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Aiming for multibody dynamics on stable humanoid motion with Special Euclideans groups

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***Abstract*— This paper deals with alternative humanoid robot dynamics modelling, using the screw theory and Lie groups called the special Euclidean group (*SE(3)*). The dynamic models are deduced analitically. The inverse dynamics model is obtained by the Lagrangian formulation under screw theory, when the Jacobian manipulator depends on the respective twist and joint angles; on the other hand, the *POE* formula drives a very natural and explicit description of the Jacobian manipulator without the drawbacks of local representation. The forward dynamics were solved by propagation method from an end-effector to the center of gravity (COG) always on the *SE(3)*. Many tests for reference dynamic walking patterns have been carried out, which are represented in simulation and experimental results. The results will be discussed in order to validate the proposed algorithms.**

I. INTRODUCTION

Humanoid robots has been an active area of research in recent years; proof of this are such current successful projects as HONDA humanoid robots [1], which have demonstrated the realibility of dynamic walking and its latest prototype, ASIMO 2 which can run up to 6km/h; furthermore, it is used as personal assistant in HONDA labs. Another important project is the HRP-3 humanoid robot designed by AIST and Kawada Industries [2]; this humanoid can carry out human cooperation tasks in outdoor environments, walk on low friction terrain, is water resistant, and so on. There are other successful humanoid projects such as Wabian 2 in Waseda University [13], HUBO in Korea [11], Johnnie in Germany [12], which cope with the problem of biped locomotion on flat surface.

The commom well known problems of biped locomotion need poweful control and modelling algorithms in order to generate stable walking patterns, by accurate kinematics and dynamics modelling of a humanoid robot. As a multibody and redundant robot, humanoid robot modelling is a complex problem, so it is necessary to develop specific algorithms in order to reduce mathematical analysis and computation time. Geometric methods for kinematics modelling considerably reduce the computation time, but the limitations are: the complexity of three dimensional motion, the infinity of solutions and singularities are not avoided. Those problems have been solved by different methods such as that proposed by Prof. Y. Nakamura [14], by projections in the null space. It is a general method for high redundant robots, but it involves complex analysis and it does not show the mechanical properties of the motion.

Thus, in this paper, the screws theory is employed because kinematics modelling describes the mechanical motion of the body analyticaly. It is possible, due that with screws, the rotations and translations are represented by free vectors in the space, which are located on each degree of freedom of the robot. Furthermore, internal singularities are avoided, because no local axis are used, only global axis, as base and end-effector ones are employed in the analysis.

For dynamics modelling, the Lagrange formulation is employed, because the Lagrangian analysis relies on energy properties of mechanical systems to compute the motion equation, so the equations obtained can be computed in close form, allowing detailed analysis of the properties of the system (Ball, R.S. [16], Murray, R.M. et al. [15].) So inverse dynamics uses the manipulator inertia matrix and the jacobian manipulator which doesn't need to derivate; the jacobian manipulator is obtained by the twist and robot geometry. This last property simplifies the mathematical analysis, because it shows a geometric description of the motion. The forward dynamics modelling is based on the study of the COG motion with the effect of external wrenches (gravity and inertial), and uses the propagation method from the end-effectors (feet, hands) to the COG. The result is the motion of the free floating base, which is the COG, when the multibody dynamics is taking into account, (Featherstone et al. [5].)

This paper is divided in the following sections: section 2 deals with a background of screws, Lie groups and Lagrangian formulation; in section 3, the Rh-1 specifications are shown. Section 4 outlines the dynamics modelling of the Rh-1 humanoid robot under Lagrangian formulation with screw theory, including the forward dynamics by the propagation method. After that, section 5 shows the simulation results on the Rh-1 humanoid robot; in the next section the experimental results are described and discussed, and finally in section 7, the conclusions of this study are summarized.

II. BACKGROUND OF SCREW THEORY AND LIE GROUPS

There are many choices of methods for the kinematics and dynamic study of humanoid robots, such as Denavit-Hartenberg parameters, quaternions, euler angles, screw theory-based study and others. The selection of the adequate method depends of the analytical complexity (highly redundant robots), computational cost and intrinsic problems of

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the mathematical and physical modelling (i.e., singularities, analytical and closed solutions).

Murray et al. [15] espouse the elegance of the "product of exponential" (POE, a sequence of screw products) formula against the Denavit-Hartenberg parameterization for robot kinematics (Barrientos et al. [17]). It is that author's opinion that the power of POE formulation lies in its geometric foundation. As such, it allows more freedom in reduction of the complexity of the transformation matrices. The POE construction is straightforward since the user does not have to remember a set of rules relating consecutive joint parameters. The directions of the axis rotations or displacements are simply taken in accordance with the spatial station frame. In addition, the user works with physical link lengths rather than distances between coordinate axes. The biggest practical advantage is that the POE formulation uses only two frames to describe the forward kinematics as opposed to ***n*** frames for the Denavit-Hartenberg parameterization, so the POE does not suffer from singularities due to the use of local coordinates. Finally, the POE approach likely provides a better platform for determining an analytical solution for the inverse kinematics because the screws describe the rigid body motion in a geometric way. Beyond that, many similarities exist between the two approaches. In each case, the user must work with a set of 4 x 4 matrices. From a calibration standpoint, characteristics that affect the accuracy of the Denavit-Hartenberg (D-H) parameterization enter into the POE formulation in similar ways. Ultimately, both methods provide the same information with similar amounts of computational complexity. The Denavit-Hartenberg parameterization has been the standard in robotics for many years, and it appears the switch to the POE formulization is slow in coming.

A. Screw motion

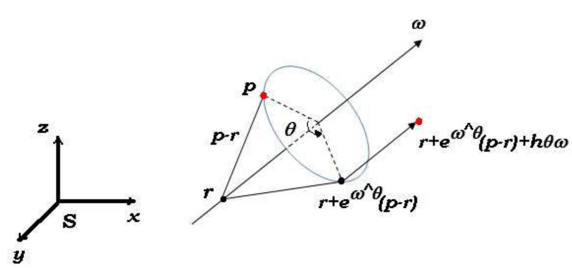

Fig. 1. Screw motion of "p".

The Chasles theorem says that every rigid body motion can be made by a rotation around an axis combined with a translation parallel to that axis. This is a "***screw motion***". The infinitesimal version of a screw motion is the Lie algebra ***se(3)***; this is a TWIST $\hat{\xi}$.

The screw theory has the following advantages:

- It allows a global description without singularities due to the use of local coordinates (e.g., Euler angles, Denavit-Hartenberg). It is possible to use only two coordinate frames, the base "S" and the end-effector "H" ones.
- A truly geometric description of rigid motion to facilitate the kinematics analysis. A very natural and explicit description of the "Jacobian Manipulator" which has none of the drawbacks of the local representation of the traditional Jacobian.
- The same mathematical treatment for the different robot joints: revolute and prismatic types.

For the above reasons, the "Screw theory" is used for kinematics and dynamics modelling.

B. Lagrangian for dynamics modelling in SE(3)

Thus, the Lagrange formulation (Lagrangian) under the lie groups and screw theory has been developed, because it gives us a natural description of a Jacobian manipulator and accurate dynamic computation. Future improvements will be included, by the Boltzmann-Hamel equations for non-holonnomic systems (Bloch, A.M. et. al '09, [19]), our first approach doesn't cover the control optimization aspects.

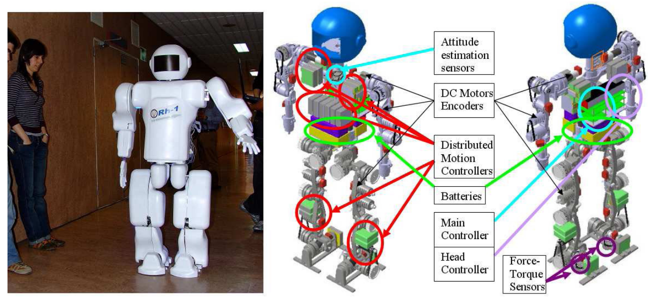

Fig. 2. Rh-1 humanoid robot on the hall, and hardware distribution.

III. RH-1 HUMANOID ROBOT

The Rh-1 humanoid robot developed in the Roboticslab of the University Carlos III of Madrid (Fig. 2) is about 1.35m tall and weight 50 Kg (Fig. 3). It has 21 degrees of freedom (see Table I) distributed in its legs, arms and waist. The joint angular ranges of the Rh-1 humanoid robot are quite similar to human, except for some range due to mobility requirements. Cantilever type structure is used on the hip joints to obtain wide range motion. The structure is designed with aeronautical aluminium, which is a strong, yet light material. The mechanical transmission system is composed of a flat harmonic drive and belt transmission in order to create a compact design. It has onboard hardware, inertial sensors, a camera including batteries, and a wireless connection with the work station; its autonomy is about 30 minutes and it can perform with an external supply. The software is custom-designed for integrating the hardware devices, which include communications, sensor management and control system. The cover was designed mainly to hide and protect the hardware, and also to improve the aesthetics of the humanoid robot. This first prototype can walk up to 0.7 Km/h, as we will show in the experimental results. Furthermore, it can respond to voice and gesture commands by using a USB microphone and a camera placed on its head. Its camera can compute the orientation and distance of the user, in order to compute the suitable motion for approaching him. The Rh-1 can speak to the user saying things, such as greeting him or saying his name; recognize the user by the vision system, which interacts with a face database.

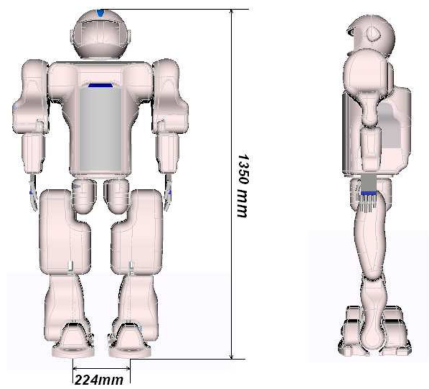

Fig. 3. Rh-1 humanoid robot dimensions.

TABLE I
RH-1 HUMANOID ROBOT DEGREES OF FREEDOM.

Link		Number of DOF	Total
Head		-	-
Waist		1 (Yaw)	1
Arm		4	4x2
	Shoulder	1 (Pitch)	
		1 (Roll)	
	Elbow	1 (Pitch)	
	Wrist	1 (Roll)	
Leg		6	6x2
	Hip	1 (Yaw)	
		1 (Roll)	
		1 (Pitch)	
	Knee	1 (Pitch)	
	Ankle	1 (Pitch)	
		1 (Roll)	
			21

IV. DYNAMICS MODELLING

Multibody dynamics modelling in robotics is a complex and high cost computation problem. Hence, the robotics researchers have proposed few approaches, such as Featherstone et al. [5], Kwatny et al. [6], Murray et al. [15], Park et al. [7]. The Lagrangian formulation gives an approach at the energy level, so it includes all the system properties. On the other hand, multibody dynamics is analized in order to solve the forward dynamics. These approaches under screw theory describe the robot motion geometrically and solve the dynamic problem analitically. In this section, the inverse and forward dynamics approaches are described in the case of high redundant robots, that is, in a humanoid robot.

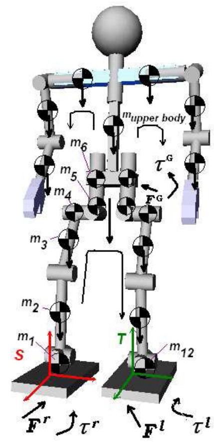

Fig. 4. Rh-1 mass distribution for inverse dynamics modelling.

A. Inverse dynamics

In order to compute the joint torques and dynamics constraints, a dynamic model is proposed [23].

With the frame attached to the COG of the link "***i***", we define the coordinates with respect to the "***S***" frame as (see Fig. 4):

$$T_1 = \begin{pmatrix} t_{1x} \\ t_{1y} \\ t_{1z} \end{pmatrix}, T_2 = \begin{pmatrix} t_{2x} \\ t_{2y} \\ t_{2z} \end{pmatrix}, \ldots, T_{12} = \begin{pmatrix} t_{12x} \\ t_{12y} \\ t_{12z} \end{pmatrix} \quad (1)$$

After that, the system for analysis is as follows:

$$\begin{aligned} g_{st_1}(0) &= \begin{bmatrix} Id & T_1 \\ 0 & 1 \end{bmatrix} \\ g_{st_2}(0) &= \begin{bmatrix} Id & T_2 \\ 0 & 1 \end{bmatrix} \\ &\vdots \\ g_{st_{12}}(0) &= \begin{bmatrix} Id & T_{12} \\ 0 & 1 \end{bmatrix} \end{aligned} \quad (2)$$

And the generalized inertia matrix for each link "***i***" being as follows:

$$M_i = \begin{bmatrix} m_i.Id & & 0 & \\ & I_{xi} & 0 & 0 \\ 0 & 0 & I_{yi} & 0 \\ & 0 & 0 & I_{zi} \end{bmatrix} \quad (3)$$

When the intentity *(3x3)* matrix is denoted by ***Id***. Next, the Jacobian body manipulator is expressed by:

$$J_1 = J^b_{st1}(0), J_2 = J^b_{st_2}(0), \ldots, J_{12} = J^b_{st12}(0) \quad (4)$$

The inertia matrix of the humanoid robot could be computed as:

$$Mm(\theta) = J_1^T.M_1.J_1 + J_2^T.M_2.J_2 + \ldots + J_{12}^T.M_{12}.J_{12} =$$

$$\begin{bmatrix} M_{11} & \dots & M_{112} \\ \vdots & \vdots & \vdots \\ M_{121} & \dots & M_{1212} \end{bmatrix} \tag{5}$$

With the last computations, it is possible to compute the inertial generalized torque vector (Γ'') as:

$$Mm(\theta).\ddot{\theta} = \Gamma'' \tag{6}$$

On the other hand, the potential effect could be computed by:

$$V(\theta) = m_1.g.h_1(\theta) + m_2.g.h_2(\theta) + \ldots + m_{12}.g.h_{12}(\theta) \tag{7}$$

Where, $h_i(\theta)$ is the vertical component of COG_i for any set of "θ" joint angle configuration.

$$g_{sti}(\theta) = e^{\xi_x^r \wedge \theta_x^r}.e^{\xi_y^r \wedge \theta_y^r}...e^{\xi_1 \wedge \theta_1}.e^{\xi_2 \wedge \theta_2}...g_{sti}(0) \tag{8}$$

And the potential generalized torque (Γ') for all the joints is expressed by:

$$N\left(\theta, \dot{\theta}\right) = \frac{\partial V}{\partial \theta} = \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ \vdots \\ N_{12} \end{pmatrix} = \Gamma' \tag{9}$$

Finally, the total joint torques are obtained with the sum of inertial generalized torque vector ("Γ'' ", eq. 6) plus the potential generalized torque ("Γ' ", eq. 9), such as the following:

$$\Gamma = \Gamma'' + \Gamma' \tag{10}$$

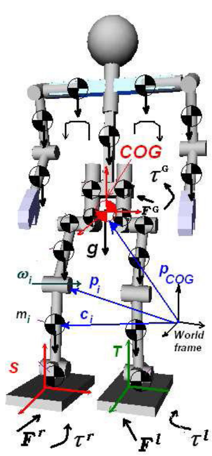

Fig. 5. Rh-1 mass distribution for forward dynamics modelling.

B. Forward dynamics

To solve the forward dynamics, the motion (translation and rotation like a "screw") of the COG is obtained from the following actions (see Fig. 5):

1) External forces
2) Inertial and gravitational effects
3) Internal interactions

So, at first the objective is to compute:

1) The link's spatial acceleration (a_i)
2) The link's angular acceleration (ε_i)
3) The joint's angular acceleration (α_i)

After that, by integrating the above accelerations (i.e. Euler, Runge Kutta methods) the spatial, joint and COG motion patterns are obtained.

Now the link motion is analized as following. We define in the COG the base frame free floating. The legs and arms are trees from the base frame. The spatial and angular velocity of each link ("***i***") can be computed as:

$$w_i = w_{COG} + R_{COG}.\omega_i.\dot{q}_i \tag{11}$$

$$v_i = v_{COG} + p_i \times R_{COG}.\omega_i.\dot{q}_i \tag{12}$$

Being R_{COG} the attitude matrix of the COG with respect to the world frame (3x3).

In order to build the generalized inertia matrix, its components can be computed as following:

At first, the component due to the rotational effect is summarized by "I_{ww_i}".

$$I_{ww\_i} = I_{ww}^a + m_i.\hat{c}_i^a.\hat{c}_i^{a\prime} \tag{13}$$

Where:

$$I_{ww}^a = R_i.I_i.R_i'$$

$$c_i^a = R_i.c_i + p_i$$

Being R_i the attitude matrix (3x3), and c_i the center of gravity of the link "***i***" (3x1); thus, c_i^a is the center of gravity of link "***i***" with respect to the world frame.

Next, the component due to the translational effect is summarized by "I_{vv_i}".

$$I_{vv\_i} = m_i.Id \tag{14}$$

Where "***Id*** is the identity matrix (3x3).

Finally, the component due to the translational and rotational effect "I_{wv_i}".

$$I_{wv\_i} = m_i.\hat{c_i^a} \tag{15}$$

In the other hand, as we have used a free floating model, with base in the COG, the reaction wrench from the floor and gravity effect must be taken into account. Those effects are modeled by the total wrench on each link as:

$$F_i = \begin{pmatrix} w_i \times \mathrm{P}_i \\ v_i \times \mathrm{P}_i + w_i \times L_i \end{pmatrix} - \begin{pmatrix} f_{g\_i} \\ \tau_{g\_i} \end{pmatrix} - \begin{pmatrix} f_{r\_i} \\ \tau_{r\_i} \end{pmatrix} \tag{16}$$

when "P_i" and "L_i" are the linear and angular momentum of the *i-th* link. The gravity and reaction wrenches are: $(f_{g\_i}, \tau_{g\_i})^T$, $(f_{r\_i}, \tau_{r\_i})^T$.

Thus, the linear and angular momentum of each link "***i*** can be computed as follows:

$$\mathrm{P}_i = m_i.\left(v_i + w_i \times c_i^a\right)$$
$$L_i = m_i.c_i^a \times v_i + I_{ww\_i}.w_i$$

The gravity effect:

$$f_{g\_i} = \begin{pmatrix} 0 \\ 0 \\ -m_i.g \end{pmatrix}$$
$$\tau_{g\_i} = c_i^a \times f_{g\_i}$$

The reaction floor effect, from a given stifness of floor "K_f" and damper "K_d" coefficients is computed as following:

$$f_{r\_i} = \begin{cases} \begin{pmatrix} -K_d.v_{x\_i} \\ -K_d.v_{y\_i} \\ -K_f.p_{z\_i} - K_d.v_{z\_i} \end{pmatrix} & \forall \quad p_z < 0 \\ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \forall \quad p_z > 0 \end{cases}$$
$$\tau_{r\_i} = p_i \times f_{r\_i}$$

An special attention should be taken into account, when the robot is walking, about the force reaction on its soles. The reaction floor effect on each foot during walking is a critical fact for controlling the humanoid robot, so this reaction is modelled with the previous equations when the *i-th* link corresponds to the left or right foot.

Now, the generalized inertial matrix of the robot is the following:

$$I_{tot\_i} = \begin{pmatrix} \sum_2^k \left(I_{vv\_i} - H_{v\_i}\right) & \sum_2^k \left(I_{wv\_i} - H_{wv\_i}\right)' \\ \sum_2^k \left(I_{wv\_i} - H_{wv\_i}\right) & \sum_2^k \left(I_{ww\_i} - H_{w\_i}\right) \end{pmatrix} \tag{17}$$

Being H_{v_i}, H_{wv_i} and H_{w_i} inertial matrices projected on angular and linear velocity directions, that is on "ω_i " and "$p_i \times \omega_i$ "

The generalized acceleration vector (a_{tot_i} of dimension (6x1)) is obtained by:

$$a_{tot\_i} = \; - I_{tot\_i} \backslash F_i \tag{18}$$

Finally, the joint (α_i) and link (a_i, ε_i) accelerations are:

$$\alpha_i = \frac{\Gamma_i - H'_{v\_i}.a_{COG} - H'_{w\_i}.\varepsilon_{COG}}{Hd_i} \tag{19}$$

When:

$$Hd_i = f(H'_{v\_i}, H'_{w\_i})$$

$$a_i = \begin{cases} \begin{pmatrix} a_{tot\_i}^{11} \\ a_{tot\_i}^{21} \\ a_{tot\_i}^{31} \end{pmatrix} & \forall \quad i = 1 \\ a_{COG} + \bar{v}_i + \tilde{v}_i.\alpha_i & \forall \quad i \neq 1 \end{cases} \tag{20}$$

And:

$$\bar{v}_i = \frac{d}{dt}\left(\tilde{v}_i\right).\dot{q}_i$$
$$\tilde{v}_i = R_{COG}.\omega_i$$

$$\varepsilon_i = \begin{cases} \begin{pmatrix} a_{tot\_i}^{41} \\ a_{tot\_i}^{51} \\ a_{tot\_i}^{61} \end{pmatrix} & \forall \quad i = 1 \\ \varepsilon_{COG} + \bar{w}_i + \tilde{w}_i.\alpha_i & \forall \quad i \neq 1 \end{cases} \tag{21}$$

Being:

$$\bar{w}_i = \frac{d}{dt}\left(\tilde{w}_i\right).\dot{q}_i$$
$$\tilde{w}_i = p_i \times R_{COG}.\omega_i$$

V. SIMULATION RESULTS

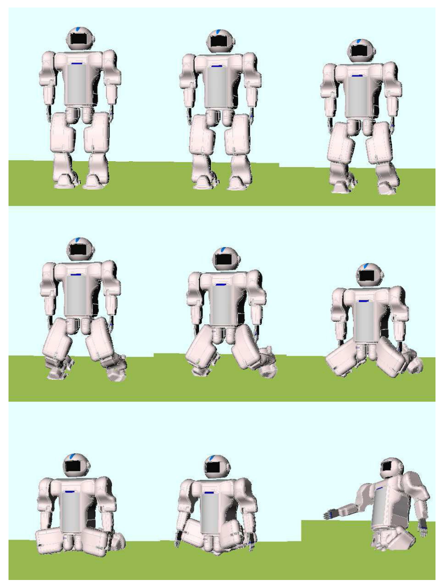

Fig. 6. Rh-1 dynamics simulation, without references, under the gravity field.

The whole body dynamics simulation allows us to design the robot structure, and to set up the control parameters, [23]. Because, the real robot performance is simulated by taking into account the mass, inertial properties, and external wrenches, (that is, the gravity field, the force and torque reactions.) We obtained link geometry and inertial parameters, from the CAD data. As an example shown in Fig. 6, the Rh-1 humanoid robot, without any joint reference (i.e. torques, joint patterns), falls down under the gravity field. As shown in the last snapshot, the contact points taken into account are between robot and floor, and not between robot covers. Thus when the robot falls down, each cover could overlap other one. In Fig. 8, the some tests of dynamic walking are shown. In this case, the control parameters have been tuned for the next control levels: local joint control and the whole body control, because the dynamic model give us humanoid realistic behaviour such as, actual robot attitude, Zero Moment Point (ZMP), surface wrench reaction; and the control loop is tuned to compensate their deviations from the reference patterns. So, stable walking motion is obtained and it is tested previously in the dynamic simulator. This simulation is achieved by integrating the accelerations from the equations (20), (21), (19). Forward dynamics and integration by the Euler method takes around 16 seconds, and with the Runge-Kutta takes around 40 sec, running Matlab in PC: AMD Athlon(tm) 64 Processor 3200+, 2.02 GHz, 1.00 GB RAM. In the table II is summarized the computation time of forward dynamics, that is for Rh-1 humanoid robot (21 DOFs) and PUMA robot (6 DOFs), in order to compare the effectiveness of our approach.

TABLE II

TIME COST COMPUTING FORWARD DYNAMICS

Method / N^o DOF	Time (sec)
Screws+Euler / 21 DOF	16
Screws+Runge-Kutta / 21 DOF	40
D-H+Runge-Kutta / 6 DOF	30

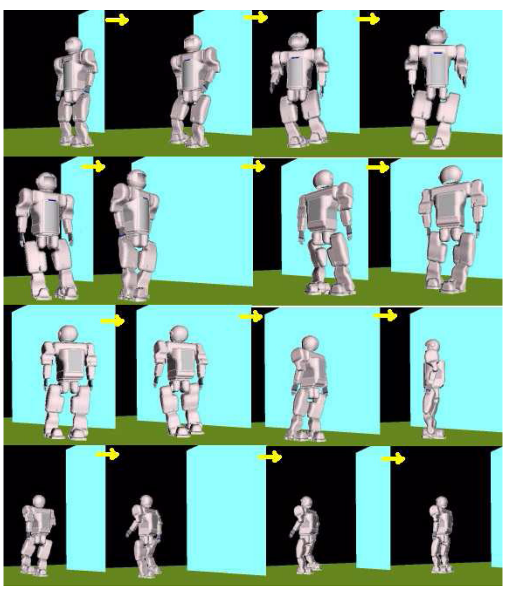

Fig. 7. Humanoid robot walking and turning.

As shown in Fig. 7, walking and turning motion is developed. Smooth patterns are successfully tracked, because the accurate dynamic model approaches to the robot performance, so the control parameters could be tunned and them compensate the wholebody dynamics effects, during walking motion.

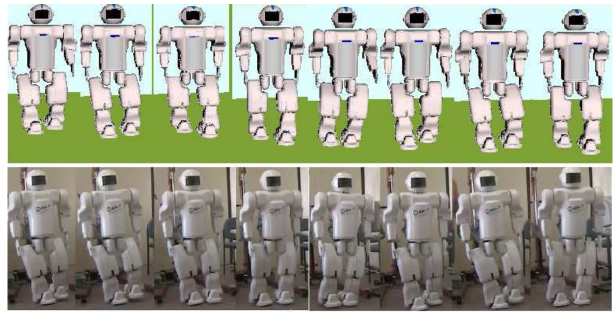

Fig. 8. Rh-1 walking forward simulation and real tests.

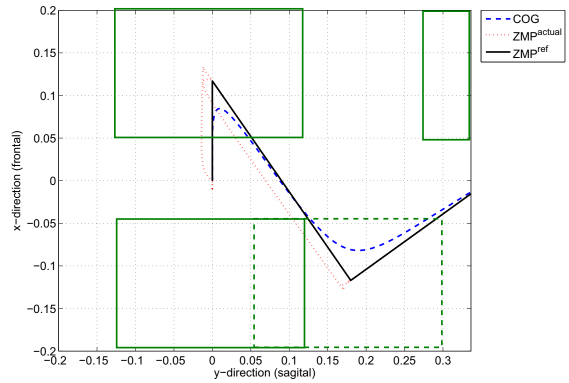

Fig. 9. Horizontal view of walking patterns. The ZMP reference (black line), actual ZMP (red dotted line) and the COG (blue dashed line) are shown. In the single support phase, the dynamic walking pattern allows us to fix the ZMP around the middle of the support foot, while the COG is near the foot boundary.

VI. EXPERIMENTAL RESULTS

A dynamic walking pattern is tested and validated, because the ZMP is inside of the support polygon while robot is walking (Fig. 8, 9.) In this feature, the joint patterns obtained remain the predicted stability, with the action of the whole-body control loop. The control loop compensates additional terrain imperfections and structural robot elasticity.

The reference walking patterns are computed from a preview controller of ZMP, which is proposed by Kajita et al. [10]. It is robust and it performs well in our robot too. The proposed dynamic model allow us to predict the reference joint patterns, in order to avoid humanoid falling down. At this stance our approach is successfully validated in the Rh-1 humanoid robot platform. The theorical proof is shown in the Fig. 9, where the actual ZMP (red dotted line) tracks the reference ZMP (black bold line) in an acceptable stability range. That is, the actual ZMP remains in the support polygon during the single support phase.

In Fig. 10, the simulation and actual measuring of surface reaction forces, are shown, for both right and left feet while the humanoid is walking. Except by the small differences between simulation and actual measures, due to the sen-

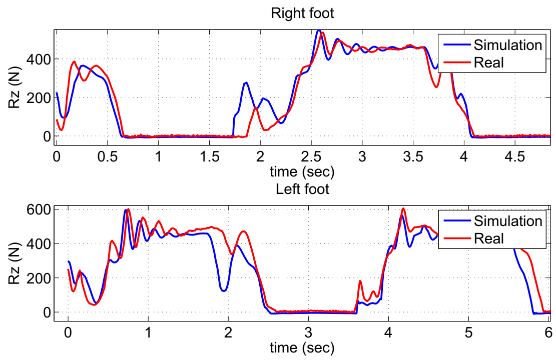

Fig. 10. Surface reaction forces while walking. Right foot reaction and Left foot reaction obtained from dynamic model (blue line), actual measuring (red line). Similariy on plots validate the proposed algorithms.

sors noise, and non-linearities; our proposed approach is validated, towards to obtain stable dynamic walking taking into account mulibody dynamics. The approach is applied at simulation level, that is off-line only for tunning the control parameters. Also, this approach could be used in more complex robots, with more degrees of freedom, or human models.

VII. CONCLUSIONS

We propose and validate the dynamic modelling of humanoid robots using the screw theory and Lagrange formulation, which have the following advantages:

- To compute the Jacobian is not necessary to derivate, only a set of matrix transformations is needed (including the twists).
- The Lagrange method allows us to solve the equations in a closed form and it is possible to make detailed analyses of the system's properties.

The forward dynamics have been solved by screws too. The propagation method, (from contact point to COG), allows us to preview the dynamic performance of the robot with free floating base.

The force reaction was modelled and validated sucessfully, by both simulation and experimental tests. That is, suitable walking paterns and control parameters could be tuned with the proposed multibody dynamic model.

Current works are focused on increasing the contact points to improve the multibody dynamics, in the other hand, optimal control of mechanical systems will be included in our improvents.

VIII. ACKNOWLEDGMENTS

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