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Multirobot Consensus while Preserving Connectivity in Presence of Obstacles with Bounded Control Inputs

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Abstract—**In the existing literatures of multirobots, it is usually assumed that the networked robots remain connected in topologyduring the task execution. In pracice, however, it is not easy to guarante connectivity of the networked robots in a clustered environment. Failure to maintain connectivity may decrease the performance of the networked robots or even fail the task. In this paper, we propose a multirobot motion coordination strategy that can maintain multirobot connectivity as well as guarante obstacle avoidance. A potential function is proposed to generate bounded control inputs for networked robots. The efficiency of the proposed apporach is demonstrated in both simulation and experiment performed on multirobot consensus tasks.**

I. Introduction

Distributed controls of networked robots have received considerable attention in recent years to solve numerous problems such as consensus network [1]-[4], formation [5]-[9], and coverage problem [10] [11], etc. Among these studies, a basic assumption is that all the networked robots remain entirely connected during the task implementation. In practice, however, due to limited sensing and communication capabilities of robots, it is difficult to guarantee connectivity of the networked robots, which may lead to failure of the group tasks.

Several approaches have been proposed to solving connectivity problem of the networked robots. The three main approaches reported in the literatures are: geometrical constraint technique, spectral graph theory method, and artificial potential field method. The geometrical constraint technique was pioneered by Ando [12], which was extended to the second order system [13]. Through measuring robustness of the local connectivity of the networked robots, global connectivity could be achieved [14]. With the graph theory method, the connectivity problem was divided into two branches. One is to maximize the second smallest eigenvalue of the graph Laplacian matrix to guarantee connectivity [15]-[17]. The other is to have connectivity rely on notions of algebraic graph theory and gossip algorithm [18]. The artificial potential field method enables the system to converge to the desired configuration while preserving connectivity by the potential field force [3] [7] [19]-[24]. The most practical way of this method is to assign an appropriate weight to each communication link [2]. The weight is characterized as the tension force, which reaches infinity whenever the communication breaks. Recently, a navigation function was introduced to achieve network connectivity with bounded control inputs [25].

A common problem of the artificial potential field force approach and the weighted graph approach is the use of unbounded potential fields to force the robots to maintain connectivity whenever the robot tends to leave the sensing or communication zone between each other. In practical applications, however, unbounded input is impossible, i.e., the motor cannot generate an infinitely large torque to the robots.

In this paper, as motivated by [2] [25], we study toward a solution to multirobot consensus problem by a bounded potential field force. Compared to [25] [2], our approach makes a particular contribution in achieving rendezvous task while maintaining connectivity in presence of obstacles. Moreover, our approach uses bounded control inputs, which make it easier to be applied in practice. A navigation-function-like potential function is proposed, with the potential field modeled by integrating consensus requirement, connectivity maintenance, and obstacle avoidance, simultaneously. The control law assigned to each robot is negative gradient of the proposed potential field. Under this control law, the networked robots can achieve consensus in the presence of obstacles. When the initial configurations of the networked robots are connected, the control law enables the underlying network to remain connected during motion evolution.

II. Problem Formulation

Consider n robots in a space $W \in \Re^2$. Denote the coordinate of the ith robot as $q_i \in \Re^2$. The dynamics of the ith robot is represented as follows

$$\dot{q}_i = u_i, \quad i \in \{1,\ldots,n\} \tag{1}$$

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where $u_i \in \Re^2$ denotes the control input, which is the velocity of the ith robot. Introduce the position vector $\mathbf{q} = \left[q_1^T, \ldots, q_n^T\right]^T$ and the control input vector $\mathbf{u} = \left[u_1^T, \ldots, u_n^T\right]^T$, where $\mathbf{q} \in \Re^{2n}$ and $\mathbf{u} \in \Re^{2n}$. Then the dynamics of all the networked robots can be rewritten as

$$\dot{\mathbf{q}} = \mathbf{u} \tag{2}$$

Similarly, positions of obstacles can be represented by a vector $\mathbf{qo} = \left[qo_1^{\;T}, \cdots qo_\varsigma^{\;T}, \cdots qo_M^{\;T}\right]^T$, where $\varsigma \in \{1, \ldots, M\}$

Due to limited sensing or communication capability of the robots, we suppose that the ith robot can communicate with the others only when they are located within a reachable area, which can be denoted by a circle centered by q_i and with the radius of r.

To preserve connectivity of the networked robots, in a similar manner to [14], we can define an information flow graph I according to the task requirement. For simplicity, we assume that the task requirement is symmetric.

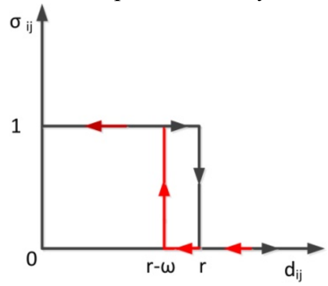

Fig.1 Dynamic adding and removing communication links.

Definition 1 (Information Flow Graph) The information graph, denoted by $I(t) = (V, E'(t))$, is a dynamic undirected graph consisting of a set of vertices $V = \{1, \cdots, n\}$ as indices of the robots, and a set of edge expressed by

$$E'(t) = \left\{(i,j) \middle| d_{ij}(t) < r \text{ and } \sigma_{ij}(t) = 1\right\},$$

where $\sigma_{ij} = \sigma_{ji} \in \{0,1\}$ is a symmetric indicator that determines whether or not the information available in the edge (i,j) should be taken into account. A hysteresis function σ_{ij} is used to create new links and remove unused links, which can be calculate as (Fig. 1)

$$\sigma_{ij}(t^+) = \begin{cases} 0, & \left(\sigma_{ij}(t^-) = 1 \cap r - \omega \le d_{ij} \le r\right) \cup d_{ij} \ge r \\ 1, & \left(\sigma_{ij}(t^-) = 0 \cap r - \omega \le d_{ij} \le r\right) \cup d_{ij} < r - \omega \end{cases} \tag{3}$$

where $\omega > 0$ is a switching threshold.

Based on [2], we define the neighborhood relationship of the ith robot in the information graph I as follows

$$N_I(i) = \left\{j \middle| (i,j) \in E'(t)\right\} \tag{4}$$

Definition 2 (Graph Connectivity) *A graph C is connected if there exists a path, i.e., a sequence of distinct vertices, such that any vertex can be reached by the other ones.*

Define I_C as the set containing all the connected graphs with n robots. The problem of connectivity control can then be formulated as follows.

Problem 1 (Graph Formulation of Connectivity Control) *Given I_C, determine the bounded control input $\mathbf{u}$, such that when $I(0) \in I_C$ in the initial time, $I(t) \in I_C$ for all the time*

III. Networked Control Design

A. Connectivity constraint

Consider a networked robot system, as shown in Fig. 2, where the dashed cycles denote the communication ranges of the robots. In order to maintain the communication links between robot i and its neighbors $i-1$ and $i+1$, robot i should stay in the space marked by $\overline{F}_i$, which is enclosed with the solid curve in Fig. 2.

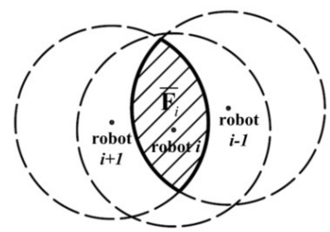

Fig.2 An example of connectivity constraint.

The connectivity constraint can be expressed as

$$G_c(q_i) = \prod_{j \in N_I(i)} \frac{1}{2}\left(r^2 - d_{ij}^{\;2}\right) = \prod_{j \in N_I(i)} \beta_{ij} \tag{5}$$

If the ith robot remains inside of $\overline{F}_i$, it connects to all of its neighbors in $N_I(i)$, and $G_c(q_i) > 0$. We further have the following definition.

Definition 3 (Connectivity Space $\overline{F}_i$)

$$\overline{F}_i = \left\{q_i \middle| G_c(q_i) > 0\right\} \tag{6}$$

B. Free work space

The free work space F_i is defined as a subset of robot positions that meet the connectivity constraint $G_c(q_i) > 0$, with additional consideration of obstacle avoidance. In Fig. 3, the obstacle is denoted by a disk. The free work space of robot i is enclosed by the solid curve, which is marked by F_i, excluding the area with the obstacle. Note that the difference between F_i and $\overline{F}_i$ is that $\overline{F}_i$ does not consider obstacle avoidance and F_i does.

Denote $O(i)$ as the set of obstacles encountered by the ith robot. For the obstacle $\varsigma \in O(i)$ centered at $q_o(\varsigma)$ with the radius $\rho_o(\varsigma)$, the distance between robot i and obstacle ς is denoted as $d_{i\varsigma} = \left\|q_i - q_o(\varsigma)\right\|$. Denote $G_o(q_i)$ as constraint for the obstacle avoidance, expressed as

$$G_o(q_i) = \prod_{\varsigma \in O(i)} \frac{1}{2}\left(d_{i\varsigma}^2 - \rho_o^{\ 2}(\varsigma)\right) = \prod_{\varsigma \in O(i)} \alpha_{i\varsigma} \tag{7}$$

If the ith robot does not hit any obstacle, we have $G_o(q_i) > 0$.

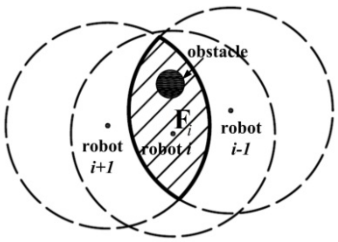

Fig. 3 The free work space of robot i.

In order to maintain the robots to be within their free work spaces, both constraints for connectivity (5) and obstacle avoidance (7) should be taken into account simultaneously. Denote $G(q_i)$ as the free work space that combines two types of the constraints, expressed by

$$G(q_i) = G_c(q_i) G_o(q_i) = \prod_{j \in N_I(i)} \beta_{ij} \prod_{\varsigma \in O(i)} \alpha_{i\varsigma} \tag{8}$$

If the ith robot remains inside of F_i, we have $G(q_i) > 0$.

Definition 5 (Free Robot Work Space)

$$F_i = \{q_i | G(q_i) > 0\} \tag{9}$$

A. Consensus control

Define $\varphi_i \in [0,1]$ as a potential function, expressed by

$$\varphi_i = \frac{\gamma(q_i)}{\left(\gamma^k(q_i) + G_c(q_i) G_o(q_i)\right)^{\frac{1}{k}}} = \frac{\gamma(q_i)}{\left(\gamma^k(q_i) + G(q_i)\right)^{\frac{1}{k}}} \tag{10}$$

where k is a positive parameter with a low bound, and $\gamma(q_i)$ is a criteria function, expressed as

$$\gamma(q_i) = \sum_{j \in N_I(i)} \frac{1}{2}\left\| q_i - q_j \right\|^2 \tag{11}$$

The consensus control aims to minimize $\gamma(q_i)$. To achieve this objective, we design the control input to be negative gradient of the potential function φ_i, i.e.,

$$u_i = -K_i \frac{\partial \varphi_i}{\partial q_i} \tag{12}$$

where $K_i > 0$ is a positive control gain.

Substituting (8), (10) into (12) yields

$$\begin{aligned} \frac{\partial \varphi_i}{\partial q_i} &= \left(\gamma^k(q_i) + G(q_i)\right)^{-\frac{1}{k}-1} \\ &\times \left(G(q_i) \nabla_{q_i} \gamma(q_i) - \frac{\gamma(q_i)}{k} \nabla_{q_i} G(q_i) \right) \end{aligned} \tag{13}$$

where $\nabla_{q_i}(\cdot)$ denotes the partial derivation of $(\cdot)$ with respect to q_i.

Further, consider the following equation

$$\begin{aligned} & G(q_i) \nabla_{q_i} \gamma(q_i) - \frac{\gamma(q_i)}{k} \nabla_{q_i} G(q_i) \\ &= \sum_{j \in N_I(i)} \left(\underbrace{\bar{\beta}_{ij} \left(\beta_{ij} + \frac{\gamma(q_i)}{k} \right) \prod_{\varsigma \in O(i)} \alpha_{i\varsigma}}_{\pi_{ij}} \right) (q_i - q_j) \\ &- \sum_{\varsigma \in O(i)} \left(\underbrace{\frac{\gamma(q_i)}{k} \bar{\alpha}_{i\varsigma} \prod_{j \in N_I(i)} \beta_{ij}}_{\delta_{i\varsigma}} \right) (q_i - q_\varsigma) \end{aligned} \tag{14}$$

where $\bar{\beta}_{ij} = \prod_{\substack{l \in N_I(i) \\ l \neq j}} \beta_{il}$ and $\bar{\alpha}_{i\varsigma} = \prod_{\substack{m \in O(i) \\ m \neq \varsigma}} \alpha_{im}$.

Substituting (13) and (14) into (12), the control input becomes

$$u_i = -\left(\sum_{j \in N_I(i)} \nu_i \pi_{ij} (q_i - q_j) - \sum_{\varsigma \in O(i)} \nu_i \delta_{i\varsigma} (q_i - q_\varsigma) \right) \tag{15}$$

where $\nu_i = K_i \left(\gamma^k(q_i) + G(q_i)\right)^{-\frac{1}{k}-1}$.

Rewrite the close-loop equation (15) as

$$\dot{\mathbf{q}} = -(L_M \otimes I_2)\mathbf{q} + (L_D \otimes I_2)\mathbf{q} + (L_O \otimes I_2)\mathbf{q_o} \tag{16}$$

where L_M is a *Metzler* matrix whose elements can be defined as

$$L_{M\,ij} = \begin{cases} \sum_{j \in N_I(i)} \nu_i \pi_{ij}, & i = j \\ -\nu_i \pi_{ij}, & j \in N_I(i) \\ 0, & j \notin N_I(i) \end{cases} \tag{17}$$

L_D is a diagonal matrix with the diagonal element as

$$L_{D\,ii} = \sum_{\varsigma \in O(i)} \nu_i \delta_{i\varsigma} \tag{18}$$

Introduce a *Weighted Robot-Obstacle Adjacent* matrix, denoted by L_O, calculated below

$$L_{O\,i\varsigma} = \begin{cases} -\nu_i \delta_{i\varsigma}, & \varsigma \in O(i) \\ 0, & \varsigma \notin O(i) \end{cases} \tag{19}$$

Theorem 1 *Consider n networked robots with dynamics (1), which are all located in the free work space at the initial time. Under the control law (15), we have* $G(q_i) > 0$, $\forall t > 0$, *which indicates that the underlying graph stays connected and all the robots are obstacle free.*

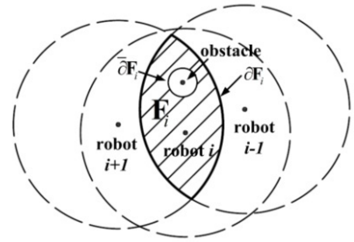

Fig. 4 The boundaries of the free work space.

Proof: As shown in Fig. 4, F_i is formed by two kinds of boundaries. The outside boundary ∂F_i is formed by the communication ranges of the neighbors of robot i, and the inside boundary $\bar{\partial} F_i$ is formed by the obstacle inside $\bar{F}_i$.

Consider the ith robot and $G(q_i) > 0$ at the initial time. In the following, we will prove that $G(q_i) = 0$ only occurs when there exists at least one neighbor $j \in N_I(i)$ at the outside boundary ∂F_i, or the ith robot itself locates at the inside boundary $\bar{\partial} F_i$, which is, however, impossible to happen.

Suppose that at a point q_i, $G(q_i) = 0$. It then follows that

$$\frac{\partial \varphi_i}{\partial q_i} = \left(\gamma^k(q_i)\right)^{-\frac{1}{k}-1} \times \left(-\frac{\gamma(q_i)}{k} \nabla_{q_i} G(q_i)\right) \quad (20)$$

The partial derivate of G_i with respect to q_i is

$$\begin{aligned} \nabla_{q_i} G(q_i) = &- \sum_{j \in N_I(i)} \prod_{\varsigma \in O(i)} \alpha_{i\varsigma} \bar{\beta}_{ij} (q_i - q_j) \\ &+ \sum_{\varsigma \in O(i)} \prod_{j \in N_I(i)} \bar{\alpha}_{i\varsigma} \beta_{ij} (q_i - q_\varsigma) \end{aligned} \quad (21)$$

When only one robot $j \in N_I(i)$ locates at the outside boundary ∂F_i, we have $\beta_{ij} = 0$ and $\bar{\beta}_{ij} \neq 0$. When the ith robot locates at the inside boundary $\bar{\partial} F_i$ formed by $\varsigma \in O(i)$ and none of its neighbors locates at ∂F_i, we have $\alpha_{i\varsigma} = 0$ and $\bar{\alpha}_{i\varsigma} \neq 0$. In either case, we have $\frac{\partial \varphi_i}{\partial q_i} \neq 0$ from (20) and (21). It then follows that at the point q_i, the negative gradient of φ_i is normal to the surface $G_i = 0$ and towards the set $G_i > 0$.

When the ith robot itself locates at the inside boundary $\bar{\partial} F_i$ formed by $\varsigma \in O(i)$ and at least one of its neighbors $j \in N_I(i)$ locate at the outsider boundary ∂F_i, we have $\beta_{ij} = \alpha_{i\varsigma} = 0$. When more than one neighbors, $j \in N_I(i)$ and $l \in N_I(i)$, locate at the outside boundary ∂F_i, we have $\beta_{ij} = \bar{\beta}_{ij} = 0$. In either case, we have $\frac{\partial \varphi_i}{\partial q_i} = 0$ from (20) and (21). Since $\varphi_i \in [0,1]$ and $G_i = 0$, it follows that $\varphi_i(q_i) = 1$ from (10), which means that φ_i reaches the maximum value at q_i. Since the initial condition $G_i > 0$ holds, it was proved that no open set of initial conditions could be attracted to the maxima of φ_i along $-\frac{\partial \varphi_i}{\partial q_i}$ [25].

The above analysis shows that it is impossible to have $G(q_i) = 0$. Therefore, $G(q_i) > 0$ always holds as time $t \to \infty$. ■

Lemma 1 For potential function φ_i, there exists a positive lower bound on parameter k and a positive upper bound on ε, such that for $k \geq N_1(\varepsilon)$ and $\varepsilon < \varepsilon_0$, φ_i has a unique minimum at $q_i = q_j$, $\forall j \in N_I(i)$.

The proof of this Lemma is rather extensive to be included here. Interested readers can refer to [26] [27].

Theorem 2 *Consider the networked robots with dynamics (1), which all locate in the free work space in the initial time. The robots can achieve rendezvous successfully the under the controller (15), as time tends to infinity.*

Proof: Define φ_i as a Lyapunov function candidate

$$V = \varphi_i \quad (22)$$

Taking the time derivation of V yields

$$\dot{V} = \dot{q}_i^T \nabla \varphi_i = -K_i (\nabla \varphi_i)^T \nabla \varphi_i = -K_i \|\nabla \varphi_i\|^2 \leq 0 \quad (23)$$

$\dot{V} = 0$ holds in the set of critical points $\psi(i) = \{q_i | \nabla_{q_i} \varphi_i = 0\}$. According to Lemma 1, the set of critical points contains only one minimum $q_i = q_j$, $\forall j \in N_I(i)$, which is the state in agreement. When time tends to infinity, there is $q_i = q_j$, $\forall j \in N_I(i)$. From Theorem 1, the underling graph always stays connected when time tends to infinity. The system then converges to the equilibrium $q_i = q_j$, $i, j \in \{1, \cdots, n\}$. ■

IV. SIMULATIONS

Simulations were performed on a group of nine robots to demonstrate the proposed approach in solving the problem of achieving rendezvous with bounded control inputs, while maintaining the network connectivity and avoiding obstacles. As shown in Fig. 5, the nine mobile robots are denoted by the small points. In the simulation, the proposed control algorithm (15) was applied with dynamic communication topology. For comparison purpose, the initial positions were exactly the same as in [2]. Unlike [2], here there were two obstacles in the workspace. The parameter of the potential function was $k = 3$, and the feedback control gains were chosen as $K_1 = K_2 \cdots = K_9 = 2$. The sampling period was set to 0.1 second in the simulation.

Fig. 5 illustrates the motion rendezvous evolution under the control law (15). The dotted circles denote the communication ranges of the robots. The solid lines denote the communication links among the robots. As expected, no communication links were broken during the maneuver, and the rendezvous task was achieved without hitting any obstacles.

V. EXPERIMENTS

We further carried out experiments on a group of three mobile robots to verify the proposed control method. The three robots are P3DX mobile robots denoted by robot 1, 2 and 3, respectively. The robots communicate with each other via 54.0MHz wireless access point. Each robot is equipped with a sonar system to detect obstacles. The robot motion state is measured using motor encoder and estimation scheme [28]. The three robots were controlled with the rendezvous control law (15). At the beginning, the robots were placed with the initial configurations of robot 1 (0m, 2.4m, 0°), robot 2 (1.8m, 0m,0°) and robot 3 (3.6m, 3m, 0°). The parameter of

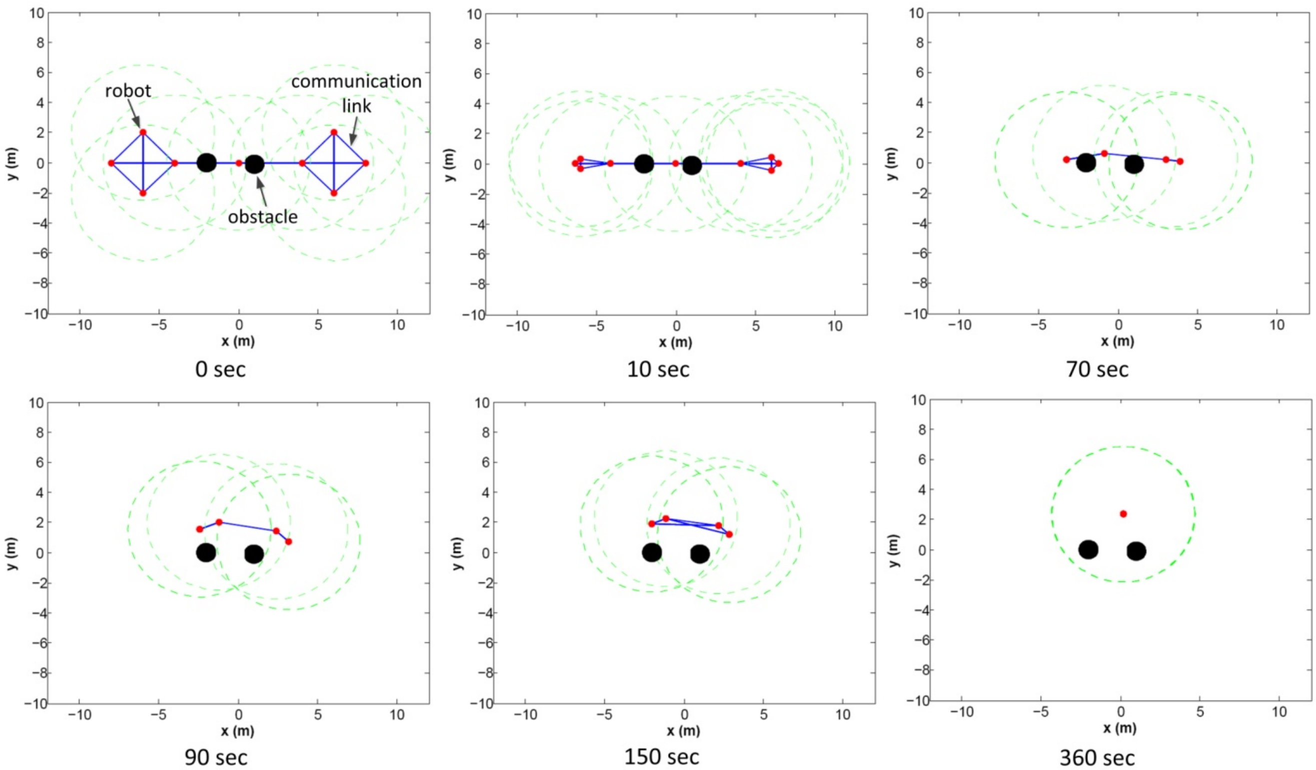

Fig. 5 Evolution of a rendezvous task with nine robots. After 360 seconds, all the robots reached to an agreement position.

the potential function was $k = 3$ and the control gains were $K_1 = K_2 = K_3 = 1$.

Fig. 6 illustrates the rendezvous process collected from the video. Fig. 6 (a) shows the robots at the beginning, and Fig. 6 (b) ~ (f) show the robots at different times. The two long lines denote x and y coordinate axes. These figures demonstrate that the proposed control law can drive robots to achieve the expected multirobot rendezvous task. Fig. 7 (a) ~ (c) illustrate the bounded control inputs of the three robots. The solid lines denote the control inputs in x direction and dotted lines denote control inputs in y direction.

VI. Conclusion

In this paper, we propose a distributed control approach to solving multirobot consensus problem in presence of obstacles while maintaining connectivity with bounded control inputs. The control law is designed to be the negative gradient of a navigation potential function, which is modeled by integrating consensus requirement, connectivity maintenance and obstacle avoidance, simultaneously. Comparing to the existing approaches, the proposed controller considers obstacle avoidance and requires bounded control input only, which makes it more feasible to practical applications. Both simulation and experiment are performed to demonstrate the effectiveness of the proposed approach.

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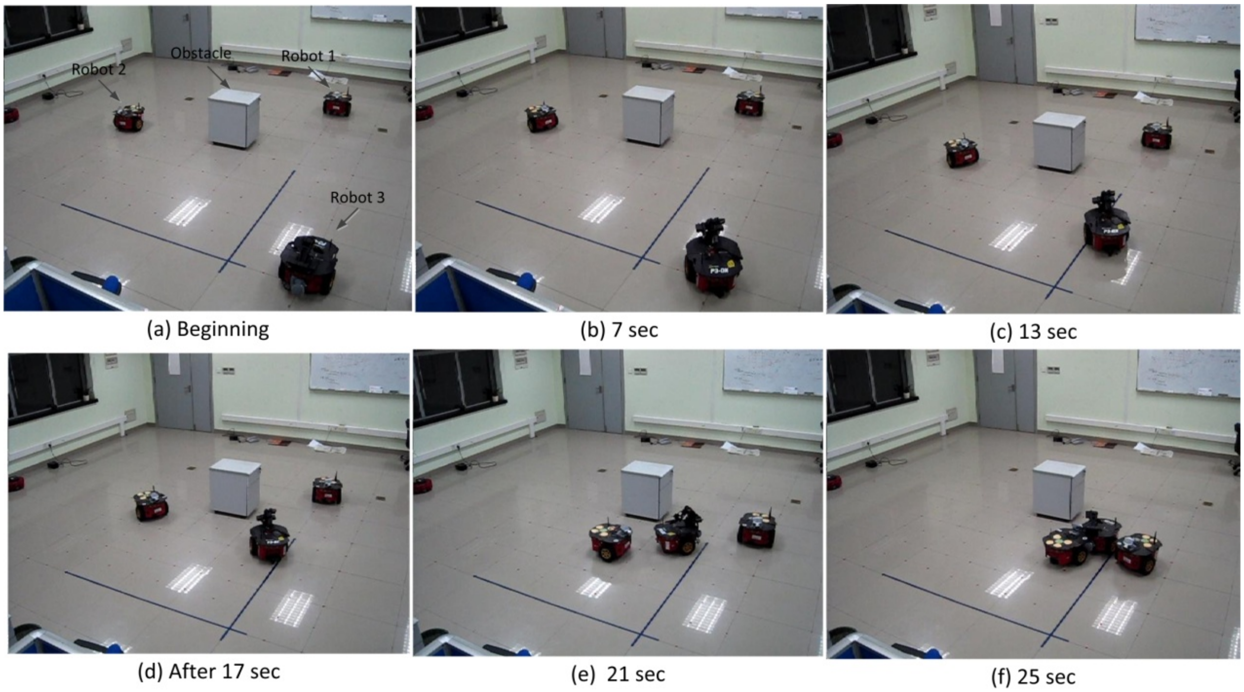

Fig. 6 Evolution of a rendezvous task with three robots. (a) At the beginning, robots started at the initial configuration. (b) After 7 seconds. (c) After 13 seconds. (d) After 17 seconds. (e) After 21 seconds. (f) After 25 seconds, all the robots reached to an agreement position.

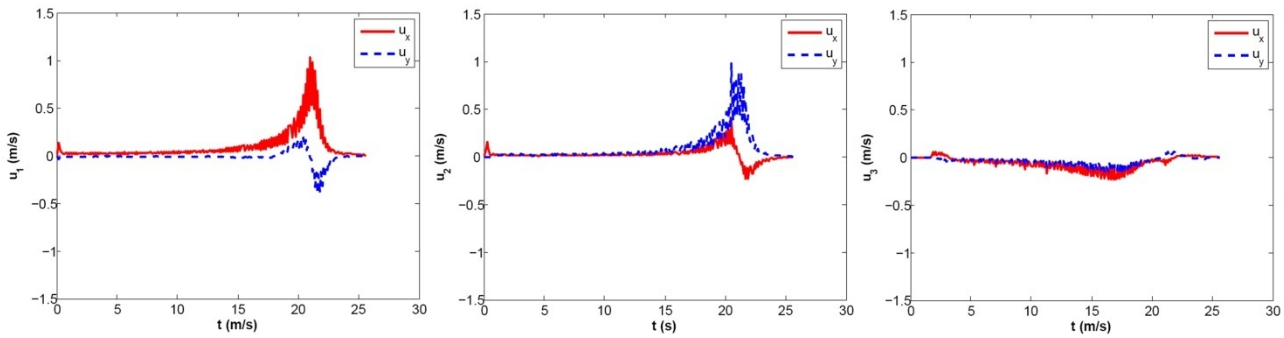

Fig. 7 The velocities of the three robots. (a) The velocities of robot 1. (b) The velocities of robot 2. (c) The velocities of robot 3.

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