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Resource Constrained Multirobot Task Allocation with A Leader-Follower Coalition Method

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Abstract—**This paper investigates the multirobot task allocation (MRTA) problem for a group of heterogeneous mobile robots. The robots and tasks are characterized by resources as required by task execution. The robots are required to generate optimal solutions for the MRTA problem while forming coalitions to meet the resource constraints imposed by tasks. A leader-follower based coalition methodology is developed, with detailed discussions on leader selection, coalition forming and refinement algorithms. It is shown that the resource constrained task allocation problem can be well resolved by the proposed leader-follower coalition algorithms. Simulations performed on a mobile robot group demonstrate the effectiveness of the proposed approach.**

I. Introduction

Multirobot task allocation (MRTA) problem, which aims for mapping the grouped robots to accomplish a set of tasks, has been a hot research topic in recent years. A well-known fault tolerant MRTA architecture is ALLIANCE [1], which utilized motivational behaviors to solve SR-ST-TA [1] type allocation problems under dynamic environment. Some other typical SR-ST MRTA frameworks with either instantaneous assignment or time-extended assignment can be found in [3]-[8]. M+ proposed in [3] utilized the negotiation amongst the robots in team to implement the tasks. A behavior-based methodology was incorporated into the broadcast of local eligibility (BLE) [4] to allocate tasks. The approach in [5] implemented the tasks under a resource efficient manner using a publish/subscribe methodology, while [6] and [7] assigned dynamic roles to the robots for implementing the tasks. Robots were assumed to be self-interested agents in [8], where a market-based approach was proposed to maximize the individual robot profits. A cooperation strategy was designed for unmanned air vehicles for multi-target interception in [9].

Some other MRTA approaches can be found in [10]-[12]. In [10], a fault tolerant adaptive scheme which focuses on accommodating task uncertainties was proposed. In [11], a stochastic algorithm for allocating a swarm of homogeneous robots to several tasks was presented. The research in [12] focused on the task representation methods, and studied the complex task trees which could be solved by many possible ways.

Forming coalitions to solve MRTA problem has been a hot research topic in recent years [13]-[19]. Coalition based MRTA architectures partition the robots into different sub-team and assign a task to each sub-team. These sub-teams are called coalitions [17].

Coalition based methodology aims for solving ST-MR type task allocation problems. Forming coalitions for solving task allocation problems was first proposed in [13], where agents were motivated to maximize benefits of the system as a whole. Tightly coordinated task allocation method, which could also be viewed as coalition based allocation, was originated in [14]-[16]. In [14], a combinatorial bid allocation method based on the robot and task capabilities was proposed. The Hoplites proposed in [15] utilized a market-based method for solving the tight coordination problems. In [16], heterogeneous robots were dynamically picked up to implement tightly coordinated tasks.

Inspired by [13], the research in [17] specialized the coalition algorithm in multirobot domain, and demonstrated the effectiveness of multiagent coalition algorithm on a group of mobile robots. Also inspired by [13], the researches in [18] and [19] coordinated the sensing and motion scheme amongst the robots to synthesize multirobot behaviors in accomplishing the tasks, and generated hierarchical frameworks for the MRTA problems.

In most of existing works, robots are assumed to be able to produce utilities for every task automatically, and the MRTA frameworks focused on developing optimal solutions to maximize the total utilities (or minimize the total costs) of the robot group. However, resource constraints between robots and tasks have not been extensively considered yet when implementing the MRTA frameworks. Due to limited ability of robots, the robot coalitions may fail in the task execution when robots are subject to insufficient resources.

This paper reports our most recent research on using a leader-follower coalition method to solve the resource constrained MRTA problem. The resource-based MRTA concept was inspired from our numerous robotic applications and experiments, e.g., multirobot localization and mapping using

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[1] In this paper, we adopt the notations proposed in [2], where ST: single-task robots, MT: multi-task robots; SR: single-robot tasks, MR: multi-robot tasks; IA: instantaneous assignment, TA: time-extended assignment.

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different resources [20][21], multirobot formations performed on a group of heterogeneous robots [22][23], and model identification of micro air vehicles [24]. With the leader-follower strategy [23], one robot is designated as the leader, and the other robots are controlled to follow their respective leaders with given relationships.

The contributions of this paper are twofold. First, we propose a quantitative resource based modeling framework for robots and tasks, and analyze the resource constraints when robots implement tasks. Second, we incorporate the leader-follower strategy into the coalition methodology, based on which a leader-follower coalition method for the MRTA problem is generated. Simulations performed on multirobot systems demonstrate the effectiveness of the proposed leader-follower coalition method.

The remainder of the paper is organized as follows. Section II presents the resource-based modeling frameworks. Section III describes the leader-follower coalition method. The analysis on the solution qualities is presented in section IV. Simulation results are given in section V. Finally, conclusion of this paper is given in section VI.

II. Resource Constrained System Modeling

Consider the ST-MR-IA type MRTA problems. The issues to be investigated in this paper can be briefly categorized as follows.

Assume that the robot group is comprised of several mobile robots r_i, where $i = 1,2,\ldots,I$ is the index of robots and I is the number of robots in the group. Assign the robot group to implement several tasks and each task is denoted by t_j, where $j = 1,2,\ldots,J$ is the index of tasks, and J is the total number of tasks to be implemented. The MRTA problem aims in mapping the I robots into the J tasks.

In this paper, we assume that robots possess different resources and are all heterogeneous with each other, and further, the same kind resource may be different on different robots. Since resources possessed by each individual robot are different and limited, robots are required to work cooperatively and form coalitions to accomplish every single task.

A. Resource-based Modeling Methodology

Denote s_k as one kind of resource required in performing the tasks, where $k = 1,2,\ldots,K$ representing the index of resource, and K is the total number of the resource types.

For every kind of resource s_k, there exists a corresponding set $\mathcal{P}_k$ that contains all the functional elements related to the resource s_k. $\mathcal{P}_k$ is expressed as

$$\mathcal{P}_k = \{(p_l)_k \mid (p_l)_k > 0, l = 1,2,\ldots,L_k\} \tag{1}$$

where $(p_l)_k$ denotes the maximum value of l-th functional element of the resource s_k, l is the index for representing different functional elements of the resource s_k, and L_k is the total number of functional elements in the set $\mathcal{P}_k$.

We then model every robot r_i by the resources on this robot. Let $\mathcal{N}_i$ represents the number of different resources on r_i, expressed as

$$\mathcal{N}_i = \{n_{i,k} \mid n_{i,k} \geq 0, k = 1,2,\ldots,K\} \tag{2}$$

where $n_{i,k}$ represents the number of the resource s_k on r_i. Further, let $\mathcal{PS}_i$ represents the value of all the resource functional elements on r_i, expressed as

$$\mathcal{PS}_i = \bigcup \mathcal{P}_{i,k} = \{\mathcal{P}_{i,k} \mid k = 1,2,\ldots,K\} \tag{3}$$

where $\mathcal{P}_{i,k}$ describes the value of the functional elements of the resource s_k on robot r_i, i.e.,

$$\mathcal{P}_{i,k} = \{(p_l)_{i,k} \mid (p_l)_{i,k} \geq 0, l = 1,2,\ldots,L_k\} \tag{4}$$

where $(p_l)_{i,k}$ is the value of the l-th functional element of the resource s_k on r_i, which is quantified within $[0,1]$ by the maximum value $(p_l)_k$ of s_k in (1).

In a similar manner, for the task t_j, we model the task imposed resource constraints as $\mathcal{N}_j$ and $\mathcal{PS}_j$, where

$$\mathcal{N}_j = \{n_{j,k} \mid k = 1,2,\ldots,K\} \tag{5}$$

where $n_{j,k}$ denotes the number of the resource s_k required by the task t_j. $\mathcal{PS}_j$ is the constraints imposed on the value of the resource functional elements, expressed as

$$\mathcal{PS}_j = \bigcup \mathcal{P}_{j,k} = \{\mathcal{P}_{j,k} \mid k = 1,2,\ldots,K\} \tag{6}$$

and

$$\mathcal{P}_{j,k} = \{(p_l)_{j,k} \mid (p_l)_{j,k} \geq 0, l = 1,2,\ldots,L_k\} \tag{7}$$

Fig. 1 gives an example of the resource modeling based on four kinds of resources: robot arm, camera, sonar, and robot motion ability. Denote w_{lift} as the lift weight of robot arm, which corresponds to s_1; α_{view}, d_{view} and f_{sampling} as the view angle, view distance and sampling frequency of camera, which corresponds to s_2; d_{det} as the detect range of sonar sensor, which corresponds to s_3; $v_{\max}$, $\omega_{\max}$ as the maximal linear and angular velocities of every robot, which corresponds to s_4. Also in Fig. 1, we find that one robot r_i, which possesses 1 robot arm, 1 camera and 14 sonar sensors, is quantified by s_1 - s_4 and expressed by $\mathcal{N}_i$ and $\mathcal{PS}_i$. One task t_j, where 2 robot arms and 3 cameras are required in the task implementation, is expressed by $\mathcal{N}_j$ and $\mathcal{PS}_j$.

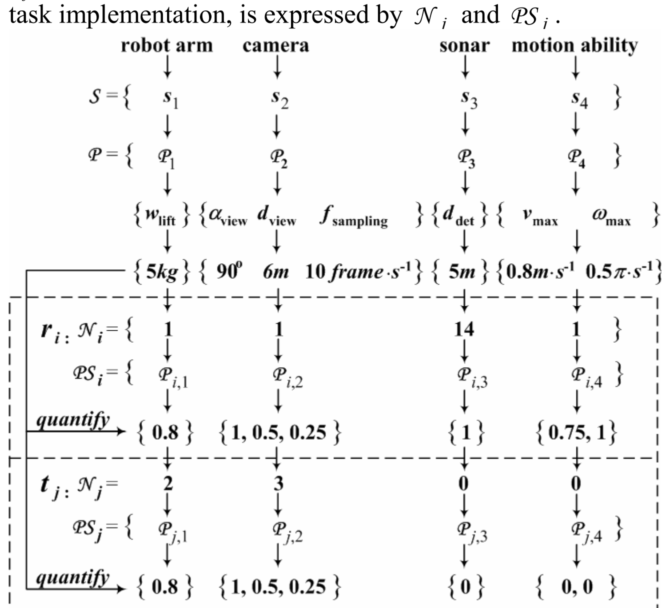

Fig. 1. Resource modeling

For simplicity, we assume that when the value of every functional element $(p_l)_k$ in $\mathcal{P}_k$ increases, the utility of s_k in performing the relevant task increases. For example, increase of w_{lift} implies that the robot can manipulate heavier objects, and increase on f_{sampling} implies that the robot can process images more efficiently. Also, we arrange all the resources from the most expensive and rare resources to the most common resources in the modeling method. Such arrangement facilitates the leader selection process in the coalition organization process.

B. Resource Utility

Consider one resource s_k on robot r_i. We claim that the resource s_k is useful for t_j, if $\mathcal{P}_{i,k}$ in (4) can satisfy the constraints imposed by $\mathcal{P}_{j,k}$ in (7). Particularly, for all $(p_l)_{i,k} \in \mathcal{P}_{i,k}$ in (4) and $(p_l)_{j,k} \in \mathcal{P}_{j,k}$ in (7), if

$$(p_l)_{i,k} \geq (p_l)_{j,k} \tag{8}$$

the resource s_k satisfies the resource constraints imposed by t_j, i.e., the resource s_k on r_i is useful for t_j.

Define

$$f\left((p_l)_{i,k}\right) = 1 - e^{-(p_l)_{i,k}} \tag{9}$$

to indicate the utility for implementing tasks of the *l*-th functional element of s_k on robot r_i. Further, define the utility of the resource s_k as

$$u_{i,k} = \frac{n_{i,k}}{L_k} \times \sum_{l=1}^{L_k} f\left((p_l)_{i,k}\right) \tag{10}$$

Note that only those resources that satisfy (8) can be utilized in (9) and (10). The utility of r_i for t_j can then be calculated by

$$u_i(t_j) = \sum_{k=1}^{K} \lambda_{j,k} u_{i,k} \tag{11}$$

where $\lambda_{j,k} \in [0,1]$ is the weight parameter for measuring the relative importance of different resources in performing t_j.

Remarks: An important property of $f\left((p_l)_{i,k}\right)$ in (9) is that when $(p_l)_{i,k}$ increases to certain extent, further increase of $(p_l)_{i,k}$ does not result in large increase to the utility function (similar to the size function in [17]).

III. Leader-Follower Coalition Methodology

In this paper, we solve the MRTA problem in an online manner, i.e., the tasks are online received and the allocation method must be produced online. Further, at every single instant, we assume that the tasks are prioritized. In this sense, the robot group needs only to handle the task with the highest priority, which helps pick up the most important tasks to implement first with the limited resources.

We adopt leader-follower strategies in the coalition organization process. The leaders, which possess the key resources (the most expensive or rare resources), play the key roles in the coalition organization process.

Denote the robot coalition organized by the leader r_i for task t_j as $C_i(t_j)$, where the subscript i implies that the coalition is led by r_i. $C_i(t_j)$ is constituted by a two-member set and expressed as follows

$$C_i(t_j) = \{\mathcal{L}_i(t_j), \mathcal{F}_i(t_j)\} \tag{12}$$

where $\mathcal{L}_i(t_j)$ and $\mathcal{F}_i(t_j)$ are the leader and follower set of $C_i(t_j)$, respectively. Since only one leader is allowed in every coalition, we have

$$\mathcal{L}_i(t_j) = \{r_i\} \tag{13}$$

The leader-follower coalition algorithm is iteratively implemented until all the tasks are allocated. During each allocation phase, the allocation of t_j is done with the following procedures: preliminary coalition forming; coalition refinement; coalition submission and task allocation.

A. Preliminary Coalition Forming

All leaders and their preliminary coalitions $C_i(t_j)$ are formed in the following two steps.

A.1 Nominate all the robots, which possess at least partial of key useful resource as required to perform t_j, as leaders. Construct the preliminary coalitions $C_i(t_j)$.

A.2 For the remaining robots, if their Euclidian distance with one leader r_i is less than a monitoring distance d, add this robot into the follower set $\mathcal{F}_i(t_i)$ of $C_i(t_i)$.

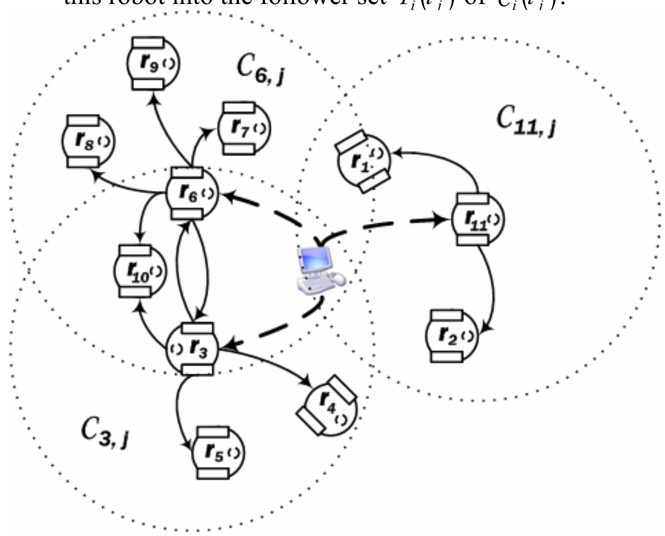

Fig. 2. Preliminary coalition forming

Fig. 2 shows an example of the preliminary coalition forming. Suppose r_3, r_6 and r_{11} are nominated as the leaders. The dotted circles represent the monitoring distance. By checking the distance amongst the robots, the leader-follower coalitions led by r_3, r_6 and r_{11} for t_j are given as follows

$$C_3(t_j): \mathcal{L}_3(t_j) = \{r_3\}, \mathcal{F}_3(t_j) = \{r_4, r_5, r_6, r_{10}\}$$
$$C_6(t_j): \mathcal{L}_6(t_j) = \{r_6\}, \mathcal{F}_6(t_j) = \{r_3, r_7, r_8, r_9, r_{10}\}$$
$$C_{11}(t_j): \mathcal{L}_{11}(t_j) = \{r_{11}\}, \mathcal{F}_{11}(t_j) = \{r_1, r_2\}$$

B. Coalition Refinement

Every preliminary coalition $C_i(t_j)$ is then refined to generate the submitted coalition $SC_i(t_j)$. The coalition refinement procedure aims in adding the necessary robots, which can provide useful resource for t_j, from $C_i(t_j)$ to $SC_i(t_j)$.

Typically, $SC_i(t_j)$ includes three members

$$SC_i(t_j) = \{\mathcal{L}_i^{SC}(t_j), \mathcal{F}_i^{SC}(t_j), u_i^{SC}(t_j)\} \tag{14}$$

where $\mathcal{L}_i^{SC}(t_j)$ and $\mathcal{F}_i^{SC}(t_j)$ are the leader and follower set of $SC_i(t_j)$, respectively, and $u_i^{SC}(t_j)$ is the coalition utility of $SC_i(t_j)$ submitted for bidding t_j.

The coalition refinement procedure is mainly implemented with the following steps:

B.1 For the task t_j, generate the useful resources and calculate the resource utilities for every robot in $C_i(t_j)$.

B.2 Move the leader robot r_i from $\mathcal{L}_i(t_j)$ to $\mathcal{L}_i^{SC}(t_j)$, use (8)-(11) to calculate the utility $u_i(t_j)$ of r_i for t_j, and add $u_i(t_j)$ into $u_i^{SC}(t_j)$.

B.3 For every kind of resource s_k, iteratively perform the following steps on $C_i(t_j)$ and $SC_i(t_j)$:

B.3.1 If robots in the current $SC_i(t_j)$ have sufficient useful resource s_k as required by the task constraint $n_{j,k}$ in $\mathcal{N}_j$, check the next type of resource s_{k+1}. Otherwise, check the remaining robots in $C_i(t_j)$ for s_k.

B.3.2 Calculate the utility for performing t_j of the remaining robots in $C_i(t_j)$. Since the task requirements imposed by $s_1,\ldots,s_{k-1}$ have already been satisfied, only the utilities of $s_k,\ldots,s_K$ are counted.

B.3.3 Check the resource s_k amongst the remaining robots in $C_i(t_j)$, following the decreasing order of the utilities calculated in step B.3.2. If one robot has s_k, move this robot from $\mathcal{F}_i(t_j)$ to $\mathcal{F}_i^{SC}(t_j)$, and add the newly calculated utility of this robot into $u_i^{SC}(t_j)$. Return to step B.3.1.

B.3.4 If all robots in this coalition cannot provide sufficient s_k for t_j, it is a conclusion that this robot coalition fails to submit a successful coalition due to insufficient useful resource s_k.

B.4 After checking all the resource, submit $SC_i(t_j)$ to the centralized server to bid for t_j.

C. Coalition Submission and Task Allocation

All successful submitted coalitions that satisfy the task resource constraints are gathered by the centralized server to allocate the task. Define $u(t_j)$ as the coalition utility which receives the task, and $I(t_j)$ as the number of robots involved in the task t_j. The principle to allocate the task is:

$$\begin{aligned}&\text{Maximize}: u(t_j)\\&\text{Subject to}: I(t_j) \text{ is minimized}\end{aligned} \tag{15}$$

(16) implies that when the coalitions with the smallest size are considered, t_j is allocated to the coalition which could produce maximal utility.

Remarks: The computational cost of the leader-follower coalition method is $O(I^2JK)$, since for every single task, the preliminary coalition algorithm yields a computational cost at $O(IJ)$, and the coalition refinement algorithm yields a cost at $O(I^2JK)$. The communication cost between the centralized server and the robots is $O(IJ)$, which mainly occurs when the tasks are announced to the robot group. The computational and communication cost of the proposed leader-follower coalition method is acceptable when compared to most existing MRTA methods [10].

IV. Solution Quality Analysis

Generally speaking, for all the robots attending J tasks, the task assignment problem can be globally optimally solved if the total utility is maximized, i.e.,

$$\begin{aligned}&\text{Maximize}: \sum_{j=1}^{J} u(t_j)\\&\text{Subject to}: I(t_0)+\sum_{j=1}^{J} I(t_j)=I\end{aligned} \tag{16}$$

where the robots without suitable tasks are allowed to be idle, and $I(t_0)$ is the number of robots in the idle task t_0. Furthermore, it is generally accepted that ST-MR-IA type MRTA problems can be viewed as a set partitioning problem (SPP), which is *NP*-hard [10]. In this paper, as revealed by step B.3.2 of the leader-follower coalition algorithm, when one robot is added into a coalition, the utilities of the remaining robots in this coalition may vary, which implies that the proposed MRTA problem can not be reduced to a simple transportation problem (TP), and it is sophisticated to obtain the global optimal solution for the proposed MRTA problem.

One possible solution for the MRTA problem is to generate all feasible coalitions and refine every single coalition. However, such algorithm is computationally difficult, which costs a computation burden of $O(2^I JK)$, and is much heavier when compared to the leader-follower method $O(I^2JK)$. To effectively solve the MRTA problem, we impose two extra constraints as follows to generate practical optimal solutions.

a) Always try to minimize the number of robots attending each task. Small robot coalitions imply that the robots, which are more suitable for the task, are chosen to attend the coalition. It must be noted that small robot coalitions will not decrease the solution robustness, since we can design more strict task-imposed constraints to enhance the robustness of the solutions.
b) The coalition tasks with higher priorities always receive better robot coalitions for implementation. Under the condition that the coalition size is minimized, always assign the task to the coalition with higher utility.

Proposition 4.1: For a given robot group, the coalition forming, refinement, submission and allocation algorithms described in section III can generate optimal result among greedy solutions in the sense that

a) The size of the robot coalition performing each coalition task, i.e., $I(t_j)$, is the smallest.
b) The highly prioritized coalition tasks are always greedily assigned to the robot coalitions with the higher utility.

Proof:

a) From step B.4 of the coalition refinement algorithm, we conclude that every submitted coalition is feasible to perform the task.
b) During the task allocation process, the tasks are assigned to the robot coalitions with the smallest size. Since first, from step B.3 of the coalition refinement algorithm, only the robots with the task required resource are added into the submitted coalitions. Second, when allocating the tasks, we always assign tasks to the submitted coalitions with the smallest size.
c) From step B.3 of the coalition refinement algorithm, every submitted coalition always includes the robots with higher utilities. Furthermore, robot coalitions with the

highest utilities are selected to perform the tasks. This implies the leader-follower coalition algorithm follows the greedy mechanism [10].

Based on the above analysis, we then conclude that the proposed leader-follower coalition methodology lead to optimal results among greedy solutions for the resource constrained task allocation problems.

■

V. Simulation

To verify the effectiveness of the proposed method, simulations are performed on a group of 10 heterogeneous mobile robots. The resources required in these tasks and the maximum value of the resource functional elements are listed as follows.

$$\begin{cases} s_1 = \text{robot arm},\ \mathcal{P}_1 = \{w_{\text{lift}} = 8kg\} \\ s_2 = \text{bumper},\ \mathcal{P}_2 = \{w_{\text{push}} = 20kg\} \\ s_3 = \text{container},\ \mathcal{P}_3 = \{w_{\text{carry}} = 60kg\} \\ s_4 = \text{camera}, \\ \mathcal{P}_4 = \{\alpha_{\text{view}} = 180^0, d_{\text{view}} = 6m, f_{\text{sampling}} = 10\,frame \cdot s^{-1}\} \end{cases}$$

Table I gives 3 tasks. t_1 is a task for area search and object manipulation, where one robot arm (s_1), one container (s_3) and two cameras (s_4) are required. Robot arm is the key resource for t_1; t_2 is a task for area exploration, where two cameras (s_4) are required, and camera is the key resource for t_2 ; t_3 is a task for cargo transportation, where one bumper (s_2), two containers (s_3) and two cameras (s_4) are required. Bumper is the key resource for t_3. The robot resources are given in table II.

The three tasks are assigned sequentially to the robot group. From table II, we find that 3 robots (r_1 , r_8 and r_{10}) have robot arms (s_1). When applying the preliminary coalition forming algorithm on the robot group for t_1, 3 preliminary coalitions led by r_1 , r_8 and r_{10} are generated

$$C_6(t_1): \mathcal{L}_6(t_1) = \{r_6\}, \mathcal{F}_6(t_1) = \{r_4, r_3, r_1, r_2, r_{10}\}$$
$$C_8(t_1): \mathcal{L}_8(t_1) = \{r_8\}, \mathcal{F}_8(t_1) = \{r_7, r_5, r_9, r_1, r_{10}\}$$
$$C_{10}(t_1): \mathcal{L}_{10}(t_1) = \{r_{10}\}, \mathcal{F}_{10}(t_1) = \{r_5, r_8, r_9, r_1, r_6\}$$

TABLE I
Task Resource Number and Value Requirements

		s_1	s_2	s_3	s_4
t_1	$\mathcal{N}_1$	1		1	2
	$\mathcal{PS}_1$	0.625		0.5	0.4, 0.4, 0.33
	$\mathcal{A}_1$	1		0.9	0.8
t_2	$\mathcal{N}_2$				2
	$\mathcal{PS}_2$				0.4, 0.4, 0.33
	$\mathcal{A}_2$				1
t_3	$\mathcal{N}_3$		1	2	2
	$\mathcal{PS}_3$		0.6	0.33	0.2, 0.2, 0.33
	$\mathcal{A}_3$		1	0.9	0.8

After applying the coalition refinement algorithm to these three coalitions, $C_6(t_1)$ fails to submit a successful coalition due to insufficient useful resource s_4 . $C_8(t_1)$ and $C_{10}(t_1)$ successfully generate the submitted coalitions as follows

$$\mathcal{SC}_8(t_1): \mathcal{L}_8^{SC}(t_1) = \{r_8\}, \mathcal{F}_8^{SC}(t_1) = \{r_1, r_7\}, u_8^{SC}(t_1) = 1.48$$
$$\mathcal{SC}_{10}(t_1): \mathcal{L}_{10}^{SC}(t_1) = \{r_{10}\}, \mathcal{F}_{10}^{SC}(t_1) = \{r_1, r_5, r_8\}, u_{10}^{SC}(t_1) = 1.55$$

Since $\mathcal{SC}_8(t_1)$ is comprised of 3 robots, $\mathcal{SC}_{10}(t_1)$ is comprised of 4 robots, and the utility of these two submitted coalitions is close, which implies that $\mathcal{SC}_8(t_1)$ is more suitable for t_1. t_1 is then assigned to $\mathcal{SC}_8(t_1)$, and the three-robot coalition is sent to implement the task.

We then allocate t_2 to the remaining robots. From table II, we find that 4 robots (r_2, r_4, r_5 and r_9) among the remaining robots have cameras (s_4), but the camera on r_2 does not satisfy the resource constraint imposed by t_2. Three preliminary coalitions led by r_4, r_5 and r_9 for t_2 are formed

$$C_4(t_2): \mathcal{L}_4(t_2) = \{r_4\}, \mathcal{F}_4(t_2) = \{r_2, r_3, r_6\}$$
$$C_5(t_2): \mathcal{L}_5(t_2) = \{r_5\}, \mathcal{F}_5(t_2) = \{r_9\}$$
$$C_9(t_2): \mathcal{L}_9(t_2) = \{r_9\}, \mathcal{F}_9(t_2) = \{r_5\}$$

The submitted coalitions for t_2 are listed as follows

$$\mathcal{SC}_5(t_2): \mathcal{L}_5^{SC}(t_2) = \{r_5\}, \mathcal{F}_5^{SC}(t_2) = \{r_9\}, u_5^{SC}(t_2) = 0.7$$
$$\mathcal{SC}_9(t_2): \mathcal{L}_9^{SC}(t_2) = \{r_9\}, \mathcal{F}_9^{SC}(t_2) = \{r_5\}, u_9^{SC}(t_2) = 0.7$$

TABLE II
Robot Resource Number and Functional Element Values

		s_1	s_2	s_3	s_4			s_1	s_2	s_3	s_4
r_1	$\mathcal{N}_1$	1		1		r_6	$\mathcal{N}_6$		1	1	
	$\mathcal{PS}_1$	0.375		0.5			$\mathcal{PS}_6$		0.6	1	
r_2	$\mathcal{N}_2$		1		1	r_7	$\mathcal{N}_7$				1
	$\mathcal{PS}_2$		1		0.25, 0.25, 1		$\mathcal{PS}_7$				1, 0.75, 0.33
r_3	$\mathcal{N}_3$			1		r_8	$\mathcal{N}_8$	1			1
	$\mathcal{PS}_3$			1			$\mathcal{PS}_8$	0.625			0.4, 0.4, 0.5
r_4	$\mathcal{N}_4$				1	r_9	$\mathcal{N}_9$		1		1
	$\mathcal{PS}_4$				0.75, 1, 0.33		$\mathcal{PS}_9$		0.3		0.4, 0.4, 0.5
r_5	$\mathcal{N}_5$			1	1	r_{10}	$\mathcal{N}_{10}$	1			
	$\mathcal{PS}_5$			0.33	0.4, 0.4, 0.5		$\mathcal{PS}_{10}$	1			

$C_4(t_2)$ fails to submit a successful coalition due to insufficient useful resource s_4. t_2 is then allocated to $SC_5(t_2)$. t_3 is finally allocated to the remaining robots, and only 5 robots (r_2, r_3, r_4, r_6 and r_{10}) are left. From table II, we find that r_2 and r_6 have useful bumpers (s_2) for t_3. The preliminary coalitions for t_3 are listed as follows

$$C_2(t_3): \mathcal{L}_2(t_3) = \{r_2\}, \mathcal{F}_2(t_3) = \{r_4, r_3, r_6, r_{10}\}$$
$$C_6(t_3): \mathcal{L}_6(t_3) = \{r_6\}, \mathcal{F}_6(t_3) = \{r_4, r_2, r_3, r_{10}\}$$

The submitted coalitions $SC_2(t_3)$ and $SC_6(t_3)$ are

$$SC_2(t_3): \mathcal{L}_2^{SC}(t_3) = \{r_2\}, \mathcal{F}_2^{SC}(t_3) = \{r_3, r_6, r_4\}, u_2^{SC}(t_3) = 2.44$$
$$SC_6(t_3): \mathcal{L}_6^{SC}(t_3) = \{r_6\}, \mathcal{F}_6^{SC}(t_3) = \{r_3, r_4, r_2\}, u_6^{SC}(t_3) = 2.26$$

Note that $u_2^{SC}(t_3) > u_6^{SC}(t_3)$, because r_2 has more useful resource s_2 than r_6. t_3 is then allocated to $SC_2(t_3)$.

The simulation results are summarized as follows

$$t_1 \to SC_8(t_1): \mathcal{L}_8^{SC}(t_1) = \{r_8\}, \mathcal{F}_8^{SC}(t_1) = \{r_1, r_7\}$$
$$t_2 \to SC_5(t_2): \mathcal{L}_5^{SC}(t_2) = \{r_5\}, \mathcal{F}_5^{SC}(t_2) = \{r_9\}$$
$$t_3 \to SC_2(t_3): \mathcal{L}_2^{SC}(t_3) = \{r_2\}, \mathcal{F}_2^{SC}(t_3) = \{r_3, r_6, r_4\}$$

We then performed another simulation to make a comparison to the leader-follower coalition method, and also, to testify the solution qualities of the leader-follower coalition method. In this simulation, we generated all feasible coalitions for every task first, and then selected the coalition based on the optimal principle (15). The results of this simulation are given as follows

$$t_1 \to \{r_1, r_7, r_8\}$$
$$t_2 \to \{r_5, r_9\}$$
$$t_3 \to \{r_2, r_3, r_4, r_6\}$$

It is seen that these results are the same as the results of the leader-follower coalition method, which proves the optimal properties of the leader-follower coalition methodology. However, as we discussed in section IV, this method takes a computation cost of $O(2^I JK)$, which is much heavier when compared to the computational cost of the leader-follower method $O(I^2 JK)$.

VI. Conclusion

In this paper, we have proposed a resource based modeling method for addressing the MRTA problem among a group of mobile robots. Robots are assumed heterogeneous with each other, and are required to work cooperatively to form coalitions to implement tasks. A leader-follower coalition methodology, which yields optimal solutions for the proposed problem, is presented to solve the resource constrained MRTA problem. The quality of the solution is analyzed in detail. Simulations are performed on a group of heterogeneous mobile robots to demonstrate the effectiveness of the proposed leader-follower coalition methodology.

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