conference on Neural Networks*, volume 6, pages 348–353, 2000.
[17] V. Vapnik. *The nature of statistical learning theory*. Springer-Verlag, New York, 1995.
[18] J. Yang, Y. Xu, and C. S. Chen. Hidden Markov model approach to skill learning and its application to telerobotics. *IEEE Trans. on Robotic and Automation*, 10(5):621–631, 1994.