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Motion Control of an Autonomous Vehicle Based on Wheeled Inverted Pendulum Using Neural-adaptive Implicit Control

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***Abstract*—Wheeled inverted pendulum (WIP) models have been widely used in the field of autonomous robotics and intelligent vehicles. A novel transportation system, WIP-car is proposed in this paper, which is composed of a mobile wheeled inverted pendulum system, a driven chair, an acceleration pedal and a deceleration pedal, which are used to drive the chair forward or backward such that the car can be accelerated or decelerated. The neural-adaptive implicit control is designed for dynamic balance and stable tracking of desired trajectories of WIP-car. Neither the dynamics nor the dimension of the regulated system is required to be known, while the relative degree of the regulated output is assumed to be known. Under the assumption that WIP-car is feedback linearizable, adaptive neural network is introduced to cancel the inversion dynamics error. Simulation results demonstrate that the system is able to track reference signals satisfactorily with all closed loop signals uniformly bounded.**

I. INTRODUCTION

Wheeled inverted pendulum models have attracted a lot of research attention recently [4], [5], [6]. Many practical systems based on WIP models have been proposed, such as JOE [4], B2 [9], Segway [8], etc. Among these systems, the Segway PT has been proven to be a popular personal transporter. The WIP system has been successfully applied in Segway vehicles, however, one deficiency of Segway system is that drivers can only stand on the Segway vehicles during driving, which is not convenient for a long-time operation. Therefore, it is more convenient that a seat can be mounted on the vehicle such that the operation can drive WIP like a car. Another limitation of Segway is that users need stand upright during moving, especially when accelerating or climbing a slope, such that the moving forward-to-backward will be uncomfortable. To overcome these problems, a new mobile wheeled transportation system called WIP-car in Figs. 1 and 2 is introduced in this study. A novel structure including a WIP system and a driven chair by pedals is introduced into the design of WIP-car system. The acceleration/decelleration pedals are mounted under the human feet, and the pedals can drive the chair forward or backward such that the WIP-car is accelerated or decelerated.

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It is apparent that the motion of a transportation system based on wheeled inverted pendulum is governed by under-actuated configuration i.e. the number of control inputs are less than the number of degrees of freedom to be stabilized [7]. This makes it difficult to apply the conventional robotics approach for controlling the Euler-Lagrange systems. Although wheeled inverted pendulums systems are intrinsically nonlinear, it is often possible to obtain a feedback linearized model of the system. If the system is operating around an operating point, and the signals involved are small, a linear model that approximates the nonlinear system in the region of operation can be obtained. Several techniques for the design of controllers and analysis techniques for linear systems were applied. In [4], dynamics was derived using a Newtonian approach and the control was designed based on the dynamic equations linearized around an operating point. In [10], dynamic equations of the inverted pendulum were studied involving pitch and rotation angles of the two wheels as the variables of interest, and in [11] a linear controller was designed for stabilization considering robustness as a condition. In [12], a linear stabilizing controller was derived by a planar model without considering yaw. In [16], the exact dynamics of two-wheeled inverted pendulum was investigated, and linear feedback control was developed on the dynamic model.

Neural Network systems have been credited in robotics controls and applications as powerful tools capable of providing robust controllers for systems [15], [13], [19], [14]. In practical control applications, it is desirable to have systematic methods of ensuring stability, robustness, and performance of the overall system. Neural network implicit control approaches have been developed in [17], [2], and [3]. However, the above mentioned papers concern little on the under-actuated systems such as wheeled inverted pendulum transportation systems with nonholonomic constraints, which is to be investigated in the paper.

In this paper, we consider the implicit control for dynamic balance and stable tracking of desired trajectories of WIP-car, in which both the dynamics and the dimension of the regulated system may be unknown. However, the relative degree of the regulated output is assumed to be known. Under the assumption that WIP-car is feedback linearizable, adaptive neural network is introduced to cancel the inversion dynamics error. Ultimate boundedness of the tracking error is shown using Lyapunov's direct method.

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II. System Structure and Dynamics

The transportation system studied in this paper, WIP-car, as illustrated in Fig. 1, is obviously different from the Segway system. Fig. 2 shows the principle of the WIP-Car system. The following variables are in order to describe the system (refer to Figs. 1 and 2): τ_l, τ_r: the torques of the left and right wheels; α: the tilt angle of the pendulum; θ: the direction angle of the mobile platform; r: the radius of the wheels; d: the distance between the two wheels; $2l$: the length of the pendulum; m: the mass of the pendulum; M: the mass of the chair and human; m_w: the mass of each wheel; I_m: inertial moment of mobile pendulum; I_w: the inertia moment of each wheel; g: gravity acceleration.

From Fig. 1, we see that the chair is controlled by the acceleration/deceleration pedals. For the acceleration, the acceleration pedal drives the chair forward to produce the positive tilt angle of the pendulum, which can accelerate the velocity of the WIP-car, conversely, the deceleration pedal makes the chair backward to produce the negative tile angle of the pendulum, which can deceleration the velocity.

Consider the following wheeled inverted pendulum dynamics described by Lagrangian formulation:

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + D(t) = B\tau + f \qquad (1)$$

where $q = [q_1, q_2, q_3, q_4]^T = [x, y, \theta, \alpha]^T \in R^4$ is the vector of generalized coordinates with x, y as the position coordinates, θ as the heading angle, and α as the tilt angle as shown in Fig. 1. $M(q) \in R^{4\times 4}$ is the inertia matrix, $V(q,\dot{q})\dot{q} \in R^4$ is the vector of Coriolis and Centrifugal forces, $G(q) \in R^4$ is the vector of gravitational forces, $D(t) \in R^4$ is the vector of the bounded external from the environment, $B \in R^{4\times 2}$ is a full rank input transformation matrix and is assumed to be known because it is a function of fixed geometry of the system; $\tau \in R^2$ is the vector of control inputs, $f = J^T\lambda \in R^4$ denotes the vector of constraint forces, $J = [J_v, 0]^T \in R^4$ is Jacobian matrix with J_v defined later, and $\lambda \in R^1$ are Lagrangian multipliers corresponding to the nonholonomic constraints, respectively.

If q is partitioned into $q_v = [x, y, \theta]^T$ and α, we obtain

$$\begin{aligned} M(q) &= \begin{bmatrix} M_v & M_{v\alpha} \\ M_{\alpha v} & M_\alpha \end{bmatrix}, V(q,\dot{q}) = \begin{bmatrix} V_v & V_{v\alpha} \\ V_{\alpha v} & V_\alpha \end{bmatrix}, \\ G(q) &= \begin{bmatrix} G_v \\ G_\alpha \end{bmatrix}, D(t) = \begin{bmatrix} d_v \\ d_\alpha \end{bmatrix}, B(q) = \begin{bmatrix} B_v & 0 \\ 0 & B_\alpha \end{bmatrix} \\ \tau &= \begin{bmatrix} \tau_v & 0 \end{bmatrix}^T \end{aligned}$$

where M_v and M_α describe the inertia matrices for the mobile platform and the inverted pendulum, respectively, $M_{v\alpha}$ and $M_{\alpha v}$ are the coupling inertia matrices of the mobile platform and the inverted pendulum, and V_v and V_α denote the Centripetal and Coriolis torques for the mobile platform and the inverted pendulum, respectively. $V_{v\alpha}$ and $V_{\alpha v}$ are the coupling Centripetal and Coriolis torques of the mobile platform and the inverted pendulum. G_v and G_α are the gravitational torque vectors for the mobile platform and the inverted pendulum, respectively. τ_v is the control input vector for the mobile platform, and $d_v(t)$ and $d_\alpha(t)$ denote the external disturbances on the mobile platform and the inverted pendulum, respectively [4].

Assumption 2.1: The WIP-car is subjected to known nonholonomic constraints.

Remark 2.1: In actual implementation, we can adopt the methods of producing enough friction between the wheels of the mobile platform and the ground such that the assumption of nonholonomic constraints holds.

Remark 2.2: The external disturbance is time-varying but bounded, i.e., $sup_{t\geq 0}\|D(t)\| \leq c$, where c is a finite positive constant.

A. Reduced Dynamics

The vehicle subjected to nonholonomic constraints can be expressed as

$$J_v \dot{q}_v = 0 \qquad (2)$$

where $J_v = [\cos(\theta),\ -\sin(\theta),\ 0]$ is the kinematic constraint matrix. Assume that the annihilator of the co-distribution spanned by the covector fields J_v is an 1-dimensional smooth nonsingular distribution Δ on R^2. This distribution Δ is spanned by a set of smooth and linearly independent vector fields $H_1(q)$ and $H_2(q)$, i.e., $\Delta = \mathrm{span}\{H_1(q),\ H_2(q)\}$, which in the local coordinates satisfy the following relation

$$H^T J_v^T = 0 \qquad (3)$$

where $H = [H_1(q), H_2(q)] \in R^{3\times 2}$. Note that $H^T H$ is of full rank. Constraint equation (2) implies the existence of vector $\dot{\varsigma} = [\omega, v]^T \in R^2$ with ω representing the component of the angular velocity of the platform perpendicular to the line of wheel centres and v representing the magnitude of the velocity of the mid-point of the wheel centres, in other words, the heading velocity of the platform, such that

$$\dot{q}_v = R(q)\dot{\varsigma}. \qquad (4)$$

Considering (4) and its derivative, letting $\zeta = [\varsigma, \alpha]^T$, and multiplying both sides of (1) by H^T to eliminate J_v^T, the dynamics of wheeled inverted pendulum can be expressed as

$$M_1(\zeta)\ddot{\zeta} + V_1(\zeta,\dot{\zeta})\dot{\zeta} + G_1(\zeta) + D_1 = B_1\tau \qquad (5)$$

$$\begin{aligned} M_1(\zeta) &= \begin{bmatrix} R^T M_v H & R^T M_{v\alpha} \\ M_{\alpha v} R & M_\alpha \end{bmatrix} \\ V_1(\zeta,\dot{\zeta}) &= \begin{bmatrix} R^T M_v \dot{R} + R^T V_v R & R^T V_{v\alpha} \\ M_{\alpha v}\dot{R} + V_{\alpha v} R & V_\alpha \end{bmatrix}, \\ G_1(\zeta) &= \begin{bmatrix} R^T G_v \\ G_\alpha \end{bmatrix} \\ D_1 &= \begin{bmatrix} R^T d_v \\ d_\alpha \end{bmatrix}, B_1\tau = \begin{bmatrix} R^T B_v \tau_v \\ 0 \end{bmatrix}. \end{aligned}$$

By exploiting the physical properties of mobile wheeled inverted pendulum embedded in the dynamics of $M_1(q)$,

$V_1(q,\dot{q})$, and $G_1(q)$ building on Lagrangian approach, one can design a controller delivering better performance than without which will be explained below. According to the structure of the dynamics of the wheeled inverted pendulum by Lagrangian formulation, we know that:

$$\begin{aligned} M_1(\zeta) &= \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32}(\zeta_3) & m_{33} \end{bmatrix}, D_1 = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \\ V_1(\zeta,\dot{\zeta}) &= \begin{bmatrix} v_{11} & 0 & v_{13} \\ 0 & 0 & v_{23} \\ v_{31} & 0 & 0 \end{bmatrix}, B_1\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ 0 \end{bmatrix} \\ G_1(\zeta) &= \begin{bmatrix} 0 & 0 & g_3 \end{bmatrix} \end{aligned} \tag{6}$$

where m_{22}, m_{33} are unknown constants, m_{11}, m_{23}, $m_{32}(\zeta_3)$, v_{11}, v_{13}, v_{23}, v_{31}, $g_3(\zeta_3)$, and d_1, d_2, d_3 are unknown functions.

For clarity, define the tracking errors and the filtered tracking errors as $e_j = \zeta_j - \zeta_{jd}$, $r_j = \dot{e}_j + \Lambda_j e_j$, $\dot{\zeta}_j = \dot{\zeta}_{jr} + r_j$, $\ddot{\zeta}_j = \ddot{\zeta}_{jr} + \dot{r}_j$, where Λ_j is a positive number, and $j = 1, 3$.

III. Problem Formulation and Control Design

It is observed that the dynamics of wheeled inverted pendulum (5) can be represented by the nonlinear non-affine MIMO form as followings:

$$\dot{\zeta} = F(\zeta, u),\ Y = H(\zeta) \tag{7}$$

where $\zeta = [\zeta_1, \zeta_2, \zeta_3] \in R^3$ is system states, $Y \in \mathbb{R}^{n-l}$ with $n = 4$ and $l = 1$ denoting the system output, and it has derivatives up to third order. A matrix of Y and its derivatives is constructed as $[Y, Y^{(1)}, Y^{(2)}] \in \mathbb{R}^{(n-l)\times 3}$. The symbol $u \in \mathbb{R}^{n-l}$ denotes the system input, function $H:\ \mathbb{R}^3 \to \mathbb{R}^{n-l}$ is a partially unknown function, and function $F:\ \mathbb{R}^{3\times(n-l)} \to \mathbb{R}^{(n-l)}$ is a partially unknown vector field with respect to input u.

System (7) is a general description of the dynamics of the nonlinear WIP-car, for which the control input is nonaffine. We see that affine nonlinear systems and linear systems are special cases of (7), such that by designing a controller for (7), we actually include more general systems as well.

Define $\Theta_j(\zeta) = L_f^{j-1} H(\zeta)$ for $j = 1, \cdots, 3$, where $L_f H$ denotes the Lie derivative of the function $H(\zeta)$ with respect to the vector field $F(\zeta, u)$. As $n - l > 2$, we see that the system is input-output linearizable with strong relative degree such that there exists function Θ_4 independent of u and that the mapping $\Theta(\zeta) = [\Theta_1(\zeta),\ \Theta_2(\zeta), \Theta_3(\zeta)]$ has a Jacobian matrix which is nonsingular for all $z \in \Omega_\zeta$. Thus, $\Theta(\zeta)$ is a diffeomorphism on Ω_ζ. Let $\phi = [\Theta_1(\zeta),\ \Theta_2(\zeta), \Theta_3(\zeta)]$ and $\vartheta = \Theta_4(\zeta)$, then system (7) can be expressed in the normal form as follows:

$$\begin{aligned} \dot{\vartheta} &= \mathcal{Z}(\phi, \vartheta) \\ \dot{\phi}_j &= \phi_{j+1},\quad j = 1, 2 \\ \dot{\phi}_3 &= \mathcal{N}(\phi, \vartheta, u) \\ Y &= \phi_1 \end{aligned} \tag{8}$$

where $\mathcal{Z}(\phi, \vartheta) = [\phi, L_f\Theta_{\rho+1}(\zeta), \cdots, L_f\Theta_{n-l}(\zeta)]$, $\mathcal{N}(\phi, \vartheta, u) = L_f^3 H$ is C^1 for $(\phi, \vartheta, u) \in R^{(n-l)\times(n-l+1)}$ and $\zeta = \Theta^{-1}(\phi, \vartheta)$, for $(\phi, \vartheta, u) \in V = \{(\phi, \vartheta, u) | (\phi, \vartheta) \in \Theta(\Omega_\zeta), \mathcal{U} \in \Omega_u\}$.

The control objective is that the output Y can track a desired output $Y_d(t)$ such that the tracking error converges to a neighborhood of zero, i.e.$\|Y(t) - Y_d(t)\| \le \delta$, with a small δ, while all the states and the control are bounded. The reference trajectory $Y_d(t)$ is given by the following reference model

$$\begin{aligned} &\dot{\phi}_{di} = \phi_{d(i+1)}, 1 \le i \le \rho - 1 \\ &\dot{\phi}_{d\rho} = \mathcal{F}_d(\phi_d) \\ &Y_d = \phi_{d1} \end{aligned} \tag{9}$$

where $\rho \ge 3$ is a constant index, $\phi_d = [\phi_{d1},\ \phi_{d2},\ \cdots,\ \phi_{d\rho}] \in \mathbb{R}^{(n-l)\times\rho}$ are the state matrix of the reference system, $Y_d \in \mathbb{R}^{n-l}$ is the system output and $F_d : \mathbb{R}^{(n-l)\times\rho} \to \mathbb{R}^{n-l}$ is a known function.

Assumption 3.1: The reference trajectory $Y_d(t)$ and its third derivatives remain bounded.

Assumption 3.2: The zero dynamics of system (8) is established by $\dot{\vartheta} = \mathcal{Z}(0, \vartheta)$ and they are exponentially stable. In addition, there exist Lipschitz constants p_1 and p_2 for $\mathcal{Z}(\phi, \vartheta)$ such that

$$\|\mathcal{Z}(\phi, \vartheta) - \mathcal{Z}(0, \vartheta)\| \le p_1\|\phi\| + p_2,\ \forall\ (\phi, \vartheta) \in\ \Theta(\Omega_\zeta)$$

Under Assumption (3.2), by the converse Lyapunov theorem, there exists a Lyapunov function $V_0(\vartheta)$ which satisfies the following inequalities:

$$\gamma_1\|\vartheta\|^2 \le V_0(\vartheta) \le \gamma_2\|\vartheta\|^2 \tag{10}$$

$$\frac{\partial V_0}{\partial\vartheta}\mathcal{Z}(0, \vartheta) \le -\lambda_a\|\vartheta\|^2 \tag{11}$$

$$\|\frac{\partial V_0}{\partial\vartheta}\| \le \lambda_b\|\vartheta\| \tag{12}$$

where γ_1, γ_2, λ_a, λ_b are positive constants.

Lemma 3.1: [20](Global Implicit Function Theorem) Assume that $f : R^n \times R^m \to R^m$ is a continuous mapping and it is continuously differentiable in the second variable $u \in R^m$. If $|[\frac{\partial f(x,u)}{\partial u}]_{ii}| - \sum_{j\ne i}|[\frac{\partial f(x,u)}{\partial u}]_{ij}| \ge d, \forall(x, u) \in R^n \times R^m, i = 1, \cdots, m$ for a fixed constant $d > 0$, then there exists a unique mapping $g : R^n \to R^m$ such that $f(x, g(x)) = 0$. Moreover, this mapping g is continuous, then the obtained g is also continuously differentiable.

Remark 3.1: By using the Global Implicit Function Theorem, we take neural network as a function approximator which emulates a given nonlinear function up to a small error tolerance, where an analysis of Lyapunov functions for semi-globally uniformly ultimate boundedness will be involved.

Define vectors ϕ_d and Υ as $\phi_d = [Y_d,\ Y_d^{(1)}, Y_d^2], \phi_d \in R^{(n-l)\times 3}$, $\Upsilon = \phi - \phi_d$, and define the filtered tracking error as $\mathbf{r} = \Upsilon[\Lambda, 1]^T$, $\mathbf{r} \in R^{n-l}$, where the vector $\Lambda = [\lambda_1,\ \lambda_2,]$ so that $s^2 + \lambda_2 s + \lambda_1$ is Hurwitz and as a result, $\Upsilon \to 0$ as

$\mathbf{r} \to 0$. Then, the time derivative of the filtered tracking error can be written as

$$\dot{\mathbf{r}} = \mathcal{N}(\phi, \vartheta, \mathcal{U}) - Y_d^{(3)}(t) + \Upsilon[0 \ \ \Lambda]^T \tag{13}$$

where $\mathbf{r} = [\mathbf{r}_1, \ \mathbf{r}_2, \ \cdots, \ \mathbf{r}_{n-l}]^T$.

Add and subtract $\hat{\mathcal{N}}(\phi, \vartheta, u)$ on the right hand side of equation (13), we obtain

$$\begin{aligned} \dot{\mathbf{r}} &= \mathcal{N}(\phi, \vartheta, u) - \hat{\mathcal{N}}(\phi, \vartheta, u) + \hat{\mathcal{N}}(\phi, \vartheta, u) \\ &\quad - Y_d^{(3)}(t) + \Upsilon[0 \ \ \Lambda]^T \\ &= \Delta + \nu - Y_d^{(3)}(t) + \Upsilon[0 \ \ \Lambda]^T \end{aligned} \tag{14}$$

where Δ is modeling error: $\Delta = \mathcal{N}(\phi, \vartheta, u) - \hat{\mathcal{N}}(\phi, \vartheta, u)$, and ν is the pseudo control, which is defined as $\nu = \hat{\mathcal{N}}(\phi, \vartheta, u)$. According to [1], the pseudo control is chosen as

$$\nu = -K_p \mathbf{r} + Y_d^{(3)} - \Upsilon[0 \ \ \Lambda]^T + \nu_{dc} - \nu_{ad} \tag{15}$$

where K_p is diagonal positive, $Y_d^{(3)}$ is the third derivative of the reference output, ν_{dc} is the output of a linear dynamic compensator, and ν_{ad} is the adaptive control signal designed to cancel Δ. Substituting (15) into the output dynamics (14), we have

$$\dot{\mathbf{r}} = -K_p \mathbf{r} + \nu_{dc} - \nu_{ad} + \Delta \tag{16}$$

According to [1], the following linear dynamic compensator is introduced

$$\dot{\eta} = A_c \eta + b_c e(t), \ \ v_{dc} = c_c \eta + d_c e(t) \tag{17}$$

where $\eta \in R^{n-1}$ and η needs to be at least of dimension $n-1$. We can obtain the augmented tracking error dynamics

$$\dot{X} = \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\eta} \end{bmatrix} = \mathcal{A} \begin{bmatrix} \mathbf{r} \\ \eta \end{bmatrix} + \mathcal{B}[v_{ad} - \Delta] \tag{18}$$

$$\mathcal{A} = \begin{bmatrix} K & -bc_c \\ 0 & A_c \end{bmatrix} + \begin{bmatrix} -bd_c & 0 \\ b_c & 0 \end{bmatrix} \tag{19}$$

$$\mathcal{B} = \begin{bmatrix} b & 0 \end{bmatrix}^T$$

where

$$b = \begin{bmatrix} 0 & 0 & 0 & \ldots & 1 \end{bmatrix}^T, c_c = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \end{bmatrix}^T$$

Theorem 3.1: [18] The LTI system $\dot{x} = A_m x$ is asymptotically stable if and only if, given any symmetric positive-definite matrix Q, there exists a symmetric positive-definite matrix P, which is the unique solution of the so-called Lyapunov equation $PA_m + A_m^T P = -Q$.

A. Neural Network Approximation

In this paper, the following RBFNN [17] [18] is used to approximate the continuous function $h_{rbf}(Z) : R^q \to R$: $h_{rbf}(Z) = W^T S(Z)$, where the input vector $Z \in \Omega \subset R^q$, weight vector $W = [w_1, w_2, ..., w_l]^T \in R^l$, the NNs node number $l > 1$, and $S(Z) = [s_1(Z), ..., s_l(Z)]^T$, with $s_i(Z)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_i(Z) = \exp\left[-(Z - \chi_i)^T (Z - \chi_i)/\beta_i^2\right], \quad i = 1, 2, ..., l \tag{20}$$

where $\chi_i = [\chi_{i1}, \chi_{i2}, ..., \chi_{iq}]^T$ is the center of the receptive field and β_i is the width of the Gaussian function. It has been proven that RBFNN can approximate any continuous function over a compact set $\Omega_Z \subset R_q$ to arbitrary any accuracy as [17], [18] $h_{rbf}(Z) = W^{*T} S(Z) + \varepsilon, \quad \forall Z \in \Omega_Z$, where W^* is an ideal constant weight vector, and ε is the approximation error. The stability results obtained in NN control literature are semiglobal in the sense that, as long as the input variables Z of the NNs remains within some pre-determined compact set $\Omega_Z \subset R^q$ where the compact set Ω_Z can be made as large as desired, there exists controller(s) with sufficiently large number of NN nodes such that all the signals in the closed-loop remain bounded.

Assumption 3.3: On the compact set Ω_z, the ideal NN $\|W^*\| \leq w_m$.

There exists an ideal constant weight W^* such that $|\varepsilon| \leq \varepsilon^*$ with constant $\varepsilon^* > 0$ for all $Z \in \Omega_Z$. The ideal weight vector W^* is an artificial quantity required for analytical purposes. W^* is defined as the value of W that minimizes $|\varepsilon|$ for all $Z \in \Omega \subset R^q$, i.e. $W^* = \arg\min_{W \in R^l} \{\sup_{Z \in \Omega_Z} |h(Z) - W^T S(Z)|\}$. In general, the ideal NN weight, W^*, is unknown though constant, while its estimate, $\hat{W}$, is used for controller design as will be shown later.

Considering the unknown continuous function Δ can be approximated by the neural network $\Delta = W^T S(x) + E(x)$, where the input vector $x = [\phi, \theta, u]^T$ are the input variables to the neural networks; $S(Z) \in R^p$ is a vector of known continuous basis functions, with p denoting the number of neural nodes; $W \in R^p$ is adaptable weights; and $E(x)$ is the approximation error which is bounded over the compact set Ω_x, i.e., $\|E(Z)\| \leq \bar{\varepsilon}$, where $\bar{\varepsilon} > 0$ is an known constant.

B. Control Design and Its Stability

In (18), the adaptive signal is chosen to be

$$\nu_{ad} = \hat{W}^T S(x) - \eta(t) \tag{21}$$

where $\hat{W}$ is estimate of W^* that is updated according to the following adaptation laws

$$\dot{\hat{W}} = -\Gamma S(x) \mathcal{B}^T P X \tag{22}$$

where P is the solution of the Lyapunov equation $\mathcal{A}^T P + P\mathcal{A} = -Q$ for some $Q > 0$, and $\mathcal{A}$ can be found in (19), and

$$\eta(t) = -KX/\|X\| \tag{23}$$

with the positive constant K satisfying $K \geq \bar{\varepsilon}$. Consider the following Lyapunov function candidate $V = \frac{1}{2} X^T P X + \tilde{W}^T \Gamma^{-1} \tilde{W}$, where $\tilde{W} = \hat{W} - W$, its time derivative is given by

$$\dot{V} = -\frac{1}{2} X^T Q X + X^T P B[\nu_{ad} - \Delta] + \tilde{W}^T \Gamma_i^{-1} \dot{\tilde{W}} \tag{24}$$

Considering $X = \tilde{X} + \hat{X}$, we have

$$\begin{aligned}\dot{V} &\leq -\frac{1}{2}\lambda_{min}(Q)\|X\|^2 + \tilde{W}^T\Gamma_i^{-1}\dot{\tilde{W}} \\ &\quad + X^T P\mathcal{B}[\nu_{ad} - W^{*T}S(x) - E(x)] \\ &\leq -\frac{1}{2}\lambda_{min}(Q)\|X\|^2 + X^T P\mathcal{B}\tilde{W}^T S(x) \\ &\quad - X^T P\mathcal{B}E - X^T P\mathcal{B}\eta(t) + \tilde{W}^T\Gamma_i^{-1}\dot{\tilde{W}} \quad (25)\end{aligned}$$

Consider (22) and (23), we can obtain

$$\begin{aligned}\dot{V} &\leq -\frac{1}{2}\lambda_{min}(Q)\|X\|^2 + X^T P\mathcal{B}\tilde{W}^T S(x) \\ &\quad + \|X\|\|P\mathcal{B}\|\|E\| - K\|P\mathcal{B}\|\|X\| + \tilde{W}^T\Gamma_i^{-1}\dot{\tilde{W}} \\ &\leq -\frac{1}{2}\lambda_{min}(Q)\|X\|^2 + \tilde{W}^T(\Gamma_i^{-1}\dot{\tilde{W}} + S(x)\mathcal{B}^T PX) \\ &\leq -\frac{1}{2}\lambda_{min}(Q)\|X\|^2 < 0 \quad (26)\end{aligned}$$

From the previous stability analysis, we know that ϕ is bounded. We assume that

$$\|\phi\| \leq \|\phi\|_{max} \quad (27)$$

where $\|\phi\|_{max}$ is a positive constant. For all the above, because of the approximation of NNs, $\|X\|$ converges to a small set including the origin as $t \to \infty$. According to Assumption 3.2, there exists a Lyapunov function $V_0(\vartheta)$. Differentiating $V_0(\vartheta)$ yields

$$\dot{V}_0(\vartheta) = \frac{\partial V_0}{\partial \vartheta}\mathcal{Z}(0,\vartheta) + \frac{\partial V_0}{\partial \vartheta}[\mathcal{Z}(\phi,\vartheta) - \mathcal{Z}(0,\vartheta)]. \quad (28)$$

Noting (10)-(11), and (28) can be written as

$$\dot{V}_0(\vartheta) \leq -\lambda_a\|\vartheta\|^2 + \lambda_b\|\vartheta\|(L_\phi\|\phi\| + L_f). \quad (29)$$

Noting (27), we have $\dot{V}_0(\vartheta) \leq -\lambda_a\|\vartheta\|^2 + \lambda_b\|\vartheta\|(L_\phi\|\phi\|_{max} + L_f)$. Therefore, $\dot{V}_0(\vartheta) \leq 0$, whenever $\|\vartheta\| \geq \frac{\lambda_b}{\lambda_a}(L_\phi\|\phi\|_{max} + L_f)$. By letting $L_\vartheta = \frac{\lambda_b}{\lambda_a}\|\vartheta\|(L_\phi\|\phi\|_{max} + L_f)$, it can be shown that ϑ is bounded.

IV. Simulation

The wheeled inverted pendulum shown in Fig. 1 is subjected to the following constraints: $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$. Using Lagrangian approach, we can obtain the reduced dynamics for $q_v = [x, y, \theta]^T$, α, $J = [\sin\theta, -\cos\theta, 0, 0]$, and $\dot{\zeta} = [\omega, \upsilon, \dot{\alpha}]^T$ as

$$\begin{aligned} M_1 &= \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & \frac{1}{2}(m+M)lc\alpha \\ 0 & \frac{1}{2}(m+M)lc\alpha & (m+M)l^2 + I_m \end{bmatrix}, \\ V_1 &= \begin{bmatrix} \frac{1}{2}(m+M)l^2 s2\alpha\dot{\alpha} & 0 & \frac{1}{2}(m+M)l^2\omega s2\alpha\dot{\alpha} \\ 0 & 0 & -(m+M)ls\alpha\dot{\alpha} \\ -\frac{1}{2}(m+M)l^2\omega s2\alpha & 0 & 0 \end{bmatrix} \\ G_1 &= \begin{bmatrix} 0 & 0 & -(m+M)gls\alpha \end{bmatrix}^T \end{aligned}$$

where $c\alpha = \cos\alpha$, $s\alpha = \sin\alpha$, $m_{11} = \frac{d^2}{4r^2}(2M_w r^2 + 2I_w + \frac{4r^2}{d^2}I_m + 4\frac{(m+M)r^2l^2}{d^2}\sin^2\alpha)$ and $m_{22} = \frac{1}{r^2}(2M_w r^2 + 2J_w + (m+M)r^2)$. In the simulation, we choose the parameters $I_w = 0.5kgm^2$, $M_w = 0.2kg$, $I_m = 2.5kgm^2$, $M = 50.0kg$, $m = 5.0kg$, $l = 1.0m$, $d = 0.5m$, $r = 0.5m$, $\zeta(0) = [-0.2, 0, \pi/18]^T$, $\dot{\zeta}(0) = [0.0, 0.1, 0.0]^T$. The disturbances from environments on the system are introduced as $1.0\sin(t)$, $1.0\cos(t)$ in the simulation model. The desired trajectories are chosen as $\theta_d = 0.2t$ rad, $\alpha_d = 0$ rad, and the initial velocity is 0.1m/s. The system state is observed through the noisy linear measurement channel, and zero-mean Gaussian noises are added to the state information. All noises are assumed to be mutually independent. The noises have variances corresponding to a 5% noise to signal radio. The neural-adaptive network control is without any knowledge of system dynamics under the random noise inputting to the controllers. The input vector is $Z_1 = [\ddot{\zeta}_{1r},\ \dot{\zeta}_{1r},\ \dot{\zeta}_3,\ \zeta_3]^T \in R^4$. Neural networks $\hat{W}_1^T S_1(Z_1)$ contains 32 nodes, with centers $\mu_l(l = 1, ..., l_1)$ evenly spaced in $[-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0] \times [-1.0, 1.0]$. Neural networks $\hat{W}_3^T S_3(Z_3)$ contains 729 nodes, with centers $\mu_l(l = 1, ..., l_2)$ evenly spaced in $Z_3 = [\ddot{\zeta}_{3r},\ \dot{\zeta}_{3r},\ r_3,\ \dot{\zeta}_1,\ \dot{\zeta}_3,\ \zeta_3]^T \in R^6$, $[-1.0, 1.0] \times [-0.1, 0.1] \times [-1.0, 1.0] \times [-3.0, 3.0] \times [-1.0, 1.0] \times [-1.0, 1.0]$. The design parameters of the above controllers are: $k_{1P} = 200.0$, $\Lambda_1 = 5.0$, $k_{2P} = 6000.0$, $\Lambda_2 = 10.0$, $\gamma_{11} = \gamma_{12} = 3000$, $\gamma_{31} = \gamma_{32} = 3500$, $\hat{W}_1(0) = (0.1)$, $\hat{W}_2(0) = (0.1)$.

The direction angles tracked by three control approaches are shown in Fig. 3, and the input torques are shown respectively in Fig. 6, the tilt angles for the dynamic balance and the stable velocities under three control approaches are shown in Figs. 4 and 5, respectively. From these figures, even if without the prior knowledge of the system, we can obtain good performance by the proposed neural network control.

V. Conclusions

In this paper, a transportation system consisting of a MWIP system and a drivable chair is proposed and an adaptive neural network implicit control design has been carried out for dynamic balance and stable tracking of desired trajectories of WIP-car, in the presence of unmodelled dynamics, or parametric/functional uncertainties.

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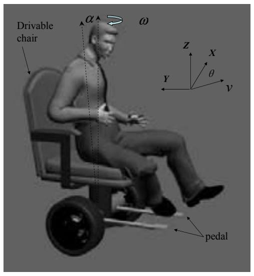

Fig. 1. A transportation system based on wheeled inverted pendulum

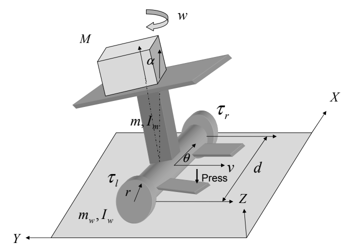

Fig. 2. Mobile wheeled inverted pendulum with drivable chair

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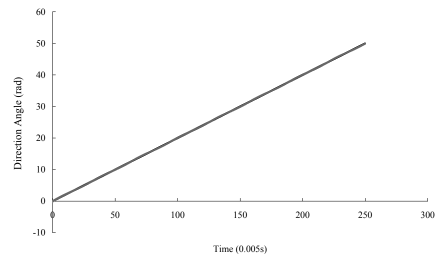

Fig. 3. Tracking the direction angle by the neural-adaptive network

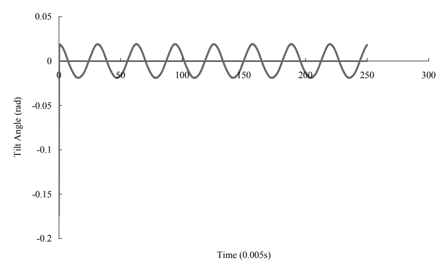

Fig. 4. Tracking the desired tilt angle by neural-adaptive network

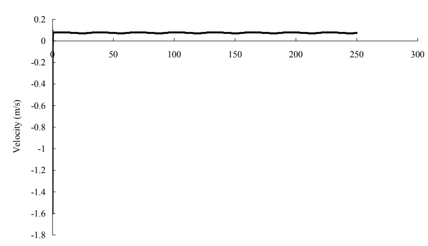

Fig. 5. The stable velocity by the neural-adaptive network control

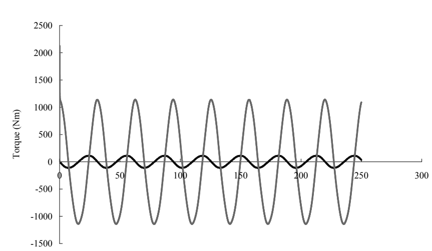

Fig. 6. Input torques by neural-adaptive network control