A State Exchange Approach in Real Conditions for Multi-Robot Cooperative Localization

Romuald Aufrère, Nadir Karam, Frédéric Chausse and Roland Chapuis

Abstract—In this paper, a state exchange based multi-robot localization is proposed in particular experiments in real conditions. The goal of such an approach is to combine the data coming from several mobile communicating robots in order to i) update and maintain in each robot an optimal map of the whole fleet, and ii) improve all the poses estimation taking into account that vehicles can detect and localize their neighbors. The approach is based on a state exchange procedure and is real applications dedicated. So, the problems of data incest, communication lattencies or losses has been solved and the robot detection association has been addressed. This paper presents the overall principle of the approach and shows several results obtained in real-time situations with real Cyebas vehicles.

I. INTRODUCTION

Localization is one of the most important tasks in robotics. In the case of a single robot system, this is usually performed by fusing the proprioceptive data and exteroceptive data [8]. The accuracy of the localization can significantly affect the quality of the task execution. In recent years, multi-robot systems have been studied in order to exploit the information provided by all robots to increase the localization accuracy of the whole fleet. However, the multi-robot systems introduce new localization problems. Kurazume et. al in [7] have demonstrated that the communication of relative poses information leads to the reduction of the localization uncertainty. Cooperative localization is obtained by fusing proprioceptive and exteroceptive data of each vehicle with information received from the other members of the group. Recently, Mourikis et. al also show in [12] that for a robot group of a certain size, the accuracy of multi-robot cooperative localization depends only on the accuracy of the proprioceptive and exteroceptive sensors embedded on the robots and is independent of the number of relative position measurements.

On this topic of the multi-robot cooperative localization, two main approaches are listed in the literature: centralized and decentralized approaches. The first one [10] is based on a central system which distributes the information to all the robots of the group. These works are only applied in simulation situations and never take into account the constraints of the real applications. Indeed, with this approach the evolution area of the robots is limited because the robots must be permanently in contact with the central system. Moreover, a fault in the central system introduces a failure of the whole localization system.

The second approach is based on a distributed architecture. In this case, all the robots exchange their information without need of a central system. Roumeliotis et. al introduced in [14] a distributed approach for collective localization based on a centralized Extended Kalman Filter (EKF). In [13], a new approach to reduce the memory and processing requirements by distributing data and computations amongst the robots is presented. In [4], the authors developed an egocentric approach using a video camera and laser to fuse (particles filter) the information collected by all the robots. However, the distributed approaches can also be separated in two parts: i) data exchange approach and ii) state exchange approach.

For the first one, the robots exchange sensor information. The collective localization is obtained by updating the global state of the group with the collected observations. Madhavan et. al present in [9] an approach which was experimented with an heterogeneous group of mobile robots in an outdoor environment. The robots exchange the relative positioning information and the information describing their position (GPS information and proprioceptive data). In [3], the authors describe an approach based on data exchange (relative position estimated by camera and laser) but without knowing the identity of the robots that provide such data. The major contribution of this mutual localization approach is the process of data association to find the robot identity. A genetic algorithm is presented in [2]. This algorithm use the fitness-sharing technique for both maintaining evolutionary niches over time and augmenting the selection pressure of individuals. For the moment, only simulation cases are presented by the authors and no measure of uncertainty of the estimation is given. The disadvantage of approaches based on data exchange is the large quantity of transmitted information on the communication network. This quantity is even larger in the case of an heterogeneous group of robots. In order to use the incoming information for the update of the group state, the robots must exchange not only the sensor data but also the error model for instance.

The second kind of approach is the exchange of the global state vector of each vehicle. This approach can reduce the amount of transmitted information but it can be faced to the Data Incest problem: the fusion of interdependent states leads to quick convergence to an inaccurate value. This Data Incest problem has been studied in [11] for general data fusion approaches but not specifically for the approaches on the multi-robot cooperative localization. To avoid and solve
this problem, A. Howard et al [4] maintain a dependency tree to update the historical dependence of states. This approach has however some limitations. The dependency tree assumes that states are only dependent on the state that was last used to update them. The authors then consider that these distributions are independent of all other distributions. This assumption is restrictive as circular updates can still occur.

Roumeliotis et al describe in [15] an original approach to resolve this data incest problem. In this approach, the robots exchange between them only a part of the group state and the relative positioning measure in order for each robot to update its own version of the group state. This solution is not optimal because this approach doesn’t take into account all the information available in each robot.

At our knowledge in the case of multi-robot cooperative localization, no satisfactory solution has been found to this Data Incest problem. This paper presents an original approach able to give a solution to this problem.

Moreover, all the cited approaches based on state exchange suppose also that the robots are able to localize and identify the other members of the group. In the case of multi-robot localization, direct identification of the detected robots is difficult to perform. Kato et al in [6] was the first to deal with this anonymous mutual localization problem. In [1], the authors present an approach based on disambiguating multiple hypotheses to manage this identity problem but without taking into account the localization parameters.

This paper proposes also a solution to these ambiguous situations. Moreover, the proposed approach is able to manage the communication losses and communication delays (solution not describe in this paper).

After an overview of the state of the art for the multi-robot localization given above, the rest of the paper is organized as follows. In Section II, the proposed approach is described with one first part describing the group estimation in each robot and a second part presenting the collective localization. In Section III, real results are reported. Finally, in Section IV conclusions of this work are drawn and future works are suggested.

II. OUR STATE EXCHANGE APPROACH

Before to develop an efficient approach for a real collective multi-robot localization, we think that the following points should be taken into account:

• The approach must integrate optimally all available information but avoiding the data incest problem.
• The approach must be used in real situations and so should take into account the functional limits of the sensors, the communication losses and communication delays.
• The approach must run in real time.
• The approach must run in an open environment where other robots (not members of the group) could evolve.

In this section, the state exchange approach taking into account the points listed above is presented. With this approach, the detected robots can not be directly identified.

To summarize this approach, every robot maintains an estimation of the group state using its own sensors. This estimation is shared with the other members of the group. As those estimations are totally independent, their fusion is achieved in each robot but does never lead to any data incest. To describe our approach, let an heterogeneous group of \( N \) robots \( R_i \) (\( N \) is unknown). Through out this paper, the following assumptions will be considered:

- Each robot can be able to localize approximatively itself in an absolute reference (GPS positioning for example).
- Each robot can be able to localize approximatively its neighborhood robots relatively to its own position, but can not identify them only using data sensors.
- The robots need to be equipped with communication devices in order to exchange information.
- Communication and sensor information can be affected with delays or can be temporarily unavailable.

The goal is to maintain in every robot the most accurate pose estimation of the other group members taking into account sensor and communication constraints. This is obtained by performing collective localization where the state of the group is viewed as a single vector. Every robots estimate the group state pose using its own sensor data and exchange it (if possible) with the other members of the group. Each robot obtains a global group state by fusing its group state estimation and the received ones. Thus, this global group state combines sensors data of the considered robot with sensors data of all communicating robots.

The presentation of this approach will be divided in two parts. In the first part, the localization algorithm executed in a single robot (estimation of its pose and the poses of other robots) is detailed. In the second part, the proposed collective localization approach is presented.

A. Group State Estimation

This section describes the algorithm running in one robot of the group which is called self robot \( R_s \), the same algorithm is executed in the other robots \( R_{oi} \) with \( i=1...M \) where \( M=N-1 \) is the number of other robots of the group. The state of the robot \( R_s \) is represented by the vector \( x_s=[x_s,y_s,\phi_s,v_s]^T \) and its covariance matrix \( P_{s,s} \), where \( x_s \) and \( y_s \) are the coordinates of the robot \( R_s \) in an absolute reference, \( \phi_s \) its orientation and \( v_s \) is the module of its velocity. The state of the other robots is represented by the vector \( x_{oi}=[x_{oi},y_{oi},\phi_{oi},v_{oi}]^T \) and its covariance matrix \( P_{oi,oi} \), where \( i=1...M, x_{oi} \) and \( y_{oi} \) are the coordinates of the other robot \( R_i \) in the same absolute reference, \( \phi_{oi} \) its orientation and \( v_{oi} \) is the module of its velocity. To perform the collective localization, the group is represented, in robot \( R_s \) by a single system \( W_s=(\bar{X}_s,P_s,\text{id}_s) \) where \( \text{id}_s \) is the robot \( R_s \) identifier, \( \bar{X}_s \) is the state vector of the group and \( P_s \) its covariance matrix as in equations (1) and (2).

\[
\bar{X}_s = [\bar{x}_s, \bar{x}_{o1}, \bar{x}_{o2}, ..., \bar{x}_{om}]^T
\] (1)

\[
X_s = [x_s, y_s, x_{o1}, y_{o1}, x_{o2}, y_{o2}, ..., x_{om}, y_{om}]^T
\] (2)
where \( m_s \) is the number of robots detected by the robot \( R_s \). The robot \( R_s \) maintains an estimation \( W_s \) of the group state. The state \( W_s \) is only updated with proprioceptive and exteroceptive sensors data of the robot \( R_s \). The data fusion is done with an Extended Kalman Filter (EKF).

1) Group state evolution (in the robot \( R_s \)): The motion of the robots can be modeled by the function \( f_s \) (bicycle model) for the self robot \( R_s \) (equation (4)) because it knows the input \( u^m_s \) and the function \( f_o \) (constant speed kinematic model) for the other robots \( R_o \) of the group (equation (5)). We assume that the error affecting the encoders data \( u_o \) can be modeled by:

\[
u_s \sim \mathcal{N}(\mu^m_s, Q_s)
\]

where \( \mu^m_s \) is the measured data and \( Q_s \) the covariance of the noise affecting it.

The evolution equation of the group state vector and its covariance matrix can be written as following

\[
\begin{align*}
\dot{\overline{x}}_{k+1} &= f_s(\overline{x}_k, u^m_k) \\
\overline{P}_{k+1} &= f_o(\overline{x}_k) + \overline{B}_s + \overline{Q}_s \\
\overline{P}_{o,k+1} &= f_o P_{o,k} f_o^T + \overline{B}_o
\end{align*}
\]

where \( k \) represents the time, \( \overline{x}_{k+1} \) and \( \overline{P}_{k+1} \) respectively the predicted \( \overline{x}_k \) and \( \overline{P}_k \), \( f_s(\cdot, \cdot) \) and \( f_o(\cdot) \) are respectively the Jacobian of the function \( f_s \) with respect to the state \( \overline{x}_k \) and \( u_k \), \( f_o(\cdot) \) is the Jacobian of the function \( f_o \) with respect to the state \( \overline{x}_o \) and \( B_s \) and \( B_o \) are the covariances of the noise affecting the motion model of the robot \( R_s \) and the other robots \( R_o \) respectively.

The obtained state is the prediction of the group state at time \( k+1 \) using the group state estimation at time \( k \) and the encoders data. This state will be updated with the exteroceptive sensors information.

2) Group state update (in the robot \( R_s \)): At the beginning of the application, the group state \( \overline{x}_s \) contains only the robot \( R_s \) with its pose estimation vector \( \overline{x}_s \) and its covariance matrix \( \overline{P}_{s} \) as in equation (8). There is no other robot pose estimation in the group state (\( m_o = 0 \)).

\[
\overline{x}_s = [\overline{x}_s], \quad \overline{P} = [\overline{P}_{s}]
\]

When the robot \( R_s \) detects an other robot \( R_{o,i} \), it can measure its relative pose \( \overline{z}_r \) noise according to the model:

\[
\overline{z}_r \sim \mathcal{N}(\overline{z}^m_r, B_r)
\]

with \( \overline{z}^m_r = (\Delta x, \Delta y, \Delta \phi) \) the measured relative pose and \( B_r \) the covariance of the noise affecting it.

As the group state of the robot \( R_s \) contains only its own pose estimation, the state of the detected robot can be added to the group state without any ambiguity. In order to keep the inter dependencies between the pose estimations of the two robots, the group state estimation is updated as following.

- The group state is extended with an initial robot state \( (\overline{x}_r, \overline{P}_{o,0}) \) set to an arbitrary values with a large covariance as in equations (10) and (11).

\[
\overline{x}_{s,k+1} = \begin{bmatrix} \overline{x}_{s,k+1} \\ \Delta \overline{x}_{s,k+1} \end{bmatrix}
\]

\[
\overline{P}_{s,k+1} = \begin{bmatrix} \overline{P}_{s,k} & 0_{4 \times 4} \\ 0_{4 \times 4} & \overline{P}_{o,0} \end{bmatrix}
\]

where \( 0_{n \times n} \) is an \( n \times n \) dimension null matrix.

- The extended state is updated with the measured relative information \( \overline{z}_{k+1} \) as in equations (12),(13) and (14).

\[
\overline{x}_{s,k+1} = \overline{x}_{s,k+1} + K_{k+1}(\overline{z}_{s,k+1} - \overline{P}_{o,k+1})
\]

\[
\overline{P}_{s,k+1} = (I - K_{k+1})\overline{P}_{s,k+1}
\]

The update matrix \( H \) is given by the equation (15).

\[
H = \begin{bmatrix} -I_{3 \times 4} & I_{3 \times 4} \end{bmatrix}
\]

where \( I_{3 \times 4} = [I_{3 \times 3} \ 0_{3 \times 1}] \) and \( I_{n \times n} \) is an \( n \times n \) dimension identity matrix.

From now on, when the robot \( R_s \) detects an other robot \( R_o \), as it can’t identify it directly, it must compare the pose of the robot \( R_o \) with the estimated poses of the other \( m_o \) robots present in the group state \( W_s \). The estimated pose of the detected robot \( \overline{z}_o \) and its covariance matrix \( \overline{P}_{o} \) is compared to the poses \( (\overline{x}_{s,k+1}, \overline{P}_{o,k+1}) \) in the group state \( (\overline{x}_{s,k+1}, \overline{P}_{o,k+1}) \). Where \( \overline{z}_o \) and \( \overline{P}_{o} \) note respectively \( \overline{z}_{o,i} \) and \( \overline{P}_{o,i} \) without the speed component \( v_o \). This comparison is performed by computing the Mahalanobis distance between the pose estimations as in equation (5)

\[
d^2 = (\overline{z}_{o,i} - \overline{z}_{o,k+1})^T(\overline{P}_{o} + \overline{P}_{o,k+1})^{-1}(\overline{z}_{o,i} - \overline{z}_{o,k+1})
\]

where \( i = 1 \ldots m_o \). The comparison process generates a set of \( m_o \) Mahalanobis distances which correspond to the \( m_o \) other robots in the group state estimation. These distances take into account the inaccuracy of the track by using the covariance matrices \( \overline{P}_{o} \) and \( \overline{P}_{o,k+1} \).

- If the minimum distance is lower than a specified threshold, it means that the detected robot already exists in the group state. The relative measure \( \overline{z}_{o,i} \) is used to update the pose of the corresponding robot according to equations (12),(13) and (14) with an update matrix \( H \) as in equation (17)

\[
H = \begin{bmatrix} -I_{3 \times 4} & 0_{3 \times (4(D-1))} & I_{3 \times 4} & 0_{3 \times (4(m_o-D))} \end{bmatrix}
\]

where \( 1 \leq D \leq m_o \) is the position of the robot which corresponds to the minimum Mahalanobis distance in the group state. In this case, the number of robots in the group state \( m_o \) does not change.
• If the minimum distance is higher than the specified threshold, it means that the detected robot is not yet in the group state, it is then added according to equations (10) and (18) and updated according to equations (12),(13) and (14), with the update matrix $H$ as in equation (19).

$$P_{s_{k+1}}^- = \begin{bmatrix} P_{s_{k+1}}^- & 0_{4(m_s+l)+1}^T \\ 0_{4(m_s+l)+1} & P_{0,0} \end{bmatrix}$$ (18)

$$H = \begin{bmatrix} -I_{3\times 4} & 0_{3\times (4m_s)} & I_{3\times 4} \end{bmatrix}$$ (19)

In this case the number of robots in the group state is incremented ($m_s = m_s + 1$).

If the pose of the $m_s$ detected robots can not be measured and updated during several iterations, the error affecting their pose estimation becomes too large, and it can lead to wrong matchings (the Mahalanobis distance between the pose estimation of a detected robot and a pose estimation with a too large covariance is regularly lower than the specified threshold). After the group update process, the pose estimations which have an error higher than a maximum allowed error are removed from the group state estimation.

The process Group state evolution and Group state update enable the robot $R_s$ to maintain a group state estimation which describes its view of the environment. Those processes are executed into each one of the robots of the group. The collective localization is obtained by combining those group state estimations.

### B. Collective Localization

Remember here that the purpose of the collective localization is to increase the position accuracy of all the robots of the group from information provided by all robots. For that the robots of the group exchange their group states in which are represented the poses estimation of every robot and its neighborhood in an absolute common reference. The robot $R_s$ sends its group state estimation to the other robots and receives theirs. The robot $R_s$ can fuse its group state with the received ones to obtain the most accurate group state estimation. In this case, knowing that all the estimated states are independent, the data incest problem is avoided.

Let us consider the self robot $R_s$ which maintains the group state $W_s = (X_s, P_s, \mathbf{id}_s)$ and receives $L \leq M$ group states $W_l = (X_l, P_l, \mathbf{id}_l)$ with $l=1...L$ from other communicating robots present in the communication area.

The global group state estimation $W^* = (X^*, P^*, \mathbf{id}^*)$ is obtained by fusing the $L$ received group states $W_l$ with the self group state $W_s$ as in equation (20).

$$W^* = \mathcal{F}(W_s, W_1, W_2, ..., W_l, ..., W_L)$$ (20)

$W_l$ is the group states received from the other robot $R_{id_l}$ and $\mathcal{F}$ the fusion function. $\mathbf{id}^*$ in the global group estimation represents a set of identifiers of the robots.

The fusion process is based on an extended Kalman filter. Moreover, this process manage the robot matching in order to combine the received states and to find the correspondences between the robots in $W^*$ and the robots in $W_l$.

The global state $W^* = (X^*, P^*, \mathbf{id}^*)$ returned by the fusion function is shaped as in equations (21) and (22).

$$X^* = \begin{bmatrix} \hat{X}_{s_1}^* \\ \hat{X}_{s_2}^* \\ \vdots \\ \hat{X}_{s_{N_{s}}}^* \end{bmatrix} \quad \mathbf{id}^* = \begin{bmatrix} \mathbf{id}_{s_1} \\ \mathbf{id}_{s_2} \\ \vdots \\ \mathbf{id}_{s_{N_{s}}} \end{bmatrix}$$ (21)

$$P^* = \begin{bmatrix} P^*_{s_1} & P^*_{s_1,s_2} & \cdots & P^*_{s_1,s_{N_{s}}} \\ P^*_{s_2,s_1} & P^*_{s_2,s_2} & \cdots & P^*_{s_2,s_{N_{s}}} \\ \vdots & \vdots & \ddots & \vdots \\ P^*_{s_{N_{s}},s_1} & P^*_{s_{N_{s}},s_2} & \cdots & P^*_{s_{N_{s}},s_{N_{s}}} \end{bmatrix}$$ (22)

$X^*$ is the global group state vector and $P^*$ is its covariance matrix. The vector $\mathbf{id}^*$ contains the identifiers of the robots poses estimations in the global group state.

In order to avoid the data incest problem, the global state estimation will never be exchanged between the robots of the group. Indeed, this global state is dependent on all estimated states in the different robots. Thus, the data exchange between the robots are only the state $W_s$ and $W_l$ estimated independently in each robot.

Inspired by the method presented in [8], our approach also manages the communication problems (losses and delays). Thus, to fuse the received states correctly, it is necessary to make evolve those states to the present time before the fusion. Due to lack of space, this part is not presented in this paper. Moreover, thanks to its principle, this approach may allow the consolidation of two fleets of robots.

In the next section, results in real situation are presented in order to show that this approach meets to the specifications listed initially (integrate all available information, manage the communication losses and delays, etc).

### III. Results

This collective localization approach was already implemented in simulation for the case of a group of four robots [5]. Lastly, this approach was tested in a real case where two robots evolve in a convoy: a first robot (called head robot) defines a trajectory and a second one (called back robot) tracks the first one. The experimental platforms are Cycabs (figure 1). These robots can be driven in automatic mode or manual mode.

The sensors embedded in the head robot are:

- Odometrs which describe the movements realized by each of the four wheels.
- Low cost GPS to localize with an accuracy contained between 3 and 9 meters depending on the configuration of the satellites and the robot environment.
- RTK GPS to localize the robot in the same reference that the low cost GPS. This RTK GPS will be used as a reference to evaluate the performances of the collective localization approach.
In the back robot, the embedded sensors are:
- Odometers which describe the movements realized by each of the four wheels.
- RTK GPS which permits to localize the robot in an absolute reference.
- A laser rangefinder used to detect and estimate the relative position of the head robot.

Moreover, the robots are equipped with communication devices to exchange their group state estimation.

The scenario of the experimentation is the following: the head robot is localized with its odometer sensors and with the data provided by the low cost GPS using an Extended Kalman Filter. The RTK GPS is only used as a true reference to compare the fusion results and estimate the approach performances. The back robot is localized with its odometer sensors and RTK GPS also with an Extended Kalman Filter. In the one hand, the back robot detects the head robot thanks to the laser rangefinder (simple obstacle detection algorithm). On the other hand, this robot builds and updates a version of the group state with its own position and the one of the head robot. The experimentations was realized on the site an university campus. Figure 2 represents the trajectory followed by the convoy. The speed of robots is about equal to 2.5m/s.

Figure 3 shows the results of the approach in the head robot. The green curve represents the trajectory provided by the low cost GPS, the red curve is the reference trajectory provided by the RTK GPS embedded in the head robot and the blue curve characterizes the trajectory of the head robot obtained with its global fused state. In this figure, we can see that the estimated trajectory (blue curve) is almost superimposed to the reference trajectory (red curve).

In order to score the performance of our approach, the localization error was characterized with two parameters. The first one is an Euclidean distance between the mean value of the estimate position after the fusion and the reference position given by the RTK GPS (figure 4). The second one is a Mahalanobis distance also between the estimate position after the fusion and the reference position given by the RTK GPS (figure 5).

The Euclidean distance, figure 4, is approximately 20cm while the initial error with the low cost GPS is about 4m.

To ensure data integrity (real position included in the uncertainty ellipse defined by the covariance matrix of the estimated pose), the Mahalanobis distance should be less than a threshold defined by the $\chi^2$ law. This threshold for a vector with two dimensions and for a probability of presence equal to 95.4% is 6.15. Figure 5 shows that the calculated Mahalanobis distance is approximately 4 on the whole trajectory. In order to illustrate the maximum errors, figure 6 represents a zoom of a part of the trajectory.

The peak of the error found in the time interval $I_1$ in figures 4 and 5 corresponds to a jump of the RTK GPS used.
detected robots are not identified. This collective localization approach was already tested in simulation and this paper presents the real experimental results in real situation with 2 robots. These results show the localization error obtained after the fusion of independent state is about 20cm for an initial value with the low cost GPS about 4m. Moreover, these results show the integrity of the estimated position despite few failures of detection due to the simplicity of the obstacle detection algorithm.

We are currently planning to make a real environment experimentation with more robots in order to show all the potential of this approach in particular to identify the robots, to manage the communication delays, to manage the appearance and disappearance of robots in the group. In these new experimental conditions, the main problems are based on the management of ambiguity due to the multiple detections.

IV. Conclusion

This paper describes a cooperative approach for the collective localization of an heterogeneous group of robots while taking into account all the available information in each robot. Moreover, this approach manage the data incest problem. To take advantage of the interdependencies between the robots poses, the group is viewed as a single system which contains the poses of the detected other robots. Each robot updates its group state with its own sensor data. When two robots meet, they exchange their views of the environment. The collective localization is obtained by fusing the received views with an Extended Kalman Filter. Moreover, this approach is able to manage the situations where the

Fig. 5. Mahalanobis distance between the estimate position after the fusion and the reference position given by the centimeter GPS.

Fig. 6. Zoom on the trajectories.

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