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Development of a Low Backlash Crown Reducer

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***Abstract*— In this paper, we describe a novel low backlash reducer using precessing crown gears. we call this mechanism Crown Reducer. The features of this reducer are low backlash, high reduction ratio, advantage in miniaturizing. These features greatly contribute to design various small mechanisms such as a finger joint of a robot hand. The crown gear pair is meshing in face to face, one acts as a rotor and the other as a stator. The rotor gear has N teeth and the hub is connected to the output shaft. The spokes of rotor gear are elastically deformed according to gears rolling. The stator gear fixed on a housing has $N-1$ teeth (or $N+1$). The meshing of crown gear pair with several shapes of rack tooth are simulated, and proper parameters of rack to improve high rigidity of teeth are calculated. Finally, backlash and minimum input torque to rotate are evaluated using a prototype reducer.**

I. INTRODUCTION

The research team formed by the authors is developing robot hands capable of replicating human actions [1]. In general, there are four requirements for the construction of a finger joint in a robot hand: high torque for stable grabbing, miniaturization for embedding in a finger joint, light weight for safety, and minimum backlash for accurate control. The recent trend is to embed all necessary components, including an actuator, in a robot hand to increase its versatility as an end-effector [2][3]. DC motors are used as these actuators. On the other hand, lightweight ultrasonic motors are also employed for finger joints of robot hands because they do not have backlash while generating a large torque [4]. However, they do present some problems, such as short service life and complex driving circuit. For these reasons, adopting a reducer with low backlash for a DC motor is considered a reasonable solution to develop an actuator for finger joints. In addition, small-diameter motors used in robot hands generally tend to operate at a high revolving speed, thus necessitating a high reduction ratio. Several reducers have been developed in the past to realize low backlash and high reduction ratio [5][6]. Among these, wave-motion gears such as a harmonic drive are commonly used. However, unlike other transmission mechanisms, these gears require the motor to have a larger starting torque. In addition, the nested structural combination of a wave generator, flexspline, and circular spline makes it difficult to miniaturize the mechanism. This paper proposes the use of a crown gear reducer developed by the authors

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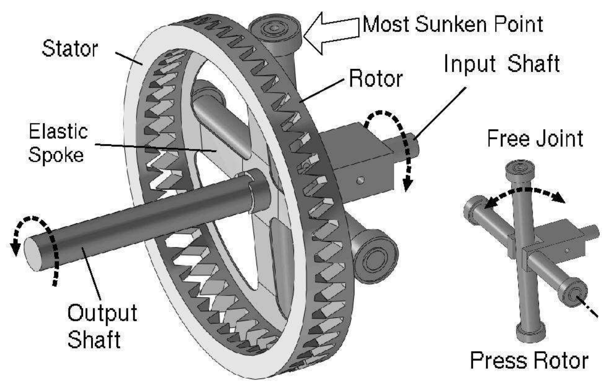

Fig. 1. Overview of the proposed reducer

as a new solution to these problems. This reducer achieves a high reduction ratio while having low backlash. Its simple mechanism is also advantageous in miniaturization. There are certain conditions for reducing backlash in the design of this reducer. Those conditions are also discussed in this paper. Finally, a report on a prototype reducer is presented.

II. PRINCIPLE AND PROPERTIES OF THE CROWN REDUCER

An overview of the reducer is presented in Fig. 1. The mechanism consists of two crown gears, one with one tooth less than the other, facing each other and engaged. The rotor, which has N teeth, is connected to the output shaft through elastic spokes. The spokes need to be rigid against twisting force but flexible in along the shaft length. The stator, which has N_s teeth, is fixed to the housing. Here, the difference between N_s and N is one. The input shaft of the press rotor is connected to the rotor shaft of the motor. In addition, the press rotor has a free joint in order to adjust the tilt of the pushing point automatically. This makes it possible to press the entire rotor against the stator with an adequate force.

When the rotor is pressed against the stator using this mechanism, the gears engage with each other after tilting a bit. By rotating the press rotor around the input shaft, the engaging point also moves round and causes the output shaft to rotate. Assuming the rotational speed of the output shaft to be $n_{\rm out}$ and that of the input shaft $n_{\rm in}$, the following equation hold true:

$$\frac{n_{\rm out}}{n_{\rm in}} = 1 - \frac{N_s}{N} \tag{1}$$

Since N_s is either $N-1$ or $N+1$, the reduction ratio is either N or $-N$. Hence, the number of teeth of the rotor

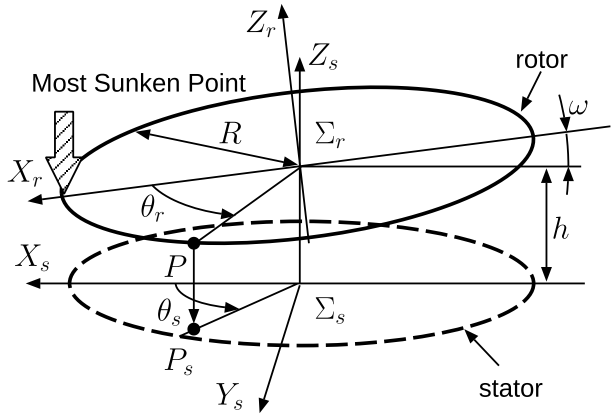

Fig. 2. Model of the rotor and stator alignment

is equal to the reduction ratio, and this makes it possible to achieve twice the reduction ratio for the same number of teeth compared to wave-motion gears, thus enabling further miniaturization. In addition, the gear that has should have more teeth (rotor or stator) determines the rotating direction. Although a ball bearing is inserted between the rotor and the press rotor because they rub against each other, it does not eliminate friction completely. Therefore, if the output shaft rotates in the same direction as the input shaft, the total distance of rubbing motion between the rear surface of the rotor and the ball bearing will be smaller per rotation, which clearly translates into higher efficiency. If the gear must be made extremely small to prohibit the use of a ball bearing, this mechanism will be more advantageous because the frictional force cannot be overlooked. In contrast, with a common wave-motion gear, the output shaft can only rotate in the opposite direction of the input shaft because the rotor is placed inside the stator.

Similar to the proposed reducer is the disk-shaped wave-motion reducer [7]. However, this mechanism requires the entire rotor including the teeth to be elastic, as is the case with any cylinder-type wave-motion reducer. On the other hand, the proposed reducer does not require the teeth to be elastic, although the spokes of the rotor must be elastic. Therefore, the material for the toothed gear can be selected freely depending on the required strength or wear resistance.

III. Model of Teeth Contact

This section discusses the engagement of the rotor and the stator. The geometrical representation of the proposed mechanism is provided in Fig. 2. Here, the circumferences of the rotor and the stator are defined as reference circles. The solid circle in Fig. 2 is the reference circle of the rotor and the dotted circle is that of the stator. Both have the same radius R. Here, Σ_s is the coordinate system relevant to the stator and Σ_s is the one relevant to the rotor. The rotor is pressed against the stator with a downward force along the Z_s axis, and the rotor and stator are most close at the point shown by the hatched arrow in Fig. 2. This point is where the $X_s - Z_s$ plane and the reference circle of the rotor intersect. Assume that some teeth are in contact at an angle ω, which is the angle between the rotor and the stator, and that the distance between the centers of rotor and stator reference circles is h along the Z_s axis. The point P on the rotor is at an angle θ_r from the X_r axis. The point P_s is a vertical projection of P on the $X_s - Y_s$ plane, and we assume this point P_s to be at an angle θ_s from the X_s axis. Incidentally, note that this point P_s is a little away from the stator reference circle and towards the center. The coordinates of P relative to Σ_s are expressed as follows:

$$P = R_y(\omega)R_z(\theta_r)\begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix} \quad (2)$$

Here,

$$R_y(\omega) = \begin{pmatrix} \cos\omega & 0 & \sin\omega \\ 0 & 1 & 0 \\ -\sin\omega & 0 & \cos\omega \end{pmatrix} and$$
$$R_z(\theta_r) = \begin{pmatrix} \cos\theta_r & -\sin\theta_r & 0 \\ \sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R_y(\omega)$ and $R_z(\theta_r)$ are rotational vectors for transferring the point P from Σ_r to Σ_s.

Although a variety of tooth forms can be considered, we consider rack teeth in this paper for simplicity. The models of teeth contact for $N_s = N + 1$ and $N_s = N - 1$ are shown in Fig. 3 and Fig. 4, respectively. These figures represent the appearances of teeth contact viewed from the outside of reference circles in the direction of the Σ_s origin. Although each tooth would, in reality, have a curved surface similar to a cylinder, it is expanded into a plane. At the same time, both figures assume that the Z_s and Z_r axes overlap each other and angle ω is very small. This model is reasonably accurate if ω is sufficiently small and N is sufficiently large.

Assume the tooth height to be h_t and the tooth pressure angle to be α. If the teeth are set up without an interval between them, the pitch is $2\pi R/N$ and the maximum tooth height is

$$h_{t\max} = \frac{\pi R}{N \tan\alpha} \quad (3)$$

Here, we introduce a constant h_r and define the tooth height as

$$h_t = h_r h_{t\max} \quad (4)$$

When the tooth forms of the rotor and the stator are the same, the teeth would not engage if $h_r < 0.5$, where the width of the tooth tip is greater than the width of the groove. Since $h_t \leq h_{t\max}$, the value of h_r is in the range of 0.5 to 1.0. We assume the same tooth form is used for the stator. However, the pitch of teeth on the stator is either $2\pi R/(N + 1)$ or $2\pi R/(N - 1)$. Using (2), the height of point P with respect to Σ_s, P_z, can be expressed as follows:

$$P_z = h - R\sin\omega\cos\theta_r \quad (5)$$

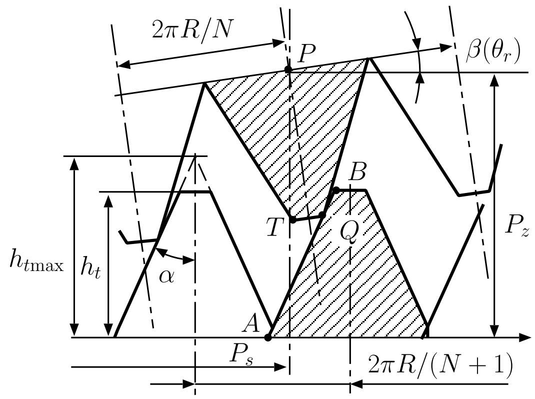

Fig. 3. Model of the teeth contact ($N_s = N + 1$)

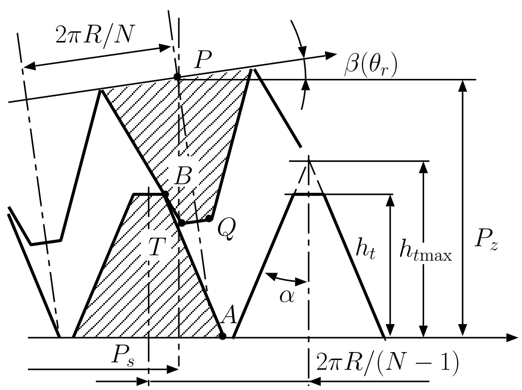

Fig. 4. Model of the teeth contact ($N_s = N - 1$)

The position of P_s with respect to Σ_s is

$$P_s = R\theta_s \tag{6}$$

θ_s can also be obtained by using (2) as follows:

$$\theta_s = \arctan 2(\sin\theta_r, \cos\omega\cos\theta_r) \tag{7}$$

Here, note that $\arctan 2$ in (7) is a function obtained by expanding the domain of $\tan^{-1}$ to $[-\pi, \pi]$. The shaded teeth in Fig. 3 and Fig. 4 are in contact. In addition, Q and T on Fig. 3 indicate the tip of the tooth on the rotor, and its position can be obtained from P and angle β. Here, β can be obtained as follows:

$$\sin\beta = \frac{d}{d\theta_r}\left(\frac{P_z}{R}\right) = \sin\omega\sin\theta_r \tag{8}$$

In addition, the base and tip of stator teeth A and B can be obtained from the pitch and the number of teeth. We examine the conditions of contact using these points A, B, Q, and T. Note that in Fig. 3 ($N_s = N+1$), the edge of the rotor tooth is in contact with the slope of the stator tooth. On the other hand, in Fig. 4 ($N_s = N-1$), this relationship is reversed and the edge of the stator tooth is in contact with the slope of the rotor tooth. If the relationship between the stator and rotor is reversed in Fig. 4, the teeth contact will be almost identical to the one shown in Fig. 3 but with one less number of teeth. For this reason, this paper only elaborates on the model shown in Fig. 3.

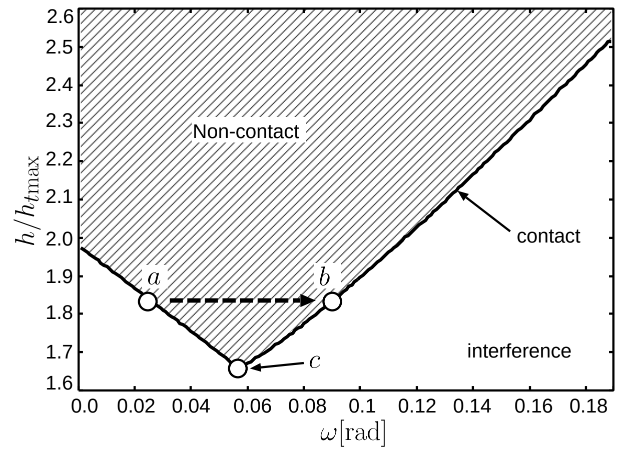

Fig. 5. Contact condition between rotor and stator in case of $h_r = 1.0$

IV. Calculation of Teeth Contact

In the model described in the previous section, the variables that determine the relative positions of the rotor and stator are h and ω. We examine the conditions of contact by assigning different values to these variables. The parameter that determines the form of a rack tooth is h_r. Contact conditions are examined for various values of h_r.

A. Conditions of Contact (the minimum value of h)

It is known from experiments using triangular teeth ($h_r = 1.0$) that ω and h are uniquely determined by pushing the press rotor towards the stator [8]. This section discusses this phenomenon using computational results.

Let $N = 49$, $N_s = 50$, and $h_r = 1.0$ and assign different values for h and ω as a case study for searching the contact point. The search was performed by incrementing ω by 0.0002 rad in the range $[0.01, 0.19]$ and h by 0.00001 mm in the range $[0, 2.5h_{t\max}]$. A graph (Fig. 5) is obtained by plotting the distribution of contact and/or non-contact with ω on the horizontal axis and $h/h_{t\max}$, which is a normalization of h with constant $h_{t\max}$, on the vertical axis. The solid line shows that there is both contact and no contact in the shaded area. The area below the solid line is not practical due to interference between teeth. Points a, b, and c in Fig. 5 are on the solid line, meaning some tooth on the rotor and another tooth on the stator are in contact. Now, let us assume that the rotor and the stator are at point a (h_a, ω_a). If h is fixed at h_a, ω cannot be less than ω_a because of teeth interference. However, ω can take any value in the range of $\omega_a < \omega < \omega_b$, where none of the teeth are in contact. In other words, there will be a backlash. Here, the variations of ω result from the inclination of the press rotor and input axis. In contrast, if h is fixed at point c (h_c, ω_c), which is the lowest point on the solid line, then the value of ω is uniquely determined and

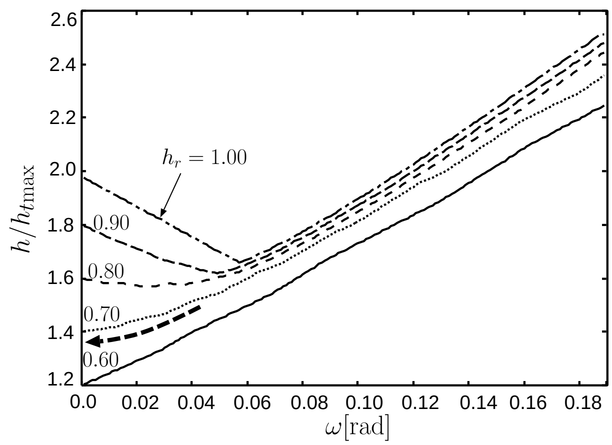

Fig. 6. Distance between rotor and stator for teeth contact in various tooth heights

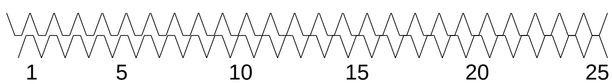

Fig. 7. An example of contact condition without meshing (h_r=0.7, $\omega = 0$)

backlashes due to slack between teeth cannot happen. Based on these conditions, one of the conditions for achieving low backlash in the case of $h_r = 1.0$ is to let the teeth engage at a point in the ω axis where h takes the minimum value.

The $\omega - h$ distribution when rack teeth (h_r = 0.6, 0.7, 0.8, and 0.9) are included, is shown in Fig. 6. The case of $h_r = 0.5$ is excluded because the region in which the teeth would engage is almost non-existent. For the h_r values of 0.6 and 0.7, there is no point where h takes the minimum value. Hence, for $h_r = 0.7$, for example, the rotor would be pushed against the stator to the point of $\omega = 0$, as shown by the broken arrowed line, where the rotor and the stator become parallel to each other. In this case, however, the teeth are lined up as shown in Fig. 7 and only the tips are in contact without engaging. To summarize these conditions, there must be a point where h takes the minimum value in the $\omega - h$ distribution for a given tooth form and teeth must be designed to get in contact with the minimum values of h and ω at that particular point.

B. Evaluation of h_r and Contact Condition

The teeth engagement at the minimum value of h (point c in Fig. 5) for $h_r = 1.0$ is shown in Fig. 8 for the range $0 < \theta_s < \pi$. According to this figure, the 12th tooth is in contact. Since the mechanism is symmetric in the $X_s - Z_s$ plane, there is another contact on the other side. Hence, the rotor and the stator engage each other symmetrically on the $X_s - Z_s$ plane at two locations as if one is clamped by the other. This prevents the occurrence of backlashes.

The distance between teeth on the rotor and stator (hereafter referred to as teeth distance) is shown in Fig. 9. The teeth distances right before and after the engaged teeth are very small and less than 0.01 mm. If there were many teeth

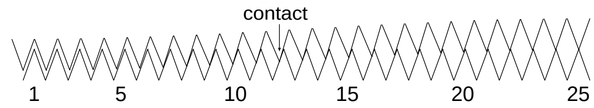

Fig. 8. A meshing condition of triangular teeth (h_r=1.0)

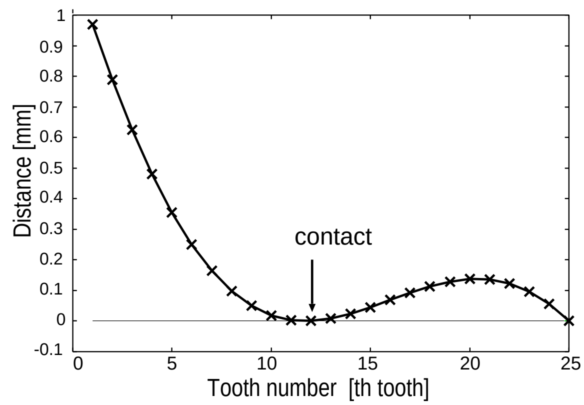

Fig. 9. Distance of the teeth (h_r=1.0)

with such a small teeth distance, a little elastic deformation would cause many teeth to be involved in the engagement and allow the transfer of a large torque. In addition, although a little turn of the press rotor would not make the condition shift from the one shown in Fig. 9, the transition of engagement condition would be smooth because adjacent teeth are very close to the state of contact to begin with.

For this reducer to work, since the spokes need to be deformed at all times to maintain the tilt angle ω. the energy is wasted. Therefore, it is desirable to (i) minimize the angle ω and (ii) increase the number of teeth (N_c) with a teeth distance of less than 0.01 mm for smooth operation. In order to evaluate these (i) and (ii) as distinct factors, the values of ω and N_c, as well as the minimum value of h (h_m) for reference, were calculated by changing the value of h_r from 0.8 to 1.00. These values are shown in Table I. According to the table, there is a tendency that ω is likely to decrease when h_r decreases. Hence, among the minimum h_r values, smaller values of h_r are preferable if the evaluation factor (i) is emphasized. For example, Fig. 10 and Fig. 11 show the

TABLE I
INCLINATION ANGLE OF ROTOR AND THE NUMBER OF SIMULTANEOUS MESHING TEETH ON NO BACKLASH CONTACT

h_r	ω [rad]	ω[deg]	N_c	h_m [mm]
0.80	0.024008	1.375557	2	13.52824
0.82	0.037241	2.133752	2	13.71735
0.84	0.037613	2.155066	2	13.83032
0.86	0.048844	2.798555	4	13.92505
0.88	0.048865	2.799758	5	13.95074
0.90	0.049117	2.814197	6	13.97819
0.92	0.049516	2.837058	7	14.00969
0.94	0.051166	2.931596	5	14.06545
0.96	0.052181	2.989751	4	14.11720
0.98	0.054943	3.148002	4	14.22147
1.00	0.056997	3.265688	4	14.31650

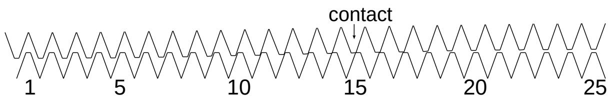

Fig. 10. A meshing condition of rack teeth (h_r=0.8)

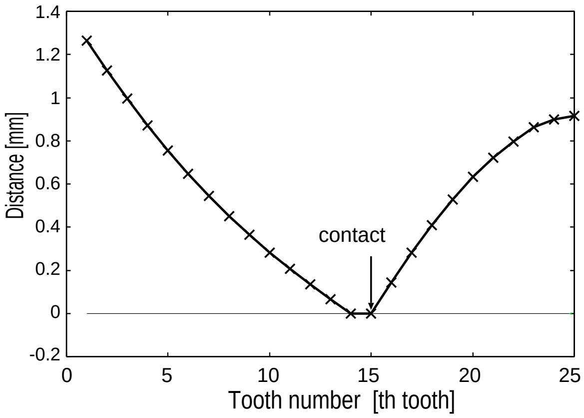

Fig. 11. Distance of the teeth (h_r=0.80)

engagement condition and teeth distance, respectively, for the h_r value of 0.80. In comparison with triangular teeth, the value of ω decreases to less than half its original value. However, N_c is small and the engagement is shallow as depicted in Fig. 10. Furthermore, the valley at the minimum value is shallow as shown in Fig. 6, and this translates to unstable ω. On the other hand, if the evaluation factor (ii) is prioritized, the h_r value of 0.92 yields the maximum value for N_c. The engagement condition and teeth distance are shown in Fig. 12 and Fig. 13. At $h_r = 0.92$, N_c takes the maximum value in Table I. Since the valley at the minimum value for $h_r = 0.90$, which is very close to the value of 0.92, is deep as shown in Fig. 6, it is reasonable to assume that the engagement is stable at $h_r = 0.92$. Due to these reasons, an h_r value of 0.92 is considered optimum.

V. Prototype and Evaluation

The prototype reducer is shown in Fig. 14. The gear diameter ϕ is 100, the number of teeth N is 49 and N_s is 50, and the form h_r is 0.92. The gears are made out of polyacetal. Ball bearings are set at the tips of four press rotor rods to reduce friction with the rotor.

A. Backlash and Starting Torque

As the press rotor of reducer is pushed against the stator (h becomes smaller), the starting torque and the state of backlash changes. This relationship is shown in Fig. 15. Here, the measurement method used to determine backlash on the prototype reducer is as follows. (i) With input axis fixed, apply a torque that is sufficiently larger than the

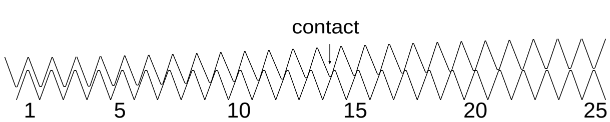

Fig. 12. A meshing condition of rack teeth (h_r=0.92)

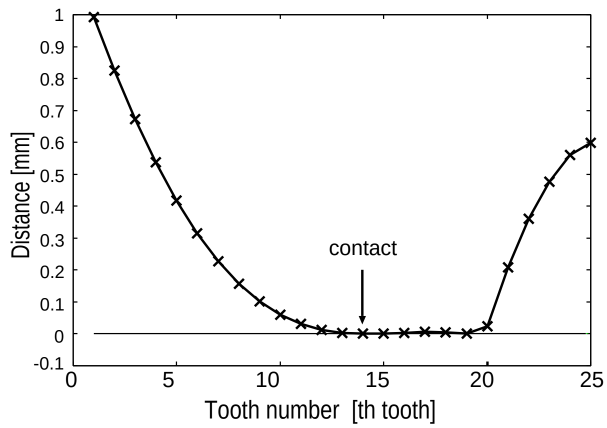

Fig. 13. Distance of the teeth (h_r=0.92)

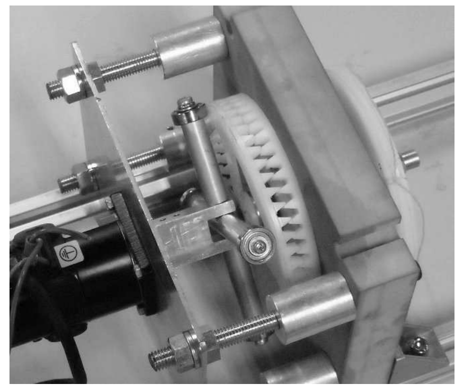

Fig. 14. A prototype reducer

inherent friction in the mechanism to the output axis. (ii) Measure the angle of output axis e_1 after removing the torque. (iii) Apply the same torque in the opposite direction. (iv) Measure the angle of output axis e_2 after removing the torque. (v) Define the difference between e_1 and e_2 as the backlash shown in Fig. 15. A rotary encoder with a resolution of 960000 per rotation is used to measure the angle. In the case of Fig. 15, it was not possible to reduce the backlash even if the rotor is pressed hard to the extent that a starting torque of more than 16 mNm would be necessary. Hence, the minimum backlash for this mechanism is 0.002 deg. On the other hand, if a backlash of 0.01 deg is allowed, a starting torque of approximately 4 mNm is sufficient to operate the gear. As illustrated in this example, there is a trade-off relationship between the starting torque and backlash. In other words, Fig. 15 can be utilized to select appropriate starting torque and backlash for a particular application in assembling a reducer. In contrast, the starting torque of the harmonic drive CSF-32-50 is 310 mNm, which is similar to the prototype reducer in size of the gear diameter and reduction ratio. Although the conditions of these reducers are not exactly same, it seems that the prototype reducer achieves a extremely small starting torque.

B. Transmission Error

Define the input angle of the reducer as θ_i and the output angle as θ_o. Then the transmission error e of this reducer is

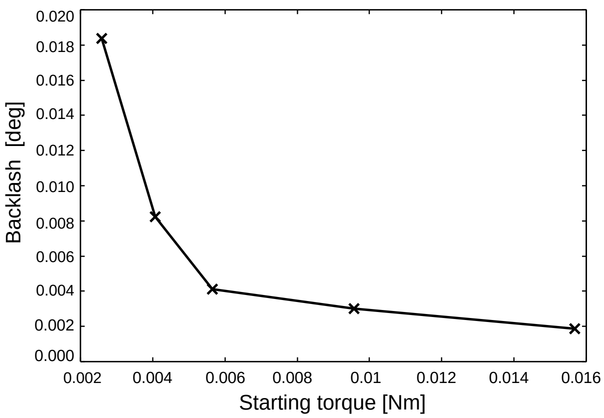

Fig. 15. Backlash against the starting torque

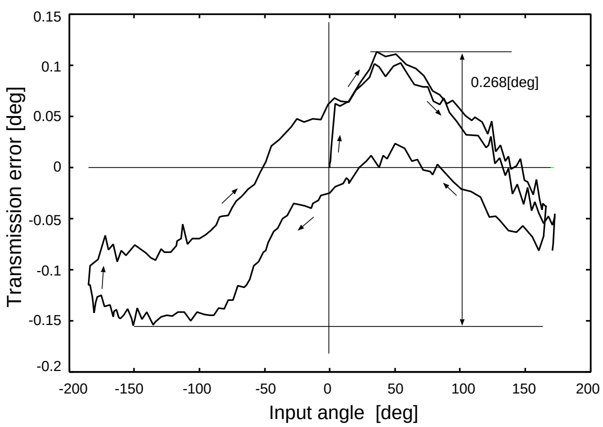

Fig. 16. Transmission error against input angle of the reducer

represented by the following equation:

$$e = \theta_o - \frac{\theta_i}{N} \tag{9}$$

The transmission errors for different input angles are shown in Fig. 16. The test was performed by changing the input angle from 0 deg to ± 180 deg in the order indicated by the arrow in Fig. 16. Judging from these results, the prototype reducer has a maximum error of 0.268 deg. This error is caused by the hysteresis loop present in the inclination of the press rotor and the rotor. It is likely that this hysteresis loop will be minimized by increasing the rigidity of directions of the press rotor thrusting and decreasing the rigidity of directions of the rotor being pressed.

VI. CONCLUSIONS AND FUTURE WORKS

This report proposes a new reducer mechanism based on a unique principle for the purposes of achieving low backlash, smaller starting torque, and miniaturization for the use in robot joints. At the same time, a rack tooth model of the proposed reducer is presented and its engagement conditions are examined by computation. According to the results, in order to make a low backlash reducer using this mechanism, the parameter h must have a minimum value and the press rotor needs to be fixed at this value. In addition, the paper examines the engagement conditions of various tooth forms by defining a parameter h_r to determine the form of rack tooth. In particular, the parameter h_r is evaluated in conjunction with the tilt angle ω, which is a factor in reducing the energy loss, and the number of teeth that are likely to be in contact simultaneously, N_c, which determines the smoothness of operation and strength. As a result, the h_r value of 0.92 is considered advantageous as a whole. A prototype gear mechanism of $h_r = 0.92$ was built and the relationship between the starting torque and backlash was presented. According to this, backlash at a practical starting torque is confirmed to be sufficiently small.

In the early days of prototyping, it was obvious that the accuracy of processing and assembling of gears was problematic. However, such a problem did not hinder the reducer to operate with a low backlash. From this fact, it is presumed that this reducer has an advantage over other mechanisms when the precision of processing becomes relatively low as a result of miniaturization. Miniaturization of this reducer will open the door to a variety of applications including medical forceps and joint mechanisms of miniature robots. Incidentally, the gears contact each other at the edge of a tooth. Therefore, future study will focus on the tooth shape with different surfaces in order to increase the relative curvature as much as possible. Also, miniaturization up to the outer diameter ϕ of 12 including the housing (ϕ = 10, N = 50, and N_s = 49, material = NAK) has been achieved to the date. Ultimately, miniaturization up to ϕ = 3 is set as a target.

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