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Evaluation of Grasps for 3D Objects with Physical Interpretations using Object Wrench Space

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Abstract— **In this paper, we present a practically useful and intuitive grasp quality measure that takes into account the shapes of object geometries and the torque limits of finger actuators. The proposed grasp quality measure is defined by the distance between the convex hulls of the absolute grasp wrench space (a-GWS) and the object wrench space (OWS), where a-GWS and OWS are, respectively, created by the active wrenches from the robot fingers with limited torque bounds and the uniform distribution of unitary normal disturbances on the surface of a polyhedral object. The computational algorithm for the grasp quality measure also yields the information of which spots of the object are fatal under the disturbance, which makes the algorithm practically useful. We demonstrate the validity of the proposed measure through numerical examples.**

I. Introduction

When grasping an object, we are often interested in knowing: to what amount the grasp can resist the disturbance and which spot of the grasped object is geometrically weak to the disturbance. In order to answer these primary concerns, the geometry of the object must be thoroughly incorporated into the grasp analysis. In fact, however, we are able to find only a few works taking the object geometry into account for grasp analysis such as [1]–[3], which motivated the present research in this paper.

A great number of works on grasp have been carried out more than three decades due to its importance not only in the robotic manipulation but also in design of fixtures for manufacturing. In the earlier times of the grasp research, the fundamental issue seemed to be the examination of whether a grasp is a force-closure or not [4]–[6]. Then researchers tried to find a way to evaluating what finger configurations and/or contact locations yield the best result in resisting the maximum amount of disturbance or producing an dexterous manipulation of the grasped object, which necessitated a proper grasp quality measure.

Research works on grasp quality measure could be categorized by: whether the grasp quality measure is dependent on or independent of tasks. As a task-independent grasp quality measure, the maximum sphere completely contained in the convex hull of grasp wrench space (GWS) was proposed by Kirkpatrick [7], and later the meanings of the convex hulls generated by the union and the Minkowski sum of primary grasp wrenches were investigated by Ferrari and Canny [8]. Handling the friction cone was a big difficulty in analyzing grasps of 3D objects since the friction cone in 3D space imposes strong nonlinear conditions [9]. Liu et al [10] proposed a method of linearization of the friction cone by approximating the friction as a polygonal pyramid with a finite number of sides. The task-dependent (in another term, the task-oriented) grasp quality measure has not been much investigated, compared with the task-independent one. The task-dependent grasp quality measure takes into consideration a particular condition of a task or a disturbance condition; for example, the disturbance from gravitational force due to self-weight acts only in the downward direction to the earth surface. Li and Sastry [11] introduced the concept of task ellipsoid to select the optimal grasp by taking into account the required wrench directions. Pollard [1] defined the object wrench space (OWS) by collecting the wrenches generated by a set of force applied on the surface of the object. Later, Borst et al. [2] jointly utilized OWS and the task ellipsoid to measure the grasp quality for discretized arbitrary 3D objects. Haschke et al. [12] formulated the problem of computing grasp quality measure in terms of linear matrix inequality. Strandberg and Wahlberg [3] also addressed a method of grasp quality evaluation; however, the measure value of the method is not given by a scalar value. As a relatively newer research issue, the robustness of the grasp stability has been studied. Prattichizzo et al. [13] proposed the grasp robustness measure to cope with some uncertainties in the grasp configuration. Roa and Suarez [14] recently studied the allowable range of error for fingertip location.

In spite of previous many researches, the existing grasp quality measures does not seem to appeal the human's intuition in various grasps situations. So, in this paper, we intend to propose an enhanced grasp quality measure which may convey a clearer physical meaning and is easily interpretable by human being. The outline of the rest of the paper is as follows. Some useful wrench spaces pivotal to the current grasp analysis are defined in Section 2; after that, our grasp quality measure is addressed as a main issue in Section 3; simulation results are presented in Section 4; and finally concluding remark is made in Section 5.

II. Useful Wrench Spaces

A. Absolute Grasp Wrench Space (a-GWS)

Consider an object which admits n-number of contact forces $\{\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_n\}$ acting in the normal directions of the object surface at positions $\{\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_n\}$, respectively, as shown in Fig.1. Each contact force produces a torque with respect to the object's center of mass (COM) such that

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$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{f}_i$; thus, a complete set of torques produced by all the contact forces is $\{\boldsymbol{\tau}_1, \boldsymbol{\tau}_2, \cdots, \boldsymbol{\tau}_n\}$. We assume that each robot finger contacts the object at a point with friction, so the friction cone means the appropriate region of all the admissible forces by a normal force. We denote $\mathrm{C}(\mathbf{p}_i)$ as the friction cone at $\mathbf{p}_i$ and $\mathrm{C}_a(\mathbf{p}_i)$ as the approximated friction cone by a pyramid with s-number of edges. If $\mathbf{f}_{ij}, i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, s$, denotes the j-th primitive force of $\mathrm{C}_a(\mathbf{p}_i)$, an arbitrary force within the friction cone such that $\mathbf{f}_i^a \in \mathrm{C}_a(\mathbf{p}_i)$ can be written as

$$\mathbf{f}_i^a = \sum_{j=1}^{s} \alpha_{ij} \mathbf{f}_{ij}, \tag{1}$$

with $\alpha_{ij} \geq 0$ and $\sum_{j=1}^{s} \alpha_{ij} \leq 1$. Note that the extreme case, $\sum_{j=1}^{s} \alpha_{ij} = 1$, implies that $(\mathbf{f}_i^a \cdot \mathbf{n}(\mathbf{p}_i))\mathbf{n}(\mathbf{p}_i) = \mathbf{f}_i$, where $\mathbf{n}(\mathbf{p}_i)$ represents the unit normal vector outward from the object surface at $\mathbf{p}_i$. If we define the primitive wrench, $\mathbf{w}_{ij}$, associated with $\mathbf{f}_{ij}$, as

$$\mathbf{w}_{ij} \triangleq \begin{bmatrix} \mathbf{f}_{ij} \\ \boldsymbol{\tau}_{ij} = \mathbf{r}_i \times \mathbf{f}_{ij}, \end{bmatrix}, \tag{2}$$

the generic form of wrench that can be produced by the robot fingers is written as

$$\mathbf{w} = \sum_{i=1}^{n} \sum_{j=1}^{s} \alpha_{ij} \mathbf{w}_{ij} \tag{3}$$

with $\alpha_{ij} \geq 0$ and $\sum_{j=1}^{s} \alpha_{ij} \leq 1$. The absolute grasp wrench space (a-GWS) means a space that is spanned by the primitive wrenches, $\{\mathbf{w}_{11}, \mathbf{w}_{12}, \cdots, \mathbf{w}_{1s}, \cdots, \mathbf{w}_{ns}\}$ created by the physical contact forces without normalization. By conjoining all the primitive wrenches, we can construct a convex hull of a-GWS such that

$$\mathcal{H}_{\text{a-GWS}} \triangleq \text{ConvexHull}\left(\bigoplus_{i=1}^{n} \{\mathbf{w}_{i1}, \mathbf{w}_{i2}, \cdots, \mathbf{w}_{is}\}\right), \tag{4}$$

where $\bigoplus$ denotes the Minkowski sum of wrenches. The geometry of convex hull $\mathcal{H}_{\text{a-GWS}}$ consists of vertices, $\mathbf{v}_i, i = 1, 2, \cdots, l$, where $l \leq s^n$. Obviously, in the force-closure grasp, $\mathcal{H}_{\text{a-GWS}}$ must contain the origin. Due to the nature of a-GWS, the volume of of $\mathcal{H}_{\text{a-GWS}}$ is the reachable wrench space that is generated by the physical robot hand with limited actuator torques. This, in return, implies the bound of external disturbances that the given grasp can resist. Hence, a possible grasp measure may be expressed by

$$M = \min ||\mathbf{w}||, \text{ subject to } \mathbf{w} \in \partial\mathcal{H}_{\text{a-GWS}} \tag{5}$$

where $\partial\mathcal{H}_{\text{a-GWS}}$ denotes the boundary of $\mathcal{H}_{\text{a-GWS}}$.

B. Object Wrench Space (OWS)

From the intuitive point of view, the grasp quality should be dependent on the shape of the object, and it seems desirable to incorporate the surface geometry of the object into computing grasp measures to achieve a more accurate result. However, measure M in (5), for instance, is computed without considering the geometry of the object to be grasped.

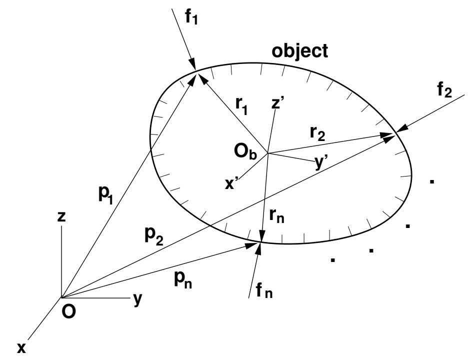


Fig. 1. Multi-point contacts for grasping a rigid body

One prominent way to incorporate the geometry of object into the grasp quality measure is to use the concept of the object wrench space (OWS), which is defined as a vector space spanned by a set of wrenches generated by a set of distributed forces on the surface of an object. The set of distributed forces implies all the possible external disturbances to be imparted on the surface of the object. Although the concept of OWS was first defined in [1], a rigorous use of OWS in defining a task-oriented grasp quality measure was done by Borst et al. [2]. As addressed in [2], the generic form of object wrench is given by

$$\mathbf{w} = \sum_{i=1}^{m} \gamma_i \mathbf{w}_i, \quad \gamma_i \geq 0, \text{ and } \sum_{i=1}^{m} \gamma_i = 1, \tag{6}$$

where m is the number of corners of the polygonal object, and $\mathbf{w}_i$ denotes any wrench produced by the force, $\mathbf{u}_i$, of the unit magnitude within the friction cone, acting on the i-th corner, satisfying

$$||\mathbf{u}_i - \mathbf{u}_i \cdot \mathbf{n}_i|| \leq \mu ||\mathbf{u}_i \cdot \mathbf{n}_i||,$$

where $\mathbf{n}_i$ and μ, respectively, denote the normal direction of the i-th facet on the discretized object and the friction coefficient. The wrench in (6), which spans OWS, is generated by any combination of forces whose L_1 norm (i.e., sum of magnitudes) is 1, in order to normalize the effect of the disturbance. The problem about their OWS is that there can be infinite choices of $\mathbf{u}_i$ even at a single corner. To reduce the complexity, they applied the sampling method to choose $\mathbf{u}_i$'s and, by using the sampled wrenches, created an ellipsoid of OWS, which is an approximate of the exact convex hull of OWS. However, still there are some drawbacks, related with this approach: (i) the number of samples should be sufficiently large, while it is not clear how many samples are sufficient, (ii) even if a sufficient number of samples is employed, the convex hull of OWS is, after all, approximated by an ellipsoid, with which an asymmetric object may not work well, and (iii) the very combination of disturbances that results in the measure value (i.e., the maximum sum of allowable disturbances) is not likely to happen in practice and conveys little physical meaning.

Upon this observation, we propose a modified OWS that is generated from a distribution of unitary external forces acting only in the normal to the surface, as shown in Fig.2. This is equivalent to ignoring the tangential components of disturbances in creating the proposed OWS, just as there is no friction on the surface. This is especially true when we are dealing with disturbances in practical cases of hard objects. Due to this setup, the number of wrenches that constitute the OWS becomes limited to a finite number; therefore, we can manage to create the exact convex hull of the OWS without relying on the approximated ellipsoid. Besides, we are able to find a representative single disturbance, at a particular point on the surface, that is the most critical to the stability of grasp, rather than a combination of disturbances on the surface. This makes the measure value more useful in practice and conveys a clearer physical meaning.

For an object with m-number of surface facets, the OWS is created by a set of elementary wrenches,

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{u}_k \\ \mathbf{l}_k \times \mathbf{u}_k, \end{bmatrix}, k = 1, 2, \cdots, m, \tag{7}$$

where $\mathbf{l}_k$, $\mathbf{u}_k$, and $\mathbf{z}_k$ denote the position of of the center of the k-th facet, the unit normal external force at $\mathbf{l}_k$, and the wrench produced by $\mathbf{u}_k$, respectively. The number of wrenches, m, in our OWS should be much smaller than that from [2]. With the set of elementary wrenches, the convex hull of OWS is defined as

$$\mathcal{H}_{\text{OWS}} \triangleq \text{ConvexHull}\left(\{\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_m\}\right). \tag{8}$$

Since $\mathcal{H}_{\text{OWS}}$ is created by the union of $\mathbf{z}_i's$, this convex geometry is composed of vertices $\bar{\mathbf{z}}_j,\ j = 1, 2, \cdots, d \leq m$ belonging to the set of elementary wrenches in OWS.

Note that the convex hull of our OWS is an exact one, made from m number of elementary wrenches, that accommodates information on the shape of geometry. Objects with different surface geometries have their own shapes of the convex hulls of OWS. For similar objects with different geometric scales, the shapes of the convex hulls show the scale difference only in the τ-axes. For instance, for a 2D object with the unit square shape and a similar one scaled by two as shown in Fig.3, the convex hulls of OWS's have a difference only in the τ_z axis, while the projections of the convex hulls onto f_x-f_y plane show the identical shapes. The convex hull of OWS for any object possesses $\mathbf{0}$ as an interior point. This is because the surface of an object in 3D space is closed, and thus any force applied on the surface of the object has some combination of surface forces resulting in the equilibrium. Since the same principle can be applied to the torque quantity, the convex hull of OWS always has $\mathbf{0}$ as an interior point, without regard to the shape of an object.

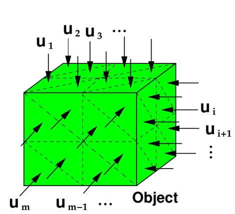
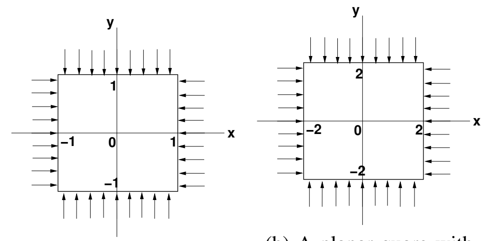


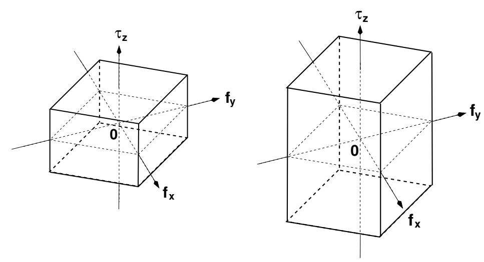
Fig. 2. A distribution of forces that produce OWS



(a) A planar unit quare with force distribution

(b) A planar quare with two times of a unit square with force distribution.

Fig. 3. 2-D objects with distributions of forces



(a) Convex hull of OWS for a unit cube

(b) Convex hull of OWS for a cube with side of 2 unit.

Fig. 4. Convex hulls of OWS's for objects in Fig.3

III. Grasp Quality Measure

A. Maximum Contact Force

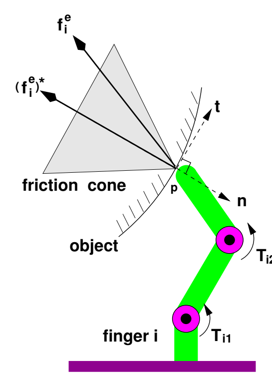
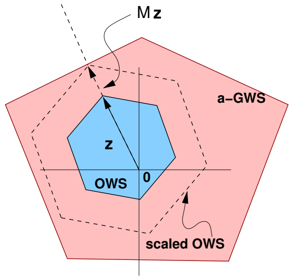


Fig. 5. Determination of the finger force direction

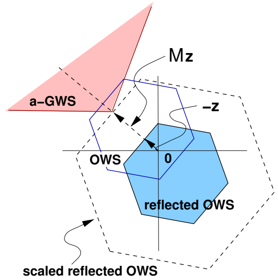
Since the magnitude of the contact force determines the size of $\mathcal{H}_{\text{a-GWS}}$, it is naturally the first step to determine at each fingertip the maximum magnitude of the contact force that a real finger, driven by actuators with limited torque bounds, can impart at a given configuration. To solve this problem, consider the following static force-torque relation:

$$\mathbf{J}_i^T(\boldsymbol{\theta}_i)\mathbf{f}_i^e = \mathbf{T}_i \ \Rightarrow\ \mathbf{f}_i^e = \left[\mathbf{J}_i^T(\boldsymbol{\theta}_i)\right]^{-1}\mathbf{T}_i,\ i = 1, 2, \cdots, n, \tag{9}$$

where $\mathbf{J}_i(\boldsymbol{\theta}_i) \in \mathbb{R}^{n_x \times n_{qi}}$, $\mathbf{f}_i^e \in \mathbb{R}^{n_x}$, $\mathbf{T}_i \in \mathbb{R}^{n_{qi}}$, $\boldsymbol{\theta}_i \in \mathbb{R}^{n_{qi}}$, respectively, denote the Jacobian matrix, the force at the finger tip, the joint torque, and the joint angle with n_{qi} and n_x being the number of joints and the dimension of workspace, all associated with the i-th articulated finger.



(a) Grasp measure under force-closure condition



(b) Grasp measure under non-force-closure condition

Fig. 6. Grasp quality measure using a-GWS and OWS

From this setting, we need to solve the following optimization:

$$\begin{aligned}&\text{Maximize} \quad -\mathbf{n}(\mathbf{p}_i)\cdot\mathbf{f}_i^e,\\ &\text{s.t.} \left\{\begin{array}{l}\mathbf{n}(\mathbf{p}_i)\left[(\mathbf{n}(\mathbf{p}_i)\cdot\mathbf{f}_i^e\right]=\mathbf{f}_i^e\\ \mathbf{f}_i^e=\left[\mathbf{J}_i^T(\boldsymbol{\theta}_i)\right]^{-1}\mathbf{T}_i\\ -\beta_{ij}\le T_{ij}\le\beta_{ij}\end{array}\right\}, \\ &\text{for} \quad i=1,2,\cdots,n,\ j=1,2,\cdots,n_{qi},\end{aligned} \tag{10}$$

where $\mathbf{p}_i$ is the contact point coincident with the i-th finger tip position, T_{ij} represents the joint torque and $\beta_{ij} > 0$ is a constant torque limit, associated with the j-th joint in the i-th finger. In the above equation, the equality condition constrains that $\mathbf{f}_i^e$ remains in the normal to the surface; otherwise, the fingertip force could be slanted from the normal vector, and, if so, the active tangential component of fingertip force would work as a disturbance, which causes a loss of some reachable wrenches in a-GWS. As shown in Fig. 5, as the direction of contact force approaches the boundary of the friction cone, the margin to resist the disturbance in the tangential direction becomes reduced. The inequality condition in (10) simply restricts the joint torque. Once (10) is solved, the maximum normal contact force, $\mathbf{f}_i^{e*}$, becomes available and works as the contact force that generates the primitive wrenches in a-GWS.

B. Grasp Quality Measure

A desirable grasp quality measure must be physically meaningful and continuous through all ranges of grasp conditions. To meet these desirable features, our grasp measure is conceptually defined by: (i) the maximum scale of $\mathcal{H}_{\text{OWS}}$ that is completely contained in $\mathcal{H}_{\text{a-GWS}}$ when $\mathbf{0}\in\mathcal{H}_{\text{a-GWS}}$, and (ii) the negative of minimum scale of $\overline{\mathcal{H}_{\text{OWS}}}$ that begins to touch the $\mathcal{H}_{\text{a-GWS}}$ when $\mathbf{0}\notin\mathcal{H}_{\text{a-GWS}}$, where $\overline{\mathcal{H}_{\text{OWS}}}$ represents the convex hull that is symmetric with $\mathcal{H}_{\text{OWS}}$ about point $\mathbf{0}$. Fig.6 shows the schematic diagrams of the measure.

Mathematically, the proposed grasp quality measure is obtained by the following formulas:
(i) $\mathbf{0}\in\mathcal{H}_{\text{a-GWS}}$ (Force-closure):

$$M=\min_{k=1,2,\cdots,d}\ \max\quad \rho_k \tag{11}$$

$$\text{subject to}\left\{\begin{array}{l}\rho_k\bar{\mathbf{z}}_k=\sum_{j=1}^{l}\lambda_j\mathbf{v}_j\\ \sum_{j=1}^{l}\lambda_j=1\\ \rho_k\ge 0,\lambda_j\ge 0.\end{array}\right\}. \tag{12}$$

(ii) $\mathbf{0}\notin\mathcal{H}_{\text{a-GWS}}$ (Non-force-closure):

$$M=-\min\sum_{k=1}^{d}\rho_k \tag{13}$$

$$\text{subject to}\left\{\begin{array}{l}\sum_{k=1}^{d}\rho_k(-\bar{\mathbf{z}}_k)=\sum_{j=1}^{l}\lambda_j\mathbf{v}_j\\ \sum_{j=1}^{l}\lambda_j=1\\ \rho_k\ge 0,\lambda_j\ge 0.\end{array}\right\}. \tag{14}$$

By the formulations, a positive value of M represents a force-closure grasp, while a negative value of M means a non-force-closure grasp. For a marginal case, M is zero.

In the case of non-force-closure grasp, we calculate the measure with the wrenches inverted to opposite directions such that $-\bar{\mathbf{z}}_k$, as stated in (13) and (14). This setup is intended to search for the minimum wrench in OWS that, if added to a-GWS in the reverse way, stretches the boundary of a-GWS to pass through the origin such that

$$\mathbf{0}\in\partial\mathcal{H}^*_{\text{a-GWS}},$$

where

$$\mathcal{H}^*_{\text{a-GWS}}\triangleq\text{ConvexHull}\left(\mathcal{H}_{\text{a-GWS}}\bigoplus\left\{\sum_{k=1}^{d}\rho_k^*\bar{\mathbf{z}}_k^*\right\}\right),$$

where ρ_k^* and $\bar{\mathbf{z}}_k^*$ denote the solution of (13) and (14), respectively. Physically, the wrench, $\sum_{k=1}^{d}\rho_k^*\bar{\mathbf{z}}_k^*$, implies the required minimum external wrench that renders a non-force-closure grasp to be a force-closure.

We should remark that the proposed grasp quality measure in (11) – (14) has a similar form to Q-distance proposed by Zhu and Wang [15]. Q-distance is mathematically defined by the L_2 distance measure between a test polyhedral set, the so-called Q, and the conventional convex hull of GWS. The difference of the grasp quality measure between ours and Q-distance is explained as follows: (1) The physical meaning of the measure by Q-distance is rather ambiguous since the set Q is an abstract set, while our measure simply exhibits a clear and concrete physical meaning by using the convex hulls of the Minkowski sum based a-GWS and the union based OWS. (2) The measure for non-force-closure grasp is computed using the reflected OWS so that the measure value can take a consistent physical as for the force-closure grasp. (3) Moreover, the wrench direction to the minimum Q-distance does not show a clear meaning, while the wrench direction to the minimum of the proposed measure is associated with the most fatal spot on the object surface to the disturbance.

C. Physical Meaning of Grasp Quality Measure

According to the definition of the proposed grasp quality measure, measure M for a force-closure grasp is the maximum scale of $\mathcal{H}_{\text{OWS}}$ that is completely contained in $\mathcal{H}_{\text{a-GWS}}$. Since $\mathcal{H}_{\text{OWS}}$ is constructed by the wrenches from the distributed unitary normal forces exerted on the surface, the scaled $\mathcal{H}_{\text{OWS}}$ by M amount is equivalent to the convex hull generated by the wrenches from the distributed normal forces that are scaled by the same amount. Therefore, M is physically the maximum magnitude of external disturbance that the robot hand can resist. Furthermore, $\mathbf{z}^*$ such that $M\mathbf{z}^* = \partial\mathcal{H}_{\text{a-GWS}}$ is, to the current grasp, the weakest directional wrench that is produced by a single unitary normal force on the surface. Since each unitary normal force is related with the location of application on the object surface, we can identify where is the weakest spot to the disturbance and how large the robot hand can resist the disturbance. Therefore, the proposed measure is fully characterized by a single force $M\mathbf{u}^*$, where $\mathbf{u}^*$ generates the wrench, $\mathbf{z}^*$.

A single force representation of the grasp quality measure in the force-closure grasp is attractive by appealing to the human's intuition and works good for identifying the weakest locations of an object at a given contact configuration.

Contrary to the case of the force-closure grasp, the measure value in a non-force-closure grasp means the minimum scale of $\overline{\mathcal{H}_{\text{OWS}}}$ that begins to intersect with $\mathcal{H}_{\text{a-GWS}}$. This scale value implies the minimum amount (L_1 norm) of helping forces exerted on the object surface that transform the grasp into a force-closure one. Unfortunately, the measure value of the non-force-closure grasp, in general, is not computed at the vertex of $\overline{\mathcal{H}_{\text{OWS}}}$ because the two convex geometries are located outside of each other; hence, the measure cannot be characterized by a single representative force on the surface but by a particular distribution of forces, whose sum of absolute magnitudes is equal to the measure value.

The grasp measure will be continuous under a smooth variation of a grasp condition such as the gradual change of contact points or the change of finger configuration.

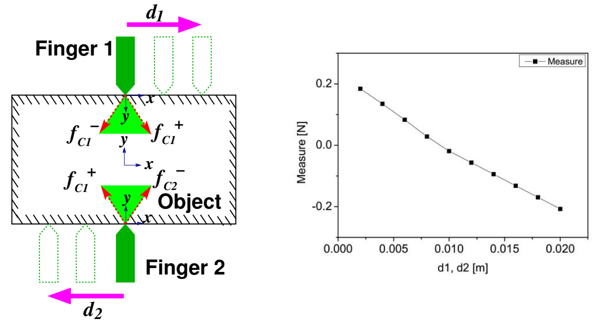
IV. SIMULATION

A. Procedures of simulation

In the first step of the simulation, a target polygonal object is loaded, followed by a generation of OWS and its convex hull. Next, the robot hand's palm is placed at the pre-defined position and orientation with respect to the object frame, and contact points are sampled in accordance with the grasp taxonomy. The inverse kinematics routine is then executed for each finger to locate the fingertip onto the assigned contact point; by employing the determined joint configuration, the maximum normal contact force is computed from (10). Then, we create the a-GWS and the corresponding convex hull. Finally, using the convex hulls of OWS and a-GWS, we determine the grasp quality measure by solving (11) – (14).

B. Simulation results

Three illustrative simulation results are shown in this subsection: the first two are intended to show the validity of the grasp quality measure, and the last one is to demonstrate the effect of the finger configurations to the grasp quality measure. For 3D grasp simulations, a 3-fingered robot hand having totally 9 DOFs – 3 DOFs for each finger like a Barrett hand – is used as a hand model. The torque limit of each actuator in the robot hand is identically set as $-1Nm \leq T_{ij} \leq 1Nm,\ i = 1,2,3,\ j = 1,2,3$, for simplicity. In 2D grasp simulation, each contact force is set to $1N$, without considering the effect of finger configurations.



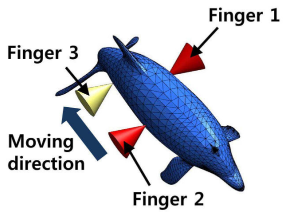
(a) Test schematic (b) Grasp quality measure

Fig. 7. Grasp quality measure for 2D grasp

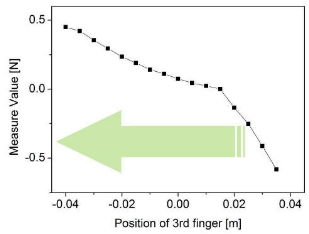
In the first simulation, we validate our grasp quality measure algorithm by using a 2D rectangular object (0.12m× 0.06m). As shown in Fig. 7 (a), two fingers in the opposite direction is moved gradually so that the grasp becomes unstable, and the grasp measure value shown in Fig. 7 (b) exhibits the amount of corresponding grasp stability.

In the second simulation, we test the grasp quality measures in grasping two 3D objects, that is, a dolphin and a rook. We consider the contact positions and finger configurations as variables. As shown in Fig. 8(a), a dolphin is grasped using the pinch method, where only the position of finger 3 is gradually varying. In case of the dolphin, we change the position of finger 3 starting from overlapped position with finger 2 to the final position shown in the figure. Next, as shown in Fig. 8(c) and (e), a dolphin and a rook are grasped using the spherical method, where the three fingers, approaching from the top, are symmetrically grasping the objects. In these tests, while fixing the first and the second fingers, we change the third finger angle from 120° to 240°. As shown in Fig.8(b), (d) and (f), the measure values for all the cases are continuously varying with the different contact positions. Those measure values represent the physical force quantity in Newton. In particular, the measure values for the force-closure cases imply single forces at the most fatal locations on the object surface.

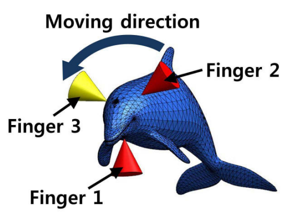
Finally, the effect of finger configuration to the grasp quality measure is investigated. For identical contact points with an identical object, we differentiate the relative positions of the palm and the object, which entails the changes in finger configurations. The maximum contact forces at fingertips increase as the finger configurations become bent due to the reduction of the distance between the palm and the object. This observation agrees to the behavior of human's hand. Because the larger contact forces produce a larger convex hull of a-GWS, it is natural that the grasp quality becomes enhanced. (Please refer to Fig.9.) In this example, the weakest spots under disturbance where the measure value



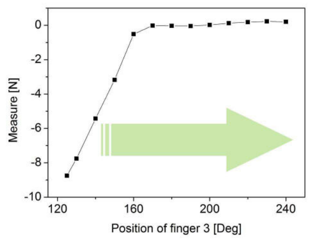
(a) Pinch grasp for a dolphin



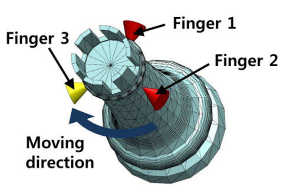
(b) Grasp measure for pinch grasping a dolphin



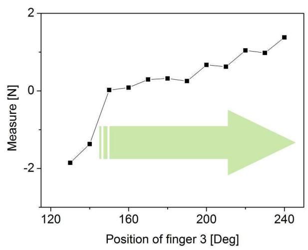
(c) Spherical grasp for a dolphin



(d) Grasp measure for spherical grasping a dolphin



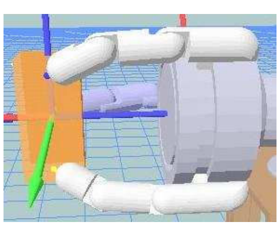
(e) Spherical grasp for a rook



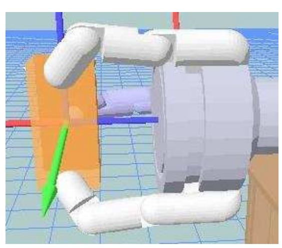
(f) Grasp measure for spherical grasping a rook

Fig. 8. Grasp quality measures for 3D grasps

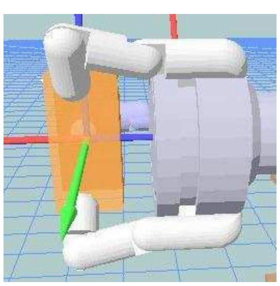
is calculated are the front two upper corners and symmetrical two corners on the opposite side.



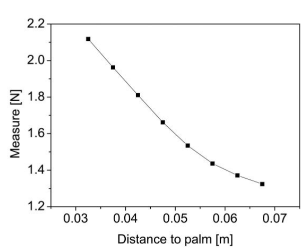
(a) Configuration of at distance: 0.07m



(b) Configuration of at distance: 0.05m



(c) Configuration of at distance: 0.03m



(d) Grasp measure vs.palm-object distance

Fig. 9. Effect of finger configurations to grasp quality measure

V. Conclusion

In this paper, we proposed a practical method to evaluating grasp quality using OWS and a-GWS, leading to a physically meaningful measure value. We suggested a way to creating a suitable OWS and a-GWS, by which we made the conventional OWS and GWS become simplified and more useful. Ultimately, a mathematical closed-form formulation of the grasp quality measure for both force-closure and non-force-closure grasps was established using the linear programming. We verified the validity of the proposed grasp quality measure through numerical simulations.

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