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The Application of the Grey-based Taguchi Method to Optimize the Global Performances of the Robot Manipulator

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Abstract— **This paper presents the procedure and the results of the multi-objective optimization of the design of a seven degrees of freedom (7DOF) robot manipulator for better global performance, which pertains to the global conditioning, global manipulability, and structural length indices. The Taguchi method was used to reduce the time needed to analyze the global performance of the robot manipulator. Grey relational analysis was performed to deal with the multiple performances of the manipulator by converting multiple responses into a single response with a Grey relational grade. Analysis of variance (ANOVA) was done to analyze the effect of the link parameters on the performance of the robot manipulator, and to determine which link parameter affects the performance of the robot manipulator. The results of the optimization of the prototype robot manipulator and the comparison of the optimization results with those of an earlier study are presented herein.**

I. INTRODUCTION

THE use of robot manipulators for a wide range of applications has been increasing, such as for professional use (e.g., defense and surveillance robots, environmental robots, medical robots, and manufacturing robots) and personal use (e.g., home service robots, silver robots, and entertainment robots) with the development of sensor and actuator technology. Service robot manipulators can help people by doing their ordinary house work; industrial robot manipulators significantly raise productivity; and military robot manipulators can work in dangerous environments. For these reasons, many researches are currently being conducted on robot manipulators, and various robot manipulators have been developed.

One of the important issues concerning robot manipulators is what design factor affects their performance [1-2]. The choice of a robotic mechanism depends on the task or application to be carried out, and the mechanism is consequently determined by the link dimension and configuration of the manipulators. The choice is generally made by the designer based on his experience and intuition, but he should consider the performance ability of the robot manipulator according to its use before making a choice.

Several performance indices that represent performance ability have been proposed for robot manipulators. Many researchers used the manipulability [3] and the condition number [4] for measuring the robot manipulator performance, but these performance indices can be used only for measuring the performance of specific tasks.

Service robots perform various tasks in a changing environment, unlike industrial robots, which perform predefined tasks in a designated area. Therefore, the global performance indices [5] and structural length index [6] must be applied to the service robot design instead of the local performance indices such as the manipulability and condition number. The global performance indices represent global behavior and condition of robot manipulators to the volume of the reachable workspace. The structural length index represents the ratio of the total link length of the robot manipulator to the volume of the reachable workspaces.

The first objective of this study was to optimize the link length of the prototype robot manipulator so that it would have better global conditioning, global manipulability, and structural length indices. In the earlier study, the micro genetic algorithm [7], which is one of the past optimization algorithms, was used to optimize the link length. It still takes too much time, however, to analyze global performance indices and optimize link length. To achieve the first objective of this study in a short time, the Taguchi method [8-9], which has an L25 orthogonal array, was used to optimize the link length of the prototype robot manipulator with fewer experiments. The Taguchi method is a powerful tool for the efficient analysis of experiment results so that the quality of such results can be improved quickly at a low cost. The limitation of the traditional Taguchi method, however, is that it can be used to optimize only a single performance index. Grey relational analysis, on the other hand, which is based on the Grey system and which was proposed by Deng [10], can handle multiple performance indices. Thus, the Grey-based Taguchi method was used to optimize the link length that had better global performance, instead of the traditional Taguchi method. The second objective of this study was to find the links that affect the global performance of the robot manipulator, using ANOVA.

The design parameters of the prototype robot manipulator are explained briefly in the next section. The kinematic

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analysis of the prototype robot manipulator is described in section 3. Section 4 covers the basic concept of the condition number, manipulability, global performance indices, and structural length index. The setting condition of the micro genetic algorithm that was used to optimize the link length, and the results of the optimization based on this algorithm, are presented briefly in section 5. The approach that uses the Taguchi method with Grey relational analysis to optimize the link length of the robot manipulator to get better global performance is described in section 6. The results of the ANOVA and the optimization, and the comparison of the Grey-based Taguchi method and the micro genetic algorithm that was carried out in an earlier research are discussed in section 7. Finally, the conclusions are presented in section 8.

II. THE PROTOTYPE ROBOT MANIPULATOR

The robot manipulator in this study was designed for home service, such as for handling some household objects or serving dishes with a mobile platform. Therefore, robot manipulators for service robots perform various tasks in a changing environment, unlike industrial robots that perform predefined tasks in a designated area. Accordingly, the prototype robot manipulator has seven actuators and three links, similar to the human arms, and its design is based on the target application and the specifications of the mechanical components, such as the motor and gearbox.

The design of the prototype robot manipulator is shown in Fig. 1. The motion range and initial link length of the prototype robot manipulator are shown in Table I.

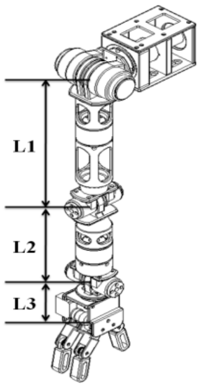

Fig. 1. Prototype robot manipulator

TABLE I
DESIGN PARAMETERS OF THE PROTOTYPE ROBOT MANIPULATOR

Joint No.	Range (Degree)
1	-170 ~ 170
2	-100 ~ 100
3	-170 ~ 170
4	-120 ~ 120
5	-170 ~ 170
6	-100 ~ 100
7	-170 ~ 170
Link No.	**Length (mm)**
Link 1	260
Link 2	160
Link 3	85

III. KINEMATIC ANALYSIS

A. Forward Kinematics

The Denavit-Hartenberg (D-H) convention provides a simple way of modeling the robot links and joints that comprise the manipulator configuration. For the manipulator shown in Fig. 2, the necessary coordinate frames were assigned based on the D-H convention. The accompanying parameters are provided in Table II (the "D-H Table").

In Table II, α_i is the angle between axes z_{i-1} and z_i at about x_i; a_i is the distance between axes z_{i-1} and z_i along x_i; d_i is the distance between axes x_{i-1} and x_i along z_{i-1}; and θ_i is the angle between axes x_{i-1} and x_i at about z_{i-1} [11].

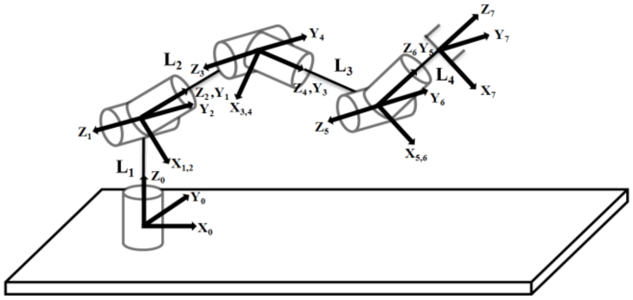

Fig. 2. Configuration of the prototype robot manipulator

TABLE II
DENAVIT-HARTENBERG LINK PARAMETERS

Joint No.	Link Parameters			
	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	160	θ_1
2	$-\pi/2$	0	0	θ_2
3	$\pi/2$	0	L1	θ_3
4	$-\pi/2$	0	0	θ_4
5	$\pi/2$	0	L2	θ_5
6	$-\pi/2$	0	0	θ_6
7	$\pi/2$	0	L3	θ_7

B. Jacobian Matrix

The Jacobian matrix pertains to the transformation matrix that maps the velocity of the end effector in the Cartesian space into the joint velocities. It can be expressed as

$$\dot{x} = J\dot{q} \tag{1}$$

The Jacobian matrix is commonly used to determine singularity and to measure the performance of a manipulator.

In this study, a 6x7 geometric Jacobian matrix [11] was used to measure the condition number and the manipulability for the global performance indices in terms of the joint space.

IV. DEFINITION OF THE PERFORMANCE INDICES

A. Condition Number

The condition number of the Jacobian matrix is used for numerical analysis to estimate the error generated in the solution of a linear system of equations by the error on the data [12]. Using the condition number, it is possible to measure the accuracy of the Cartesian velocity of the end effector. The condition number is defined as

$$\mathrm{k} = \frac{\sigma_{\max}(J)}{\sigma_{\mathrm{mim}}(J)} \tag{2}$$

where $\sigma_{max}(J)$ is the maximal and $\sigma_{min}(J)$ is the minimal singular value of the Jacobian matrix of the robot manipulator.

The condition number is also useful in designing a manipulator with a high capability for gross motion in the limited subset of the workspace. In general, however, it does not ensure the conditioning of the manipulator in the overall workspace.

B. Manipulability

Yoshikawa proposed kinematic manipulability as a measure of dexterity. Its concept indicates how close the manipulator is to having a singular configuration. The manipulability of manipulators is defined as

$$\mathrm{w} = \sqrt{\det(JJ^T)} \tag{3}$$

It is useful for the design of a manipulator that has fast recovery ability from the escapable singular points. A well-designed manipulator has higher manipulability.

C. Global Performance Indices

The aforementioned performance indices are the local performance indices for measuring the performance of specific tasks or motions. They cannot sufficiently indicate the performance of various tasks. Thus, the global performance indices have to be considered to indicate the performance of the manipulator for its entire reachable workspace.

To measure the global behavior of the robot manipulator, the global performance indices were proposed by G. Gosselin and J. Angeles. They are defined as

$$\eta = \frac{A}{B} \tag{4}$$

where

$$A = \int_W \left(\frac{1}{p} \right) dW \tag{5}$$

and

$$B = \int_W dW \tag{6}$$

where p is the condition number or 1/p is the manipulability at a particular point in W, the workspace of the manipulator; and the denominator B is the volume of the workspace.

In the case of general manipulators, the workspace is not known in the Cartesian space, and it is easier to describe the workspace in the joint space. It is defined as

$$A = \int_R \left(\frac{1}{p} \right) \ |\Delta| \ d\theta_n \ldots d\theta_2 d\theta_1 \tag{7}$$

and

$$B = \int_{\mathrm{R}} |\Delta| \, d\theta_n \ldots d\theta_2 d\theta_1 \tag{8}$$

where R denotes the workspace in the joint space and Δ denotes the determinant of the Jacobian matrix.

The numerical way of calculating the global performance indices in the joint space was used instead of the analytical way in the Cartesian space, because of the complexity of the 7DOF robot manipulator that was used in this study.

If dθ in (7) and (8) is given a small value, the number of sampling points will increase. As value of dθ getting smaller and smaller to get accurate results, the time required for getting global performance indices will be longer and longer. To choose a reasonable dθ, the graphs that represent the global performance indices with respect to the number of sampling points were used, as shown in Fig. 3.

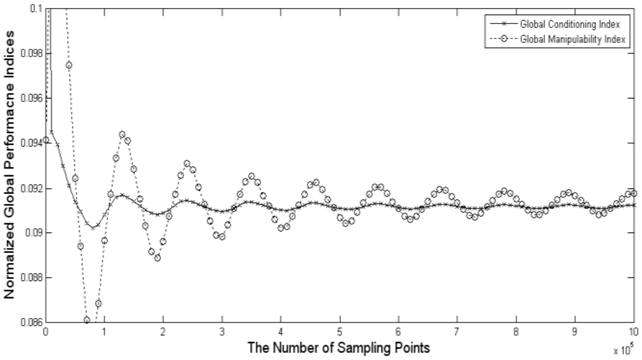

Fig. 3. Graphs for the global performance indices

In the graph, both global performance indices converge on the specific value as the number of sampling points increases. When the number of sampling points exceeds a million, the variation of the global performance indices will be within 0.1%. Thus, a million sampling points were used in this study based on the variation of the global performance indices.

D. Structural Length Index

The objective of a well-designed robot manipulator is to maximize the volume of reachable workspaces. In general, a longer manipulator has a larger reachable workspace, and the value of the reachable workspace volume also depends on the configuration of the robot manipulator. In other words, two robot manipulators with the same link length can have different reachable workspaces. Therefore, the structural length index can be used to measure the reliable reachable workspace as a global performance index. This is based on the ratio of the robot manipulator link length sum to the cube root of the reachable workspace volume. A good robotic design has a small link length sum with a large reachable workspace volume. The structural length index is defined as

$$Q_L = \frac{L}{\sqrt[3]{V}} \tag{9}$$

where V is the volume of the reachable workspace of the robot manipulator and L is the link length sum of the robot manipulator, as follows

$$L = \sum_{i=1}^{n} (a_i + d_i) \qquad (10)$$

where a_i and d_i are the link length and the joint offset, respectively.

The procedure for calculating the volume of the robot manipulator's reachable workspace is summarized as follows. First, it is necessary to estimate the size of the reachable workspace along the x, y, and z axes using the Monte Carlo method [13-14]. Then it is necessary to decide on the size of resolutions Δx, Δy, and Δz by considering the calculation capability of the computer. The small cubes that have the volume (Δx×Δy×Δz) from the cell represent the reachable location of the end effector of the robot manipulator. Mathematically, it can be described with a 3D matrix [15]. In this study, the resolution of each axis was set at 10 mm. In other words, one cube had a $10mm^3$ volume. Second, sufficient end effector position data are generated through forward kinematics with a random joint variable, as shown in Fig. 4. Third, the position data are converted into the cell described as the 3D matrix. If the cube in the cell has at least one position datum, value "1" is assigned to the element of the 3D matrix; and if not, value "0" is assigned to it, as shown in Fig. 5. Finally, the cube volume of "1" elements in the reachable workspace can be determined by the number of "1" elements.

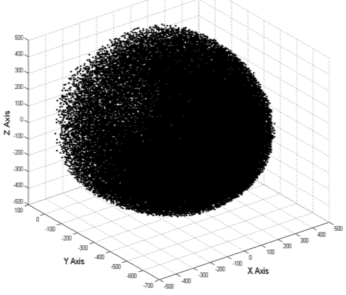

Fig. 4. Position data of the end effector through forward kinematics

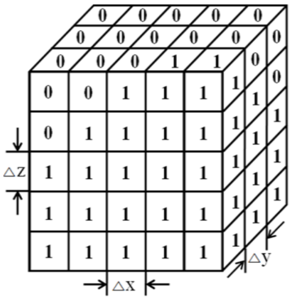

Fig. 5. Cubes for building the reachable workspace

V. EARLIER STUDY USING THE MICRO GENETIC ALGORITHM

In the earlier study, the micro genetic algorithm was used to optimize the link length of the robot manipulator because it could save the time required for optimization compared to other methods. The micro genetic algorithm was proposed by Krishnakumar to reduce the operation time. The micro genetic algorithm uses random creation instead of mutation of the genetic algorithm and needs a smaller population than the genetic algorithm. Therefore, the micro genetic algorithm shows fast convergence.

The population size was set at 5 and the maximum number of the generation was set at 20. The initial and optimal values of the link lengths and the global performance indices are shown in Table III.

TABLE III

INITIAL AND OPTIMAL VALUES THROUGH THE MICRO GENETIC ALGORITHM

Index	Link1	Link2	Link3	GCI	GMI	SLI
Initial	260	160	85	0.0015	0.1163	0.6643
Optimal	236	203	69	0.0016	0.1367	0.6600

Link1-3: Length of Links 1-3 (mm), GCI: Global Conditioning Index, GMI: Global Manipulability Index ($x10^8$), SLI: Structural Length Index

VI. THE GREY-BASED TAGUCHI METHOD

A. The Grey-based Taguchi Method

The first objective of this study was to find the optimal link length of the 7DOF manipulator so as to minimize its structural length index and maximize its global conditioning and manipulability indices. Calculation of a full factorial design was almost impossible, since it would have taken too much time to calculate the multiple performance indices. Thus, the optimization was performed using the Grey-based Taguchi method, because of its ability to dramatically reduce the number of experiments required to gather the necessary data [16] and to handle multiple performance indices.

B. Design of the Experiments

The design of the experiments (DOE) in this study included three adjustable link parameters at five levels. The minimum value of the link parameters at the first level was set based on the limitation of the mechanical constraints, such as the diameter and length of the motor and gearbox, which had to satisfy the payload. The maximum value of the link parameters at the last level was set by adding 30% of the prototype link length to the initial link length, and the values of the other levels were set by dividing them equally between the first level and the last level, because setting uniform experiment points at each level is helpful in understanding the response tendency. These values are listed in Table IV.

TABLE IV

LINK PARAMETERS AND THEIR LEVELS

Link No.	Level 1	Level 2	Level 3	Level 4	Level 5
Link 1	230 mm	257 mm	284 mm	311 mm	338 mm
Link 2	140 mm	157 mm	174 mm	191 mm	208 mm
Link 3	50 mm	65 mm	80 mm	95 mm	110 mm

C. Orthogonal Array and Experiment Results

The choice of a relevant orthogonal array for the experiments depended on the following criteria. The first criterion was the number of factors and interactions of interest; the second, the number of levels for the factors of interest; and the third and last, the desired experiment resolution or the cost and time limitations.

The L25 orthogonal array, which has four columns and 25 rows, was selected after considering the factors and levels. The experiment layout for the link parameters using the L25 orthogonal array is shown on the left side of Table V. The results of the experiment on the global conditioning, global manipulability, and structural length indices that corresponded to the link length in each experiment are shown on the right side of Table V.

TABLE V

L25 ORTHOGONAL ARRAY AND RESULTS OF EACH EXPERIMENT

Exp.	Link Parameter			Result		
No.	Link 1	Link 2	Link 3	GCI	GMI	SLI
1	1	1	1	0.0021	0.0794	0.6726
2	1	2	2	0.0018	0.0921	0.6645
3	1	3	3	0.0016	0.1058	0.6587
4	1	4	4	0.0014	0.1204	0.6545
5	1	5	5	0.0013	0.1360	0.6516
6	2	1	2	0.0018	0.0964	0.6731
7	2	2	3	0.0016	0.1113	0.6654
8	2	3	5	0.0013	0.1272	0.6572
9	2	4	1	0.0018	0.1440	0.6706
10	2	5	4	0.0014	0.1620	0.6573
11	3	1	3	0.0016	0.1152	0.6738
12	3	2	5	0.0013	0.1325	0.6632
13	3	3	4	0.0014	0.1508	0.6644
14	3	4	2	0.0016	0.1701	0.6706
15	3	5	1	0.0017	0.1906	0.6735
16	4	1	4	0.0014	0.1359	0.6746
17	4	2	1	0.0017	0.1557	0.6888
18	4	3	2	0.0016	0.1766	0.6786
19	4	4	5	0.0012	0.1986	0.6637
20	4	5	3	0.0014	0.2217	0.6688
21	5	1	5	0.0012	0.1584	0.6754
22	5	2	4	0.0013	0.1810	0.6769
23	5	3	1	0.0016	0.2047	0.6909
24	5	4	3	0.0014	0.2295	0.6758
25	5	5	2	0.0015	0.2556	0.6781

Link1-3: Level of Links 1-3, GCI: Global Conditioning Index,
GMI: Global Manipulability Index ($\times 10^8$), SLI: Structural Length Index

D. Grey Relational Analysis

The link length of the robot manipulator that yielded better global performance indices was optimized through Grey relational analysis [17-20]. First, using Grey relational analysis, the results of the experiment on the global performance indices were normalized in the range of zero to one. Next, the Grey relational coefficient was calculated from the normalized experiment data to express the relationship between the desired and the actual experiment data. Then the Grey relational grade was computed by averaging the Grey relational coefficient that corresponded to each process response.

First, the linear normalization of the DOE data according to the type of performance characteristic is performed as follows.

In the case of a "the-higher-the-better" characteristic, such as with the global conditioning and manipulability indices, the experiment value of $y_i(k)$ can be normalized as follows

$$x_i(k) = \frac{y_i(k) - \min\ y_i(k)}{\max\ y_i(k) - \min\ y_i(k)} \tag{11}$$

On the other hand, in the case of a "the-smaller-the-better" characteristic, such as with the structural length index, the experiment value of $y_i(k)$ can be normalized as follows

$$x_i(k) = \frac{\max\ y_i(k) - y_i(k)}{\max\ y_i(k) - \min\ y_i(k)} \tag{12}$$

where $x_i(k)$ is the normalized value, and min $y_i(k)$ and max $y_i(k)$ are the minimal and maximal values, of $y_i(k)$ in the i_{th} experiment for the k_{th} characteristic. The larger values yielded better performance, and the ideal value should be equal to one ($x_0(k) = 1$). The results of the data processing are shown on the left side of Table VI.

The Grey relational coefficient determines the relationship between the ideal and the actual normalized values. It is defined as:

$$\xi_i(k) = \frac{\Delta_{\min} + \zeta\ \Delta_{\max}}{\Delta_{0i}(k) + \zeta\ \Delta_{\max}} \tag{13}$$

where $\Delta_{0i} = \| x_0(k) - x_i(k) \|$ and ζ is the distinguishing coefficient set between zero and one. In this study, it was set at $\zeta = 0.5$. $\Delta_{\min}$ is the smallest value of Δ_{0i}, and $\Delta_{\max}$ is the largest. The results of the Grey relational coefficients are shown on the right side of Table VI.

TABLE VI

DATA PROCESSING RESULT AND GREY RELATIONAL COEFFICIENT

Exp.	Data Processing			Grey Relational Coefficient		
No.	GCI	GMI	SLI	GCI	GMI	SLI
1	1.0000	0.0000	0.4644	1.0000	0.3333	0.4828
2	0.7195	0.0722	0.6708	0.6406	0.3502	0.6030
3	0.4720	0.1497	0.8195	0.4864	0.3703	0.7348
4	0.2584	0.2327	0.9242	0.4027	0.3945	0.8684
5	0.0760	0.3213	1.0000	0.3511	0.4242	1.0000
6	0.6765	0.0965	0.4514	0.6072	0.3563	0.4768
7	0.4653	0.1810	0.6475	0.4832	0.3791	0.5865
8	0.1330	0.2711	0.8572	0.3658	0.4069	0.7778
9	0.6818	0.3669	0.5142	0.6111	0.4413	0.5072
10	0.1921	0.4689	0.8543	0.3823	0.4849	0.7743
11	0.4082	0.2033	0.4343	0.4579	0.3856	0.4692
12	0.1119	0.3013	0.7032	0.3602	0.4171	0.6275
13	0.2304	0.4050	0.6729	0.3938	0.4566	0.6045
14	0.4681	0.5148	0.5157	0.4846	0.5075	0.5080
15	0.5445	0.6310	0.4414	0.5233	0.5754	0.4723
16	0.1855	0.3206	0.4140	0.3804	0.4239	0.4604
17	0.6060	0.4331	0.0523	0.5593	0.4687	0.3454
18	0.4353	0.5516	0.3118	0.4696	0.5272	0.4208
19	0.0471	0.6765	0.6916	0.3441	0.6071	0.6185
20	0.2439	0.8080	0.5614	0.3981	0.7226	0.5327
21	0.0000	0.4483	0.3939	0.3333	0.4754	0.4520
22	0.1260	0.5766	0.3562	0.3639	0.5415	0.4371
23	0.4802	0.7111	0.0000	0.4903	0.6338	0.3333
24	0.2193	0.8521	0.3827	0.3904	0.7717	0.4475
25	0.2995	1.0000	0.3251	0.4165	1.0000	0.4256

GCI: Global Conditioning Index, GMI: Global Manipulability Index,
SLI: Structural Length Index

In the last step, the Grey relational grade is calculated by averaging the Grey relational coefficients in each experiment. The Grey relational grade γ_i can be obtained as follows

$$\gamma_i = \frac{1}{n} \sum_{k=1}^{n} \xi_i(k) \tag{14}$$

where γ_i is the Grey relational grade in the i_{th} experiment, and n is the number of performance characteristics (in this study, n = 3). The Grey relational grade refers to how close the grade given in the experiments is to the ideal. The Grey relational grades and their order in the experiment results are shown in Table VII.

TABLE VII
GREY RELATIONAL GRADE AND ITS ORDER

Exp. No.	Grey Relational Grade	Order
1	0.6054	2
2	0.5313	8
3	0.5305	9
4	0.5552	4
5	0.5918	3
6	0.4801	18
7	0.4829	17
8	0.5168	13
9	0.5199	12
10	0.5472	6
11	0.4376	23
12	0.4683	20
13	0.4850	16
14	0.5000	14
15	0.5237	10
16	0.4216	24
17	0.4578	21
18	0.4725	19
19	0.5233	11
20	0.5511	5
21	0.4203	25
22	0.4475	22
23	0.4858	15
24	0.5365	7
25	0.6140	1

VII. RESULTS OF THE OPTIMIZATION

A. Results of the Grey-based Taguchi Method

Based on the results of the Grey relational grade, the main effect graph was plotted using the mean of the grade at each link parameter and level. The mean of the Grey relational grades at each link parameter and level are summarized in Table VIII.

TABLE VIII
MEAN OF THE GREY RELATIONAL GRADE
AT EACH LEVEL AND LINK PARAMETER

Link No.	Level 1	Level 2	Level 3	Level 4	Level 5
Link 1	0.5628	0.5094	0.4829	0.4853	0.5008
Link 2	0.4730	0.4776	0.4981	0.5270	0.5656
Link 3	0.5185	0.5196	0.5077	0.4913	0.5041

Total mean of the Grey relational grade: 0.5082

The optimal link length was determined by selecting the level that had the highest Grey relational grade at each link parameter. Fig. 6 shows the main effect graph using the mean of the grades, where the line indicates the total mean of the Grey relational grade. The graph shows the optimal level of each link parameter: the length of link 1 in level 1, the length of link 2 in level 5, and the length of link 3 in level 2.

The initial and optimal values of the link lengths and global performance indices are shown in Table IX.

TABLE IX
INITIAL AND OPTIMAL VALUES IN THE GREY-BASED TAGUCHI METHOD

Index	Link1	Link2	Link3	GCI	GMI	SLI
Initial	260	160	85	0.0015	0.1163	0.6643
Optimal	235	208	65	0.0016	0.1360	0.6599

Link1-3: Length of Links 1-3 (mm), GCI: Global Conditioning Index, GMI: Global Manipulability Index (x10^8), SLI: Structural Length Index

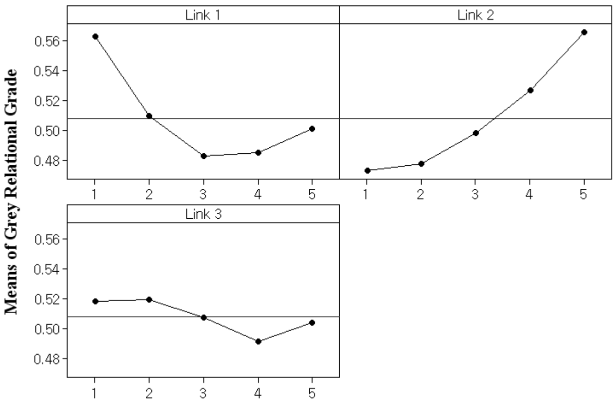

Fig. 6. Main effect graph for the Grey relational grade

ANOVA was performed to determine which link parameter significantly affected the performance of the robot manipulator. This was accomplished by separating the total variability of the Grey relational grades, which was measured by the sum of the squared deviations from the total mean of the Grey relational grade, into the contributions by the link parameter and the error. The percentage contribution of each link parameter to the total sum of the squared deviations can be used to evaluate the importance of the link parameter that affects the performance characteristic. In addition, the F-test, which was named after Fisher [21], can be used to determine which link parameters significantly affected the performance characteristic. Usually, the link parameter has a significant effect on the performance of the manipulator when the value of F is large. The ANOVA results are shown in Table X. The results show that Link 2 affected most the performance indices of the robot manipulator, followed by Link 1.

TABLE X
ANOVA RESULTS

Link Parameter	Degree of Freedom	Sum of Squares	Mean Square	F	Contri-bution
Link 1	4	0.0210	0.0053	4.44	31.14%
Link 2	4	0.0296	0.0074	6.25	43.84%
Link 3	4	0.0027	0.0007	0.57	3.98%
Error	12	0.0142	0.0012		21.04%
Total	24	0.0676			100%

B. Comparison of the Results of the Grey-based Taguchi Method and the Micro genetic Algorithm

In an earlier study, the micro genetic algorithm was used to optimize the link length of the robot manipulator. As a result, the total length of the manipulator increased by 0.59%. The global conditioning index improved by 4.06%; the global manipulability index, by 17.53%; and the structural length index, by 0.64%, as shown in Table XI.

In this study, the Grey-based Taguchi method was used to optimize the link length of the robot manipulator. As a result, the total length of the manipulator decreased by 0.40%. The global conditioning index improved by 5.50%; the global manipulability index, by 16.93%; and the structural length index, by 0.66%, as shown in Table XI.

The results of the comparison of the micro genetic algorithm and the Grey-based Taguchi method showed that the average improvements of the performance indices in the two methods were almost the same, but that the Grey-based Taguchi method is 4 times faster than the micro genetic algorithm because it uses less function calls. In other words, using the Grey-based Taguchi method is more effective in terms of time than using the micro genetic algorithm.

TABLE XI

COMPARISON OF THE INITIAL AND OPTIMAL VALUES

Index	Initial Value	Optimum MGA	Optimum GBT
Length of Link 1	260 mm	236 mm	230 mm
Length of Link 2	160 mm	203 mm	208 mm
Length of Link 3	85 mm	69 mm	65 mm
Total length	505 mm	508 mm	503 mm
Global Conditioning $(x10^{-2})$	0.1541	0.1604	0.1626
Global Manipulability $(x10^{8})$	0.1163	0.1367	0.1360
Structural Length	0.6643	0.6600	0.6599
Average Improvement	-	7.41%	7.70%
Number of Function Calls	-	101 times	25 times

MGA: Micro genetic algorithm, GBT: Grey-based Taguchi method

VIII. CONCLUSION

This paper reported how the Grey-based Taguchi method can be used to optimize the link length of the robot manipulator with better global conditioning, global manipulability, and structural length indices. The Grey-based Taguchi method integrated the L25 orthogonal array for the experiments and the Grey relational analysis to deal with multiple performance characteristics. This method was used to optimize the link length of the robot manipulator with the Grey relational grade. The link parameter that affected most the performance indices of the robot manipulator was also investigated using ANOVA.

Two optimization techniques for the link length of the prototype robot manipulator with better the global performance indices were performed to find the efficient method. There was a 7.70% average improvement in the global performance indices of the robot manipulator. It took only a quarter of the operation time for the Grey-based Taguchi method to get almost the same results as those of the micro genetic algorithm. This results show that the Grey-based Taguchi method is a powerful tool in dealing with optimization problems such as too many cost functions or a too long analysis time. This method can be used not only robotic field but other fields. Then it is possible to improve efficiency of solving optimization problem.

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