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Integral Nested Super-Twisting Algorithm for Robotic Manipulators

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Abstract— **A controller, based on integral nested sliding modes and super-twisting algorithm, is proposed for n-link robotic manipulator tracking problem. This controller has the robustness of nested sliding modes against matched and no matched perturbations, the capability of integral sliding modes to reduce the sliding functions gains, and the softness of control signals of super-twisting algorithm. The performance of the proposed algorithm is compared with integral nested sliding mode control via simulation. For this purpose, the application of both controllers in a two-link planar robot manipulator is presented.**

I. Introduction

THE usual objective in robotic manipulators control is to command a desired response for the motion of its end effector. In order to design the controller, the kinematic or dynamic model of the manipulator must be obtained. Several dynamic model-based controllers have been designed to solve the robotic manipulator trajectory control problem as Computed Torque [1], Lyapunov Stability [2] and Passivity [2]. Common assumptions for the aforementioned strategies are that the model of the manipulator is completely known and the plant is not perturbed. Usually, these conditions are not fulfilled for practical implementations.

Hence, the robustness of the controller to uncertainties and disturbances is fundamental to successfully solve the control objective. In [3]-[4] adaptive controllers were designed based in dynamic model of the manipulator. The neural control approach was studied in [5], where a controller composed of a linear feedback part and a neural control part was proposed. A fuzzy controller that ensures robust and global stability is proposed in [6], where a fuzzy control is obtained by blending nonlinear sub-controls designed for each fuzzy set and obtained via Lyapunov's direct method.

Among robust control methodologies for robotic manipulators, Sliding Mode Control (SMC) [7]-[10] is one of the most effective approaches because its robustness to matched perturbations. In addition, SMC is obtained from a simple procedure, which impacts in a low computational cost of a real implementation. However, standard SMC is not robust against unmatched disturbances.

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In this work we design a controller on the basis of Nested Sliding Mode (NSM) [8], Integral Sliding Mode (ISM) and Super-Twisting algorithm [9] in order to achieve robustness to matched and unmatched perturbations, and ensure output tracking in a robotic manipulator. This Integral Nested Super-Twisting (INST) algorithm can guarantee the robustness of the system throughout the entire response starting from the initial time instance and reduce the sliding functions gains in comparison with NSM. In addition, the control signals of the proposed controller are smoother than in Integral Nested Sliding Mode Control (INSMC) [10], which presents high-frequency components in its control signal.

The structure of the document is defined as follows. First, the dynamics of an n-link robotic manipulator and its structural properties are formulated. Then, an INST algorithm for robotic manipulators is designed. The simulation results are obtained applying the proposed controller in a two-link planar robotic manipulator. In addition, a comparison between INST algorithm and INSMC is presented to verify the improvement in the performance of the proposed control strategy. Finally, some conclusions are given in section V.

II. Problem Formulation

Consider a non perturbed n-joint robotic manipulator system described by the following model:

$$M(q)\ddot{q} + C(q,q)\dot{q} + g(q) = \tau \tag{1}$$

where $q(t)$ is an $n\times1$ vector of joint angular positions, τ is the $n\times1$ vector of applied joint torque, $M(q)$ is the $n\times n$ manipulator inertia matrix, $C(q,q)$ is the $n\times1$ vector of centripetal and Coriolis torques and $g(q)$ is the $n\times1$ vector of gravitational torques. This model has the following important properties:

1) $M(q)$ is a symmetric positive definite matrix for all $q\in\Re^n$.

2) There exists a unique matrix $C(q,q)$ such that $M(q)-2C(q,\dot{q})$ is skew symmetric.

Defining $y_1 = q$, $y_2 = \dot{q}$ as the state variables and adding a perturbation term $\lambda(y_1,y_2,t)$ due to external disturbances, parameters variation and model uncertainties, we obtain the following state-space representation

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= f_2(y_1, y_2) + b_2(y_1)u + \lambda(y_1, y_2, t) \end{aligned}, \tag{2}$$

where y_1 is the output of the system and $u = \tau$ is the vector of the torques applied to the joints of the robot, $f_2(y_1, y_2)$ and $b_2(y_1)$ are continuous vector functions.

Throughout the development of the controller we will use the following assumptions:

A1) The unmatched $\lambda_u(y_1, t)$ and matched $\lambda_m(y_1, y_2, t)$ perturbation terms, which will be defined later, are bounded by known positive scalar functions:

$$\|\lambda_u(y_1, t)\| < \beta_1(y_1, t)$$
$$\|\lambda_m(y_1, y_2, t)\| < \beta_2(y_1, y_2, t)$$

A2) The sign function can be approximated by the sigmoid function as shown by the following limit:

$$\lim_{\varepsilon \to \infty} sigm(\varepsilon, S) = sign(S)$$

Figure 1 shows the approximation for various values of ε, which defines the slope of the sigmoid function for every s. The sigmoid function used in this work is

$$sigm(\varepsilon, s) = \tanh(\varepsilon s)$$

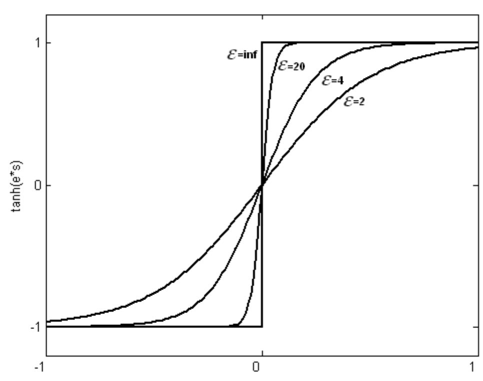

Fig. 1. Sigmoid function for various values of parameter ε.

A3) $rank[b_2(y_1)] = n$, where n denotes the degrees of freedom of the manipulator.

Let $y_{1ref}(t)$ be a twice differentiable function, but with unknown derivatives, which represents the desired trajectory of the joint positions vector. The considered problem is to design an Integral Nested Super-Twisting Algorithm that ensures output trajectory tracking in presence of the perturbations of the system due to external disturbances, parameters variation and model uncertainties.

III. INST ALGORITHM FOR ROBOTIC MANIPULATORS

Let $y_{1ref}(t)$ be a twice differentiable function, but with unknown derivatives, and define the output tracking error as

$$e_1 = y_1 - y_{1ref}(t). \tag{3}$$

Then, its derivative yields

$$\dot{e}_1 = y_2 + \lambda_u(y_1, t) \tag{4}$$

where $\lambda_u(y_1, t)$ is the unmatched term defined by the following equality

$$\lambda_u(y_1, t) = -\dot{y}_{1ref}. \tag{5}$$

Defining the pseudo-sliding function $s_1 \in \Re^n$ for the first block (4) as

$$s_1 = e_1 + z_1, \; z_1(0) = -e_1(0) \tag{6}$$

where z_1 is the integral variable that will be defined later, the dynamics of s_1 can be obtained of the form

$$\dot{s}_1 = y_2 + \dot{z}_1 + \lambda_u(y_1, t). \tag{7}$$

Considering y_2 as virtual control in (7), we propose

$$y_{2ref} = y_{2,0ref} + y_{2,1ref} \tag{8}$$

where $y_{2,0ref}$ is the nominal part of the control and $y_{2,1ref}$ is the control which will be designed to reject the perturbation term [7]. To obtain y_2 and replace it in (7) we must define the error variable and the sliding function for the second block as

$$e_2 = y_2 - y_{2ref}, \quad s_2 = e_2 + z_2, \quad z_2(0) = -e_2(0), \tag{9}$$

with z_2 as the integral variable. From the equation (9) we obtain

$$y_2 = s_2 + y_{2ref} - z_2. \tag{10}$$

Then, using (8) and (10), the first transformed block (7) becomes as

$$\dot{s}_1 = s_2 - z_2 + y_{2,0ref} + y_{2,1ref} + \dot{z}_1 + \lambda_u. \tag{11}$$

Now, choosing $\dot{z}_1$ of the form

$$\dot{z}_1 = -(s_2 - z_2 + y_{2,0ref}) \tag{12}$$

with the initial condition $z_1(0) = -e_1(0)$, and defining $y_{2,0ref}$ as follows

$$y_{2,0ref} = -c_1 e_1 \tag{13}$$

where $c_1 > 0$, the dynamics for z_1 and s_1 are represented as

$$\begin{aligned} \dot{z}_1 &= -e_2 - y_{2,0ref} \\ \dot{s}_1 &= y_{2,1ref} + \lambda_u \end{aligned}. \tag{14}$$

The second part $y_{2,1ref}$ of the virtual control is selected of the form

$$y_{2,1ref} = -k_1 sigm(\varepsilon_1 s_1) \tag{15}$$

where $k_1 > 0$.

Proceeding with the second block, its dynamics can be obtained by differentiating (9) along the trajectories of the system (2) as:

$$\dot{s}_2 = f_2(y_1, y_2) + b_2(y_1)u - \dot{y}_{2ref} + \dot{z}_2 + \lambda_m \quad (16)$$

where $\dot{y}_{2ref}$ is defined as

$$\begin{aligned} \dot{y}_{2ref} &= -k_1\varepsilon_1 P y_{2,1ref} - c_1 y_2 \\ P &= diag\{1-\tanh^2(\varepsilon_1 s_{11}) \quad \dots \quad 1-\tanh^2(\varepsilon_1 s_{1n})\} \end{aligned} \quad (17)$$

with $s_1 = \begin{bmatrix} s_{11} & \dots & s_{1n} \end{bmatrix}^T$, and the matched perturbation term λ_m is given by

$$\lambda_m = -(c_1 + k_1\varepsilon_1 P)\lambda_u + \lambda(y_1, y_2, t). \quad (18)$$

Designing $u = u_0 + u_1$ we obtain

$$\dot{s}_2 = f_2(y_1, y_2) + b_2(y_1)u_0 + b_2(y_1)u_1 - \dot{y}_{2ref} + \dot{z}_2 + \lambda_m \quad (19)$$

and we choose $\dot{z}_2$ as follows

$$\dot{z}_2 = -f_2(y_1, y_2) - b_2(y_1)u_0 + \dot{y}_{2ref} \quad (20)$$

with

$$z_2(0) = -e_2(0) \quad (21)$$

to ensure sliding mode occurrence from initial instance. Then, choosing

$$u_0 = b_2(y_1)^{-1}\left(-f_2(y_1, y_2) + \dot{y}_{2ref} - c_2 e_2\right) \quad (22)$$

where $c_2 > 0$ and using (20), the equation (19) is reduced to

$$\dot{s}_2 = b_2(y_1)u_1 + \lambda_m. \quad (23)$$

To induce sliding mode in (23) we design the second part u_1 of the control law using the super-twisting algorithm as

$$\begin{aligned} u_1 &= b_2(y_1)^{-1}\left[-\sigma N sign(s_2) + \mu\right] \\ \dot{\mu} &= -\Sigma sign(s_2) \end{aligned} \quad (24)$$

where $N = diag\left\{|s_{21}|^\rho, \dots, |s_{2n}|^\rho\right\}$ with $s_2 = \begin{bmatrix} s_{21} & \cdots & s_{2n} \end{bmatrix}^T$, and σ, ρ, Σ are designing parameters. Using (13), (15), (22) and (24), the dynamics of the variables s_1 and s_2 are derived as follows

$$\begin{aligned} \dot{s}_1 &= -k_1 sigm(\varepsilon_1 s_1) + \lambda_u \\ \dot{s}_2 &= -\sigma\|s_2\|^\rho sign(s_2) + \mu + \lambda_m \end{aligned} \quad (25)$$

while the tracking errors, e_1 and e_2, dynamics are obtained from (4), (8)-(10), (13), and (22), respectively, of the form

$$\begin{aligned} \dot{e}_1 &= -c_1 e_1 + e_2 + y_{2,1ref} + \lambda_u \\ \dot{e}_2 &= -c_2 e_2 + b_2(y_1)u_1 + \lambda_m. \end{aligned} \quad (26)$$

Now, establishing the conditions for the super-twisting algorithm as

$$\begin{aligned} &\rho = 1/2, \quad \beta_2(y_1, y_2, t) = \alpha|s_2|^\rho \\ &\sigma > 2\alpha, \quad \Sigma > \sigma\frac{5\alpha\sigma + 4\alpha^2}{2(\sigma - 2\alpha)} \end{aligned} \quad , \quad (27)$$

and the following set of conditions, for the part of the controller based in INSM, as

$$\begin{aligned} &k_1 > \frac{\beta_1}{1-\delta}, \ 1 > \delta > 0 \\ &\|\dot{\lambda}_u\| \le d_u\|\dot{s}_1\|, \ d_u > 0 \end{aligned} \quad (28)$$

we can enunciate a theorem as follows:

Theorem 1. *If the assumptions A1), A2) and A3) hold, the conditions (29) and (30) are satisfied, and the control law*

$$u = b_2(y_1)^{-1}(-f_2(y_1, y_2) + \dot{y}_{2ref} - c_2 e_2 - \sigma\|s_2\|^\rho sign(s_2) + \mu)$$
$$\dot{\mu} = -\Sigma sign(s_2)$$

is constructed; then a solution of the error dynamics (28) is asymptotically stable.

Proof:

Defining a candidate Lyapunov function [11] for the dynamics of s_2 as

$$V_2(s_2) = \Psi^T P \Psi \quad (29)$$

where

$$\Psi^T = \left[|s_2|^{1/2} sign(s_2) \quad , \quad s_2\right]$$
$$P = \frac{1}{2}\begin{bmatrix} 4\Sigma + \sigma^2 & -\sigma \\ -\sigma & 2 \end{bmatrix}.$$

It is demonstrated in [11] that the derivative of (29) is given by

$$\dot{V}_2 = -\frac{1}{|s_2|^{1/2}}\Psi^T Q \Psi + \frac{\lambda_m}{|s_2|^{1/2}} q_1^T \Psi$$
$$Q = \frac{\sigma}{2}\begin{bmatrix} 2\Sigma + \sigma^2 & -\sigma \\ -\sigma & 1 \end{bmatrix}$$
$$q_1^T = \left[\left(2\Sigma + \frac{\sigma^2}{2}\right) \quad \frac{-\sigma}{2}\right], q_2^T = \begin{bmatrix} -\sigma & 2 \end{bmatrix}$$

and that $\dot{V}_2$ is negative definite under conditions (27). So, we can conclude that s_2 converges to zero in finite time.

Proceeding with first block - dynamics of s_1 - we can define a candidate Lyapunov function as

$$V_1 = \tfrac{1}{2} s_1^T s_1. \quad (30)$$

The derivative of (30) is of the form

$$\dot{V}_1 = s_1^T\left(-k_1 sigm(\varepsilon_1 s_1) + \lambda_u\right).$$

It is demonstrated in [10] that s_1 converges to a vicinity of zero bounded by

$$\Omega = \frac{\ln\left(\frac{2-\delta}{\delta}\right)}{2\varepsilon_1}.$$

and that $\dot{s}_1$ converges to zero in finite time if **A1**, **A2** and condition (28) hold. Therefore, from (14) and (23) we obtain

$$\begin{aligned} y_{2,1ref} &= -\lambda_u \\ b(y_1)u_1 &= -\lambda_m \end{aligned}.$$

Hence the system (26) can be reformulated as

$$\begin{aligned} \dot{e}_1 &= -c_1 e_1 + e_2 \\ \dot{e}_2 &= -c_2 e_2 \end{aligned} \tag{31}$$

and, if $c_1 > 0$ and $c_2 > 0$, a solution of (31) tends asymptotically to zero, provided then

$$\lim_{t\to\infty} e_1(t) = 0.$$

and Theorem 1 is proved.

IV. Simulations

The controller designed in this work was applied to a two-link planar robot manipulator with perturbations due to external disturbances, model uncertainties, parameters variation and the load that the robot manipulates. Also, the INSMC [10] defined as

$$u = b_2(y_1)^{-1}(-f_2(y_1,y_2) + \tilde{y}_{2ref} - c_b e_2) - K_b b_2(y_1)^{-1} sign(s_2)$$

where

$$\begin{aligned} \tilde{y}_{2ref} &= -K_a \varepsilon P_1 y_{2,1ref} - c_a y_2 \\ P_1 &= diag\{1-\tanh^2(\varepsilon s_{11}) \;\; \dots \;\; 1-\tanh^2(\varepsilon s_{1n})\} \end{aligned}$$

was simulated for comparison purposes.

The following terms define the state-space model, as in (2), of the robot manipulator with two degrees of freedom

$$\begin{aligned} f_2(y_1,y_2) &= -M(y_1)^{-1} N(y_1,y_2)^{-1} \\ b_2(y_1) &= M(y_1) \end{aligned},$$

$$M(y_1) = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix},$$

$$N(y_1,y_2) = \begin{pmatrix} -L_1L_2M_2\left(2y_2(1)y_2(2) - y_2(2)^2\right) \\ L_1L_2M_2 y_2(1)^2 \sin(y_1(2)) \end{pmatrix},$$

$$\begin{aligned} m_{22} &= L_2^2 M_2 \\ m_{12} = m_{21} &= m_{22} + L_1L_2M_2\cos(y_1(2)), \\ m_{11} &= L_1^2(M_1+M_2) + 2m_{12} - m_{22} \end{aligned}$$

where L_1, L_2, M_1 and M_2 are the lengths and masses of the first and second links, respectively. The values of these manipulator parameters used in both simulations were

$$M_1 = 10\,kg,\;\; M_2 = 1\,kg,\;\; L_1 = 1\,m,\;\; L_2 = 1\,m;$$

To fulfil all the design conditions, the control parameters for INST algorithm were adjusted to

$$K_1 = 5,\;\; \Sigma = 6,\;\; \sigma = 80,\;\; c_1 = 15,\;\; c_2 = 10,\;\; \varepsilon_1 = 6,$$

and the control parameters for INSMC were set to

$$K_a = 7,\;\; K_b = 35,\;\; c_a = 4,\;\; c_b = 4,\;\; \varepsilon = 20.$$

The perturbations terms used in both simulations are

$$\lambda_u = \begin{bmatrix} 4 \\ 2\sin(.5t) \end{bmatrix} \quad \lambda_m = \begin{bmatrix} 5\cos(2t) \\ 5+3\sin(t) \end{bmatrix},$$

and the references for the angular joint positions are

$$y_{1ref} = \begin{bmatrix} 2\sin(2t) & 2+3\cos(t) \end{bmatrix}^T.$$

The results obtained from simulation can be evaluated based on figures 2-11. The tracking response for joint 1 is shown in Fig. 2 and Fig. 3, for INST and INSMC respectively. It can be noted that the objective of control is fulfilled for both cases. However, the performance for INST is less oscillatory than INSMC. This can be seen too in the tracking response for joint 2 in Fig. 4 and Fig. 5. The performance of the controller is satisfactory, since the proposed algorithm rejects the external disturbances, model uncertainties and parameters variations in the model.

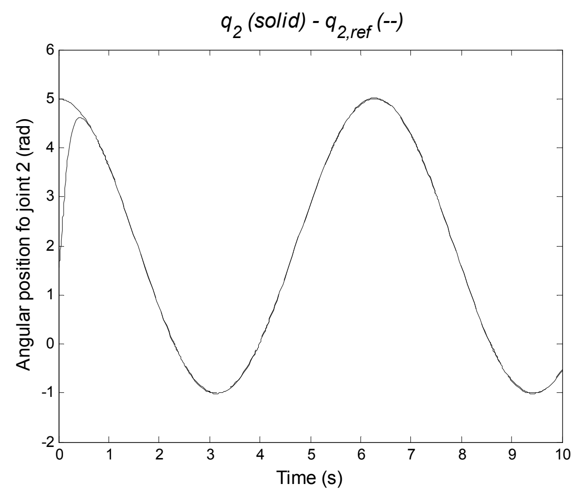

Fig. 2. Tracking response for joint 2 (INST).

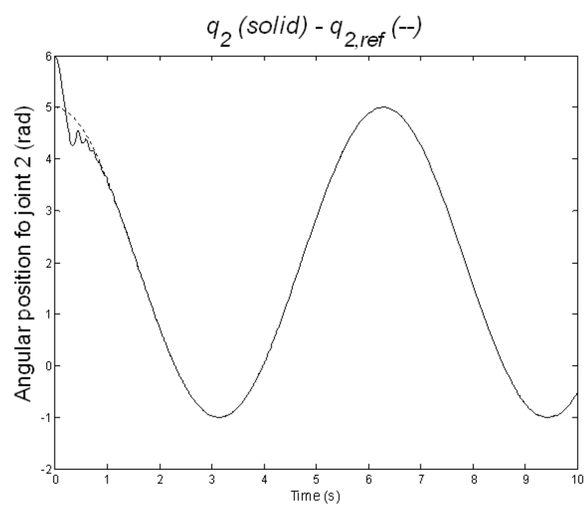

Fig. 3. Tracking response for joint 2 (INSMC).

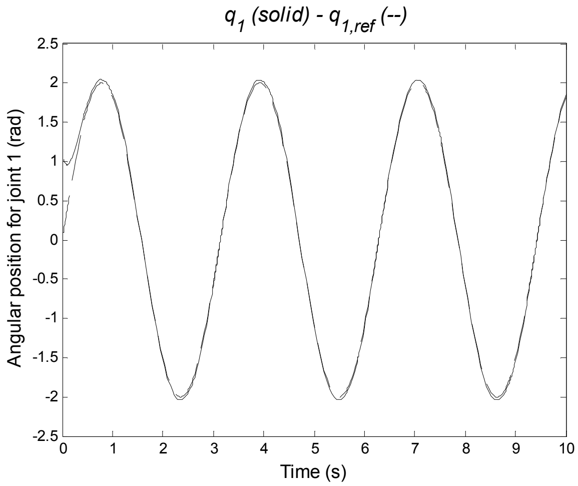

Fig. 4. Tracking response for joint 1(INST).

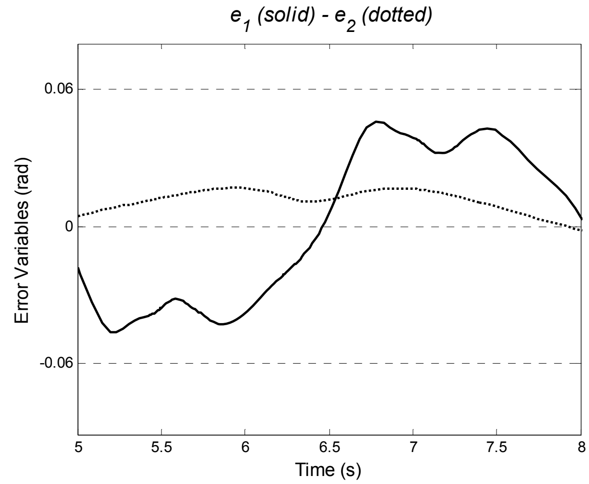

Fig. 7. Zoom in to the tracking errors (INST).

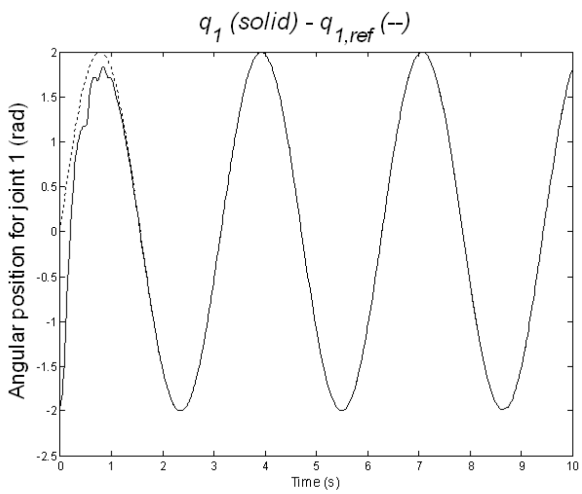

Fig. 5. Tracking response for joint 1(INSMC).

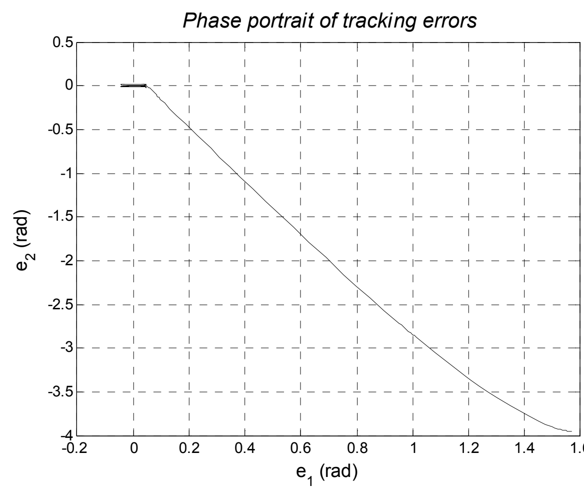

Fig. 8. Phase portrait of the tracking errors (INST).

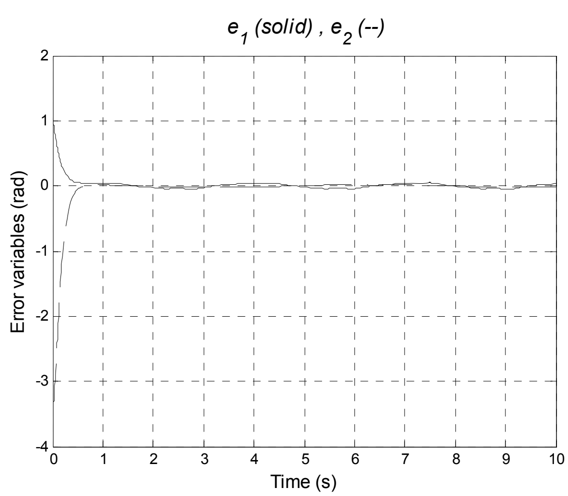

Fig. 6. Tracking errors (INST).

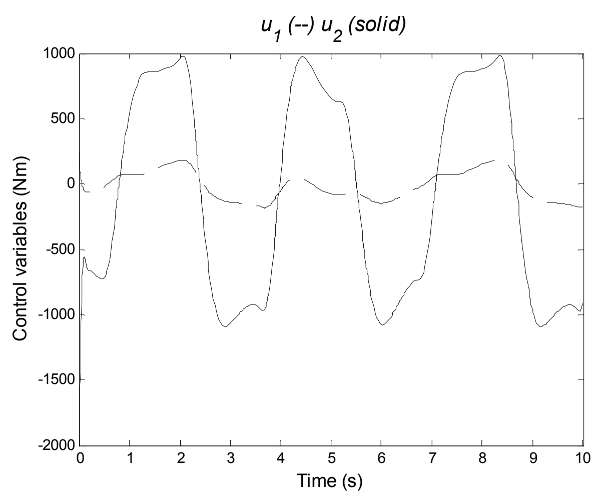

Fig. 9. Input controls for joint 1 and joint 2 (INST).

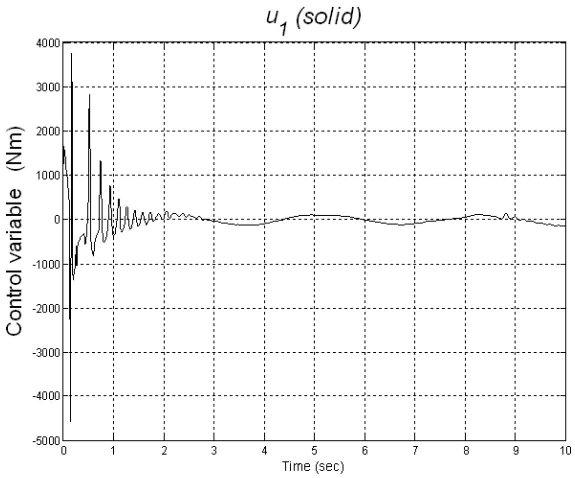

Fig. 10. Input controls for joint 1 (INSMC).

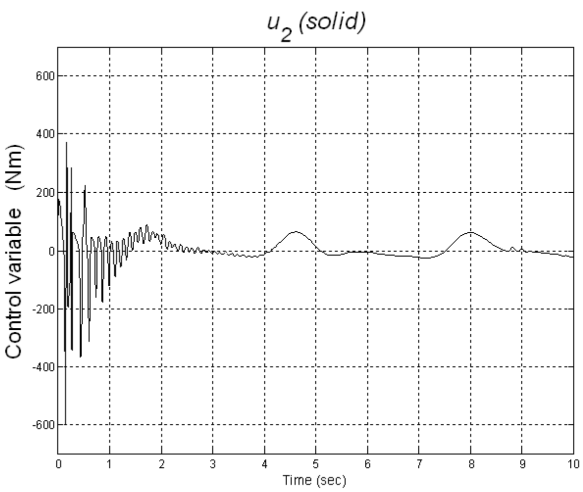

Fig. 11. Input controls for joint 2 (INSMC).

The convergence of the tracking errors is shown in Fig. 6, and in more detail in Fig. 7, where the errors converge to a neighbourhood of zero. This also can be observed in Fig. 8 where the phase portrait of the tracking errors is shown. These figures also allow us to observe that the settling time for the error variables is low.

The control variables, for INST algorithm, can be observed in Fig. 9. The control variables for INSMC are depicted in Fig. 10 and Fig. 11. Clearly, the control variables for the proposed controller are smoother than INSMC. Moreover, the magnitude of the control signals is lower.

V. CONCLUSIONS

An Integral Nested Super-Twisting (INST) algorithm for rigid robotic manipulators is designed, by the combination of nested, integral and super-twisting SMC concepts.

The proposed algorithm is robust against matched and no matched perturbations due to external disturbances, model uncertainties and parameters variations; and the inclusion of the Super-Twisting algorithm in the control law results in smooth control variables. This is a good feature, since high frequency components in the control signals may causes undesirable effects on the plant when a real-time implementation of the controller is developed.

The INST controller demonstrates a satisfactory performance in output tracking problem of robotic manipulators, moreover it obtains a reduced steady tracking error in comparison with standard SMC.

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