On Contact Models for Assembly Tasks: Experimental Investigation Beyond the Peg-in-Hole Problem on the Example of Force-Torque Maps


Abstract—Force guided assembly is attractive but the implementation is challenging when uncertainties are present. In these cases, contact models that guide the assembly process are attractive. This article elaborates the usage of static contact models, which map displacements to force-torque vectors of particular contact situations (force-torque map). This model, a map of discrete points, contains force and torque values that correspond to position errors. The inversion of this map, using forces and torques measured from an assembly attempt, yields correction movements in order to accomplish the assembly iteratively.

A hypothesis stating the dependency of the model’s quality on its injectivity is investigated. This aspect is studied thoroughly in so-called redundancy maps, which reveal regions of considerable ambiguity of the model. Experimental results are presented, which validate the hypothesis about the dependency of the convergence of the assembly process on the ambiguity of the initial position. In addition to the peg-in-hole problem, which has become a standard scenario to validate force guided assembly, the scope of this article also covers force guided assembly of more complex parts. Here, the analysis gives evidence to believe that it is unlikely that the implementation convergences acceptably, which is validated by experimental results.

I. INTRODUCTION

Automated assembly in an industrial context demands for robustness of the assembly strategy employed. In order to automate assembly of complex parts successfully, the compensation of geometrical uncertainties is an important issue. There are strategies where the geometrical uncertainties may be treated by artificial compliance [1]. In cases where such additional compliance is undesired, the integration of sensors is of superior importance to improve automated assembly [2].

The Collaborative Research Center SFB 562 is concerned with the elaboration of this idea towards industrial applications on parallel robots. Parallel robots are mechanisms that are based on closed kinematic chains. If well-designed, their performance is superior to serial robots [3]. The SFB 562 presented an implementation of the task frame formalism [4], where hybrid control in conjunction with a primitive-oriented way of programming is at the core [5].

A. Strategies for Automated Assembly

Traditionally, assembly tasks have been specified by positioning commands only, which is reflected by industrial robot programming languages. Here, the robustness of the assembly process relies on the clearance between the parts in assembled configuration and the compliance of the chain of more or less flexible elements. If the stiffness of the kinematic system is too high, the compliance can be adjusted explicitly by mounting a flexible joint. If the clearance becomes very low and additional compliance in the kinematic chain is not desirable, sensor integration helps to improve robustness of the assembly. Four assembly strategies are in the focus of the SFB 562:

1) Object Placing in the Task-Frame Formalism: The task-frame formalism simplifies the parametrization of a sequence where an object shall be placed on a surface. A contact planner\(^1\), to which a certain contact force is given as setpoint, controls the approach direction. Torque controllers keep the reaction torques at zero. The task frame is attached to the hand frame.

2) Camera based Assembly: Camera systems may guide assembly sequences, either offline by pose estimation or online by visual servoing. The SFB 562 implemented a vision-based system that automatically plans the sequence of actions [6]. The robot system then executes this sequence and uses sensor-guidance from the built-in force-torque sensor to accomplish its task. Unfortunately, the image data is often sensitive against improper or uncontrollable lighting. This renders its industrial application questionable in some applications.

3) Graph-like Contact Models: Laugier [7] proposed contact informations for the planning of sensor-based movements in contact situations. In [8] a graph-based description of geometric constraints for a planar shaft-in-hole problem is considered. There, the trajectory is adapted suitably. The nodes of the graph represents possible contact situations (e.g. point-edge contact, edge-plane contact). The edges represent transitions from one contact situation into another. As soon as the contact between two parts is established, the most likely situation is estimated [9]. Based hereon a strategy is

\(^1\) A control scheme where the system is velocity controlled as long as no contact force is measured. A force controller takes over control as soon as the contact is established.
chosen automatically to proceed to the desired configuration (cf. [10], [11]).

d) Other Contact Models for Assembly: Consider a shaft insertion problem (peg-in-hole) where uncertainty is present. If no sensor data is available, a “blind” search strategy is a promising, but cumbersome method [12]. It was shown that contact models are helpful to find the geometrical alignment when uncertainties of considerable level are present [13]–[15]. These models are intended to direct the search for the desired relative displacement in order to cut down the number of attempts that are required.

Force-torque maps are static contact models of assembly scenarios. Whenever a contact is established and contact forces and torques are measurable, the contact configuration may be estimated from this model. This estimate is used to start a new attempt for assembly at a corrected position. A continuous search strategy may be employed, where the peg is dragged across the surface in order to find the hole [16]. For various reasons, this method may not be recommendable. A discrete method is favorable in this situation, making quasi-static attempts at discrete positions. This article focuses on this discrete method. The estimation of the displacement has been implemented successfully using neural methods [16] or bayesian particle filters [17]. The attempts may be enhanced by superimposed perturbation which is especially helpful for online fine-tuning of the trajectory [18], [19]. Since force-torque maps require many data points, their experimental acquisition means considerable downtime for the production system. Attempts were made to overcome this drawback, which resulted in proposals to obtain force-torque maps offline from CAD data [20].

B. Contents of the Article

There are numerous publications investigating the peg-in-hole / shaft insertion problem. In contrast, force-guided assembly of more complex geometries based on contact models is treated only rarely. This article is dedicated to broaden the insights in contact models for more complex parts. In the examples presented, the direction of insertion is assumed to be known. Hence, there remain degrees of freedom in the orientation and the relative displacement.

The following section states a definition of force-torque maps (FTM). Based on this definition so-called redundancy maps (RM) are introduced, giving a measure for non-injective points in a force-torque map. This analysis is augmented and illustrated by experimental results from a peg-in-hole problem. These results motivate the subsequent proposal and analysis of a strategy that compensates points which are not bijective. Finally, these methods are applied to a triangular and a hexagonal shape in order to retrieve the major conclusions of this article.

II. Definition of Force-Torque Maps

Consider a robot to which a force-torque sensor is attached ($n$: dof of robot, $m$: dof of the sensor). The robot is commanded to assemble two parts following the approach direction. When the two parts touch each other, contact forces $F$ and torques $T$ will be measured. These forces and torques depend on the relative displacement $x$ of the two parts. This motivates the definition of a force-torque map.

$$ FTM : \mathbb{R}^n \to \mathbb{R}^m $$

This map matches relative displacements of the two parts to forces and torques:

$$ FTM(x) = [F^T, T^T]^T. $$

A FTM may be used in an assembly situation where uncertainties are present: The contact forces and torques of a failed attempt of the assembly are used to estimate the position error by inversion of the FTM. The direct inversion is not suitable since $f(x)$ is not bijective and noise is present in the measurement. A particle filter is used instead which evaluates of candidates in order to find the point with maximum fitness [11]. These candidates are shifted so that they correspond to the correction movement of the robot. In each iteration the candidates are resampled and re-evaluated, enforcing areas where “good” estimates are expected.

III. Quality Analysis of Force Torque Maps

The application of force-torque maps may suffer from being a non-injective function. It will not be possible to distinguish between two points whenever they have similar combinations of forces and torques. This becomes severe when sensor noise is present in the measurements. In [16] this circumstance was investigated, using a k-means algorithm: It is proposed there that neighborhood of positions should result in neighborhood of force-torque vectors. Whenever this condition is poorly satisfied for a position or force-torque vector, the quality of this point is considered badly. Unfortunately, this algorithm will have difficulties at discontinuities of the force-torque maps.

In our earlier work [21] we proposed to use a so-called redundancy map (RM) for quality analysis. A RM maps $[F^T T^T]^T$ of a force-torque map to the level of redundancy which is found across the force-torque map:

$$ RM : \mathbb{R}^n \to \mathbb{R}^m \to \mathbb{N} $$

Since the FTM is a function of $x$, a RM is also a function of $x$. The function value is a measure for the number of positions which have the same force / torque vector. Since we consider force-torque maps which are built from data at discrete positions and the presence of noise in the measurements, the following function is proposed to establish the redundancy value:

$$ RM(x_i) = \sum_j \begin{cases} 1 & |FTM(x_i) - FTM(x_j)| < \epsilon \\ 0 & \text{else} \end{cases}, $$

where $|\cdot|$ is a suitable norm or $\epsilon$ a vector of suitable values. Based on practical considerations we recommend to choose $\epsilon$ in the same order of magnitude of the sensor noise². Whenever two force-torque vectors of the FTM are closer to each other than the threshold $\epsilon$ this pair of values is

²In the experiments presented in this article: $\epsilon_i = 0.01 Nm$. 
considered non-injective (redundant). In this way regions in $x$ that contain non-distuingashable data areas are marked with high redundancy values. Figure 1b, for example, shows the RM of the peg-in-hole task of fig. 1a. Obviously there are highly redundant regions and regions without redundancy.

A. Experimental Results

The convergence of the estimate will diminish in redundant regions; the escape from redundant regions will be a lucky situation because many poor position choices receive inappropriate fitness values. This hypothesis is validated by the experimental data in fig. 2: Starting from a random position $x_0$, the Hexa robot iteratively tries to accomplish the peg-in-hole task illustrated in fig. 1. The number of attempts to succeed are recorded, as well as $\text{RM}(x_0)$. After multiple repetitions, each experiment is assigned to one of three classes, dependent on $\text{RM}(x_0)^3$. Figure 2a displays the sample probability of each class versus the number of attempts taken from 155 repetitions. The comparison of the plots shows that start positions with higher redundancy values require more attempts to accomplish the assembly task.

1) Interference of PRBS: Inspired by related work where the reference position signal is superimposed by small movements [18], [19], we interfered the position reference with a pseudo-random binary signal (PRBS) during assembly attempts. The intention of this scheme is twofold: First, effects of friction shall be decreased. Taking into account, that an exact feedfoward of estimated friction is rather challenging, an artificial uncertainty is added to the reference signal for compenation of frictional effects. Here, a pseudo-random-binarysequence (PRBS, [22]) is used, which guarantees a constant power spectral density over a large frequency range. The amplitude of the PRBS has to be smaller than the value which is used for the determination of task completion in position control via skill primitives and proves best for a PRBS-periode of $1.5T_{95,\text{pos}}$. Here, $T_{95,\text{pos}}$ denotes the duration in time for the step-response of position control to attain 95% of the step-value and is specific to parametric design of position control. PRBS is superimposed to the position reference (only) at the end of the trajectory and thus guarantees reduction of slip-stick and frictional effects during assembly.

Second, the convergence of the assembly shall be improved. The PRBS can be considered as a type of local search having a small horizon in this context. Figure 2b contains

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experimental results from assembly sequences involving this scheme. In comparison to fig. 2a, where no PRBS was added, the average number of trials to succeed drops from \( \approx 4.7 \) steps to \( \approx 3.8 \) steps. It can be seen that this difference is mainly caused by experiments which required many trials before. These are located especially in highly redundant areas.

For details regarding the theory and implementation of the control involved, we refer to our earlier work containing the proposal of this scheme [21].

For the peg-in-hole problem, a straight-forward idea is to change the tilting angle to move the redundant areas. Due to imperfectness of the assembly situations it is not adviseable to attempt to touch the parts in a coplanar configuration. If coplanarity is not exactly guaranteed the virtual lever arms vary significantly. This results in distortion of the force-torque map which decreases convergence of the pose estimation unacceptably. Hence, a slightly tilted configuration is recommended to obtain the FTM. The FTM obtained experimentally is shown in fig. 1a (Tilt: \( \text{Rot}_x = \text{Rot}_y = +2^\circ \)). Figure 1d shows the FTM in the configuration of \( \text{Rot}_x = \text{Rot}_y = -2^\circ \) (symmetrical to the case previously mentioned). Consequently the RM shown in fig. 1e results from fig. 1a. The comparison of the RM in fig. 1b and fig. 1e shows that the regions of high redundancy are mostly disjoint. Hence, for the peg-in-hole problem, it is reasonable to change the tilting angle of the configuration whenever a force-torque vector which contains redundant values is detected. In order to illustrate this idea, we define the set union of multiple RM using the minimum operator:

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\text{MURM}(x) = \min (\text{RM}(x)_1, \ldots \text{RM}(x)_n).
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The minimum union of redundancy maps (MURM) is shown in fig. 1f, based on \( \text{RM}(x)|_{\text{Rot}_x=\text{Rot}_y=-2^\circ} \) and \( \text{RM}(x)|_{\text{Rot}_x=\text{Rot}_y=+2^\circ} \) (figs. 1b and 1e, respectively). It shows that the redundancy values of the peg-in-hole problem can be reduced considerably when multiple maps are used.

The strategy presented above is transferred to and studied on other parts. We choose a triangle-like and a hexagonal shape in order to examine the properties for more complex geometries. The parts have been realized in similar scale to the peg-in-hole problem in order to receive comparable results. An overview of the assembly situation of the triangle is given in fig. 3. In contrast to the peg-in-hole problem the force-torque map for the triangle not only covers translational displacements but also rotational displacements. Figure 3 only shows the configuration of \( \text{Rot}_z = 0^\circ \), for the sake of simplicity. It can already be seen from the FTM in fig. 3a that there will be regions with high redundancy. The redundancy map, which is built not only from the data at \( \text{Rot}_z = 0^\circ \) but also from data at \( \text{Rot}_z \in \{-2^\circ, -1^\circ, -0.5^\circ, 0^\circ, 0.5^\circ, 1^\circ, 2^\circ\} \), confirms that impression. The avoidance strategy of changing the tilt of the configuration to escape from redundant areas will rarely be successful. This can be seen from the MURM in fig. 3f. There is still a considerable level of redundancy. This leads to the conclusion that the strategy proposed above will rarely be successful to escape from redundant areas for the triangle. Our experiments, where no successful assembly below 50 attempts was recognized, validate this hypothesis.

Figure 4 shows the force-torque maps and the redundancy maps for two configurations of the hexagon. Again, it appears that the force-torque maps have considerable regions of redundancy, which is confirmed by the redundancy maps. In comparison to the triangle, the minimum union set of the hexagon (fig. 4f) appears to be slightly better. Unfortunately the level of redundancy is still too high to guarantee successful assembly in only few trials. Again, all experiments carried out were aborted after 50 failed trials, which can be considered as a validation of this hypothesis.
V. Remarks and Conclusions

This article is concerned with the analysis of contact models for assembly. In addition to the peg-in-hole problem, which has been studied in many publications already, the scope of this article also covers force guided assembly of more complex parts.

The theory on which FTMs rely appears very attractive for the realization of assembly tasks on first sight: Based on a map of pre-calculated force/torque vectors, the pose error of a workpiece is estimated and corrected iteratively. Indeed, geometrically robust assembly strategies that only require force sensors are very appealing, considering the challenges of concurrent strategies.

In contrast, the implementation reveals the challenges which arise from idealized pre-calcualtions and practical circumstances. At first, deviations between pre-calculated and real contact configurations impose considerable uncertainty on the estimation problem. Coplanar configurations are not advisable for contact models. It is, even in an iterative procedure, difficult to achieve coplanarity of the contact between workpiece and environment. This unintentionally causes large deviations between computed and measured forces due to non-ideal lengths of the virtual lever arms. Consequently, each iteration must consist of two steps: a coplanar assembly attempt and, if failed, a sensing procedure using a tilted configuration. Practically spoken, the contact itself is the most important and, at the same time, most challenging element of assembly using static contact models.

Secondly, the analysis of redundancy maps reveals that many data points are not injective. The amount of such points is dependent on the shape of the objects involved. This means that the corresponding contact configurations are ambiguous – the estimator will not be able to distinguish between these points. In this situation it is likely that the particle filter makes a totally wrong decision. If the correction movement points to a location with high redundancy again, the estimator is likely to make a wrong decision in the subsequent iteration as well. This causes slow convergence, or, as shown in experiments, does not converge at all. Even a particle filter, implicating the history of prior measurements, has not succeeded in this problem yet. For the peg-in-hole scenario the particle filter is able to escape from redundant regions by changing the tilting angle. This is due to the fact that redundant regions are not disjunct in opposite force-torque maps. Unfortunately, this strategy does not hold for the more complex parts investigated. A useful strategy for the avoidance of redundant regions has not been found yet.

The intention of this article is to point out frontiers related to the implementation of static contact models in assembly situations. Despite the drawbacks elaborated in this article, force-torque maps are considered as promising candidates to solve the peg-in-hole task. This article contributes advances that increase convergence of this problem. Additionally, the research presented gives evidence to believe that the more complex the geometries become, the more unlikely the successful implementation of force guided assembly sequence will become. In case that a force guidance is planned for a specific scenario, it is recommended to analyze RM at first, in order to evaluate the potential of FTM there.
(a) Force-torque map ($\theta_x = -2^\circ$).

(b) Redundancy map ($\theta_x = -2^\circ$).

(c) Experimental setup.

(d) Force-torque map ($\theta_x = +2^\circ$).

(e) Redundancy map ($\theta_x = +2^\circ$).

(f) Minimum union of redundancy maps.

Fig. 4: Assembly situation: hexagon (experimental map data).

**REFERENCES**


