

# Innovative Kinematics and Control to Improve Robot Spatial Resolution

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**Abstract**— The paper presents innovative kinematics and control of a planar redundant robot designed to improve spatial resolution by a factor 5. This result is obtained in a restricted area of the workspace using position information from external sensors. This innovation results from a clearer understanding of the factors that influence the robot micrometric behavior: axes control resolution generates a set of attainable points in the robot workspace and different spatial resolution patterns appear when introducing redundancy depending on the final axes chosen to correct the position.

## I. INTRODUCTION

CHOOSING and designing the adequate robot to perform a task is not easy. Manufacturers provide some performance criteria such as payload, workspace dimensions, acceleration and repeatability. These criteria are detailed in ISO9283 [1] or ANSI R15-05.1-1990 [2] but these norms describe numerous other criteria which are not often available for industrial robots. For instance, if we wanted to choose the right robot to perform minute assembly tasks like pick and place in electronic assembly, which criteria should be used ?

Should we consider the pose repeatability, the pose accuracy, the robot spatial resolution ? Should we choose a serial or a parallel robot ?

Optimal robot design is a key issue in robotics and a clear insight is given in [3]. Numerous methodologies exist to optimize robot kinematics. A large body of them proposes using manipulability or dexterity indexes [4]. Others are more concerned with dynamic [5]-[7] or elastostatic performance [8], [9]. The pros and cons of these different methodologies are well-known for serial robots [10]. Now recent research deals with accuracy analysis in parallel robotics [11]. One of these papers [12] is a comparison of the accuracy of a parallel and a serial robot and the methodology is based on the sensitivity. Other criteria can be used for this kind of comparison for example in relation to the stochastic ellipsoid theory based on the angular covariance matrix [13].

In this paper, we describe innovative serial robot kinematics that can greatly improve precision in a limited area of the workspace. This architecture was not discovered using a traditional approach because roboticists usually avoid redundancy and singularities. In fact, new properties of

redundancy on spatial resolution are here presented and used in an innovative control scheme to improve local precision.

In section II, we describe the traditional approach to local resolution modeling based on axis resolution. In section III, we study the SCARA robot case and describe in detail the strong variations of the spatial resolution in the workspace. In section IV, we introduce redundancy and point out two interesting properties. At the same time, we show how internal and external sensors can be used in a control strategy paving the way to different spatial resolutions. In section V, we present a redundant kinematic robot structure improving spatial resolution in a limited area of the workspace. In section VI, we explain how this architecture is efficient to improve precision on the micrometric scale.

## II. SPATIAL RESOLUTION DEFINITION

The precision of a rotational or linear axis is characterized by different indices displayed in fig.1. The control resolution  $r$  is the smallest difference between two different targets  $r = |T_{i+1} - T_i|$ . The precision error can be described by two different indices :

- The repeatability index  $Rep$  which estimates the distance error around the position mean;
- The exactitude index  $Ex$  which is the distance between the position mean and the real target.

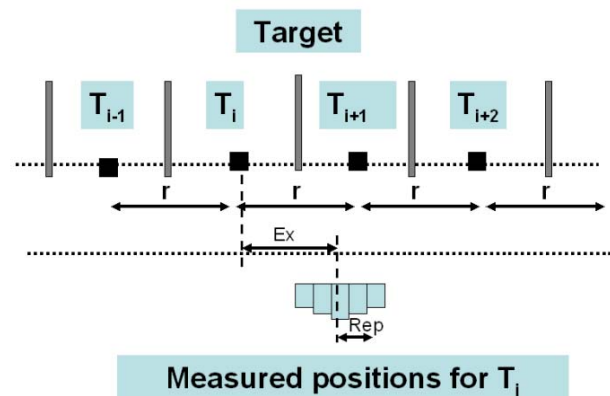


Fig.1 Resolution, repeatability, exactitude.

When the control is perfect and the target  $T_i$  is to be reached, the final position lies in the interval  $I_i = [-\frac{r}{2} + T_i; +T_i + \frac{r}{2}]$ . Some authors consider that the position distribution is constant over the interval  $I_i$ . We studied statistically this distribution and showed that in

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reality it was Gaussian [14]. From this it is possible to model repeatability with stochastic ellipsoids. This approach is highly interesting because it is possible to study all repeatability influence factors. For example, we proved that for two 6 axis serial robot studied, the load influence was far less important than the workspace location influence [15], [16].

This was not the first work on this subject. Statistical work has been proposed to quantify the influence of some factors on repeatability [17], [18]. Riemer and Edan were interested in workspace location influence [19]. Offodile and Ugwu have studied load and speed influence [20].

When controlling the robot from the angular space, the angular resolutions of each actuator are combined to create the spatial resolution patterns. For a given target  $\Theta = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$  where the resolution of each actuator is  $R = (r_1, r_2, r_3, r_4, r_5, r_6)$ , the final angular position will be:

$$\Theta \pm \frac{R}{2} = (\theta_1 \pm \frac{r_1}{2}, \theta_2 \pm \frac{r_2}{2}, \theta_3 \pm \frac{r_3}{2}, \theta_4 \pm \frac{r_4}{2}, \theta_5 \pm \frac{r_5}{2}, \theta_6 \pm \frac{r_6}{2})$$

We studied the performances of one Samsung Faraman FARA2 and one Kuka IR384 robots with respective angular resolution of  $0.01^\circ$  and  $0.001^\circ$ . Currently, we are working on an EPSON RS450. It is possible to control this robot by specifying the number of pulses. If the servo-control is perfect with nil errors, the spatial resolution is then at its best.

Let the spatial resolution SR be the set of points of the final robot position  $X = (x, y, z)$  corresponding to the target  $\Theta$  with the uncertainty R. It can be computed in two different ways.

In the first method  $2^6$  points are computed in the Cartesian space using the interval analysis and the forward kinematics function  $X = F(\theta)$  transforming joint coordinates  $\Theta$  into workspace coordinates  $X = (x, y, z)$ . The spatial resolution SR will be a convex polyhedron obtained from the convex envelop of the  $2^6$  points  $F(\Theta \pm \frac{R}{2})$ :

$$SR = Conv\{F(\Theta \pm \frac{R}{2})\}.$$

The second method consists in differentiating the forward kinematics function  $X = F(\theta)$  and using the Jacobian function mapping the joint velocity vector to the Cartesian velocity vector in the linear transformation  $dX = J(\theta)d\theta$ . This relationship can also be understood as the link between small angular and Cartesian variations. The spatial resolution SR will be a convex polyhedron obtained from the convex envelop of the  $2^6$  points  $\pm J(\Theta) \times \frac{R}{2}$ :

$$SR = Conv\{\pm J(\Theta) \times \frac{R}{2}\}$$

Whatever the method, the results will be the same because the resolution R is usually very small compared to the angular value  $\Theta$  so the first order approximation is very

precise. The two methods are only different because the first one considers the set of points and the other one, the point variations. In both cases, the following results in section III are identical.

In general, it is difficult to evaluate the evolution of size, volume, orientation of these different polyhedra. This could explain why spatial resolution has been sometimes replaced by repeatability, hoping that the repeatability sphere could approximate the polyhedron. But the repeatability index will not give any clue about spatial distribution [21].

Using this traditional approach where the resolution is constant in angular space, the variations of the spatial resolution in the workspace depend only on the lengths of the links. The final uncertainty interval at the end of a rotating arm is equal to the product of the link length by the angular resolution. If the angular resolution is constant, the longer the arm, the larger the uncertainty for the extremity's final position. This phenomenon is known as lever-arm amplification error. As robots are mostly built with rotational joints, the spatial resolution depends on the robot posture and location which determine the lever-arm length for each actuator. This function is non-linear and difficult to study in the general case.

### III. SPATIAL RESOLUTION VARIATIONS FOR THE SCARA

To illustrate the strong variations of spatial resolution, let us consider a SCARA robot with the usual four dof and the same angular resolution on the 1<sup>st</sup> and 2<sup>nd</sup> axis. The third axis is a rotation around the robot endpoint. It does not intervene for spatial resolution but only for orientation resolution. The fourth axis corresponding to the Z-translation is easy to take into account adding vertical vertices to the patterns drawn on the plane. So to illustrate the resolution variations, the representation will be drawn on the horizontal plane considering only the first and second axes. In this particular case, the spatial resolution is a parallelogram. For the three different locations in the SCARA workspace displayed on fig.2, we draw the mesh of the different parallelograms corresponding to the closest targets with the same scale.

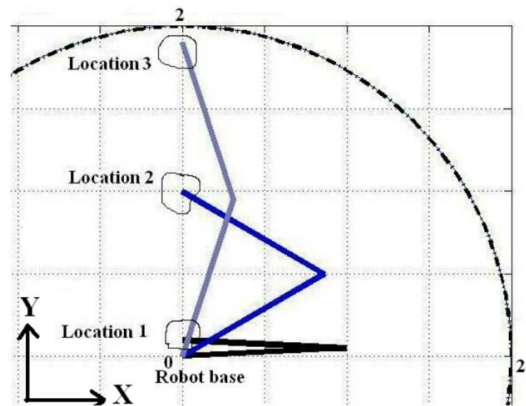


Fig.2 The three different chosen locations of the SCARA.

The parallelogram patterns are displayed on fig.3 and lead to the following conclusions:

- The surface, size and orientation of the spatial resolution parallelograms are very different according to the workspace location.
- The spatial resolution is much finer in the center of the workspace corresponding to the 1st location, compared to the other locations because the surface of the parallelogram is smaller. It means that the final position will be in a smaller area and that it is possible to reposition more precisely the robot changing slightly the target. The position can be minutely corrected in the X-direction but as the rectangle sides are in a 1/10 ratio, the resulting Y-position uncertainty is larger. Moreover, in this case, displacements are decoupled in the X and Y directions.
- For the 3rd location, the spatial resolution is better in the Y direction than in the X direction.

The main conclusion is that spatial resolution is better near the singularities of the robot. For the 1st location, this best resolution direction corresponds to the move induced by the first axis rotation. The length of this move is obtained multiplying the distance between the 1st axis and the location by the first axis angular resolution. So it is really the distance of the lever-arm that is the main parameter to take into account for resolution improvement.

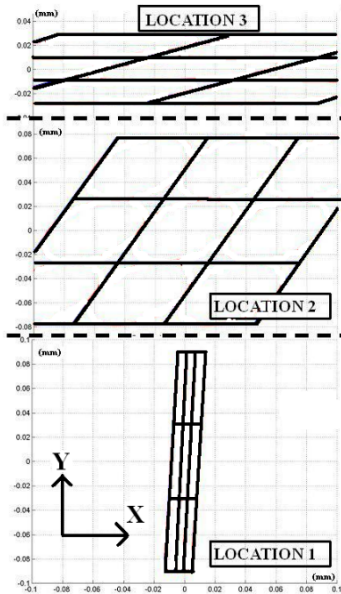


Fig.3 Parallelogram of spatial resolution for the three locations.

#### IV. IMPACT OF REDUNDANCY ON SPATIAL RESOLUTION

Redundancy is often used in robotics to give more dexterity to the robot. When some areas are difficult to attain because of obstacles or when some tasks need special orientations that are not compatible with a 6 dof robot, introducing redundancy can be a solution. Unfortunately the robot control is then less easy.

What is the impact of redundancy on spatial resolution ?

Roughly speaking, the introduction of redundancy produces both a densification and pattern modifications related to the moving axes.

#### A. Densifying Cartesian space

To illustrate this property, let us draw the spatial resolution polygons for two planar robots, SCARA2 and SCARA3. SCARA2 is the SCARA described in the preceding section. SCARA3 is a planar redundant robot with three degrees of mobility as shown in fig.4. The three links of the SCARA3 have the same length. Let  $J_{SC3}$  be the Jacobian matrix of the SCARA3.

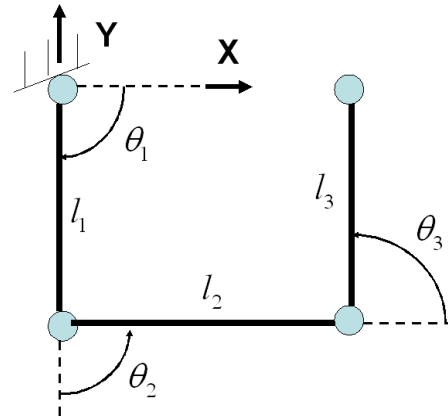


Fig.4 Geometry of SCARA3 robot and posture

Let  $r_1 = r_2 = r_3 = r$  be the resolution of angular control. The spatial resolution polygons are hexagons corresponding to the convex envelop of the  $2^3$  points:

$$\left\{ \frac{1}{2} \times J_{SC3} \times [\pm r_1, \pm r_2, \pm r_3]^T \right\}$$

One of this hexagon displayed on fig.5 is geometrically built from the variations

$$\pm \vec{u}_1 = \pm \frac{1}{2} J_{SC3} \times [r_1, 0, 0]^T$$

$$\pm \vec{u}_2 = \pm \frac{1}{2} J_{SC3} \times [0, r_2, 0]^T$$

$$\pm \vec{u}_3 = \pm \frac{1}{2} J_{SC3} \times [0, 0, r_3]^T$$

The disposition of some close polygons are shown on fig.6 corresponding to 3 translations of  $2\vec{u}_1, 2\vec{u}_2, 2\vec{u}_3$ . It illustrates the fact that a point in a given area does not correspond to a unique target. This is quite obvious because the robot is redundant but this has important consequences for the micrometric positioning performance. On the other hand, it does not mean it is easier to control the robot to reach the desired area. The choice of the right target is still a difficult problem. But when one of the three axes is locked, activating an electromechanical brake for instance, then the polygon mesh representing the different repositioning area associates again one point to one unique target.

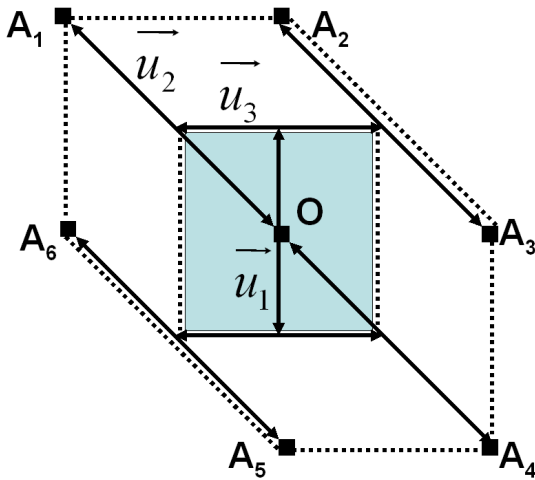


Fig.5 Hexagon for spatial resolution of the SCARA3.

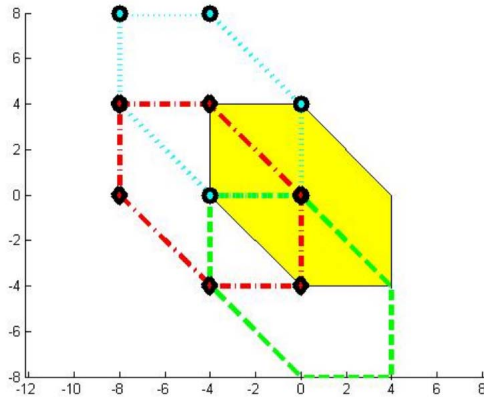


Fig.6 The mesh of spatial resolution hexagons for SCARA3

### B. Modifying polygon resolution patterns

We are now considering a new strategy consisting of controlling the SCARA3 robot using only two axes instead of three. The remaining axis is locked. The robot is no longer redundant and a new polygon mesh can be built. For instance, for the posture of fig.4, the pattern will change depending on the pair of axes chosen to drive the robot. Fig.7 illustrates this principle. If the final control uses only the 1<sup>st</sup> and 3<sup>rd</sup> axes, then the polygons are parallelograms in an orthogonal layout. If the control uses only 2<sup>nd</sup> and 3<sup>rd</sup> axes, then the orientation of the parallelogram is quite different. So it is clear that the choice of the final pair of axes has an important influence on the structure of the final parallelogram mesh.

It is then possible to choose the axes to be controlled in the last positioning step according to the performance criterion, here spatial resolution. It will be easier to correct the position using the 1<sup>st</sup> and 3<sup>rd</sup> axes because the induced displacements are orthogonal. The final uncertainty will be twice larger in the X-direction if the 2<sup>nd</sup> and 3<sup>rd</sup> axes are chosen for the control, but the final error will be the same in the Y-direction. But in both cases, the final spatial resolution

is finer than the hexagon of fig.5. The square on the left side of fig.7 corresponds to the grey square of fig.5.

At this point, we are still aware that the final polygon surface is still the same as the move is performed using the three axes and the global spatial resolution has not been improved yet. Here is the new strategy: a first rough positioning is performed using the three axes and at the end of the move, the minute final position correction is done using only two of the three axes. In this strategy, the final polygon resolution depending of the last pair of axes, is smaller than the global one and it is possible to choose a better target to improve the final position. This is the strategy that will be used in the next sections.

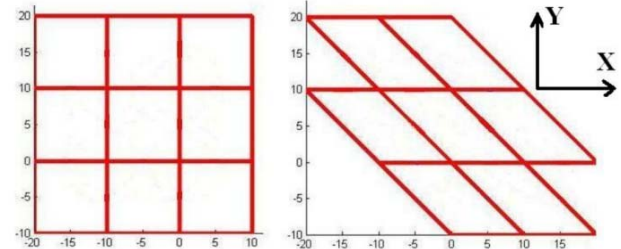


Fig.7 Spatial resolution patterns for 1st and 3rd axes control (left) or 2nd and 3rd axes final control (right).

## V. INNOVATIVE KINEMATICS AND CONTROL TO IMPROVE SPATIAL RESOLUTION

We are now considering the previous results to improve the spatial resolution for a specific location  $P_1$  “point of interest” in the workspace. A minute positioning is demanded near this point, so the final spatial resolution must be fine in two orthogonal directions. This could be done using two joint articulations whose rotation centers are as close as possible to  $P_1$ . It is for instance 1<sup>st</sup> and 2<sup>nd</sup> axes of fig.8. Then to enlarge the workspace, it is necessary to add a third axis. Consequently, the robot is now redundant, and able to grasp an object in a wider area.

The control strategy consists of two steps: in a first step, the robot endpoint is brought close to the desired target  $P_1$  using the three axes. Then the 3<sup>rd</sup> axis is locked. The position error is estimated from external sensor information and the new target is computed. In the second step, the robot comes closer to  $P_1$  using only the 1<sup>st</sup> and 2<sup>nd</sup> axes.

For the structure displayed in fig.8, let the lengths of the links be  $l_1 = 1, l_2 = 5, l_3 = 4.5$  (dm). For the angular position  $(\theta_1 = 135^\circ, \theta_2 = 90^\circ, \theta_3 = 173^\circ)$ , the location of the robot endpoint is  $P_1(-0.697, -0.058)$  (dm) very close to the 1<sup>st</sup> and 2<sup>nd</sup> axes. The parallelogram mesh resulting from the 1<sup>st</sup> and 2<sup>nd</sup> axes final control in the vicinity of the point of interest is orthogonal and consists of short-sided squares displayed on Fig.9. The scale of the drawing is obtained for a resolution of  $2 \cdot 10^{-4}$  rad. Fig.10 is a comparison of this square mesh with the hexagon of spatial resolution when the 3 axes are used. It then becomes obvious that it is easier to

correct the final position using only 1<sup>st</sup> and 2<sup>nd</sup> axes in the final step.

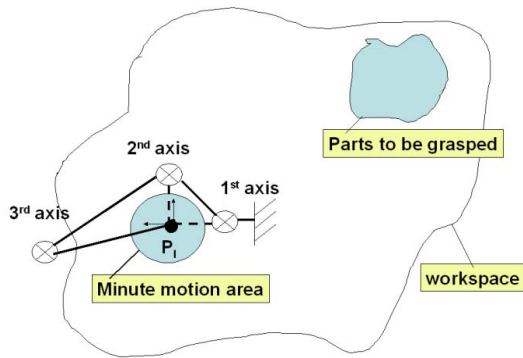


Fig.8 Innovative planar redundant structure SCARA3

This minute spatial resolution is still true if the final point moves away from  $P_1$  but stays in the vicinity of  $P_1$ . The mesh is slightly changed in orientation and the square becomes a parallelogram, but the dimensions of the parallelogram remain small as long as the final point stays in a disk centered on  $P_1$  with a radius lower than 0.35 for instance.

This point  $P_1$  is named point of interest and the vicinity of  $P_1$  is an interesting area for local precise repositioning.

The interesting area surface may seem to be quite small but in reality, other 1<sup>st</sup> and 2<sup>nd</sup> axis configurations produce more interesting surfaces and the total interesting area is a ring centered around the 1<sup>st</sup> axis and with 0.35 dm internal and 1.05 dm external radius.

This can be a disadvantage in certain applications but it can be useful in some specific tasks for instance in some electronic manufacturing operations where the assembly zone is often small and precision is limited by the camera or binocular visual field, and the pieces are grasped elsewhere in the workspace. The precision for the grasping in this example is not as high as for the final placement on the PCB.

This improvement in spatial resolution can be roughly estimated from the ratio of the third link length above the first link length, reaching a factor 5 at least in this example.

This innovative kinematics and control strategy have to be compared to a traditional approach to obtain minute spatial resolution. In fact, until now, the answer consisted in associating a micro and a macro positioner, in what was called a micro-macro approach.

One solution is for tool held by the robot to have a micro-positioner for minute spatial resolution. In this case, the robot performs the rough positioning and the tool the minute positioning.

The other solution is to put the tool on the robot and the part on a table. The minute spatial resolution is obtained by small movements of the table.

In both cases, 4 actuators are necessary for the process. The innovative solution described in this paper needs only 3 actuators. So the total cost is lower. In fact, the micro

positioning is part of the robot structure itself, hidden in its geometrical characteristics.

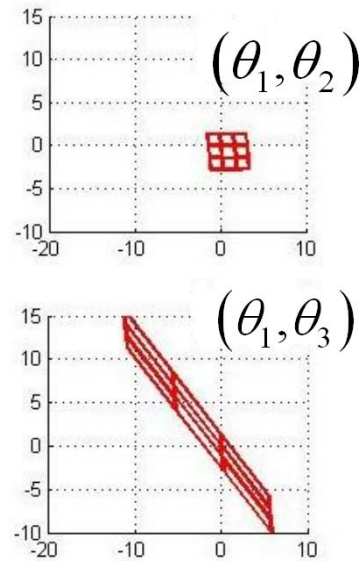


Fig.9 spatial resolution parallelogram resulting from different final control (in micrometers)

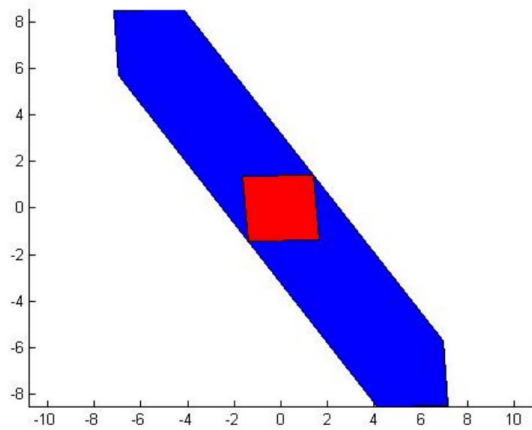


Fig.10 comparison of the spatial resolution resulting from different final control ; hexagon for the 3 axes and square for 1<sup>st</sup> and 2<sup>nd</sup> axes.

## VI. EXTERNAL SENSORS AND CONTROL STRATEGY

In the preceding section, we explained the strategy. At the end of the rough positioning, in the last step, we need to estimate the position correction before the minute last move. This last movement must be estimated from measures by external sensors. For example, these sensors can be a camera with high resolution. As the final zone for minute positioning is small compared to the total workspace area, it is possible to choose a camera with high resolution and a narrow visual field, for instance 4-micron resolution with a visual field of one  $\text{cm}^2$ .

Two important problems may appear and ruin our efforts if not tackled seriously.

The first one deals with the lost motion zone described in fig.11. It is sometimes impossible to move the robot endpoint directly by a small increment, because of friction.

If the robot endpoint is within the lost motion zone, nothing may happen or the robot could move (from S2 to F2) but the trajectory is unknown which can be very dangerous in some applications. Conversely it is possible to go directly from S1 to F1.

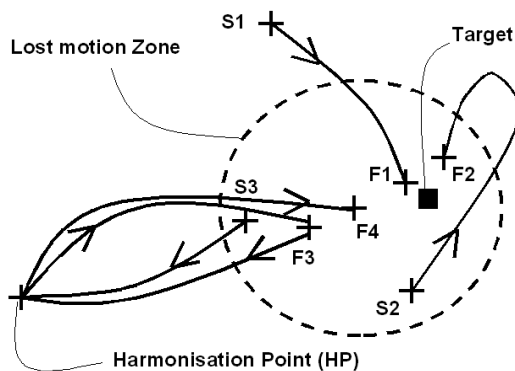


Fig.11 Corrective trajectories to come closer to the target

So what could be done when the robot endpoint is S3, in the lostmotion area, but too far from the desired target? The correct strategy is to repeat one portion of a trajectory and try to change it slightly so that the final endpoint position is very close to the preceding one. The start point of the trajectory to be repeated will be named “harmonization point” (HP), because it is from this point that every trajectory is repeated in nearly the same conditions. For instance, starting at S3, coming backwards to HP, correcting the target, arriving at F3, backwards to HP, correcting again the target, arriving at F4 show how this strategy works. For both attempts, the final target is slightly changed.

The second problem is to decide when to stop correcting the target considering that the final position is “good enough”. In general, if the position error is inferior to the spatial resolution, the result is considered satisfactory and the correction process is stopped.

## VII. CONCLUSIONS

Spatial resolution was here investigated and we have concluded that it was strongly dependent on the robot topology, architecture, posture and workspace location. Hence an innovative robot structure is detailed based on redundant kinematics and a new control strategy: a rough positioning using 3 axes, then for the final control, extra information is collected via external sensors and a minute final move is performed using only the two axes with the shorter lever-arm lengths. This innovation opens the field to great improvements in the performance of serial robot for minute micrometric positioning with promising applications in minute assembly tasks, electronic and microelectronic assembly industry, optoelectronics, medical and biological fields...

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