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Kinematic Modeling Framework for Biomimetic Undulatory Fin Motion Based on Coupled Nonlinear Oscillators

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***Abstract*—The present work is motivated by the need to develop a generic method modeling the biomimetic undulatory motion for fish robots with long fin propulsors. Combined with mechanical design of long fins proposed in current literatures, we explore the application of coupled nonlinear oscillators in the modeling of swimming gaits and propose a kinematic modeling framework. Coupled nonlinear oscillators can also be regarded as models of artificial Central Pattern Generators (CPGs) for swimming gait control of fish robots. The advantages of this method over the normal sinusoidal functions based method are discussed. The synchronization of multiple oscillators is derived, which can be utilized for the coordination of multiple joints of fish robots and the online gait transition. The framework is applied and tested in swimming motion control of an eight-DOF undulatory fin prototype. The effectiveness of the control is shown through experiments.**

I. Introduction

UNDULATORY swimming modes are adopted by many fishes as a major way of propulsion in aquatic locomotion [1]. Fish obtaining thrust through undulatory fin motion show remarkable flexibility and maneuverability in locomotion, which renders the undulatory fin an ideal model for the biomimetic propulsor design in robotic fishes [2]. Such fins consist of a set of muscles and bones and can produce backward traveling wave toward the opposite direction of heading during steady swimming [1-3]. To achieve the similar performance as real fish, mechanism of undulatory propulsor is routinely made of parallel-connected fin rays covered by flexible membrane materials, which resembles actual fins of real fishes. Each biomimetic fin ray is driven by an actuator (an electric motor, for instance). Usually a significant number of degrees of freedom (DOF) are involved in the fin movements [3, 4]. Multiple fin rays oscillate coordinately under certain control schemes and expand a travelling wave on the fin membrane.

Borrowed from the legged robot, the control signals applied on the actuators of fin rays are called *gaits* [4]. The modeling of undulatory fin motion is such a process that coordinated swimming gaits are derived to produce travelling wave on the long fin. The well-developed modeling scheme is important because they directly determine the performance of swimming. In recently proposed undulatory long fin prototypes [3-6], sine or cosine functions are commonly employed as the gait generator. These gait models can successfully control the motion of long fins. However, the modeling method needs to be improved in two major aspects. Firstly, it is not suitable for online gait generation and gait tuning. The amplitude, natural frequency and the phase lag of a sine generator is defined based on the pre-selected fin motions. And these parameters cannot be online tuned for the need of adaptation to environment because this may cause the instability of motion, which will be discussed in this paper. Secondly, this modeling method is not generic. When used for different fin motions or different fin prototypes it has to be tailored to accommodate various applications.

The present work is motivated by the need to develop a generic method modeling the biomimetic undulatory motion for fish robots with long fin propulsors. Instead of using normal sinusoidal gait generators, we explore the application of coupled nonlinear oscillators in modeling of swimming gaits and propose a kinematic modeling framework. Recently, coupled nonlinear oscillators are more and more often employed to model the rhythmic movements in robotics areas such as legged walking, flying and swimming [7-10]. They can also be regarded as models of artificial Central Pattern Generators (CPGs) which are used in the inter-limb coordination for multi-DOF robots and more recently in swimming gait generation of fish robots [9, 11, 12]. Compared with the sinusoidal function based gait generator, the method discussed in this paper is expected to have the following advantages: i) the swimming gait is not pre-defined but online generated, which makes the adaptive swimming to the environment feasible; ii) different types of gait patterns can be generated to achieve versatile aquatic locomotion; iii) an analytical and generic solution to the problem of modeling the biomimetic undulatory fin motion is provided; iv) the swimming gait can be online tuned with simple control signals. These features will be illustrated through experiments on an eight-DOF long fin prototype.

The paper is structured as follows. Firstly, the mechanisms of undulatory fin design in current fish robots are discussed in Section II. In Section III, we focus on the gait generation by nonlinear oscillators. The synchronization of multiple oscillators is detailed. This property is utilized for the coordination of multiple joints of fish robots and the online gait transition. The modeling method is applied in control of an eight-DOF fish prototype to achieve undulatory swimming locomotion. The effectiveness of the control is tested through experiments, which are presented in Section IV.

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II. Mechanisms of Undulatory Fins

The modeling and control of undulatory motions are closely dependent on the actual mechanical designs of the biomimetic fins. Typical designs in current literatures are discussed in this section. Most of them are similar in structure. We categorize long fin designs into two types depending on the way the fin rays are connected. As shown by examples in Fig. 1a-b, the biomimetic fin rays are parallel-arranged on a fixed baseline and covered by a flexible membrane. Similar designs are also presented in [6, 13].

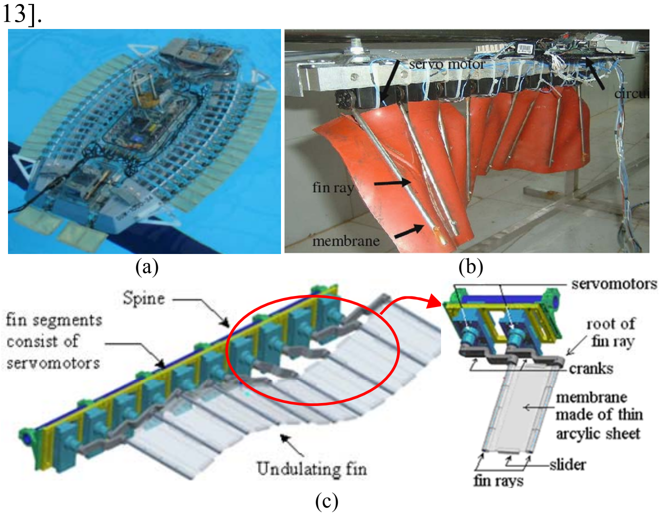

Fig. 1. Examples of fish robots with long undulatory fin propulsors: (a) Robotic Squid (version III, [3]); (b) an undulating fin propeller and the test bed [11]; (c) Design of a undulatory fin with free baseline (adopted from [14]).

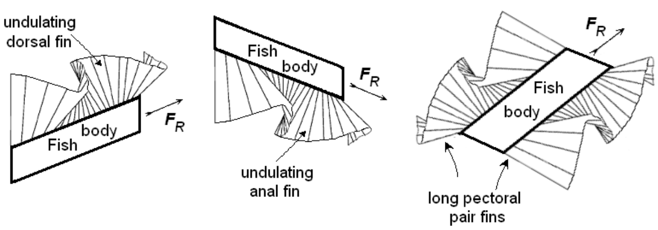

Fig. 2. Fundamental structures of undulatory fins found in amiiform, gymnotiform and rajiform swimmers respectively [4].

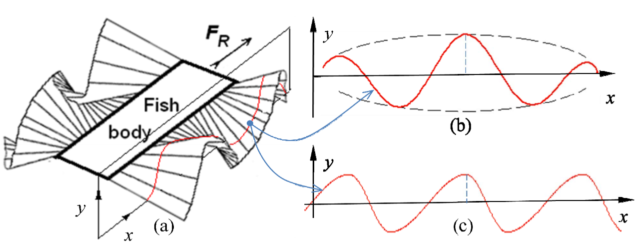

Fig. 3. Modeling of waveform on a long undulatory fin: the profile of intersection line (red curve in (a)) between fin surface and the x-y plane is normally modeled as a harmonic wave (b) or non-harmonic wave (c).

In another type of design, the fin rays are connected on a free baseline through cranks, as shown in Fig. 1c. Each crank is directly driven by a motor and baseline moves as all cranks oscillate. The membrane covering fin rays can be rigid or flexible. Fig. 1c shows a case that membrane made of rigid sliders are adopted as the propulsor which is also proved effective in terms of propulsion [4]. In this paper, we do not intend to compare the propulsive functions of them but focus on the modeling and control of motions. Configuration of long fins on fish robots follows the case in real fishes. Single or paired long fins are commonly adopted in robotic fishes. Fig. 2 shows three fundamental structures in robotic fishes with long fin propulsor.

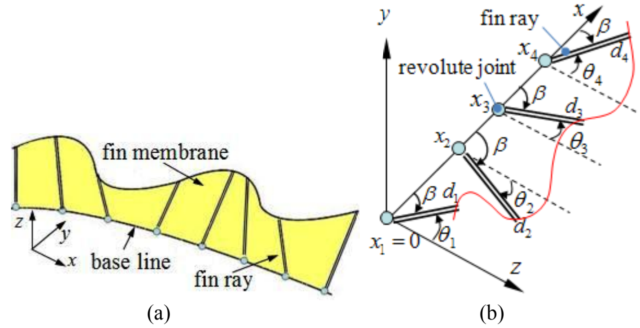

Fig. 4. (a) Mechanism schematic of the long fins [4] and (b) Fin ray arrangement on the baseline where x_i is the position of a fin ray, d_i is the length of fin ray or crank, θ_i is the oscillatory angle, i=1, 2, 3, ... and β reflects the extent of inclination.

The undulaotory motion generated by a long fin is commonly modeled as traveling wave. As illustrated by Fig. 3a, at an arbitrary time instance during steady swimming the intersection line of fin surface and the x-y plane is assumed as *harmonic* waveform (Fig. 3b) or *non-harmonic* waveform (Fig. 3c), which depends on different types of gait functions. Those gait functions control the oscillations of fin rays to produce the travelling wave on the long fin. In this paper, the term *harmonic* refers to that the frequency of fin ray in one oscillation cycle is kept constant while *non-harmonic* means in one oscillation cycle the frequency varies. Although the fin designs for different fish robots are not exactly the same, they share the same topological structure: a simple parallel mechanism Fig. 4. The methods discussed in this paper are based on this mechanism instead of a specific fish robot. Therefore, they are expected to be generic for control of different types of robots propelled by long undulatory fins.

III. Modeling of Undulatory Fin Motion Based on Coupled Nonlinear Oscillators

Harmonic gait is the most commonly used in oscillation control of fin rays and modeled by sinusoidal functions [3, 14-16]. Coordination of multiple fin ray motions is achieved through well-defined phase difference between adjacent fin rays. However, there are limitations with these functions in online motion planning. In this section we develop a framework that models the undulatory fin motion for fish robots with undulatory fin propulsors by using coupled nonlinear oscillators. The oscillator is also modified to generate non-harmonic gait pattern. We then discuss the synchronization of multiple oscillators by designing inter-couplings.

A. Gait Generator Based on Nonlinear Oscillators

High performance aquatic locomotion needs online motion planning to achieve the swimming adaptation to the environment around the fish robots. This property can hardly be achieved with sinusoidal function based gait control because the online changing of parameters (phase lag, frequency or amplitude of a sine generator) may cause discontinuity or instability of motions. For example, when a fish robot performs accelerating swimming by increasing the frequency of fin ray oscillation, the sine gait function is not suitable. To illustrate this, we record the oscillatory motion of a fin ray driven by a servomotor with rotary encoder in experiment and show the result in Fig. 5. At an arbitrary time instance t^*, the oscillation frequency is changed. It can be observed that the online changing of the frequency results in instability of motion.

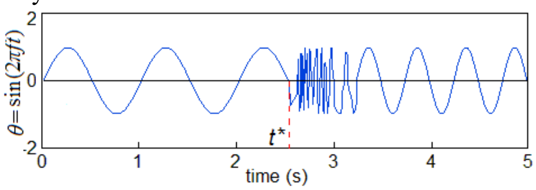

Fig. 5. An example of motion instability recorded in experiment. A fin ray is driven by a servo motor and controlled with a sine function. The angular position is shown and normalized to 1. The change of frequency is changed from 1Hz to 2Hz at t^*.

The smooth transition of swimming gaits is important for robots because it ensures there is no jerk or discontinuity in locomotion. It can be solved by using a nonlinear oscillator as the gait generator. Here the Hopf oscillator is selected, which is defined by:

$$\dot{u} = k(A^2 - u^2 - v^2)u - 2\pi f v \tag{1}$$

$$\dot{v} = k(A^2 - u^2 - v^2)v + 2\pi f u \tag{2}$$

where u and v are two state variables of the oscillator which are all functions of time t, A is a positive number that determines the amplitude of the steady state oscillation, f is the oscillation frequency and k is a positive constant which regulates the speed of convergence.

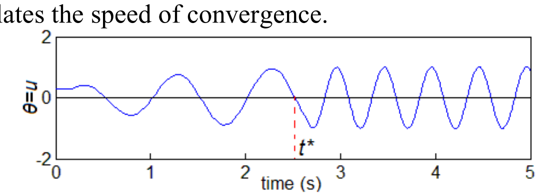

Fig. 6. An example of fin ray motion controlled by Hopf oscillator. State u of the oscillator is used as the control signal, i.e. $\theta(t)=u(t)$. The change of frequency is changed from 1Hz to 2Hz at t^*.

Fig. 6 shows the motion of a fin ray controlled by Hopf oscillator. The state of the oscillator, u, is adopted as the gait generator. It can be seen that the oscillation naturally become faster with smooth transition, which is different from the case shown in Fig. 5. Another attraction of this nonlinear oscillator is its limit cycle behavior [17, 18]. The states u and v always asymptotically converge to a limit cycle with radius A as time gets large. These properties make the nonlinear oscillator an ideal tool to model the swimming gaits in fish robots.

B. *Generation of Different Gait Patterns*

The steady states output of the Hopf oscillator have harmonic patterns (Fig. 6). Therefore it is suitable to model the swimming gait depicted in Fig. 3b. Non-harmonic gait can also be achieved with Hopf oscillator by applying adaptive frequencies in one cycle of fin ray oscillation. It can be assumed that the asymmetry of the wave shape depicted in Fig. 3c is resulted from the different frequencies involved in one cycle. The switching of frequencies depends on the shape of the non-harmonic wave. Here we assume that there are two different frequencies in the oscillation accounting for the fast rising phase and slow decline phase respectively (Fig. 3c). The uni-polar sigmoid function is employed to perform the switching of frequencies. Then the frequency is defined by:

$$f = \frac{f_{\text{nominal}}}{2\alpha} + \frac{f_{\text{nominal}}(2\alpha - 1)}{2\alpha(1-\alpha)(1+e^{-\tau v})} \tag{3}$$

where the f_{nominal} is the nominal frequency of the fin ray oscillation and it equals to the reciprocal of the period T, α is a shape ratio that determines how much time the rising phase takes in one cycle and $0.5 \le \alpha < 1$, τ is a positive time constant that tunes the speed of switching, v is a state of oscillator defined in (1) and (2). Fig. 7 shows gait patterns generated by the nonlinear oscillator. With (3), both harmonic gait (α=0.5, i.e. rising phase equals decline phase) and non-harmonic gait (α>0.5) are obtained. Equation (3) gives a solution to the problem that the shape of undulatory wave can be adjusted by tuning of only one parameter, α. The sigmoid function ensures that the switching is continuous for α. Therefore, the gait pattern can also be online tuned smoothly.

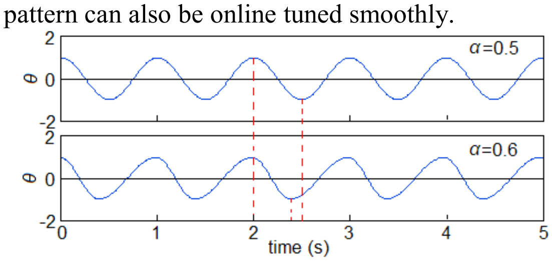

Fig. 7. The swimming gait patterns modeled by Hopf oscillator ($\theta=u$). In this example, frequency f=1Hz and τ=40. The amplitude is normalized to 1.

C. *Coordination of Multiple Fin Ray Motions*

Now we move to the next step: establish the connections for multiple oscillators to achieve coordinated swimming motion, i.e. synchronization of multiple nonlinear oscillators. In fin motion control, the motion pattern of all fin rays can be generated with the identical oscillator because those fin rays are similar in mechanical structure. Here we adopt the method used to model the CPGs for quadruped locomotion in [19] and modify it to derive the swimming gaits for biomimetic fins.

Without lose of generality, we consider the canonical form of identical nonlinear dynamical systems defined in (1) and (2) [17]:

$$\dot{\mathbf{X}}_i = F(\mathbf{X}_i) + \mathbf{P}_i = \begin{bmatrix} k(1-u_i^2 - v_i^2)u_i - 2\pi f v_i \\ k(1-u_i^2 - v_i^2)v_i + 2\pi f u_i \end{bmatrix} + \begin{bmatrix} p_{u,1} \\ p_{v,2} \end{bmatrix} \tag{4}$$

where $F(\mathbf{X}_i)$ represents a nonlinear system defined by the Hopf oscillator, $\mathbf{X}_i=(u_i, v_i)^T$ is the state vector of the i^{th} system and $\mathbf{P}_i=(p_{u,1}, p_{v,2})^T$ is a perturbation vector. The coupling of oscillators can be achieved in the way that one oscillator perturbs another and in order that a stable phase difference between the two oscillators are maintained. This phenomenon is a case of phase locking.

We firstly derive the coupling term for two Hopf oscillators and extend the couplings to multiple oscillators afterwards. For the ease of deriving, transform (4) into the phase-radius form in polar coordinates by using relations $u_i=r_i\cos\varphi_i$ and $v_i=r_i\sin\varphi_i$:

$$\dot{\varphi}_i = \omega + p_{\varphi,i} \tag{5}$$

$$\dot{r}_i = F_r(r_i, \varphi_i) + p_{r,i} \tag{6}$$

where $(r_i, \varphi_i)^T$ is the system state vector in polar coordinates, ω (ω=2πf) is the natural frequency of the unperturbed system, F_r is the dynamical system describing the evolution of r_i, $\mathbf{P}_{c,i}=(p_{r,i},\ p_{\varphi,i})^T$ corresponds to $\mathbf{P}_i$. Its two elements are components of the perturbation acting in direction of the radius and phase respectively. To simplify the problem, we assume that perturbation is one-directional: oscillator one perturbs oscillator two and there is no reverse perturbation. In this case, $\mathbf{P}_1$ equals to zero. $\mathbf{P}_2$ is unknown and needs to be derived.

In a phase locking regime, two oscillators evolve with stable phase difference φ_d:

$$\varphi_d = \varphi_2 - \varphi_1 \approx \text{const.} \tag{7}$$

Combine (5) and (7), we obtain

$$\dot{\varphi}_d = \dot{\varphi}_2 - \dot{\varphi}_1 = \omega_d + p_{\varphi,2} \tag{8}$$

where ω_d=ω_1-ω_2. In polar coordinate system, we can think that the phase difference is caused by perturbation acting in direction of phase, i.e. tangential to the limit cycle. This direction for oscillator two (unperturbed) is given by:

$$\mathbf{e}_{\varphi,2} = \frac{\dot{\mathbf{X}}_2}{|\dot{\mathbf{X}}_2|} = (-\sin\varphi_2,\ \cos\varphi_2)^T \tag{9}$$

The perturbation on the phase is obtained by

$$p_{\varphi,2} = \mathbf{P}_2^T \cdot \mathbf{e}_{\varphi,2} \tag{10}$$

The two oscillators are synchronized with a constant phase difference φ_d after a short transient phase evolving (from 0 to t_0). Therefore, in steady state we have:

$$\lim_{t\to\infty} \int_{t_0}^{t} \dot{\varphi}_d dt = 0 \tag{11}$$

The integration of (11) over time t can be done implicitly by integration over φ_2 in the steady state of the system. If we assume that the system evolves into the steady state since φ_2=Φ_0 (at t=t_0), then phase locking is maintained after φ_2=Φ>Φ_0 (t>t_0). Equation (11) can be rewritten as:

$$\dot{\varphi}_{d,res} = \lim_{\Phi\to\infty} \int_{\Phi_0}^{\Phi} \dot{\varphi}_d d\varphi_2 = \sum_{j=0}^{\infty} \int_{\Phi_0+2j\pi}^{\Phi_0+2(j+1)\pi} \dot{\varphi}_d d\varphi_2 = 0 \tag{12}$$

The phase locking with arbitrary phase difference is achieved through the coupling between two oscillators. The following coupling scheme is employed (adopted from [19]):

$$\mathbf{P}_2 = \varepsilon \mathbf{QRX}_1 = \varepsilon r_1 \left[0 \quad \cos(\varphi_1 + \gamma)\right]^T \tag{13}$$

where ε is a positive constant that determines the coupling strength, γ is the rotation angle, $\mathbf{Q}$ is the coupling matrix and $\mathbf{R}$ is the rotation matrix which are defined by:

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \text{ and } \mathbf{R} = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix}$$

By substituting (13) into (10), we obtain the perturbation $p_{\varphi,2}$ on the phase. Then substitute the result into (12), we can find the phase difference by integration:

$$\dot{\varphi}_{d,res} = 2\pi\omega_d + \varepsilon r_1 \pi(\cos\varphi_d \cos\gamma + \sin\varphi_d \sin\gamma) = 0 \tag{14}$$

By solving (14) we obtain:

$$\varphi_d = \cos^{-1}\left(-\frac{2\omega_d}{\varepsilon r_1}\right) + \gamma \tag{15}$$

It can be seen that in steady state (ω_d≈0) the phase difference is completely determined by the rotation angle γ. In order to achieve the arbitrary phase difference, we can first choose a proper γ. Substitute (15) into (13), the coupling term $\mathbf{P}_2$ is obtained:

$$\mathbf{P}_2 = \left[0 \quad \varepsilon(u_1 \sin\varphi_d + v_1 \cos\varphi_d)\right]^T \tag{16}$$

The synchronization of multiple oscillators is an extension of two mutually interacting oscillators. Based on (15) and (16), the undulatory swimming motion can be implemented on multi-DOF long fins with the following gait generator:

$$\theta_i(t) = A_i u_i \tag{17}$$

$$A_i = \frac{f_e(x_i)}{d_i} \tag{18}$$

$$\begin{bmatrix} \dot{u}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} k(1-u_i^2-v_i^2)u_i - 2\pi f v_i \\ k(1-u_i^2-v_i^2)v_i + 2\pi f u_i \end{bmatrix} + \varepsilon \begin{bmatrix} 0 \\ u_{i-1}\sin\varphi_d + v_{i-1}\cos\varphi_d \end{bmatrix}$$

$$i = 1, 2, 3, \ldots, n \tag{19}$$

$$\varphi_d = -\frac{2\pi m}{n} \tag{20}$$

$$\varepsilon = \begin{cases} 0 & i = 1 \\ \text{positive const.} & i > 1 \end{cases} \tag{21}$$

where n is the number of DOF (actuators) involved in fin movement, m is the number of waves on the long fin, A_i is the amplitude of the i^{th} oscillator. Equation (17)-(21) can be thought as model of CPG controlling the undulatory swimming. Output equation (17) generates harmonic gait control signals for fin rays. Combined with (3), the non-harmonic wave is also achieved.

IV. Experiments on an Eight-DOF Biomimetic Fin

In this section, the CPG model is applied to the control of an eight-DOF undulatory fin prototype. The purpose is to test the performance of the gait generator given by (17)-(21). The advantages of the nonlinear oscillator based approach discussed in Section I is illustrated through the experiments.

A. Experimental Setup

The experiments were conducted on a biomimetic undulatory fin prototype (Fig. 8a). Eight fin ray modules are equally spaced along the body and driven by eight servo motors respectively (n=8). The mechanism is similar as what is shown in Fig. 1c. The detailed mechanical design can be found in [4].

Equations (17)-(21) is solved in LabVIEW™ development environment and implemented with the CompactRIO™ embedded controller (Fig. 8b). Gait signals obtained from (17) are coded into 50Hz PWM signals to control the servo motor. The nonlinear differential equations are solved with the fourth-order Runge-Kutta method [20]. Given that (0, 0) is a stable equilibrium point of the Hopf oscillator the initial value should avoid this point [17]. They can be set as any values near the limit cycle.

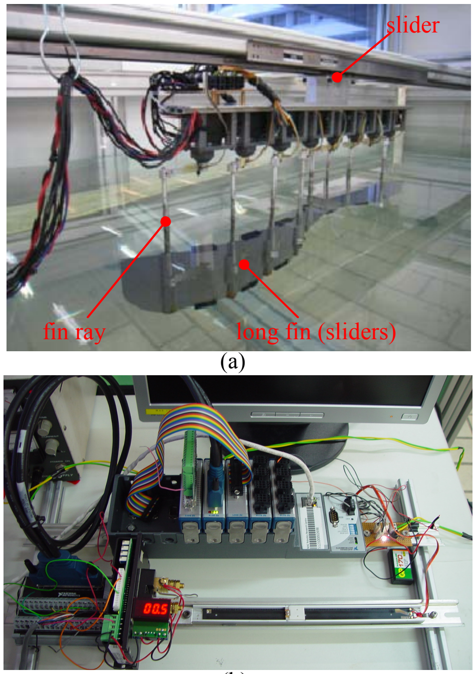

Fig. 8. Experimental setup for the undulatory fin motion control based on coupled nonlinear oscillators: (a) a biomimetic long fin prototype submerged in a water tank and (b) the CompactRIO™ embedded controller.

B. *Configuration of Gait Generators*

The CPGs defined by (19) are connected in the way that the behavior of one CPG is perturbed by the previous one in order to maintain a stable phase difference between them, which forms a one-way chain structure. For the experimental platform shown in Fig. 8a, the CPG model is used to generate eight gait signals to control the oscillation of fin rays, which can be illustrated in Fig. 9.

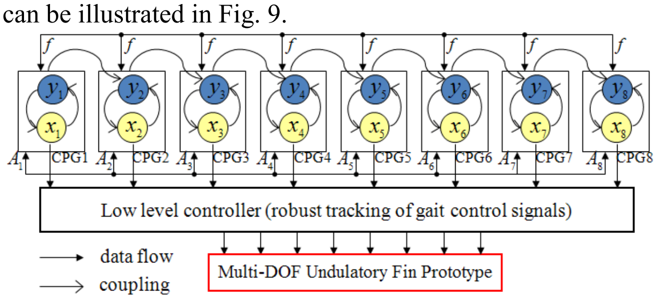

Fig. 9. Control architecture based on CPGs. The CPGs (oscillators) are connected with a serial chain structure.

In our experiments, we found that the coupling scheme given in Fig. 9 works well in steady state swimming. However, the gait transition between forward swimming gait and backward gait under this chain structure is slow. The reason is that each oscillator is synchronized with the previous one and the lag dynamics propagates and accumulates one by one. Upon application in experiment, an alternative solution is to use the coupling scheme shown in Fig. 10. All oscillators are synchronized to the first one with different stable phase lags depending on the position of the associated fin ray on the long fin. It is observed in experiments that the actual transition time is significant reduced with the revised scheme.

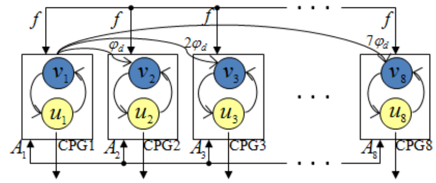

Fig. 10. A revised coupling scheme for multiple oscillators

C. *Experimental Results*

Experiments were conducted to test the swimming performance under CPG-based control. The testing includes several aspects: i) online change of wave numbers on the long fin by tuning m (Fig. 11a); ii) smooth accelerating /decelerating by increasing/decreasing the fin ray oscillation frequency f (Fig. 11b); iii) the natural switching between harmonic gait and non-harmonic gait through tuning of shape ratio α (Fig. 11c); and iv) the forward-backward swimming gait transition by flipping the sign of the phase difference φ_d (Fig. 11d).

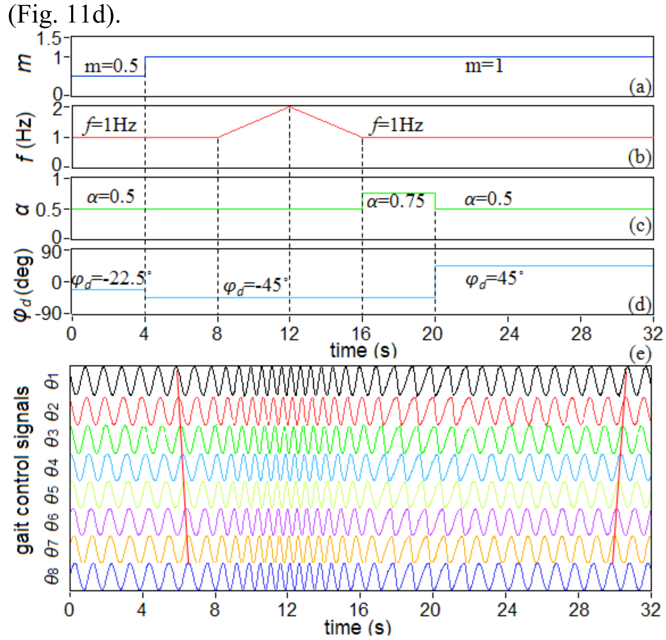

Fig. 11. The experimental swimming gait control signals generated by the coupled nonlinear oscillators (CPG model): (a)-(d) online tuning of control parametesrs and (e) the evolving of gaits for eight fin rays. The amplitudes are normalized to 1. Two red straight lines in (e) indicate the stable phase differences among fin ray oscillators. Other parameters in (17)-(21) are: n=8, ε=1 and k=10,

The evolving of the swimming gaits for eight fin rays are illustrated in Fig. 11e. When t<4s, harmonic (α=0.5) forward (φ_d<0) swimming gaits are applied to generate a half waveform (m=0.5) on the long fin. From t=4s, full length waveform is performed with the wave number jumps to one and the phase lag between two adjacent oscillators increases to 45° subsequently. We can observe a seamless evolving from Fig. 11e. During t=8s~12s, we tested the accelerating and decelerating process. The swimming reaches the highest speed at t=12s. The oscillations of all fin rays smoothly get faster, then smoothly slower. Afterwards, the gait pattern naturally transits from harmonic pattern to non-harmonic pattern by changing α. From t=20s the natural reverse change also illustrated. Finally the transition of forward gaits to backward gaits are performed staring form t=20s. The two red lines in Fig. 11e shows the difference: in forward swimming, the motion of the i^{th} oscillator lags behind the $(i+1)^{th}$ while in backward swimming the motion of the i^{th} oscillator leads the $(i+1)^{th}$.

Note that the control parameters illustrated in Fig. 11a-d are modeled with piecewise functions in experiments. It can be seen that even if those parameters are not smoothly tuned, he gaits are still continuous and smooth. This is one of the most attractive properties of the CPG based method. By applying the gait control signals shown in Fig 11e, smooth and natural locomotion on the long fin prototype is obtained in experiments.

V. CONCLUDING REMARKS

Kinematic modeling of undulatory fin motion and its application in swimming motion control of fish robots are discussed in this paper. Experiment shows that the coupled nonlinear oscillator based method has advantages in terms of the online generation of swimming gaits. We hope that this method can form a framework for modeling and control of multi-DOF undulatory fin motions. Although the hydrodynamic aspects of the locomotion, for example, the adaptation to actual flow situation and the energy efficient swimming, are not included, the method discussed in the present paper is fundamental for performance improvement in those aspects. Firstly, this method provides a simple way to online switching of motion patterns, which can be achieved by tuning corresponding parameters as illustrated in Fig. 11. Secondly, the evolving of the gaits maintains smooth and continuous as those parameters vary. These features make the closed-loop swimming control possible. For example, feedback control laws can be developed to tune the parameters in Fig. 11a-d. In this case, sensory feedback of water environment is incorporated into the modeling framework, which will be useful to achieve autonomous or adaptive locomotion. For another potential research direction, the amplitudes of all fin rays are not necessarily the same. We use the same amplitudes in the present paper just to illustrate how the model works. The model allows all amplitudes can be different and independently controlled. The real-time tuning of those amplitudes can influence the swimming efficiency, which is also an interest in future.

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