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Gait Planning for a Biped Robot by a Nonholonomic System with Difference Equation Constraints

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Abstract— **New gait planning using a nonholonomic model with difference equation constraints is proposed for biped robot walking. A model of a pivoting telescopic segment is used as the kinematic foothold selection model of a bipedal robot. The repetitive and discontinuous constraints of pivoting, expanding, and contracting make up the set of walking trajectory data. The k-step reachable region is defined as the set of the k-th state that the system can reach from the initial state, and the motion planning is solved using the Jacobian matrix of the state with regard to the input series. The difference equation constraints can be discussed as a digital control of continuous-time nonholonomic systems. The gait planning is modified based on the limiting condition for the HRP-2 humanoid robot. Energy consumption is evaluated based on the linear-pendulum model and the gait planning is optimized. The feasibility of the proposed walking planning is demonstrated through a numerical simulation and an experiment involving the HRP-2 humanoid robot.**

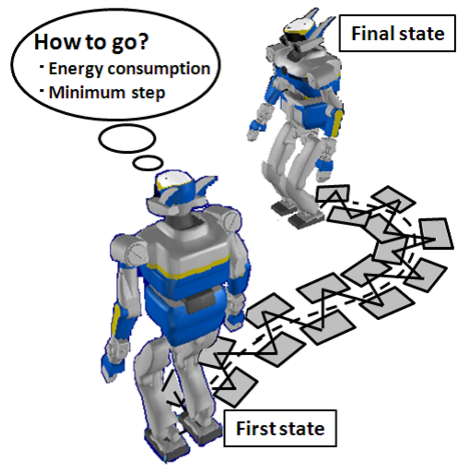

Fig. 1. Walking trajectory planning

I. INTRODUCTION

Humanoid robots are expected to be applied as human friendly robots because humanoid robots have a human-like body and can achieve human-like motion. However, a number of problems remain to be solved before the successful implementation of humanoid robots. Walking pattern generation is a special problem in humanoid robots. Humanoid robots require a walking trajectory and landing positions in order that the feet are placed in target positions. Omnidirectional wheeled robots are able to directly use the minimum trajectory to the target [1], but other robots must plan a characteristic motion according to their own mechanism. For example, nonholonomic wheeled mobile robots must control steering to track the planned trajectory by considering the nonholonomic constraint of the robot [2]. Legged robots also have to plan landing positions for the feet as shown in Fig. 1, by considering the work space of the legs, the stability of the robot, and the efficiency of the walking pattern [3][4][5]. The footstep pattern is designed as a batch process to follow a trajectory or a walking direction is given for a certain short period as omni-directional wheeled robot. In these cases, the path planning algorithm is implemented in order to create an ideal trajectory for the robot to track. On the other hand, model-based motion planning methods using non-linear optimum feedback control have been proposed for a two-link arm [6] and a vehicle control [7]. Arai proposed the concept of discrete-time nonholonomic systems for planning motion that has nonholonomic constraints [8]. This concept is implemented to create motions for a two-wheeled mobile robot and pivoting manipulation of a polyhedral object as simple examples. The above methods are able to derive the motion directly according to the robot model. Since the motion of pivoting is similar to the bipedal foot stamp trajectory, we apply the method to a walking pattern generator. If a good model can be obtained considering the actual use of humanoids based on a nonholonomic system, this model can directly specify the optimal foot placement for reaching the target position without using a path planning algorithm.

In the present paper, for the practical nonholonomic model of bipedal locomotion, we propose a model of a pivoting telescopic segment, as shown Fig. 2. The repetitive and discontinuous constraints of the pivoting of this model, expansion, and contraction make the set of walking trajectory. The difference equation constraints can be defined as the set of k-th state manipulations of the model, and the motion planning is solved using the Jacobian matrix of the state. The gait planning is modified based on the hardware limitation of the real humanoid robot HRP-2. We propose the optimization of an algorithm considering the linear-pendulum model for bipedal walking to reduce energy consumption. The proposed model and algorithm are implemented in the HRP-2 humanoid robot, and the feasibility of the proposed model and algorithm are demonstrated.

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II. GAIT PLANNING BY A NONHOLONOMIC SYSTEM WITH DIFFERENCE EQUATION CONSTRAINTS

In this section, the basis of gait planning for a bipedal robot using a nonholonomic system with difference equation

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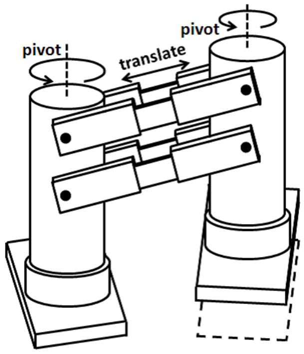


Fig. 2. Biped robot model with pivoting and telescopic motion

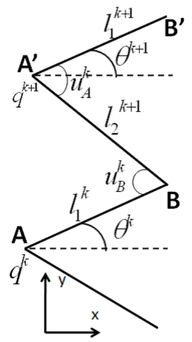


Fig. 3. Walking model of alternate pivoting

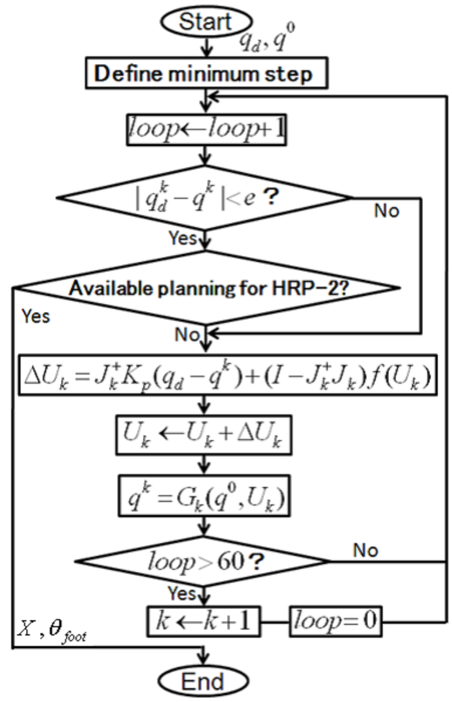


Fig. 4. Gait planning algorithm

constraints is described.

A. Bipedal robot model for kinematic foothold definition

A new bipedal model for foothold planning is shown in Fig. 2. Figure 3 shows the kinematic planning model of Fig. 2 on a planar surface. In this section, we do not consider the direction of foot placement. First, the line segment l_1^k is rotated u_B^k[rad] around B, and the length is changed to l_2^{k+1}. Next, the line segment l_2^{k+1} is rotated u_A^k[rad] around A', and the length is changed to l_1^{k+1}. The series of the manipulation of the segment is defined as one Step-of-Walking (SoW). The transition of A and B indicate the footholds of left and right legs, respectively. The equations of state are as follows:

$$\begin{aligned} x^{k+1} &= x^k + l_1^k \cos\theta^k - (l_2^k + u_{l_2}^k)\cos(u_B^k - \theta^k) \\ y^{k+1} &= y^k + l_1^k \sin\theta^k + (l_2^k + u_{l_2}^k)\sin(u_B^k - \theta^k) \\ \theta^{k+1} &= \theta^k + u_A^k - u_B^k \\ l_1^{k+1} &= l_1^k + u_{l_1}^k \\ l_2^{k+1} &= l_2^k + u_{l_2}^k \end{aligned} \quad (1)$$

where x and y indicate the position of the left foot in the global coordinates, θ is the angle of the segment from the x axis in the global coordinates, and $u_{l_1}^k$ and $u_{l_2}^k$ are the changes in length of each segment. Now, the state of $(x^k, y^k, \theta^k, l_1^k, l_2^k)$ and the input of $(u_A^k, u_B^k, u_{l_1}^k, u_{l_2}^k)$ are expressed as $\boldsymbol{q}^k$ and $\boldsymbol{u}^k$ so that Eq. 1 is expressed as $\boldsymbol{q}^{k+1} = \boldsymbol{G}(\boldsymbol{q}^k, \boldsymbol{u}^k)$. By eliminating $\boldsymbol{u}^k$ from Eq. (1), the difference equation is expressed as follow:

$$\begin{aligned} (l_2^{k+1})^2 = (x^{k+1} - x^k - l_1^k \cos\theta^k)^2 \\ +(y^{k+1} - y^k - l_1^k \sin\theta^k)^2 \end{aligned} \quad (2)$$

Equation (2) is expressed as $h(\boldsymbol{q}^{k+1}, \boldsymbol{q}^k) = 0$. However, this equation cannot be transformed to $h(\boldsymbol{q}^k, k) = 0$. This means that the bipedal mode is a nonholonomic system with difference equation constraints [8].

From Eq. (1), the individual steps can be expressed as follows:

$$\begin{aligned} \boldsymbol{q}^1 &= \boldsymbol{G}(\boldsymbol{q}^0, \boldsymbol{u}^0) &&= \boldsymbol{G}_1(\boldsymbol{q}^0, \boldsymbol{u}^0) \\ \boldsymbol{q}^2 &= \boldsymbol{G}(\boldsymbol{q}^1, \boldsymbol{u}^1) &&= \boldsymbol{G}_2(\boldsymbol{q}^0, \boldsymbol{u}^0, \boldsymbol{u}^1) \\ \vdots &= \quad \vdots &&= \quad \vdots \\ \boldsymbol{q}^k &= \boldsymbol{G}(\boldsymbol{q}^{k-1}, \boldsymbol{u}^{k-1}) &&= \boldsymbol{G}_k(\boldsymbol{q}^0, \boldsymbol{u}^0, ..., \boldsymbol{u}^{k-1}) \end{aligned} \quad (3)$$

where $\boldsymbol{q}^0$ is the initial state. Using the input series $\boldsymbol{U}_k = (\boldsymbol{u}^0, ..., \boldsymbol{u}^{k-1})$, the k-step state $\boldsymbol{q}^k$ can be represented as

$$\boldsymbol{q}^k = \boldsymbol{G}_k(\boldsymbol{q}^0, \boldsymbol{U}_k) \quad (4)$$

B. Gait planning

The input series $\boldsymbol{U}_k$ defines footholds for biped walking and realizes the state $\boldsymbol{q}^k$. Thus, the gait planning is the solution of $\boldsymbol{U}_k$ that leads $\boldsymbol{q}^k$ to the target state $\boldsymbol{q}_d$. The input series $\boldsymbol{U}_k$ can be easily obtained by solving the inverse problem of Eq. (4). The Jacobian $\boldsymbol{J_k}$ $(= \partial\boldsymbol{G_k}/\partial\boldsymbol{U_k} = (\partial\boldsymbol{G_k}/\partial\boldsymbol{u_0}, ..., \partial\boldsymbol{G_k}/\partial\boldsymbol{u_{k-1}}))$ is used to manipulate the resolved velocity control of a redundant manipulator[9], and we obtain a solution to the motion planning problem. Although the solution is derived from a type of Newton-Raphson method, the available solution for a real humanoid robot should be controlled by a redundant term. Using null-space matrix $(\boldsymbol{I} - \boldsymbol{J}^+\boldsymbol{J})$, the planned gait is modified to match the hardware limits of the robot. Since the modification does not affect the final state $\boldsymbol{q}^k$, the modification can optimize the evaluation function, which deals with obstacle avoidance or energy consumption, without changing the final state. If it is not possible to make an available planning for a robot with k-SoW, one step is added to the SoW and evaluates again. Figure 4 shows the flow of the gait planning. The inputs are the initial state $\boldsymbol{q}^0$ and the target state $\boldsymbol{q}_d$, and the outputs are the control inputs $\boldsymbol{U}_k$, the foot placements $\boldsymbol{X}$, and the angles of foot placements $\boldsymbol{\theta}_{foot}$. Here, $\boldsymbol{J}_k^+$ is the pseudo-inverse matrix of $\boldsymbol{J}_k$, $\boldsymbol{K}_p$ is the state-error feedback gain, V is the evaluation function of the planned gait, $\boldsymbol{f}(\boldsymbol{U}_k) = \boldsymbol{K}_v \partial V/\partial \boldsymbol{U}_k$, $\boldsymbol{K}_v$ is the input-error feedback gain, and the allowable error for target state

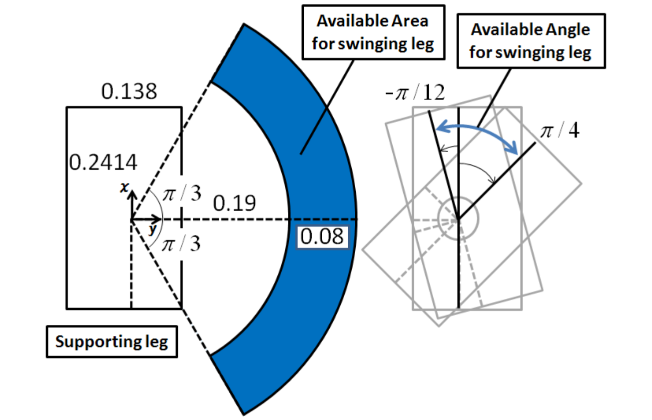


Fig. 5. Swing leg workspace and relative angle between the right and left feet

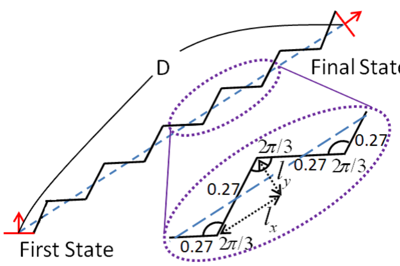


Fig. 6. Definition of minimum-step

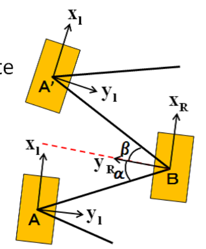
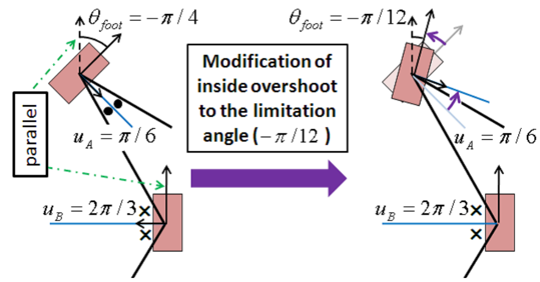
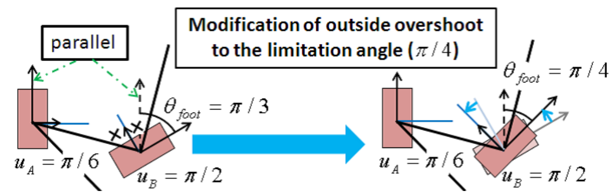


Fig. 7. Right and left foot coordinates



(a) Modification of inside overshoot



(b) Modification of outside overshoot

Fig. 8. Foot angle modification

is $e = 0.0005$.

III. Planning process considering mechanical constraints

Optimization considering hardware limitations should be implemented so that the proposed bipedal walking model (Fig. 2) is applied to an actual robot gait. In the present paper, we use constraint parameters for the HRP-2 humanoid robot [13].

A. Swing leg constraints

Figure 5 shows the available workspace and the relative angle between the right foot, which is the swing leg, and the left foot, which is the support leg. When we assume a polar coordinate system for the left foot, the available area is defined in sector form. The minimum and maximum lengths between the right and left feet are 0.19 [m] and 0.27 [m], respectively, and the available angle is $\pm\pi/3$[rad]. The swing leg foot placement angle is also limited from $-\pi/12$[rad] to $\pi/4$[rad], as defined by the yaw angle of the hip joint.

For the available area of the swing leg foot position, we define the constraints of the inputs for the alternately pivoting walking model as follows:

- $0.19 \leq l_1, l_2 \leq 0.27$ [m]
- $-2\pi/3 \leq u_A, u_B \leq 2\pi/3$ [rad]
- $-\pi/12 \leq \theta_{foot} \leq \pi/4$ [rad]

B. Initial step-of-walking and control input

The initial step of walking is easily defined kinematically without considering hardware constraints. The required walking distance (D) is assumed as the distance between the center of the initial segment and the goal segment, as shown in Fig. 6. Since, ideally, the maximum step length (l_x) is 0.27sin$(\pi/3)$ [m] when the robot uses the maximum number of steps for traveling, where $u_A, u_B = 2\pi/3$[rad], $l_1, l_2 = 0.27$[m] , the minimum number of steps (k_{min}) for walking is calculated as follows:

$$k_{min} = ROUNDUP(D/2l_x) \tag{5}$$

where $ROUNDUP()$ is the roundup function, $l_x = 0.27\ sin(\pi/3)$[m], and $l_y = 0.27cos(\pi/3)$[m], as shown in Fig. 6. Here, in order to fit the planned SoW distance to the required walking distance (D), the input constraints are not applied to the first and final step inputs ($\boldsymbol{U}^0, \boldsymbol{U}^{(k_{min}-1)}$), which are instead assigned ideal parameters.

C. Adjustment of the angle of foot placement

Figures 7 and 8 show the foot coordinates and the concept of foot angle modification. The x-axis directs the front of the robot, and the y-axis directs the inside of the robot in each coordinate. The angles α and β are defined by the y-axis and the individual segments, as shown in Fig. 7. In the first step of the foot angle definition, α and β are set to have the same value. Here, at each step, α and β are examined in order to determine whether these angles exceed the foot placement angle limitation, as described in Section III-A. When one of these angles is found to exceed the foot placement angle limitation, this angle is changed to the maximum modifiable angle. Fig. 8 is the example of foot angle modification. If the process cannot be achieved, the k-th step of walking is judged to be impractical. The SoW number k is iterated to $k+1$, and the walking pattern is replanned.

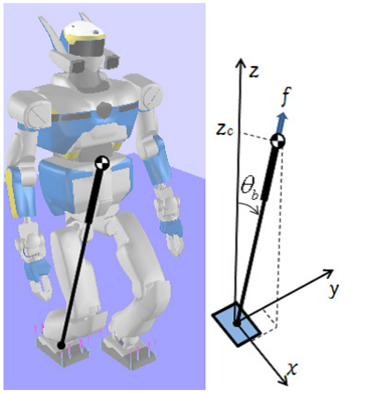


Fig. 9. Three-dimensional inverted pendulum

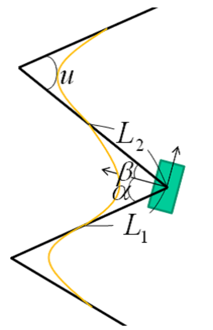


Fig. 10. CoM trajectory

D. Optimal gait planning method

The planned gait trajectory can be modified flexibly using the null-space matrix without changing the initial and final states. In this section, we optimize the walking trajectory considering energy consumption based on the linear pendulum model of bipedal walking.

Figure 9 shows the linear pendulum model for the bipedal robot[12]. The model maintains the Center of Mass (CoM) at a constant height (z_c), and the required reaction forces ($f(t)$) from the ground are calculated as follows:

$$f(t) = \frac{Mg}{\cos\theta_b} \tag{6}$$

where M is the total robot mass, g is the acceleration due to gravity, and θ_b is the incline angle from the z-axis of the foot coordinate. From Newton's equation of motion in the horizontal direction, we have

$$M\sqrt{\ddot{x}^2 + \ddot{y}^2} = f(t)\sin\theta_b \tag{7}$$

Equations (6) and (7) represent

$$f(t) = M\sqrt{\ddot{x}^2 + \ddot{y}^2 + g^2} \tag{8}$$

The energy consumption of each step ρ is expected to be as follows:

$$\rho = \int_0^T f(t)dt \tag{9}$$

where T is the single support time. Hence, the k-SoW energy consumption A is calculated as follows:

$$A = \sum_{i=1}^{2k} \rho_i \tag{10}$$

The energy consumption can be used as an evaluation function $\boldsymbol{f_A}$ corresponding to $\boldsymbol{U}_k$, as follows:

$$\boldsymbol{f_A} = \boldsymbol{K_{v_A}}\partial A/\partial \boldsymbol{U}_k \tag{11}$$

where K_{v_A} is the weighting factor matrix.

In the model, the x- and y-accelerations are calculated independently of Eqs. (6) and (7), as follows:

$$\ddot{x} = \frac{g}{z_c}x, \ddot{y} = \frac{g}{z_c}y \tag{12}$$

Equation (12) can then be solved for x and y, as follows:

$$x(t) = x(0)C(t) + T_c\dot{x}(0)S(t) \tag{13}$$
$$\dot{x}(t) = x(0)/T_cS(t) + \dot{x}(0)C(t) \tag{14}$$
$$y(t) = y(0)C(t) + T_c\dot{y}(0)S(t) \tag{15}$$
$$\dot{y}(t) = y(0)/T_cS(t) + \dot{y}(0)C(t) \tag{16}$$

where $T_c \equiv \sqrt{z_c/g}$, $C(t) \equiv \cosh(t/T_c)$, $S(t) \equiv \sinh(t/T_c)$. Figure 10 shows the CoM trajectory. The initial parameters are set as follows:

$$x(0) = -L_1\sin\alpha \tag{17}$$
$$x(T) = L_2\sin\beta \tag{18}$$
$$y(0) = L_1\cos\alpha \tag{19}$$
$$y(T) = L_2\cos\beta \tag{20}$$
$$\dot{x}(0) = \frac{x(T) - x(0)C(T)}{T_cS(T)} \tag{21}$$
$$\dot{y}(0) = \frac{y(T) - y(0)C(T)}{T_cS(T)} \tag{22}$$

where $\alpha + \beta = \theta$, $\theta = u_B(or\ u_A)$, $L_1 = l_1/2(or\ l_2/2)$, $L_1 = l_2/2(or\ l_1/2)$, and the reaction force from the ground $f(t)$ is

$$f(t) = \frac{M\sqrt{x^2(t) + y^2(t) + T_c^4g^2}}{T_c^2} \tag{23}$$

where

$$\begin{aligned} x^2(t) + y^2(t) &= L_1^2(C(t) - QS(t))^2 + P^2L_2^2S^2(t) \\ &+ 2PL_1L_2\cos\theta S(t)(C(t) - QS(t)) \\ P &= 1/(T_cS(T)) \\ Q &= C(T)/(T_cS(T)) \end{aligned} \tag{24}$$

Here, $f(t)$ can be expressed in terms of u_A, u_B, u_{l_1}, and u_{l_2}, regardless of the direction of the foot placement (α, β). In order to simplify the calculation, let the evaluation value for each step be ρ', and let the evaluation value be A'. Then, the evaluation function $\boldsymbol{f_{A'}}$ is obtained as follows:

$$A' = \sum \rho' \tag{25}$$
$$\rho' = \int_0^T f^2(t)dt$$
$$\boldsymbol{f_{A'}} = \boldsymbol{K_{v_{A'}}}\partial A'/\partial \boldsymbol{U}_k \tag{26}$$

E. Desired input for flexible walking

In order to direct the desired walking trajectory, the evaluation value for each control input is implemented as follows:

$$\begin{aligned} B = \Sigma_{t=0}^{k-1}[(u_A^t - u_{A_d}^t)^2 + (u_B^t - u_{B_d}^t)^2 \\ +(l_1^{t+1} - l_{1_d}^{t+1})^2 + (l_2^{t+1} - l_{2_d}^{t+1})^2] \end{aligned} \tag{27}$$

The evaluation function is set as $\boldsymbol{f_B} = \boldsymbol{K_{v_B}}\partial B/\partial \boldsymbol{U}_k$, and the null-space matrix can modify the input vector to obtain the values of the desired input vector $(u_{A_d}, u_{B_d}, l_{1_d}, l_{2_d})$ for as long as possible without changing the final state $\boldsymbol{q}^k$. For example, if $u_{Ad}^0 = -\pi/6$[rad],$u_{Bd}^0 = \pi/6$[rad] are given, then the first stage of the input vector can be directed to turn clockwise.

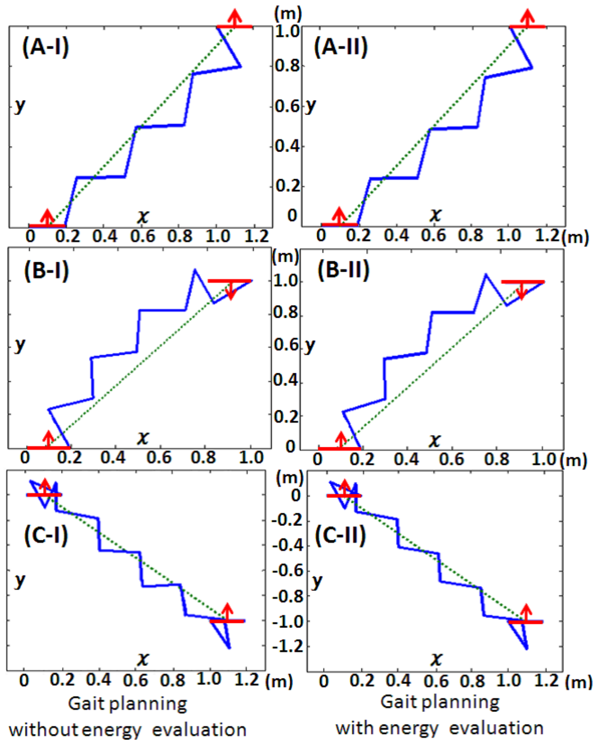


Fig. 11. Gate planning results

IV. Numerical simulation

In this section, we describe the numerical simulation of walking trajectory path planning using MATLAB.

A. Walking path planning

We implement evaluation functions $\boldsymbol{f_{A'}}$ and $\boldsymbol{f_B}$. Here, $\boldsymbol{f_{A'}}$ optimizes energy consumption, and $\boldsymbol{f_B}$ modifies the path to avoid the input constraint, as shown in Section III-A, and manages the turning direction as shown in Section III-E. In the modification using the null-space matrix, the preference order is defined as follows:
(i) Path planning within the input constraint
(ii) Path planning for the lowest energy consumption.

Figure 11 shows the planning results for three target positions. The initial state in all cases is $(x^0, y^0, \theta^0, l_1^0, l_2^0)$=(0 [m],0[m],0 [rad],0.19 [m],0.19 [m]), and the target positions are as follows:
(A)$(x_d, y_d, \theta_d) = (1$ [m],1 [m],0[rad]).
(B)$(x_d, y_d, \theta_d) = (1$ [m],1 [m],π[rad]).
(C)$(x_d, y_d, \theta_d) = (1$ [m],-1 [m],0[rad]).
In cases (B) and (C), the turning direction is commanded during the first stage of the input vector. The paths in the left-hand column are the results obtained using only evaluation functions $\boldsymbol{f_B}$, and the paths in the right-hand column are the results obtained using evaluation functions $\boldsymbol{f_{A'}}$ and $\boldsymbol{f_B}$. Although there are a few difference in foot placement, individual energy consumption is 5 percent or more.

In actual bipedal walking, the controller must prepare the double support phase to stabilize the walking pattern.

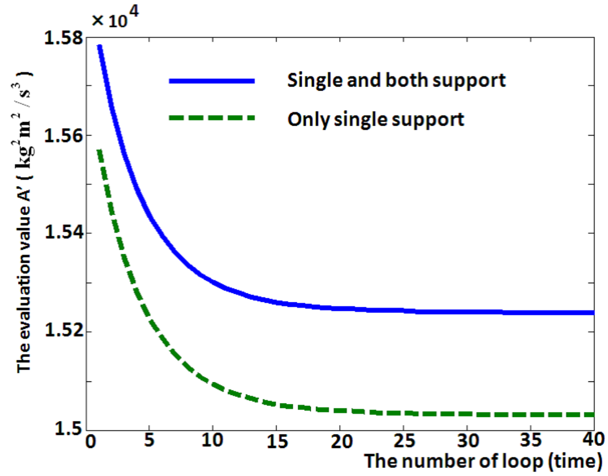


Fig. 12. Comparison of energy consumption

Thus, the actual energy consumption, including the double support phase, must be calculated. However, this increases the scale of the calculation process. Figure 12 shows the optimization process in the case of target (A). The solid line indicates the ideal energy consumption considering the double support phase. The dotted line indicates the ideal energy consumption using only the single support phase (Eq. (25)). The energy consumptions become smaller, and the directions of the gradients in the optimization process are the same. The hundred random sampling simulations are the same in content. Thus, an evaluation function that uses only the single support phase is sufficient for optimizing energy consumption.

V. Experiment

The proposed algorithm is implemented using the HRP-2 humanoid robot. The gait planning is written in the C++ language on a Linux system. The PC has a 2.4-GHz Pentium Core 2 Processor and 3.0 [GB] of memory. In this case, the planning results can be get within 100 [msec] for various places. Since the walking pattern of the present study uses the single support time 0.8 [s] and the double support time 0.1 [s], the planning algorithm can be implemented during every double support phase. We use only the Newton-Raphson method for the convergence process. However, the proposed planning process first defines a rough trajectory, and the convergence calculation is then started, so no local minimum problems occurred in the simulations of the present study. Thus, the model of energy consumption based on a linear pendulum may have a single convex hull. Figures 14 through 16 show the experimental results. Two target locations are prepared. Initial position, target (A) and (B) are (x, y, θ) = (0.0, 0.0, 0.0), (1.0, 1.0, 0.0), (1.0, 1.0, $-\pi/2$) [m, m, rad] respectively. The target positions differ only in angle. The dotted line in Fig. 12 is a straight line connecting the initial state and the goal state, and each planning result differs from the straight line. Evaluation of energy consumption and hardware limitation check work on the planning and we can get the actual use of results, as shown in Figs. 15 and 16.

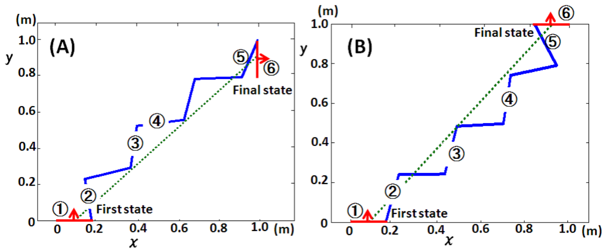


Fig. 13. Walking trajectories

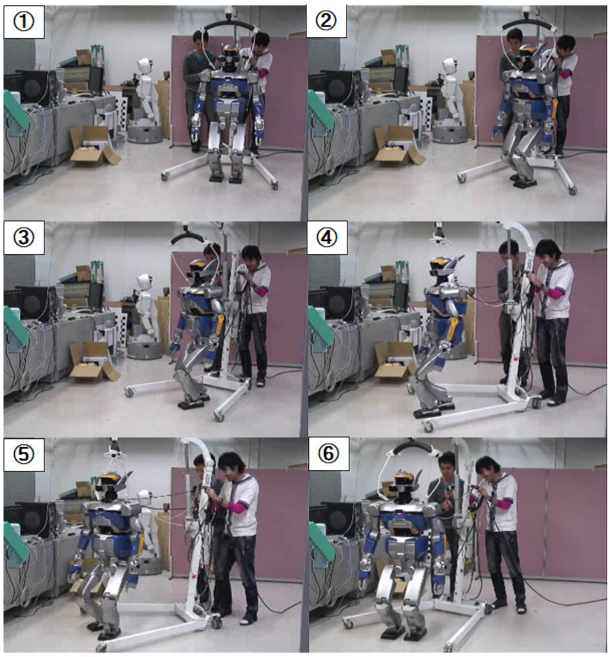


Fig. 14. Experimental results for target (A)

VI. Conclusion

A nonholonomic model for planning bipedal locomotion was proposed and the proposed model was implemented using the HRP-2 humanoid robot. Although the motion of a nonholonomic system is usually difficult to design, the data set of the discrete motion of the nonholonomic system could be manipulated by the nonlinear optimization algorithm of Newton-Raphson. A linear-pendulum model for bipedal walking was used to evaluate the energy consumption and to generate an efficient walking trajectory. The effectiveness of this model was demonstrated through a numerical simulation and an experiment using the HRP-2 humanoid robot.

In the present paper, the feasibility of motion planning using a discrete nonholonomic system was demonstrated. In the future, we intend to implement an obstacle avoidance method that achieves minimum energy consumption and more complicated environmental path planning.

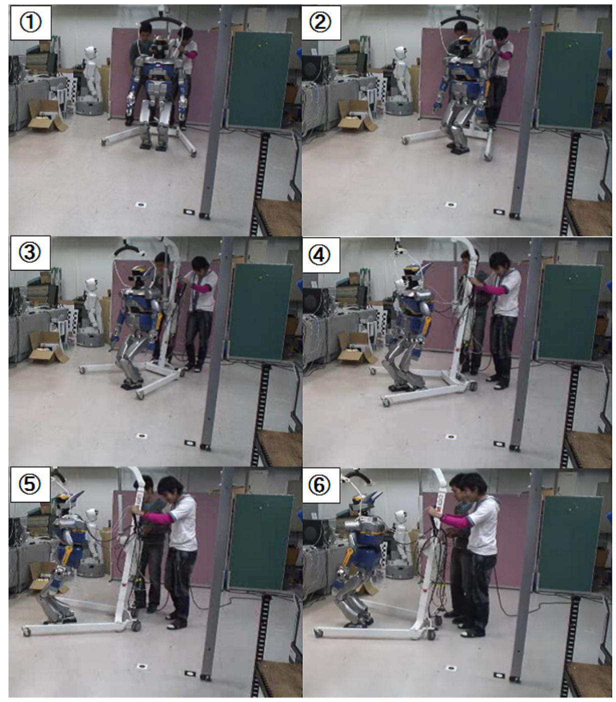


Fig. 15. Experimental results for target (B)

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