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A Simple Tractor-Trailer Backing Control Law for Path Following

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Abstract— **Backing of tractor-trailer systems is a problem addressed in many literatures. It is usually solved using various nonlinear-based control methods, which are often not easy to implement or tune. Similar to other work focused on backing a single axle trailer with a car like vehicle, we propose a two-tier controller that is simple and intuitive. However, ours is based upon curvature as opposed to hitch angle, which allows the control input to be more directly related to path specification and to handle path curvature discontinuity better. Experimental results are provided to illustrate the capability of this new algorithm applied to a full scale autonomous vehicle and trailer system in a real field environment using minimal sensing capability. Results demonstrate good performance on sloped grounds with various grades.**

I. Introduction

Backing a trailer is very common in industry and social activities. Because of trailer is not stable in backing, trailer backing can be difficult for inexperienced drivers. Even for many experienced drivers, it is still time consuming. If trailer backing can be handled automatically, a large amount of cost and time may be saved.

There are numerous works on trailer backing in the literature. (The reader is referred to [1] for a more detailed review of recent and traditional techniques.) While a wealth of traditional techniques based upon nonlinear control theory exists, these often provide complicated and impractical solutions that are difficult to implement on real systems with delay and error. In this work, we focus on a two-tier linear control system that is easy to implement and tune on a real vehicle system. In our system, we first issue curvature commands based upon path following error. In the second layer, these curvature commands are translated into steering wheel angles by a low level servo loop aimed at achieving the commanded path curvature. Feed forward steering wheel commands based upon a curvature-wheel map are then used to improve responsiveness and tracking error in the system.

Similar to [1][2], we show that the two tier controller offers advantages compared to traditional approaches. Some researchers transform the kinematic models into different forms [3][4], such as chained form, where it is easier to formulate the controller. This provides a general solution that is applicable to a system with many trailers, but normally requires complicated transformations and is sensitive to parameter inaccuracy. Other methods based on optimal control, like [5], rely heavily on a system model and requires very high computational power for constant re-planning caused by disturbances. Learning or evolution based methods are also used for this problem, such as [6][7], but they traditionally need large amounts of time and data to train the controller when implemented on actual vehicles.

While some of the aforementioned techniques are applicable to multi trailer systems, they are generally inefficient for backing one trailer.Since backing of single trailer systems is quite common to in daily life, simple and intuitive controllers have been developed specifically for this kind of tasks. Most related to our work, is that of Pradalier [1][2], which is also focused on a linear-based two-tier control method for a tractor-trailer system. Their work differs from ours, however, in that it uses different control loops in the different layers of the controller. Their first level forms a loop that creates hitch angle commands based upon linear combinations of lateral tracking error, heading error, and curvature error. The second level is a hitch angle stabilizer, which uses tractor steering angle to control hitch angle. This method is also relatively simple to implement and intuitive to tune, but our layers are more related to path specifications and better matched to actual vehicle kinematics. While a detailed comparison of the performance of the two methods is outside the scope of this paper, we believe that our technique can offer improved path following performance especially the handling of path curvature discontinuity. Since it is better matched to actual vehicle kinematics, it should result in simpler path planning as curvature is a common variable in such endeavors.

Section II discusses the kinematics model of vehicle-trailer system. Steering Controller development and tunning will be presented in Section III. Speed controller design and tuning is discussed in Section IV. Experiment results are shown in Section V. The paper concludes in Section VI.

II. Tractor-trailer Kinematic Model

The geometry of a vehicle-trailer system is shown in Fig.1, where ϕ is the angle of the front wheels with respect to the longitudinal axis of the vehicle, θ_1 is the heading of the vehicle; (x, y) is the position of the vehicle (which is defined at the center of the rear axle, C), θ_2 is the heading of trailer, ψ is the hitch angle defined as $\psi = \theta_2 - \theta_1$, (x_T, y_T) is the position of the trailer (which is defined at the center of the trailer's rear axle, Q), L is the vehicle wheel base, L_1 is the hitch length (the distance between the vehicle's rear axle and hitch point H), and L_2 is the trailer tongue length (the distance between hitch point H and the trailer's rear axle). According to [2], the standard kinematic model of the

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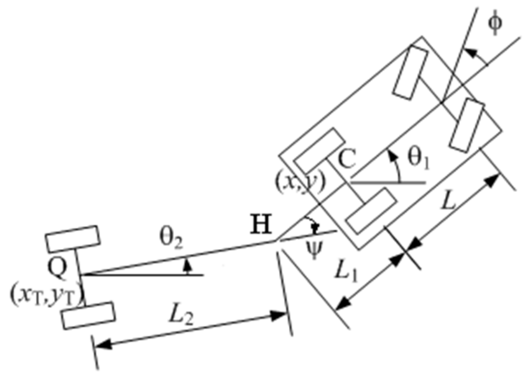

Fig. 1. Kinematics of a vehicle-trailer system

vehicle-trailer system can be described by,

$$\begin{aligned} \dot{x} &= v\cos(\theta_1) && (1)\\ \dot{y} &= v\sin(\theta_1) && (2)\\ \dot{\theta}_1 &= \frac{v\tan(\phi)}{L} && (3)\\ \dot{\psi} &= -\frac{v}{L}(\tan(\phi)+\frac{\sin(\psi)L}{L_2}+\frac{L_1\tan(\phi)\cos(\psi)}{L_2}) && (4)\end{aligned}$$

where v is the velocity of the vehicle at point C. One difficulty, however, is that (3) is not sufficiently accurate for controlling the steering response of a full size vehicle where Ackerman Steering is common. Besides, front wheel angles are not directly measured. As such, the following general vehicle-trailer model is proposed:

$$\begin{aligned} \dot{x} &= v\cos(\theta_1) && (5)\\ \dot{y} &= v\sin(\theta_1) && (6)\\ \dot{\theta}_1 &= vF_\kappa(\varphi) && (7)\\ \dot{\psi} &= -v(F_\kappa(\varphi)+\frac{\sin(\psi)}{L_2}+\frac{L_1F_\kappa(\varphi)\cos(\psi)}{L_2}) && (8)\end{aligned}$$

where φ is the angle of the steering wheel, which is easily measure and directly controlled in autonomous vehicles. Thus, $F_\kappa(\varphi)$ is a function that maps steering wheel angle to curvature of the vehicle's trajectory, which is called the "steering wheel map". The construction of this function and its inverse is discussed further in Section III.

Trailer velocity v_T is defined as the velocity of point Q. To calculate it, the velocity vector of the hitch point $\mathbf{v}_H$ needs to be determined first, which is the combination of a velocity along the vehicle and a velocity perpendicular to the vehicle.

Define $\mathbf{\Omega} = [0,0,\dot{\theta}_1]^T$ and $\mathbf{v} = [v\cos\theta_1, v\sin\theta_1, 0]^T$. Then $\mathbf{v}_H$ can be calculated as:

$$\mathbf{v}_H = \mathbf{v} + \mathbf{\Omega}\times\overrightarrow{CH} = v\begin{bmatrix}\cos\theta_1 + L_1F_\kappa(\varphi)\sin\theta_1\\ \sin\theta_1 - L_1F_\kappa(\varphi)\cos\theta_1\\ 0\end{bmatrix} \quad (9)$$

With the no-slip assumption, the trailer velocity is calculated as:

$$\begin{aligned} v_T &= \mathbf{v}_H\cdot\overrightarrow{QH}\\ &= v\left[\cos(\theta_2-\theta_1) - L_1F_\kappa(\varphi)\sin(\theta_2-\theta_1)\right]\\ &= v\left[\cos\psi - L_1F_\kappa(\varphi)\sin\psi\right] && (10)\end{aligned}$$

Finally, according to (7), (8) and (10), the trailer trajectory curvature is:

$$\kappa_2 = \frac{\dot{\theta}_2}{v_T} = -\frac{\sin(\psi)/L_2 + L_1F_\kappa(\varphi)\cos(\psi)/L_2}{\cos(\psi) - L_1F_\kappa(\varphi)\sin(\psi)} \quad (11)$$

Notice that the trailer curvature can be directly controlled by vehicle steering wheel angle φ.

III. Steering Controller Development

A. Path Tracking

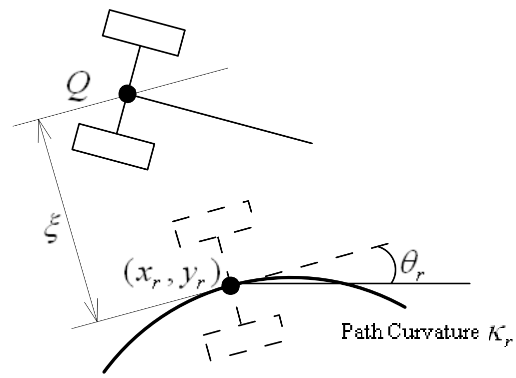

Fig. 2. Path following references and lateral error

Many of the existing path following control methods make the reference point for tracking move on the path with certain speed. But since most trailer backing operations do not have timing requirements, instant reference position (x_r, y_r) on the path can be calculated according to the position of trailer instead. In this paper, the reference position (x_r, y_r) is defined as the point on the path that has shortest distance to the trailer.

As shown in Fig. 2, reference heading θ_r is the tangential direction at the reference position. Reference curvature κ_r is the local curvature at the reference position. Lateral tracking error ξ is the distance between actual trailer position and the path and is defined as positive when trailer is on the right hand side of the path (when facing the direction of path progression). Reference velocity is calculated according to the preset velocity limit and the location of this reference position on the path.

Since curvature is to be transformed into a real control input, i.e. steering wheel angle, the tracking errors shown above must be transformed into curvature first. A simple method similar to its counterpart in [2] is as below:

$$\kappa_d = \kappa_r + k_\xi\xi + k_\theta(\theta_r - \theta) \quad (12)$$

where κ_d is the desired curvature used to follow the path while reducing the errors; k_ξ and k_θ are the control parameters for lateral error and heading error.

However, we found in practice that a good κ_ξ for small lateral errors may be too high for large lateral error and lead to overshoot if large lateral errors appear. This is because larger κ_ξ will lead to larger curvature and larger hitch angle. The curvature stabilizing capability of the system will be lower when hitch angle is bigger. Therefore, the system should be steered less aggressively when a large hitch angle is anticipated, otherwise it may end up in overshooting.

A simple alternative to account for this is as follows:

$$\Delta\theta = \mathrm{sgn}(\xi)\frac{\pi}{2}(1 - e^{-k_\xi|\xi|}) \tag{13}$$

$$\theta_e = \theta_r + \Delta\theta - \theta \tag{14}$$

$$\kappa_d = \kappa_r + k_\theta \theta_e \tag{15}$$

where $\Delta\theta$ is the heading offset; θ_e is heading error and defined in $[-\pi, \pi]$.

With this control scheme, control effort increases slower and slower when lateral error increases.

In this form, κ_ξ controls the approaching angle to the path given a lateral error. The gain κ_θ determines the aggressiveness of turning for correcting a heading error.

B. Curvature Control

According to (11), if given a desired trailer motion curvature κ_d, the desired steering angle can be calculated as:

$$\varphi_d = F_\kappa^{-1}\left(\frac{\kappa_d L_2 \cos\psi + \sin\psi}{\kappa_d L_1 L_2 \sin\psi - L_1 \cos\psi}\right) \tag{16}$$

where, φ_d is the desired steering wheel angle and will be sent to steering wheel actuator.

To compensate for possible inaccuracies of given trailer parameters and environment disturbances, we add a steering gain k_φ to (16), which makes the final control function as follow:

$$\varphi_d = k_\varphi F_\kappa^{-1}\left(\frac{\kappa_d L_2 \cos\psi + \sin\psi}{\kappa_d L_1 L_2 \sin\psi - L_1 \cos\psi}\right) \tag{17}$$

In practice, k_φ can be initially set slightly larger than 1 and tuned according to the degree of chattering when the trailer is very close to its path. The higher k_φ is, the better the system can overcome parameter inaccuracies and environment disturbances. But, very high k_φ will also result in chattering. Typical k_φ setting is 1.1 or 1.2.

C. Steering Wheel Map

$F_\kappa(\varphi)$, the relation between steering wheel angle and curvature of vehicle trajectory is generally non-linear and very hard to fully model into an accurate non-linear function. A feasible approximation method for this function and its inverse is given below.

Normally, $F_\kappa(\varphi)$ varies with vehicle velocity [8]. However, trailer backing is mostly conducted at very low velocities (lower than 0.7 m/s in our application), which is limited to a very small range. Therefore, $F_\kappa(\varphi)$ is considered as velocity-invariant in this context.

For constructing $F_\kappa(\varphi)$, the vehicle needs to be manually driven forward with fixed steering wheel angles at low speed. Then the radius of the vehicle trajectory can be extracted and turned into curvature. For a reliable steering wheel map, it is advised to obtain curvature measurement every 50 degree steering wheel angle. The function used to fit these curvature-steering wheel angle data points can be selected based on the observation of the distribution of the data points in the curvature-steering wheel angle coordinate. Otherwise, linear interpolation can be used to for the area between data points.

It is possible that $F_\kappa(\varphi)$ has multiple steering wheel angles leading the same curvature, which makes it not invertible. However, the real interest in inverting this function is to find a steering angle that can realize the given curvature. Therefore, F_κ^{-1} can be constructed as the function returning only the smallest steering wheel that can achieve the given curvature.

It has been known that steering wheel is not rigidly connected to front wheels in most commercial vehicles today. Moreover, ground tilt can affect vehicle turning radius. The approximation mentioned above may also bring in error. However, the steering wheel map for all the experiment results provided in Section V is based on test result at about 2 m/s with 5

D. Controller Tuning

Tuning steering controller gains is a two-step process.

First, in order to tune k_θ, let $\xi = 0$ in (13),which results in $\Delta\theta = 0$ such that (14) becomes $\theta_e = \theta_r - \theta$. Then select a θ_r so that θ_e is non-zero at the beginning of a test run. Finally set $\kappa_r = 0$ such that (15) becomes,

$$\kappa_d = k_\theta \theta_e \tag{18}$$

On this condition, k_θ is the only control parameter affecting system performance. All the parameters and tracking specifications that are irrelevant to the tuning of k_θ are ignored. Then various tuning techniques for proportional controller can be used.

In the second step, only k_ξ need to be tuned. In tuning k_ξ, the system should be set to track a straight path with non-zero initial lateral error. k_ξ can then be tuned using various tuning techniques for proportional controller.

IV. Speed Controller

The speed controller is designed to control vehicle speed v, which is measured by GPS. There is huge delay in vehicle engine and transmision systems, therefore vehicle longitudinal acceleration a and jerk j are also used to help stabilize vehicle velocity. Acceleration is estimated by five-point linear fit on velocity data. Jerk is estimated by five-point linear fit on acceleration data. The speed controller is designed as follow:

$$u_v(t_{now}) = \int_0^{t_{now}} [k_v(v_d - v) - k_a a - k_j j]dt \tag{19}$$

where, t_{now} is current time; v_d is the desired speed; k_v, k_a and k_j are control parameters for velocity, acceleration and jerk. u_v, defined on $[-100\%, 100\%]$, is the control effort that will be output to the throttle and brake manipulators. If $u_v \geq 0$, brake effort will be 0 and throttle effort will be u_v

Fig. 3. The autonomous vehicle and the test trailer

percentage of the full throttle. Otherwise, throttle effort will be 0 and brake effort will be $|u_v|$ of the full brake. Since the brake and throttle have different responses, (19) can be tuned separately for the throttle and brake. The two speed control branches can be switched according to whether $u_v \geq 0$.

As seen in (19), this control scheme is a variation of classic PID control law and can be tuned according to various PID tuning techniques available in existing literatures.

After brief tuning, at low speed ($v_d \leq 2$ mps), this speed controller can acheive ± 0.15 m/s accuracy with the test vehicle on flat ground and ground surface desending with 5% grade. The performance is worse when the vehicle moves on ground surface ascending with 5% grade, which is because the throttle delay is much larger than the brake delay.The velocity accuracy on ascending ground is ± 0.25 m/s.

V. Tests and Discussions

The control scheme is implemented on the Red Rover, which is a 2005 Dodge Grand Caravan with an autonomous ground vehicle system. The steering wheel angle is limited to 360 degrees. Position, heading and velocity are acquired from GPS. Hitch angle is measured from a potentiometer installed above the hitch point. The vehicle and test trailer is shown in Fig. 3. For this system, L_1 is 1.23 meters; L_2 is 2.51 meters; and L is 3 meters.

The Control period is about 0.11 seconds. Trailer position and heading are estimated from vehicle position, heading and hitch angle. Preset speed is 0.5 m/s for all the tests mentioned below.

Tests were conducted on flat ground and two tilt grounds. The local grades of the two sloped grounds are 5% and -5% on average. Different performances and behaviors were observed. For each terrain, two different tests were conducted: straight backing, and transitioning from a straight line to an arc. The arc radius was 18 meters for flat ground tests and 20 meters for sloped ground tests. All the arcs start at $x = 20$. In all tests, the trailer started from the left side and moved toward the right side.

A. Flat Ground Tests

For the flat ground test, the vehicle and trailer were positioned randomly near the path. The measured initial lateral error is -0.63 meters. Measured initial trailer heading error is -7.75 degrees, which leads the trailer away from the path when backing. Measured initial hitch angle is 0.30 degrees.

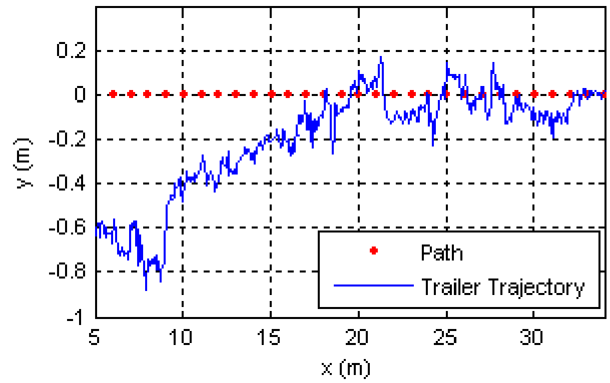

Fig. 4. Trailer trajectory of flat ground straight backing

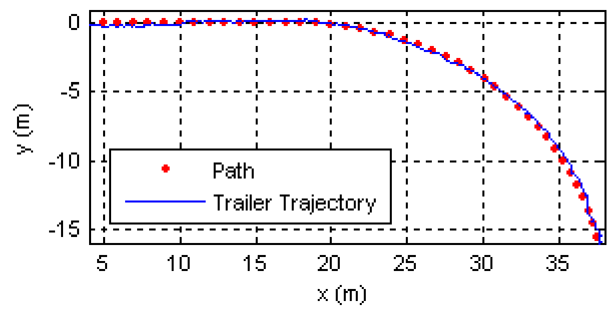

Fig. 5. Trailer trajectory of flat ground transition

It can be seen in Fig. 4 that the lateral error converged to about 0 from initial position error. The jagged appearance of the trajectory comes from noises within vehicle position, heading and hitch angle data.

The GPS positioning precision is about 0.1 meters. Therefore the lateral error may not always converge to 0. Lateral error is considered as having converged if it is within ± 0.1 meters range.

The trailer trajectory following the transition path is shown in Fig. 5. It can be seen that the curvature discontinuity at $x = 20$ does not cause any sudden rise of lateral error. The transition is handled very well, which shows that the controller has a good performance in controlling the curvature of the trailer.

B. Downhill Tests

Downhill tests were conducted on a slope with -5% grade. The slope descends along positive x axis direction.

According to Fig. 6, backing downhill increases the stability of the trailer and makes it more difficult to maneuver. The controller may not be able to correct small lateral errors when backing downhill.

Because the gravity force pulls the trailer downhill, part of the correction force passed by hitch is compromised. This

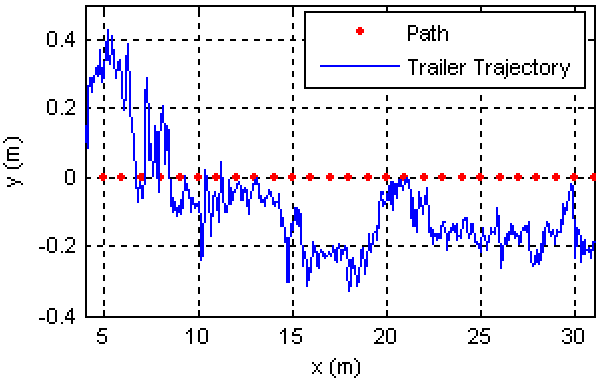

Fig. 6. Trailer trajectory of downhill straight backing

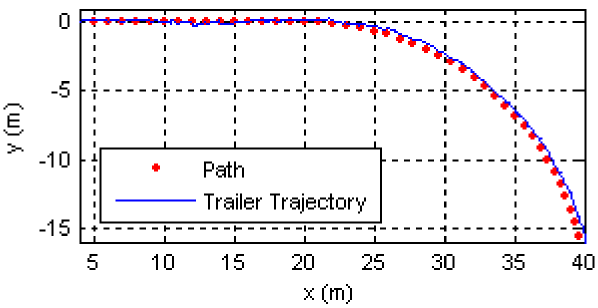

Fig. 7. Trailer trajectory of downhill transition

process may involve side slip, and is better observed with the transition path shown in Fig. 7. Compared to Fig. 5, the trailer trajectory following the arc path shown in Fig. 7 clearly is biased towards the downhill direction.

C. Uphill Tests

Uphill tests were conducted on a slope with 5% grade. The slope ascends along positive x axis direction.

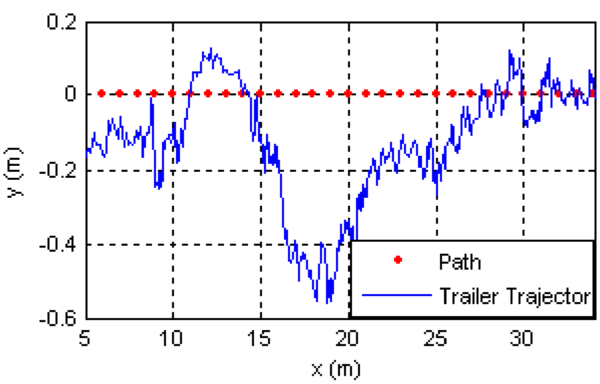

Fig. 8. Trailer trajectory of uphill straight backing

According to Fig. 8, uphill trailer backing increases the instability of the system. Big oscillations occur, which is caused by gravity. However, in this case, lateral error still converges into the 0.1 meter range.

Fig.9 shows that the trailer trajectory is biased towards the downhill direction, which is similar to the downhill case shown in Fig. 7.

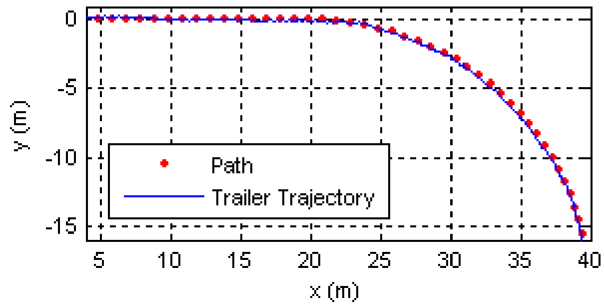

Fig. 9. Trailer trajectory of uphill transition

D. Future Work

Although an integrator at the higher level of the controller can reduce constant lateral error caused by gravity, the proper integration gain depends on the weight of the trailer and ground surface condition. The integrator also has difficulty working with changing slope grade. If the trailer weight is different from the nominal value and the path involves changing grade, such as a U-turn on a slope, an integrator may cause larger lateral errors or oscillation. Our ongoing research is to analyze the behavior of the trailer, identify and compensate the gravity bias.

VI. Conclusion

A two-level trailer backing controller is designed with simplicity in implementation and tuning. It has been proven in experiments on a full size vehicle that the controller can follow a path with good precision on flat ground. Backing on sloped ground may cause steady state lateral error. Authors are still working on a method to identify the gravity bias the sloped ground and compensate for it.

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