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Identification of Standard Inertial Parameters for Large-DOF Robots Considering Physical Consistency

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***Abstract*— The identification method for industrial manipulators considering physical consistency such as positive definiteness of inertial parameters has been developed, however it has to solve the quadratic programming with the non-linear inequality constraints. In identifying the large DOF systems like humanoid robots, the converged solution is difficult to be obtained. In this paper, we propose the method to realize physical consistency and computational stability. As inertial parameters of each link are represented with a finite number of mass points, the constraints can be approximated by linear inequalities. We also propose to solve the optimization problem, which minimizes the errors both from measured data and the priori parameters extracted from the geometric model like CAD data. The method can estimate standard inertial parameters, which is a useful notation to be used for other applications.**

I. INTRODUCTION

Identification of the inertial parameters of robots has been an active field of robotics. Needless to say, the motivation to identify the parameters is derived from the importance of understanding the dynamics of robots to perform accurate simulation or precise control. The identification technique of robotics utilizes the feature found in the equations of motion [1], [2]. The equations of motion of robots can be written in a linear form with respect to the dynamic parameters such as mass, center of mass, and moment of inertia tensor of each link. And the inertial parameters can be identified using this linearity of equations and the geometric parameters available in the design process of a robot.

Many identification methods can identify only the minimal set of inertial parameters, which describes the dynamics of the system, and is called base parameters [3], [4], [5]. The base parameters also appear linearly in the equations of motion of a robot. Thus, the classical identification problem can be solved with a linear least squares method. Nevertheless, the obtained results are not necessarily physically consistent [6], [7]; for example, some results about inertia matrix are not positive definite. And these parameters generate problems in the simulation or control requiring physical consistency. Mate et al. [8] proposed the method considering physical consistency, and also tested it on the 6-axis industrial manipulator. This method solves the quadratic programming with the non-linear inequality constraints which come from the condition of positive definiteness of the parameters.

In recent years, the technique of robotics has been applied for many other fields, and the application for legged systems like humanoid robots and humans is one example of those. For the study of physical ability performed by humanoid robots and humans, it is of course required to understand the dynamics of them. Thus, the identification process considering physical consistency is required. However, if we apply the method developed for manipulators to legged systems, there exist the following problems. First above all, the number of degrees of freedom (DOF) of the system is large, which requires a large-scale non-linear optimization. Next, it is difficult to generate stable and exciting motions used for identification because of the absence of the link fixed in the inertia frame, and insufficient data leads to a poor estimation result of some parameters. The body structure also causes gaps of identification performance among parameters, as the characteristics of every joint such as joint velocity and range of motion are different. Thus, converged solution is difficult to be obtained by the usual method.

In this paper, we propose the identification method to realize both physical consistency and computational stability for large DOF systems like legged systems. Inertial parameters of each link are represented with a finite number of mass points. Then the positive definiteness of the parameters can be replaced with linear inequalities, which always satisfy the original conditions, and the performance of approximation can be enhanced as the number of mass points increases. We also propose to solve the optimization problem, which minimizes both the error of the identification result and the error from the priori parameters extracted from the geometric model, which is obtained from CAD data of a robot or measured data of a human. If measured motions have little excitation, the priori parameters ensure the stability of computation. And the method using priori parameters can estimate standard inertial parameters. The notation of base parameters can be used for the control of robots, however the expression depends on the way to compose base parameters, and is complicated to implement and use for other applications especially in human identification, and thus the notation of standard inertial parameters is desirable.

II. IDENTIFICATION METHODS DEVELOPED FOR MANIPULATORS

The equations of the robot, composed of N rigid bodies and that has N_J DOF, is given by Eq.(2).

$$\boldsymbol{H}\ddot{\boldsymbol{q}} + \boldsymbol{b} = \boldsymbol{\tau} + \sum_{k=1}^{N_c} \boldsymbol{J}_k{}^T \boldsymbol{f}_k^{ext} \tag{1}$$

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where,

- $\boldsymbol{H} \in \boldsymbol{R}^{N_j \times N_j}$ is the inertia matrix of a robot,
- $\boldsymbol{q} \in \boldsymbol{R}^{N_j}$ is the generalized coordinates of a robot,
- $\boldsymbol{b} \in \boldsymbol{R}^{N_j}$ is the bias force vector including centrifugal, Coriolis and gravity forces,
- $\boldsymbol{\tau} \in \boldsymbol{R}^{N_j}$ is the vector of joint torques,
- N_c is the number of contact points with the environment,
- $\boldsymbol{f}_k^{ext} \in \boldsymbol{R}^6$ is the vector of external forces exerted to a robot at contact k,
- $\boldsymbol{J}_k \in \boldsymbol{R}^{6 \times N_J}$ is the basic Jacobian matrix of the position at contact k and of the orientation of the contact link with respect to generalized coordinates.

The equations of motion of multi-body systems can be written in a linear form with respect to the dynamic parameters [1], [2], and Eq.(2) can be obtained from Eq.(1).

$$\boldsymbol{Y}\boldsymbol{\phi} = \boldsymbol{f} \tag{2}$$

Where,

- $\boldsymbol{f} \in \boldsymbol{R}^{N_j}$ is equal to the right-hand side of Eq.(1),
- $\boldsymbol{\phi} \in \boldsymbol{R}^{10N}$ is the vector of constant inertial parameters such that

$$\boldsymbol{\phi} = \begin{bmatrix} \boldsymbol{\phi}_0^T & \boldsymbol{\phi}_1^T & \cdot\cdot & \boldsymbol{\phi}_{n-1}^T \end{bmatrix}^T \tag{3}$$

- $\boldsymbol{\phi}_j \in \boldsymbol{R}^{10}$ is the vector of constant inertial parameters of link j,

$$\boldsymbol{\phi}_j = [\ m_j \ ms_{j,x} \ ms_{j,y} \ ms_{j,z} \ I_{j,xx} \ I_{j,yy} \ I_{j,zz} \ I_{j,yz} \ I_{j,zx} \ I_{j,xy}\ ]^T \tag{4}$$

- m_j is the mass of the link j,
- $I_{j,xx}$, $I_{j,yy}$, $I_{j,zz}$, $I_{j,yz}$, $I_{j,zx}$, $I_{j,xy}$ are the 6 independent components of the moment of inertia matrix $\boldsymbol{I}_j \in \boldsymbol{R}^{3\times 3}$ expressed in the frame attached to link j,
- $s_{j,x}$, $s_{j,y}$, $s_{j,z}$ are the components of the vector $\boldsymbol{s}_j \in \boldsymbol{R}^3$, the center of mass with respect to the origin of the frame attached to link j.
- $\boldsymbol{Y} \in \boldsymbol{R}^{N_J \times 10N}$ is the regressor matrix or regressor, which is composed of $\boldsymbol{q}$, their derivatives $\dot{\boldsymbol{q}}$ $\ddot{\boldsymbol{q}}$, and geometric parameters like length of each link. The method to obtain $\boldsymbol{Y}$ is shown in [2].

Only the minimal set of inertial parameters that describes the dynamics of the system can be identified. This minimal set is called base parameters. It is computed symbolically or numerically from the inertial parameters $\boldsymbol{\phi}$ by eliminating those that have no influence on the model and regrouping some according to the kinematics of the system [3], [4], [5], [9]. The minimal identification model given by Eq.(5) is thus obtained.

$$\boldsymbol{Y}_B\boldsymbol{\phi}_B = \boldsymbol{f} \tag{5}$$

Where,

- N_B is the number of the base parameters,
- $\boldsymbol{Y}_B \in \boldsymbol{R}^{N_j \times N_B}$ is called the regressor matrix for the base parameters,
- $\boldsymbol{\phi}_B \in \boldsymbol{R}^{N_B}$ is the vector of the base parameters, and it is a linear combination $\boldsymbol{\phi}_B = \boldsymbol{Z}\boldsymbol{\phi}$, using the composition matrix $\boldsymbol{Z} \in \boldsymbol{R}^{N_B \times 10n}$ which can be computed from the structure of a robot.

For the identification process, we have to compute $\boldsymbol{Y}_B$ and $\boldsymbol{f}$ at every sampling time, measuring generalized coordinates, joint torques, and external forces acted on a robot. Then, we arrange T sampled regressors and forces of Eq.(5) at $t = t_1, t_2 \cdots t_T$ lengthwise, and compose the large regressor matrix $\boldsymbol{Y}_{all}$ and the large vector of forces $\boldsymbol{F}_{all}$ as below.

$$\boldsymbol{Y}_{Ball}\boldsymbol{\phi}_B = \begin{bmatrix} \boldsymbol{Y}_{B,t_1} \\ \vdots \\ \boldsymbol{Y}_{B,t_T} \end{bmatrix} \boldsymbol{\phi}_B = \begin{bmatrix} \boldsymbol{f}_{t_1} \\ \vdots \\ \boldsymbol{f}_{t_T} \end{bmatrix} = \boldsymbol{f}_{all} \tag{6}$$

After sampling along a motion, the parameter $\boldsymbol{\phi}_B$ in Eq.(6) is solved by the least squares method (LSM). However, $\boldsymbol{\phi}_B$ cannot be identified if improper data of motion is used for identification. It is thus important to sample the identification model along an adequately chosen motion that excites the system dynamics to estimate. Such motions are generally called persistent exciting trajectories [10].

The solution $\widehat{\boldsymbol{\phi}}_B$ of LSM with Eq.(6) minimizes the norm $||\boldsymbol{f}_{all} - \boldsymbol{Y}_{all}\widehat{\boldsymbol{\phi}}_B||$ that comes from the error of generalized forces estimated from identified parameters. Nevertheless, the obtained result is not necessarily physically consistent because of measurement noise and modeling error [6]. Some properties derived from physical consistency of inertial parameters are that mass and moment of inertial matrix of each link j $(1 \leq j \leq N)$ must to be positive definite [7].

$$m_j > 0, \qquad \boldsymbol{I}_{Cj} > 0 \tag{7}$$

Where, $\boldsymbol{I}_{Cj}(= \boldsymbol{I}_j - m_j[\boldsymbol{s}_j\times][\boldsymbol{s}_j\times]^T)$ is the moment of inertia matrix around center of mass $\boldsymbol{s}_j$ expressed in the frame attached to link j. Meta et.al. [8] proposed an identification method to solve the quadratic programming (QP) with the nonlinear inequality constraints Eq.(7), and also tested it on a 6-axis industrial manipulator. The solution satisfies Eq.(7) and minimizes the following evaluation function.

$$f(\boldsymbol{\phi}) = \lambda_\tau|\boldsymbol{Y}_{all}\boldsymbol{\phi} - \boldsymbol{f}_{all}|^2 + \lambda_\tau|\boldsymbol{Z}\boldsymbol{\phi} - \widehat{\boldsymbol{\phi}}_B|^2 \tag{8}$$

Where, $\boldsymbol{Y}_{all} \triangleq \boldsymbol{Y}_{Ball}\boldsymbol{Z}$, and $\widehat{\boldsymbol{\phi}}_B \triangleq \boldsymbol{Y}_{Ball}{}^{\#}\boldsymbol{F}_{all}$ is the solution of LSM.

III. Identification considering physical consistency and computational stability

A. Problems of application for legged systems

QP with the evaluation function Eq.(8) and the nonlinear inequality constraints Eq.(7) can be also applied for legged systems. However, the following problems exist, which make it difficult to obtain the converged solution.

- First above all, the number of DOF of legged systems is generally large, which require large-scale non-linear optimizations.
- As there exist no links fixed to the environment, it is difficult to separate motion planning for identification from control stability. This dynamics constraint leads to decrease the performance of persistent exciting trajectories [10].

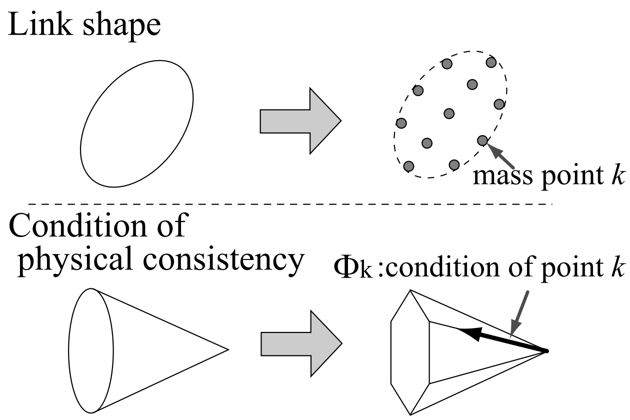

Fig. 1. A link object is approximated by a finite number of mass points. It leads that the condition of positive definiteness of inertia matrix represented by a convex cone in the space of inertial parameters is approximated by a polyhedral convex cone.

- The characteristics of every joint such like joint velocity and range of motion are significantly different, which causes gaps of identification performance among parameters.

Thus, the method considering computational stability is required. In this paper, we approximate both Eq.(7) and Eq.(8) to improve the computational stability under those problems.

B. Physical consistency based on approximation with polyhedral convex cones

First, we place a finite number of mass points in the convex hull of the link object as shown in Fig.1, in order to replace Eq.(7) with linear inequalities. $S_{\phi_j}(\subset \boldsymbol{R}^{10})$ is the set of inertial parameters $\boldsymbol{\phi}_j \in \boldsymbol{R}^{10}$ of the link j satisfying Eq.(7). S_{ϕ_j} is clearly a convex set, and $\forall a > 0$, $(a\boldsymbol{\phi}_j) \in S_{\phi_j}$ is verified, thus S_{ϕ_j} is an open set within a convex cone. Here, we approximate the parameters $\boldsymbol{\phi}_j$ of the link j using $N_{\rho,j}$ mass points as follows.

$$\boldsymbol{\phi}_j = \boldsymbol{P}_j \boldsymbol{\rho}_j \tag{9}$$

Where,

- $\boldsymbol{\rho}_j \in \boldsymbol{R}^{N_{\rho,j}}$ is the vector of mass of all points of link j such that $\boldsymbol{\rho}_j \triangleq [\ \rho_{j,1}\ \cdots\ \rho_{j,N_{\rho,j}}\ ]^T$, and $\rho_{j,k}$ is the mass of the point $k(1 \le k \le N_{\rho,j})$,
- $\boldsymbol{P}_j \in \boldsymbol{R}^{10\times N_{\rho,j}}$ is the matrix to compose $\boldsymbol{\phi}_j$ of $\boldsymbol{\rho}_j$ such as

$$\boldsymbol{P} \triangleq [\ \boldsymbol{\Phi}_1 \quad \boldsymbol{\Phi}_2 \quad \cdots \quad \boldsymbol{\Phi}_{N_{\rho,j}}\ ] \tag{10}$$

- $\boldsymbol{\Phi}_k \in \boldsymbol{R}^{10}$ is the inertial parameters of mass point k normalized by mass $\rho_{j,k}$ such that

$$\boldsymbol{\Phi}_k \triangleq \left[\ 1 \quad {}^j\boldsymbol{r}_k{}^T \quad \boldsymbol{\phi}_I([{}^j\boldsymbol{r}_k\times]^T[{}^j\boldsymbol{r}_k\times])^T\ \right]^T \tag{11}$$

- ${}^j\boldsymbol{r}_k \in \boldsymbol{R}^3$ is the position of mass point k with respect to the origin of the frame attached to link j,
- The function $\boldsymbol{\phi}_I(\boldsymbol{I}) \in \boldsymbol{R}^6$ returns the vector of inertial parameters concerning inertia matrix and is given by

$$\boldsymbol{\phi}_I(\boldsymbol{I}) \triangleq [\ I_{x,x} \quad I_{y,y} \quad I_{z,z} \quad I_{y,z} \quad I_{z,x} \quad I_{x,y}\ ]^T \tag{12}$$

Hence, $\boldsymbol{\phi}$ is represented using all $N_\rho(=\sum_j N_{\rho,j})$ mass points as follows.

$$\boldsymbol{\phi} = \boldsymbol{P}\boldsymbol{\rho} \tag{13}$$

Where, $\boldsymbol{P} \in \boldsymbol{R}^{10N\times N_\rho}$ and $\boldsymbol{\rho} \in \boldsymbol{R}^{N_\rho}$ is the following matrix and vector.

$$\boldsymbol{P} \triangleq \begin{bmatrix} \boldsymbol{P}_1 & \cdots & \boldsymbol{O} \\ \vdots & \ddots & \vdots \\ \boldsymbol{O} & \cdots & \boldsymbol{P}_N \end{bmatrix}, \quad \boldsymbol{\rho} \triangleq \begin{bmatrix} \boldsymbol{\rho}_1 \\ \vdots \\ \boldsymbol{\rho}_N \end{bmatrix} \tag{14}$$

If $\boldsymbol{P}_j$ is a full row rank matrix, then $\boldsymbol{P}$ is also full row rank, and there exists $\boldsymbol{\rho}$ to realize any $\boldsymbol{\phi}$. Thus, mass points have to be located to make each $\boldsymbol{P}_j$ full row rank. Then, Eq.(7) can be approximated as follows.

$$\boldsymbol{\rho} > \boldsymbol{0} \tag{15}$$

As we mentioned, Eq.(7) means the open set within a convex cone as Fig.1. On the other hand, the moment of inertia matrix which comes from the inertial parameters $\rho_k\boldsymbol{\Phi}_k$ consist only of the mass point k is always semi-positive definite, thus Eq.(15) means a ridge line of the convex cone as Fig.1. It means that Eq.(15) approximates the convex cone by the polyhedral convex cone. Thus, if Eq.(15) is verified, then Eq.(7) is verified, and Eq.(7) will be well approximated if the number of mass points increases. As the inertial parameters of a rigid body are originally defined by an infinite number of mass points, the positiveness of mass of each mass point also satisfies other properties of inertial parameters essentially, for example, concerning center of mass; the center of mass of each link always exists in the convex hull of the link. The optimization strategy to approximate nonlinear inequality constraints represented as a convex cone by a polyhedral convex cone is often adopted in other fields of robotics, for example in grasp analysis [11].

C. Evaluation function for computational stability

Next, we deal with the following evaluation function instead of Eq.(7).

$$\begin{aligned} g(\boldsymbol{\rho}) = (\boldsymbol{Y}_{all}\boldsymbol{P}\boldsymbol{\rho} - \boldsymbol{f}_{all})^T \boldsymbol{W}_f (\boldsymbol{Y}_{all}\boldsymbol{P}\boldsymbol{\rho} - \boldsymbol{f}_{all}) \\ + (\boldsymbol{\rho} - \boldsymbol{\rho}^{ref})^T \boldsymbol{W}_\rho (\boldsymbol{\rho} - \boldsymbol{\rho}^{ref}) \end{aligned} \tag{16}$$

Where, $\boldsymbol{W}_f \in \boldsymbol{R}^{N_jT\times N_jT}$ and $\boldsymbol{W}_\rho \in \boldsymbol{R}^{N_\rho\times N_\rho}$ are the weight matrices, and (semi-)positive definite. And $\boldsymbol{\rho}^{ref} \in \boldsymbol{R}^{N_\rho}$ is the desired value of $\boldsymbol{\rho}$ in the optimization. The solutions from LSM satisfy the first term of Eq.(16). However, the Hessian matrix of the usual evaluation function is always semi-positive definite as following reasons. Standard inertial parameters $\boldsymbol{\phi}$ (and of course $\boldsymbol{\rho}$) are not structurally identifiable as mentioned in the previous section. Moreover, if the poor motion trajectories cause the ill condition that the rank of $\boldsymbol{Y}_{Ball}$ is nearly equal to zero, identification performance of some parameters declines significantly. Thus, the second term of Eq.(16) is added to evaluate. This term makes the Hessian matrix of the evaluation function positive definite, and prevents from those ill conditions.

The evaluation function Eq.(8) is the generalized notation which includes both the evaluation of LSM and QP using Eq.(8). If $\boldsymbol{W}_f = \lambda_\tau \boldsymbol{E}_{N_jT} + \lambda_\phi \boldsymbol{Y}_{Ball}{}^{\#T}\boldsymbol{Y}_{Ball}{}^{\#}$, $\boldsymbol{W}_\rho = \boldsymbol{O}$ and $\boldsymbol{\rho}^{ref} = \boldsymbol{0}$, then Eq.(16) is equal to Eq.(8). Thus we can control both the exactness of the solution obtained from LSM and the stability of computation, choosing the weight matrices $\boldsymbol{W}_f$ and $\boldsymbol{W}_\rho$. This problem establishment uses an analogy from inverse kinematics solution with singularity robustness shown by [12].

IV. Identification of standard inertial parameters using geometric shape of links

A. Motivation to identify standard inertial parameters

Many identification methods can identify only the base parameters $\boldsymbol{\phi}_B$, as mentioned above. This notation of the minimal identification model can be used for the control of robots, but the expression depends on the way to compose base parameters, and is complicated to implement and use for other applications. For example, the base parameters obtained from human identification [13] are inadequate for the applications such as in medical field. The optimized solution obtained from only the first term of the evaluation function Eq.(8) is also meaningless in the sense of standard inertial parameters $\boldsymbol{\phi}$.

Dynamics identification usually requires the geometric parameters of the system. The parameters of a robot can be computed from CAD data, and those of a human body can be measured or estimated from literature data. In this section, we design the desired value $\boldsymbol{\rho}^{ref}$ and its weight matrix $\boldsymbol{W}_\rho$ in the second term of Eq.(8) based on the geometric model of links, in order to estimate the standard inertial parameters.

B. Design of $\boldsymbol{\rho}^{ref}$ and $\boldsymbol{W}_\rho$

First, S_j is defined as the bounding box of the link object j $(1 \leq j \leq N)$, and Σ_{Sj} as the frame attached in the bounding box S_j. The origin of Σ_{Sj} is located in the center of S_j, and each axis of Σ_{S_j} is aligned along each edge of the bounding box S_j. Then, the range of S_j can be written as $S_j = \{\ \boldsymbol{p} \mid -\boldsymbol{d}_j \leq \boldsymbol{p} \leq \boldsymbol{d}_j\ \}$, where $\boldsymbol{d}_j = [d_{j,x}\ d_{j,y}\ d_{j,z}]^T$ is equivalent to the half length of each edge of S_j. And m_j^{ref} is defined as the desired mass of each link estimated from geometric models like CAD data, and we also estimate the error $\Delta m_j (>0)$ of the desired mass. Then, the following inequality constraints concerning inertial parameters of each link are verified.

$$\boldsymbol{\phi}_{S_j}^{min} \leq \boldsymbol{\phi}_{S_j} \leq \boldsymbol{\phi}_{S_j}^{max} \tag{17}$$

Where,

- $\boldsymbol{\phi}_{S_j} \in \boldsymbol{R}^{10}$ is the vector of inertial parameters of link j expressed in Σ_{S_j}.

$$\boldsymbol{\phi}_{S_j} \triangleq {}^j\boldsymbol{B}_{S_j}{}^{-1}\boldsymbol{\phi}_j \tag{18}$$

- ${}^j\boldsymbol{B}_{S_j} \in \boldsymbol{R}^{10\times10}$ is the transformation matrix of inertial parameters [5] from $\boldsymbol{\phi}_j$ to $\boldsymbol{\phi}_{Sj}$.
- $\boldsymbol{\phi}_{S_j}^{max}$, $\boldsymbol{\phi}_{S_j}^{min} \in \boldsymbol{R}^{10}$ are the upper and lower bound defined as follows.

$$\boldsymbol{\phi}_{S_j}^{min} \triangleq -(m_j^{ref} + \Delta m_j)\begin{bmatrix} 1 & \boldsymbol{d}_j{}^T & \boldsymbol{0}^T & \boldsymbol{j}_{cj}{}^T \end{bmatrix} \tag{19}$$

$$\boldsymbol{\phi}_{S_j}^{max} \triangleq (m_j^{ref} + \Delta m_j)\begin{bmatrix} 1 & \boldsymbol{d}_j{}^T & \boldsymbol{j}_{dj}{}^T & \boldsymbol{j}_{cj}{}^T \end{bmatrix} \tag{20}$$

- $\boldsymbol{j}_{dj}, \boldsymbol{j}_{cj}$ are the upper bounds concerning components of moment of inertia matrix of S_j.

$$\boldsymbol{j}_{dj} \triangleq \begin{bmatrix} d_{j,y}{}^2 + d_{j,z}{}^2 \\ d_{j,z}{}^2 + d_{j,x}{}^2 \\ d_{j,x}{}^2 + d_{j,y}{}^2 \end{bmatrix},\ \boldsymbol{j}_{cj} \triangleq \begin{bmatrix} |d_{j,y}d_{j,z}| \\ |d_{j,z}d_{j,x}| \\ |d_{j,x}d_{j,y}| \end{bmatrix} \tag{21}$$

Standard inertial parameters $\boldsymbol{\phi}$ can be estimated from the geometric model and mass density of each link using, for example, CAD software, and $\boldsymbol{\phi}^{ref} \in \boldsymbol{R}^{10N}$ is defined as the estimated value of $\boldsymbol{\phi}$. If there exists no information about mass density, $\boldsymbol{\phi}^{ref}$ is estimated simply as $\boldsymbol{\phi}_{ref} = (\boldsymbol{\phi}_{S_j}^{max} + \boldsymbol{\phi}_{S_j}^{min})/2$. Then, $\boldsymbol{\phi}_j^{ref} \in \boldsymbol{R}^{10}$ is defined as the parameters of link j, $\boldsymbol{\phi}_{S_j}^{ref} \in \boldsymbol{R}^{10}$ as the parameters expressed in Σ_{Sj}, and $\sigma_{S_j,k}{}^2$ as the maximum value of mean square error of each element $\boldsymbol{\phi}_{Sj,k}^{ref} (1 \leq k \leq 10)$ of $\boldsymbol{\phi}_{Sj}^{ref}$. From Eq.(17), $\sigma_{S_j,k}{}^2$ can be computed as follows.

$$\sigma_{S_j,k}{}^2 = \max(\phi_{S_j,k}^{max} - \phi_{S_j,k}^{ref},\ \phi_{S_j,k}^{ref} - \phi_{S_j,k}^{min}) \tag{22}$$

Using the estimated value $\boldsymbol{\phi}_{S_j}^{ref}$ ($\boldsymbol{\phi}_j^{ref}$) and the mean error $\sigma_{S_j,k}{}^2 (1 \leq k \leq 10)$, the desired value $\boldsymbol{\rho}_{ref}$ and the weight matrix $\boldsymbol{W}_\rho$ in the second term of Eq.(8) is designed as follows.

$$\boldsymbol{\rho}^{ref} = \boldsymbol{P}^{\#}\boldsymbol{\phi}^{ref} \tag{23}$$

$$\boldsymbol{W}_\rho = \lambda_\phi \boldsymbol{P}^T\boldsymbol{N}\boldsymbol{W}_\phi\boldsymbol{N}\boldsymbol{P} + \lambda_\rho \boldsymbol{E} \tag{24}$$

Where,

- $\lambda_\rho,\ \lambda_\phi \in \boldsymbol{R}$ are the scaling factors,
- $\boldsymbol{N} \in \boldsymbol{R}^{10N\times10N}$ is the matrix representing the null-space of the composition matrix $\boldsymbol{Z}$,

$$\boldsymbol{N} \triangleq \boldsymbol{E} - \boldsymbol{Z}^{\#}\boldsymbol{Z} \tag{25}$$

- $\boldsymbol{W}_\phi \in \boldsymbol{R}^{10N\times10N}$ is the weight matrix concerning all the standard inertial parameters,

$$\boldsymbol{W}_\phi \triangleq \begin{bmatrix} \boldsymbol{W}_{\phi_1} & \cdots & \boldsymbol{O} \\ \vdots & \ddots & \vdots \\ \boldsymbol{O} & \cdots & \boldsymbol{W}_{\phi_N} \end{bmatrix} \tag{26}$$

- $\boldsymbol{W}_{\phi_j} \in \boldsymbol{R}^{10\times10}$ is the weight matrix concerning $\boldsymbol{\phi}_j$,

$$\boldsymbol{W}_{\phi_j} \triangleq {}^j\boldsymbol{B}_{S_j}{}^{-T}\ \boldsymbol{W}_{Sj}\ {}^j\boldsymbol{B}_{S_j}{}^{-1} \tag{27}$$

- $\boldsymbol{W}_{S_j} \in \boldsymbol{R}^{10\times10}$ is the weight matrix concerning $\boldsymbol{\phi}_{Sj}$ expressed in Σ_{S_j}. In this paper, we assume that the errors of $\boldsymbol{\phi}_{S_j}$ obey normal distribution, and $\boldsymbol{W}_{S_j} \in \boldsymbol{R}^{10\times10}$ is designed as the following variance-covariance matrix.

$$\boldsymbol{W}_{S_j} \triangleq \begin{bmatrix} \sigma_{S_j,1}{}^2 & \cdots & \boldsymbol{O} \\ \vdots & \ddots & \vdots \\ \boldsymbol{O} & \cdots & \sigma_{S_j,10}{}^2 \end{bmatrix} \tag{28}$$

C. Summary

From Eq.(23) and Eq.(24), Eq.(16) can be transformed as below.

$$\begin{aligned} g(\boldsymbol{\rho}) = & (\boldsymbol{Y}_{all}\boldsymbol{P}\boldsymbol{\rho} - \boldsymbol{f}_{all})^T \boldsymbol{W}_f (\boldsymbol{Y}_{all}\boldsymbol{P}\boldsymbol{\rho} - \boldsymbol{f}_{all}) \\ & + (\boldsymbol{P}\boldsymbol{\rho} - \boldsymbol{\phi}^{ref})^T \boldsymbol{N}\boldsymbol{W}_\phi \boldsymbol{N}(\boldsymbol{P}\boldsymbol{\rho} - \boldsymbol{\phi}^{ref}) \\ & + \lambda_\rho |\boldsymbol{\rho} - \boldsymbol{\rho}^{ref}|^2 \end{aligned} \quad (29)$$

The solution from the standard dynamics identification minimizes only the first term of Eq.(29), and the weight matrix $\boldsymbol{W}_f$ is equivalent to one used in LSM. If $\boldsymbol{W}_f = \boldsymbol{E}$ is chosen, then the solution minimize the error norm of $\Delta \boldsymbol{f}_{all} \triangleq \boldsymbol{f}_{all} - \boldsymbol{Y}_{all}\boldsymbol{\phi}$. And if the standard deviation $\sigma_{f,j} (1 \le j \le N)$ of each element of $\Delta \boldsymbol{f}_{all}$ is known or estimated, $\boldsymbol{W}_f = \mathrm{diag}([1/\sigma_{f,1}{}^2 \ \cdots \ 1/\sigma_{f,n}{}^2])$ is generally adopted to normalize by the variances. The second term of Eq.(29) is the error from the priori parameters $\boldsymbol{\phi}^{ref}$ obtained from geometric model, which makes it possible to estimate the standard inertial parameters. The third term makes the Hessian matrix of the evaluation function positive definite, and realizes computational stability even when it is difficult to generate persistent exciting trajectories or the priori parameters $\boldsymbol{\phi}_{ref}$ are not reliable.

We add explanations about the second term of Eq.(29). If the physical consistency and computational stability are not considered, i.e. Eq.(15) is ignored and $\lambda_\rho = 0$, the optimized solution of the evaluation function Eq.(29) is as follows.

$$\widehat{\boldsymbol{\rho}} = \boldsymbol{P}^{\#}(\boldsymbol{Z}^{\#}\widehat{\boldsymbol{\phi}}_B + \boldsymbol{N}\boldsymbol{\phi}^{ref}) + (\boldsymbol{E} - \boldsymbol{P}^{\#}\boldsymbol{P})\boldsymbol{r} \quad (30)$$

Where, $\boldsymbol{r} \in \boldsymbol{R}^{N_\rho}$ is an arbitrary vector, and $\widehat{\boldsymbol{\phi}}_B$ is the base parameters obtained from LSM as below.

$$\widehat{\boldsymbol{\phi}}_B = (\boldsymbol{Y}_{Ball}{}^T \boldsymbol{W}_f \boldsymbol{Y}_{Ball})^{-1} \boldsymbol{Y}_{Ball}{}^T \boldsymbol{W}_f \boldsymbol{f}_{all} \quad (31)$$

If we express the solution by the notation of standard inertial parameters, Eq.(32) can be obtained from $\widehat{\boldsymbol{\phi}} = \boldsymbol{P}\widehat{\boldsymbol{\rho}}$.

$$\widehat{\boldsymbol{\phi}} = \boldsymbol{Z}^{\#}\widehat{\boldsymbol{\phi}}_B + \boldsymbol{N}\boldsymbol{\phi}^{ref} \quad (32)$$

The minimal notation $\boldsymbol{Z}\widehat{\boldsymbol{\phi}}$ is equal to identified base parameters $\widehat{\boldsymbol{\phi}}_B$, thus Eq.(32) satisfies the LSM solution exactly and the parameters not to be identified from LSM are estimated from the priori parameters using the null-space of the composition matrix $\boldsymbol{Z}$. If it is difficult to generate the motion with enough excitation, we should use the matrix $\boldsymbol{Z}^{num}$ computed from the measured regressor $\boldsymbol{Y}_{all}$ numerically [9] instead of the matrix $\boldsymbol{Z}$ obtained from the kinematic structure symbolically. If using $\boldsymbol{Z}^{num}$, the parameters, which are difficult to be identified because of poor excitation, are eliminated from base parameters and can be estimated from the priori parameters $\boldsymbol{\phi}^{ref}$.

We summarize the process to design $\boldsymbol{\rho}^{ref}$ and $\boldsymbol{W}_\rho$. First, we require the bounding box S_j of each link obtained from geometric model, for example, using CAD software, and then compute $\boldsymbol{d}_j$. We also estimated the desired mass m_j^{ref}. The parameters to be designed are the estimated error of mass Δm_j of each link, the scaling factor λ_ϕ weighting the influence of $\boldsymbol{\phi}^{ref}$, and the scale λ_ρ for computational stability.

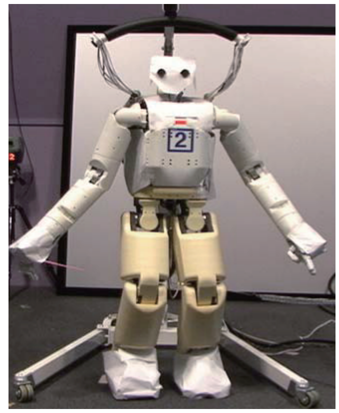

Fig. 2. The IRT project humanoid robot.

TABLE I

STANDARD INERTIAL PARAMETERS OF 6 LINKS ESTIMATED FROM THE PROPOSED METHOD. (L1:UPPER TRUNK, L2: LOWER TRUNK, L3:RIGHT UPPER LEG, L4:RIGHT FOOT, L5:LEFT UPPER ARM, L6:LEFT HAND)

Link	L1	L2	L3	L4	L5	L6
m/M	0.293	0.073	0.060	0.013	0.032	0.021
s_x^{BB}	0.234	-0.146	-0.085	0.258	0.723	0.088
s_y^{BB}	0.248	0.350	0.003	-0.082	-0.093	0.057
s_z^{BB}	0.342	0.348	-0.253	-0.512	-0.021	0.165
I_{cxx}/m	0.023	0.013	0.033	0.015	0.003	0.021
I_{cyy}/m	0.024	0.009	0.023	0.013	0.004	0.007
I_{czz}/m	0.022	0.018	0.022	0.008	0.004	0.022
I_{cyz}/m	-0.000	0.001	0.010	-0.001	-0.000	-0.004
I_{czx}/m	-0.002	-0.000	0.001	0.001	-0.000	0.000
I_{cxy}/m	-0.000	0.000	-0.001	-0.000	0.000	-0.000

V. EXPERIMENTAL RESULTS

A. Identification model for legged systems and experimental setup

In this section, we show identification results of the humanoid robot shown in Fig.2 using the proposed method. The robot has 38 DOF consisting of: 3 joints in the head, 7 in each arm, 7 in each leg, 1 in the waist, and 3 in the fingers of each hand. During experiments, the fingers were not used and maintained in a constant position, resulting in the use of 32 DOF. The robot is equipped with a gyro sensor in the upper body link, encoders in each joint, and 6-axis force sensors in both feet. In this paper, we use the identification model for legged systems [14]. The equations of motion of legged systems are given by Eq.(33). Eq.(33) is represented as a minimal identification model.

$$\begin{bmatrix} \boldsymbol{Y}_{BO} \\ \boldsymbol{Y}_{BC} \end{bmatrix} \boldsymbol{\phi}_B = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\tau}_c \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_0^{ext} \\ \boldsymbol{f}_c^{ext} \end{bmatrix} \quad (33)$$

Where,

- The upper part of Eq.(33) represents the equations of motion of the base-link, which means the root of the kinematic tree structure, and the lower part represents the equations of motion of the joints.
- $\boldsymbol{Y}_{BO} \in \boldsymbol{R}^{6 \times N_B}$ is the regressor matrix of the base-link, and $\boldsymbol{Y}_{BC} \in \boldsymbol{R}^{(N_j-6) \times N_B}$ is the regressor of the joints,
- $\boldsymbol{\tau}_c \in \boldsymbol{R}^{N_j-6}$ is the vector of joint torque,

TABLE II

STANDARD INERTIAL PARAMETERS ESTIMATED FROM LSM.

Link	L1	L2	L3	L4	L5	L6
m/M	0.321	0.057	0.072	0.016	0.026	0.029
s_x^{BB}	0.243	-0.001	-0.441	7.362	-1.225	0.287
s_y^{BB}	0.067	0.671	-0.290	16.304	-1.683	0.475
s_z^{BB}	0.339	-0.522	0.227	-0.487	2.144	0.163
I_{cxx}/m	0.024	-4.825	0.478	-5.683	0.886	-0.113
I_{cyy}/m	0.005	4.801	-0.047	-2.982	0.642	0.041
I_{czz}/m	0.027	-0.171	-0.615	-3.000	-0.625	-0.088
I_{cyz}/m	0.074	0.219	0.067	1.155	-1.090	0.057
I_{czx}/m	0.184	0.693	-0.157	1.066	-0.928	-0.079
I_{cxy}/m	0.015	0.975	-0.002	-2.481	0.224	0.049

TABLE III

STANDARD INERTIAL PARAMETERS COMPUTED FROM CAD DATA.

Link	L1	L2	L3	L4	L5	L6
m/M	0.301	0.054	0.070	0.020	0.026	0.028
s_x^{BB}	0.146	-0.011	-0.009	0.187	0.298	0.078
s_y^{BB}	0.179	0.113	-0.159	0.003	0.002	0.092
s_z^{BB}	0.233	0.134	0.030	-0.467	0.005	0.018
I_{cxx}/m	0.014	0.008	0.015	0.002	0.002	0.005
I_{cyy}/m	0.018	0.003	0.015	0.003	0.001	0.005
I_{czz}/m	0.016	0.010	0.003	0.002	0.002	0.001
I_{cyz}/m	0.000	-0.000	0.001	-0.000	-0.000	-0.000
I_{czx}/m	0.002	-0.001	0.000	0.000	0.000	0.000
I_{cxy}/m	0.000	0.000	-0.000	0.000	0.000	-0.000

- $\boldsymbol{f}_0^{ext} \in \boldsymbol{R}^6$ is the vector of total external forces exerted to the base-link, and $\boldsymbol{f}_c^{ext} \in \boldsymbol{R}^{N_j-6}$ is the vector of external joint torque.

Most common identification methods use Eq.(33) to identify $\boldsymbol{\phi}_B$, we have proposed to use only the upper-part of the identification model Eq.(33), i.e. $\boldsymbol{Y}_{BO}\boldsymbol{\phi}_B = \boldsymbol{f}_0^{ext}$ that are the equations of motion of the base-link. The feature of the base-link is that the generalized force which actuates 6 DOFs of the base-link is always zero, which means that the joint-torque measurement is unnecessary for identification using only equations of motion of the base-link. However, this method stands only if the reduction of the system to these six equations keeps unchanged the number of parameters that are structurally identifiable with the whole system. We have mathematically proven that the reduced system leads to similarly identify the whole set of base parameters [14]. For identification, we generated several types of walking motions and upper body motions to be used.

We mention the location of mass points used in the method. Eq.(7) will be well approximated if the number of mass points increases, which leads the large computation cost of the optimization. If we utilize the sparseness of $\boldsymbol{P}$ to reduce the computation cost for the optimization, the cost increases in proportion to the square of the number of mass points. Moreover, there exist the measurement and geometric modeling error in practice, thus the obtained results cannot be improved if we approximate Eq.(7) with maximum accuracy. It means that we should choose the number of points according to the computational feasibility and the practical limit of accuracy improvement. In this instance, the bounding box S_j of each link object obtained

TABLE IV

STANDARD INERTIAL PARAMETERS ESTIMATED FROM THE PROPOSED METHOD WITHOUT THE PRIORI PARAMETERS $\boldsymbol{\phi}^{ref}$ FROM CAD DATA

Link	L1	L2	L3	L4	L5	L6
m/M	0.090	0.072	0.050	0.018	0.029	0.013
s_x^{BB}	0.088	0.032	-0.221	0.079	0.067	0.144
s_y^{BB}	0.037	0.564	0.073	0.036	-0.358	0.037
s_z^{BB}	0.704	-0.031	-0.352	-0.465	0.060	-0.170
I_{cxx}/m	0.052	0.016	0.032	0.016	0.006	0.019
I_{cyy}/m	0.037	0.010	0.023	0.013	0.004	0.006
I_{czz}/m	0.032	0.020	0.023	0.009	0.006	0.021
I_{cyz}/m	0.006	0.002	0.010	-0.000	0.000	-0.003
I_{czx}/m	-0.001	-0.000	0.001	0.001	-0.000	0.000
I_{cxy}/m	0.000	0.001	-0.000	-0.001	-0.000	-0.000

from CAD data is treated as the convex hull, and we put $27(= 3^3)$ equally-spaced points in S_j.

Next, the designed parameters of Eq.(29) are mentioned. In order to minimize the root mean squares of the estimated force error, we have chosen $\boldsymbol{W}_f = \boldsymbol{E}/N_T$. From CAD data of the robot, m_j^{ref} and $\boldsymbol{\phi}^{ref}$ can be computed. We have also estimated 50 percent relative error of mass of each link and set the lower bound of absolute error as 2.0[kg]; $\Delta m_j = \min(0.5\, m_j^{ref},\ 2.0)$. And the scale factors have been chosen as $\lambda_\rho = 0.001$, $\lambda_\phi = 1$ to prevent from the ill condition of the optimization.

B. Results of identification

The standard inertial parameters estimated from the proposed method are given in Table I. Table I shows the relative mass m/M normalized by the total mass M of the robot, the center of mass s_i^{BB} from the center of the bounding box which is normalized by the size $\boldsymbol{d}_j$ of the box, and the relative components of moment of inertia matrix $I_{cij}/M[\mathrm{m}^2]$ (around the center of mass) normalized by the mass m of some typical links; upper torso(L1), lower torso(L2), right thigh(L3), right foot(L4), left upper arm(L5), left hand(L6). As we could not obtain the converged solution from QP with the nonlinear inequalities [8], we compared to the parameters $\widehat{\boldsymbol{\phi}}$ of Eq.(32) using the same weight matrix $\boldsymbol{W}_f$. It means the comparison with LSM because the parameters extracted from $\widehat{\boldsymbol{\phi}}$ are equivalent to the ones identified from LSM. The obtained results are given in Table II. The prior standard parameters from CAD data of the robot are shown in Table III, and the parameters identified without the priori parameters, i.e. $\boldsymbol{W}_\phi = \boldsymbol{O}$ in Eq.(29), are also shown in Table IV to be compared.

In Table II, all the relative mass is positive, however most of the principal moments of inertia show negative values, and they are clearly not physically consistent. Actually, all the 33 inertia matrices are non-positive definite. Furthermore, the centers of mass of the right thigh(L4) and the left hand(L6) are located outside of each link object, as some elements of s_i^{BB} are not within ± 1. On the other hand, all the results in Table I and Table IV satisfy the conditions of Eq.(7), and the centers of mass also exist in the bounding box because all the elements of s_i^{BB} are within ± 1. Hence, the proposed method shows both physical consistency and computational stability.

TABLE V

ROOT MEAN SQUARES OF 6 COMPONENTS OF EXTERNAL FORCE ERRORS ESTIMATED FROM 4 METHODS; (A) THE PROPOSED METHOD(TABLE I), (B) LSM(TABLE II), (C) THE ESTIMATION FROM THE PRIORI PARAMETERS OF CAD DATA(TABLE III), AND (D) THE PROPOSED METHOD WITHOUT THE PRIORI PARAMETERS(TABLE IV).

	σ_{Fx}	σ_{Fy}	σ_{Fz}	σ_{Nx}	σ_{Ny}	σ_{Nz}
(A)	6.73	6.03	3.74	3.00	2.69	0.73
(B)	6.69	5.63	3.60	2.49	1.92	0.80
(C)	7.27	6.23	36.03	3.53	8.78	0.74
(D)	6.74	6.11	3.69	3.13	2.57	0.75

The relative masses in Table I are close to the prior parameters in Table III, and other parameters in Table I also show the good correlations with Table III. However the parameters in Table IV show clear differences compared with the ones in Table I and Table III, which means that the proposed method can estimate the standard inertial parameters successfully.

Table V gives the root mean squares of 6 components of external force errors $\boldsymbol{f}_0^{ext} - \boldsymbol{Y}_{OB}\boldsymbol{\phi}_B$, estimated from 4 types of parameters in Table I - IV. As it can be seen from Table V, the estimated forces only from CAD data in the case (C) show significant errors especially in F_z and N_y. These errors derive from the modeling errors such as cables, electronic devices and some additional parts of hardware. On the other hand, the maximum relative difference of the root mean squares from the proposed method to the ones from LS method is less than about 40[%] in both cases (A) and (D). When we compare the case (A) using the priori parameters $\boldsymbol{\phi}^{ref}$ with the case (D) without $\boldsymbol{\phi}^{ref}$, all the relative errors of force are less than 5[%] and there is no clear difference. It means that in the proposed method the priori parameters $\boldsymbol{\phi}^{ref}$ is used mainly for the estimation of the parameters not concerning about base parameters, and have little influence on the normal identification of base parameters because of the use of the null-space of the composition matrix $\boldsymbol{Z}$ in the evaluation function Eq.(29).

Finally, Fig.3 shows the comparison of the total external forces acted on the base-link in the walking motion that is not used during the identification procedure. The red lines show the external forces $\boldsymbol{f}_0^{ext}$ measured by force sensors, the blue long dashed dotted lines mean the estimated ones $\widehat{\boldsymbol{f}}_{0,LSM}^{ext}$ from LSM, and the black dashed lines are the estimated ones $\widehat{\boldsymbol{f}}_0^{ext}$ of the proposed method. As we mentioned, we could not obtain the converged solution from the usual method [8]. From these figures, both identified parameters allow a good prediction of the generalized forces, and the proposed method slightly reduces the accuracy with respect to LSM.

VI. CONCLUSION

If we identify the inertial parameters of large DOF systems considering physical consistency, the converged solution is difficult to be obtained by the quadratic programming with the non-linear inequality constraints representing the conditions concerning about positive definiteness of moment of inertia matrix. In our proposed method, inertial parameters of each link are represented with a finite number of mass points. The original conditions of inertia matrix can be approximated by the positiveness of mass of each mass point, which can replace those conditions with linear inequalities. And the performance of approximation can be enhanced as the number of mass points increases. The positive definiteness of moment of inertia matrix is only a part of physical consistency of inertial parameters. As the inertial parameters of a rigid body are originally defined by an infinite number of mass points, the positiveness of mass of each mass point also satisfies other physical conditions essentially, for example, concerning center of mass; the center of mass of each link always exists in the convex hull of the link.

We also propose to solve the optimization problem, which minimizes both the error of the identification result and the error from the priori parameters extracted from the geometric data, which is obtained from CAD data of a robot or measured data of a human. If measured motions have little excitation, the priori parameters ensure the stability of computation. And the method using priori parameters can also estimate standard inertial parameters, although usual identification methods can identify only the base parameters. The notation of standard inertial parameters is useful to be implemented and used for other applications. In the proposed method, the priori parameters is used mainly for the estimation of the parameters not concerning about base parameters and the stability of computation, and have little influence on base parameters. It means that the method ensures the exactness of the normal identification results.

The method has been tested on a 34 DOF humanoid robot. As the converged solution from QP with the nonlinear inequalities could not be obtained, the results are compared with the standard parameters identified from LSM. The LSM solutions of all links show no physical consistency, on the other hand, all the parameters identified from the proposed method show the physical consistency. The root mean squares of estimation errors of 6-axis external force acted on the base-link are nearly equal in both methods. The maximum relative difference of the proposed method to LSM is less than 40[%], which means that the proposed method slightly reduce the accuracy with respect to the LSM.

The results are also compared with the priori parameters obtained from CAD data of the robot. The identified parameters by the proposed method are close to the priori parameters, and thus the proposed method can estimate the standard inertial parameters successfully. Additionally, in the proposed method we compare the results in the case using the priori parameters with in the case without them, and there is no clear difference of the root mean squares of estimated force errors between in both cases. Thus it is shown that the priori parameters have little influence on base parameters and the proposed method ensures the exactness of the normal identification results.

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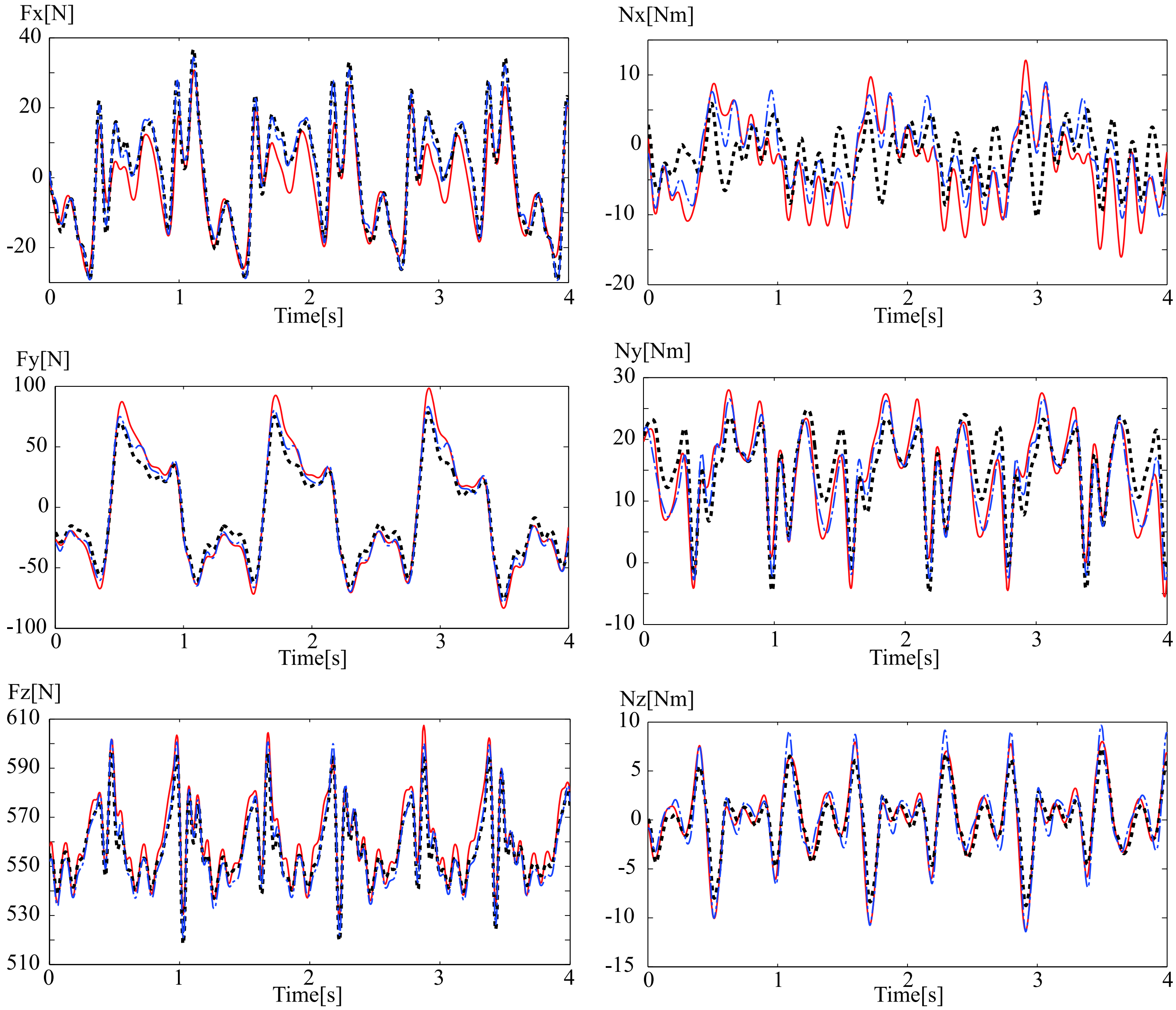

Fig. 3. Cross validation figures of the total external forces acted on the base-link of the robot in the walking motion that is not used during the identification procedure. The red lines show the measured external forces. The blue long dashed dotted lines mean the estimated ones from LSM. The black dashed lines are the estimated ones of the proposed method. Root mean square errors of 6-axis forces of both two methods are as follows :(LSM) Fx 5.34[N], Fy 5.56[N],Fz 5.21[N], Nx 3.13[Nm], Ny 2.36[Nm], Fz 1.14[Nm], (proposed) Fx 5.76[N], Fy 9.72[N],Fz 5.52[N], Nx 5.45[Nm], Ny 4.25[Nm], Nz 1.09[Nm].

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