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Basis-Motion Torque Composition Approach: Generation of Motions with Different Velocity Profiles among Joints

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***Abstract*— The basis-motion torque composition (BMC), which generates feedforward torque for precise joint-trajectory tracking of a multi-joint robot arm, has been recently suggested. It is based on four arithmetical operations of time-series torque data of several motions preliminarily obtained by the iterative learning control. The remarkable advantage of BMC is to generate desired feedforward torque without prior information of dynamics parameters. However, the class of torque generated by the BMC has been restricted. The paper presents an enhanced BMC which permits generation of motions with different motion-velocity profiles among joints of a multi-joint robot. The algorithm is presented and the validity of algorithm is confirmed through numerical simulations in the case of a two-joint robot arm under the influence of gravity. Furthermore, a class of applicable systems of BMC approach is discussed.**

I. INTRODUCTION

For precise trajectory tracking of a multi-joint robot, it is widely known that the iterative learning control (ILC) is effective. Thanks to feedforward inputs built by an iterative learning update law, the ILC achieves a desired task after repetitions of trials without any information on dynamics of a robot. The effectiveness of ILC has been verified by a lot of practical use and mathematical analyses [1]–[5]. However, as for robot control, the ILC has been often criticized in comparison to the computed torque method (CTM) because another learning process is needed according to change of task, though the problems to be solved remain even in the CTM like the difficulty of accurately evaluating all of the dynamics parameters.

Against the criticism, Kawamura *et al.* have suggested the scheme named *time-scale transformation (TST)* to generate a specified motion without any additional learning processes for change of a desired motion nor any priori knowledge of dynamics parameters [6], [7]. The TST permits generation of feedforward torque for a motion with a specified velocity profile by performing arithmetic operations of time-series torque data obtained preliminarily by the ILC. However, the class of generable torque of TST is restricted to motions determined by extending or shortening a time duration of reference motions. That is, the trajectories generated by the TST are same in configuration space. Sekimoto *et al.* have recently suggested the scheme named *motion-scale transformation (MST)* to generate a motion to a specified posture (*i.e.*, a motion with a different trajectory from stored motions in configuration space) by reusing stored motion data [8]. The MST allows generation of desired feedforward torque by arithmetic operations of stored time-series torque data under assumptions of a fundamental motion-velocity profile called a motion primitive and a dynamics structure. However, the class of generable torque of MST is restricted to motions with a velocity profile same as an assumed motion primitive. Later, by combining the TST and the MST, they has developed a strategy named *basis-motion torque*

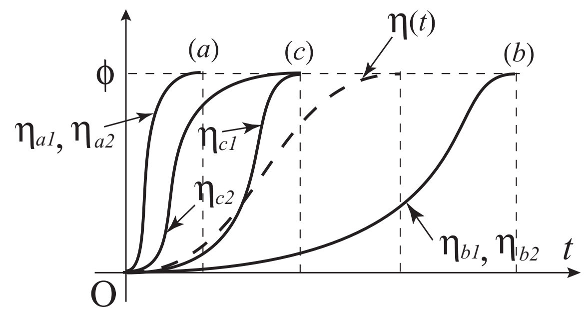

Fig. 1. Generable motions of BMC in the case of a two-joint robot arm; for a motion primitive $\eta(t)$, (a) the motion that $\eta(t)$ is shortened linearly in the time axis and the motion profiles of both joints are same, (b) the motion that $\eta(t)$ is extended non-linearly in the time axis and the motion profiles of both joints are same, (c) the motion that the motion profiles are based on the time-scale transformation of $\eta(t)$ but the motion profiles of both joints are different each other; the BMC can achieve the motions (a), (b) but cannot achieve the motion (c).

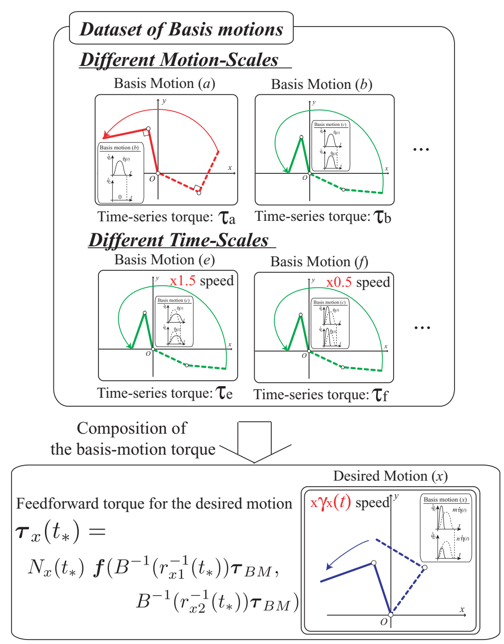

Fig. 2. An Overview of basis-motion torque composition

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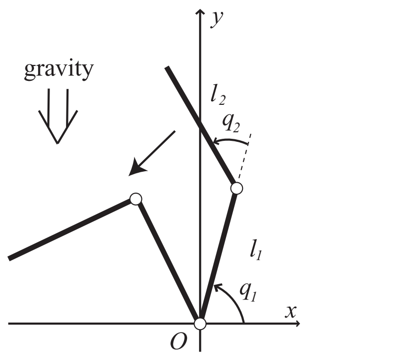


Fig. 3. Movement of a two-joint robot arm in gravity

composition (BMC) [9]. The BMC allows generation of a motion with a different trajectory in configuration space and with a different joint-velocity profile from a motion primitive. The experimental results have demonstrated that the tracking errors of angular velocities by the BMC tend to be smaller than those by the CTM, and they have supported the effectiveness of BMC. The remarkable advantage of BMC is disuse of priori information of dynamics parameters for generation of feedforward torque, despite an assumption of a dynamics structure. Unfortunately, however, the class of generable motions of BMC is still restricted. The generable motions of BMC are limited to motions that the joint-velocity profile is same at every joint (see Fig.1). Furthermore, the situation treated in the previous papers was limited in the only case of non-gravity planar movement of a two-joint robot arm. The class of applicable systems and situations of BMC has never been discussed.

The paper aims at improving the BMC so that it can generate motions with different joint-velocity profiles among joints of a multi-joint robot (see Fig.2). In the case of a two-joint robot arm under the influence of gravity, the algorithm for generation of feedforward torque is presented and the validity of algorithm is confirmed through numerical simulations. The enhanced BMC permits generation of a motion with a straight endpoint trajectory in work space. The simulation result in such a case is also illustrated. Furthermore, we point out the potential of the BMC approach that it can be applied even in the case of spatial motions of a more-than-two-joint robot.

II. PROBLEM STATEMENT

A. ASSUMPTIONS OF DYNAMICS

Our objective is to generate feedforward torque input for a multi-joint robot to track a desired joint trajectory $\boldsymbol{q}_r(t) \in \Re^2$ precisely. Let us consider movement of a robot arm with two joints under the influence of gravity as shown in Fig.3. Firstly, we assume that Lagrange's equation of motion of the robot arm including its drive systems can be described by

$$H(\boldsymbol{q})\ddot{\boldsymbol{q}}+\left\{\frac{1}{2}\dot{H}(\boldsymbol{q})+S(\boldsymbol{q},\dot{\boldsymbol{q}})\right\}\dot{\boldsymbol{q}}+B\dot{\boldsymbol{q}}+\boldsymbol{f}_c(\dot{\boldsymbol{q}})+\boldsymbol{g}(\boldsymbol{q})=\boldsymbol{\tau} \quad (1)$$

where $\boldsymbol{q} = (q_1,\, q_2)^{\mathrm{T}}$ denotes the vector of joint angles, $H(\boldsymbol{q}) \in \Re^{2\times 2}$ denotes the inertia matrix, $S(\boldsymbol{q},\, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ denotes the gyroscopic force term including centrifugal and Coriolis forces, $S(\boldsymbol{q},\, \dot{\boldsymbol{q}}) \in \Re^{2\times 2}$ denotes the skew-symmetric matrix, $B\dot{\boldsymbol{q}} + \boldsymbol{f}_c(\dot{\boldsymbol{q}}) \in \Re^2$ denotes the joint-friction force, $B \in \Re^{2\times 2}$ denotes the positive definite and diagonal matrix, $\boldsymbol{g}(\boldsymbol{q})$ denotes the gravity force, $\boldsymbol{\tau} \in \Re^2$ denotes the control input torque at joints [4]. Then, the feedforward input torque $\boldsymbol{\tau}_r$ for achieving the desired motion should satisfy

$$\boldsymbol{\tau}_r = H(\boldsymbol{q}_r)\ddot{\boldsymbol{q}}_r+\left\{\frac{1}{2}\dot{H}(\boldsymbol{q}_r)+S(\boldsymbol{q}_r,\dot{\boldsymbol{q}}_r)\right\}\dot{\boldsymbol{q}}_r+B\dot{\boldsymbol{q}}_r+\boldsymbol{f}_c(\dot{\boldsymbol{q}}_r)+\boldsymbol{g}(\boldsymbol{q}_r) \quad (2)$$

Secondly, we assume that a dynamics structure of robot is known but individual elements in dynamics are unknown. For instance, the inertia matrix $H(\boldsymbol{q})$, since every entry of $H(\boldsymbol{q})$ is a constant or a sinusoidal function of components of joint angle vector $\boldsymbol{q}$, can be represented by

$$H(\boldsymbol{q}) = \begin{bmatrix} a_{11} + 2a\cos q_2 & a_{22} + a\cos q_2 \\ a_{22} + a\cos q_2 & a_{22} \end{bmatrix} \quad (3)$$

where a_{11}, a_{22}, and a are unknown dynamics parameters. Thus, the desired feedforward input in eq.(2) is concretely defined as

$$\begin{bmatrix}\tau_{r1}\\ \tau_{r2}\end{bmatrix}=\begin{bmatrix} a_{11}+2ac_{r2} & a_{22}+ac_{r2} \\ a_{22}+ac_{r2} & a_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_{r1}\\ \ddot{q}_{r2}\end{bmatrix}+\left\{\frac{as_{r2}\dot{q}_{r2}}{2}\begin{bmatrix}-2 & -1\\ -1 & 0\end{bmatrix}+\frac{as_{r2}(2\dot{q}_{r1}+\dot{q}_{r2})}{2}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\right\}\begin{bmatrix}\dot{q}_{r1}\\ \dot{q}_{r2}\end{bmatrix}$$
$$+\begin{bmatrix}d_1 & 0\\ 0 & d_2\end{bmatrix}\begin{bmatrix}\dot{q}_{r1}\\ \dot{q}_{r2}\end{bmatrix}+\begin{bmatrix}\rho_1\mathrm{sgn}(\dot{q}_{r1})\\ \rho_2\mathrm{sgn}(\dot{q}_{r2})\end{bmatrix}+\begin{bmatrix}c_{r1}g_1+c_{r12}g_2\\ c_{r12}g_2\end{bmatrix} \quad (4)$$

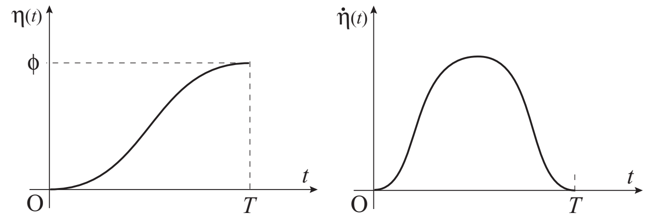
where a_{11}, a_{22}, and a denote positive constants related to the inertia matrix, d_1, d_2, ρ_1, and ρ_2 denote positive constants related to the joint-friction force, g_1 and g_2 denote positive constants related to the gravity force, $\boldsymbol{\tau}_r = (\tau_{r1}, \tau_{r2})^{\mathrm{T}}$, $s_{r2} = \sin q_{r2}$, $c_{r2} = \cos q_{r2}$, $c_{r1} = \cos q_{r1}$, $c_{r12} = \cos(q_{r1}+q_{r2})$, and $\mathrm{sgn}(\cdot)$ denotes a signum function. In the right hand side of eq.(4), the first term denotes the inertial force, the second term denotes the Coriolis force and the centrifugal force, the third and fourth terms denote the joint-friction force corresponding to $B\dot{\boldsymbol{q}}_r + \boldsymbol{f}_c(\dot{\boldsymbol{q}}_r)$ in eq.(2), and the last term denotes the gravity force.

B. ASSUMPTIONS OF MOTIONS

Motions treated in the paper are restricted to motions formulated on the basis of a smooth function over a finite time duration $t \in [0, T]$ defined by

$$\eta(t) = \phi\left[6\left(\frac{t}{T}\right)^5 - 15\left(\frac{t}{T}\right)^4 + 10\left(\frac{t}{T}\right)^3\right] \quad (5)$$

where ϕ denotes the magnitude of motion given as a positive constant and T denotes a terminal time of motion given



(a) $\eta(t)$ (b) $\dot{\eta}(t)$

Fig. 4. The profiles of motion primitive

as a positive constant. The position and velocity profiles of function are shown in Fig.4. For the sake of convenience, the function is called *a motion primitive*. Based on the motion primitive, a motion ($\boldsymbol{q}_r(t) = (q_{r1}(t), q_{r2}(t))^{\mathrm{T}}$) is described as

$$\begin{cases} q_{r1}(t) = m_r\eta(t) + q_{r01} \\ q_{r2}(t) = n_r\eta(t) + q_{r02} \end{cases} \tag{6}$$

where m_r and n_r denote constants for motion scales, and q_{r01} and q_{r02} denote constants for an initial pose of robot. Note that all of the motions treated in the paper follow the axioms of iterative learning: (A1) every motion ends in a fixed time duration T, (A2) a desired motion is given a priori over time duration $t \in [0, T]$ and the desired angles and the angular velocities belong to the class of $L^2[0, T]$ and $C[0, T]$, (A3) invariance of the system dynamics is ensured, (A4) a motion can be measured, (A5) the system dynamics are invertible (see the book [4]). The function in eq.(5) can be replaced with an arbitrary function of C^2 class.

C. TIME-SCALE TRANSFORMATION

The time-scale transformation (TST) allows generation of feedforward torque for a motion of specified speed by performing arithmetic operations of a few sets of time-series torque data obtained by the ILC. The TST generates a desired feedforward input without any additional learning process for change of target task though the class of generable torque is restricted to motions obtained by extending or shortening a time duration of reference motion [7].

Let us introduce another motion $\boldsymbol{q}_s \in \Re^2$ which is of the same path as $\boldsymbol{q}_r$ but of the different velocity profile from $\boldsymbol{q}_r$. Unless moving back, it satisfies

$$\boldsymbol{q}_s(r_s(t)) = \boldsymbol{q}_r(t) \tag{7}$$

where $r_s(t)$ denotes a time-scale function related to the time t of the motion (r). It also satisfies the conditions:

$$\begin{cases} \text{(i)} \quad r_s(0) = 0, \quad r_s(T) = T_s \\ \text{(ii)} \quad r_s(t) \in C^2 \quad \text{for} \;\; t \in [0, T] \\ \text{(iii)} \quad 0 < \frac{\mathrm{d}r_s(t)}{\mathrm{d}t} < \infty \quad \text{for} \;\; t \in [0, T] \end{cases} \tag{8}$$

Also, the velocity and acceleration of $\boldsymbol{q}_s(r_s(t))$ follow

$$\begin{cases} \boldsymbol{q}'_s(r_s(t)) \left(= \dfrac{\mathrm{d}\boldsymbol{q}_s(r_s(t))}{\mathrm{d}r_s(t)}\right) = \alpha_s(t)\, \dot{\boldsymbol{q}}_r(t) \\ \boldsymbol{q}''_s(r_s(t)) \left(= \dfrac{\mathrm{d}^2\boldsymbol{q}_s(r_s(t))}{\mathrm{d}r_s(t)^2}\right) = \alpha_s^2(t)\, \ddot{\boldsymbol{q}}_r(t) - \alpha_s^3(t)\beta_s(t)\, \dot{\boldsymbol{q}}_r(t) \end{cases} \tag{9}$$

where $\alpha_s(t)$ and $\beta_s(t)$ are defined by

$$\alpha_s(t) = 1\Big/\frac{\mathrm{d}r_s(t)}{\mathrm{d}t}, \quad \beta_s(t) = \frac{\mathrm{d}^2 r_s(t)}{\mathrm{d}t^2} \tag{10}$$

Then, since the feedforward torque for achieving the motion $\boldsymbol{q}_s(r_s(t))$ should satisfy

$$\boldsymbol{\tau}_s = H(\boldsymbol{q}_s)\boldsymbol{q}''_s + \left\{\frac{1}{2}H'(\boldsymbol{q}_s) + S(\boldsymbol{q}_s, \boldsymbol{q}'_s)\right\}\boldsymbol{q}'_s + B\boldsymbol{q}'_s + \boldsymbol{f}_c(\boldsymbol{q}'_s) + \boldsymbol{g}(\boldsymbol{q}_s), \tag{11}$$

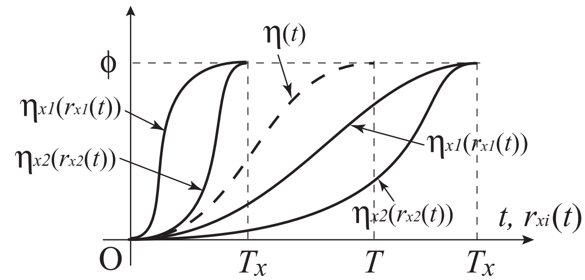


Fig. 5. The profiles of time-scale transformed motion primitive

substituting eqs.(7) and (9) into this equation yields

$$\begin{aligned} \boldsymbol{\tau}_s = {} & \alpha_s^2 H(\boldsymbol{q}_r)\ddot{\boldsymbol{q}}_r + \alpha_s^2\left\{\frac{1}{2}\dot{H}(\boldsymbol{q}_r) + S(\boldsymbol{q}_r, \dot{\boldsymbol{q}}_r)\right\}\dot{\boldsymbol{q}}_r \\ & + \alpha_s B\dot{\boldsymbol{q}}_r + \boldsymbol{f}_c(\alpha_s\dot{\boldsymbol{q}}_r) + \boldsymbol{g}(\boldsymbol{q}_r) - \alpha_s^3\beta_s H(\boldsymbol{q}_r)\dot{\boldsymbol{q}}_r \end{aligned} \tag{12}$$

It is noteworthy that the feedforward torque for achieving the motion $\boldsymbol{q}_s(r_s(t))$ can be described based on the elements of dynamics for the motion $\boldsymbol{q}_r(t)$ and the time-scale function $r_s(t)$. Thus, by referring to the dynamic property in differences of time scales, the TST allows generation of feedforward torque for an arbitrary-speed motion from feedforward torque for realizing the four-time-scale motions (exactly, three linear ones and a nonlinear one).

D. PROBLEM

Under the assumptions described above, let us consider to generate feedforward torque for achieving a motion (x):

$$\begin{cases} q_{x1}(r_{x1}(t)) = m_x\eta_{x1}(r_{x1}(t)) + q_{x01} \\ q_{x2}(r_{x2}(t)) = n_x\eta_{x2}(r_{x2}(t)) + q_{x02} \end{cases} \tag{13}$$

together with a time variable $t \in [0, T]$ for given m_x, n_x, q_{x01}, q_{x02}, $r_{x1}(t)$, and $r_{x2}(t)$. The time-scale functions ($r_{x1}(t)$ and $r_{x2}(t)$) are defined so as to satisfy the conditions in eq.(8) together with the terminal condition

$$r_{x1}(T) = r_{x2}(T) = T_x \tag{14}$$

where T_x denotes the terminal time for the motion (x). Note that the terminal times of time-scale functions must be coincident at the terminal time (see Fig.5). Also, based on the original motion primitive in eq.(5), time-scale transformed motion primitives ($\eta_{xi}(r_{xi}(t))$ for $i = 1, 2$) are defined so as to satisfy

$$\begin{cases} \eta_{xi}(r_{xi}(t)) = \eta(t) \\ \eta'_{xi}(r_{xi}(t)) \left(= \dfrac{\mathrm{d}\eta_{xi}(r_{xi}(t))}{\mathrm{d}r_{xi}(t)}\right) = \alpha_{xi}(t)\, \dot{\eta}(t) \\ \eta''_{xi}(r_{xi}(t)) \left(= \dfrac{\mathrm{d}^2\eta_{xi}(r_{xi}(t))}{\mathrm{d}r_{xi}(t)^2}\right) = \alpha_{xi}^2(t)\, \ddot{\eta}(t) - \alpha_{xi}^3(t)\beta_{xi}(t)\, \dot{\eta}(t) \end{cases} \tag{15}$$

where $\alpha_{xi}(t)$ and $\beta_{xi}(t)$ are defined by

$$\alpha_{xi}(t) = 1\Big/\frac{\mathrm{d}r_{xi}(t)}{\mathrm{d}t}, \quad \beta_{xi}(t) = \frac{\mathrm{d}^2 r_{xi}(t)}{\mathrm{d}t^2} \tag{16}$$

However, the time of each joint is mismatched in eq.(13). To adjust the mismatch, the relation between a global time variable (t_*) and local time variables (t_1 and t_2) in $r_{x1}(\cdot)$ and $r_{x2}(\cdot)$ is introduced in a definition such that

$$t_* = r_{x1}(t_1) = r_{x2}(t_2) \quad \text{for } t_* \in [0, T_x], \tag{17}$$

Then, the desired joint trajectory is represented by

$$\begin{aligned} \boldsymbol{q}_x(t_*) &= (q_{x1}(t_*), q_{x2}(t_*))^{\mathrm{T}} \\ &= (q_{x1}(r_{x1}(t_1)), q_{x2}(r_{x2}(t_2)))^{\mathrm{T}} \end{aligned} \tag{18}$$

The desired feedforward torque cannot be directly derived from eq.(4) because of unknown dynamics parameters. Even in the case, the ILC allows simultaneous acquirement of the desired motion and the desired feedforward torque after repetitions. However, an additional learning process is required to achieve another motion.

As shown in eq.(4), the time-series torque data obtained by the ILC relate to the system dynamics, though each torque datum does not directly represent the effect of each element in dynamics. Hence, on the basis of the relations between the torque data obtained by the ILC and their dynamics properties, the desired feedforward torque should be available without any additional learning processes even in the case of change of a desired motion. Now, let me consider the following proposition.

Proposition — In the case of motions of a two-joint robot arm in gravity under the assumptions described above, for given ϕ and T, a motion primitive is defined by eq.(5), and five motions are chosen adequately based on the motion primitive. Feedforward torque for achieving the motions are obtained by the ILC, and five pairs of joint trajectories and time-series toque data $((\boldsymbol{q}_a(t), \boldsymbol{\tau}_a(t)),\ \ldots,\ (\boldsymbol{q}_e(t), \boldsymbol{\tau}_e(t)))$ are prepared. Then, for a desired motion given in eq.(18) with given m_x, n_x, q_{x01}, q_{x02}, $r_{x1}(t)$, and $r_{x2}(t)$, the corresponding desired feedforward torque can be obtained from arithmetic operations of the five dataset pairs (see Fig.2).

Note that the proposition is represented in a concrete form to avoid any confusion. More general proposition is given in the latter section. For the sake of convenience, the five motions are called *basis motions*, and the torque-generation method is called *basis-motion torque composition (BMC)*. The details of algorism of the BMC are discussed in the next section.

III. BASIS-MOTION TORQUE COMPOSITION (MATHEMATICAL ANALYSIS)

In order to verify the proposition described in the previous section, the algorism of BMC is presented. By referring to eq.(4), the feedforward torque $\boldsymbol{\tau}_x(t_*) = (\tau_{x1}(t_*), \tau_{x2}(t_*))^{\mathrm{T}}$ for realizing the desired motion $\boldsymbol{q}_x(t_*)$ shown in eqs.(13) and (18) follows

$$\begin{cases} \tau_{x1}(t_*) = m_x a_{11}\eta''_{x1}(t_*) + 2m_x c_{x2}(t_*) a\eta''_{x1}(t_*) \\ \qquad + n_x c_{x2}(t_*) a\eta''_{x2}(t_*) + n_x a_{22}\eta''_{x2}(t_*) \\ \qquad - 2m_x n_x s_{x2}(t_*) a\eta'_{x1}(t_*)\eta'_{x2}(t_*) \\ \qquad - n_x^2 s_{x2}(t_*) a\eta'^2_{x2}(t_*) + m_x d_1\eta'_{x1}(t_*) \\ \qquad + \rho_1 \mathrm{sgn}(m_x\eta'_{x1}(t_*)) + c_{x1}(t_*) g_1 + c_{x12}(t_*) g_2 \\ \tau_{x2}(t_*) = m_x c_{x2}(t_*) a\eta''_{x1}(t_*) + m_x a_{22}\eta''_{x1}(t_*) \\ \qquad + n_x a_{22}\eta''_{x2}(t_*) + m_x^2 s_{x2}(t_*) a\eta'^2_{x1}(t_*) \\ \qquad + n_x d_2\eta'_{x2}(t_*) + \rho_2 \mathrm{sgn}(n_x\eta'_{x2}(t_*)) + c_{x12}(t_*) g_2 \end{cases} \tag{19}$$

where $s_{x2} = \sin q_{x2}$, $c_{x2} = \cos q_{x2}$, $c_{x1} = \cos q_{x1}$, and $c_{x12} = \cos(q_{x1} + q_{x2})$. In eq.(19), the parameters m_x, n_x, s_{x2}, c_{x2}, c_{x1}, and c_{x12} are preliminarily specified and known, but the other parameters are unknown. Now, eq.(19) can be rewritten as

$$\boldsymbol{\tau}_x(t_*) = N_x(t_*)\boldsymbol{p}_x(t_*) \tag{20}$$

where

$$N_x(t) = \left[\begin{array}{cccccccccccccc} m_x & 2m_x c_{x2} & n_x c_{x2} & 0 & n_x & -2m_x n_x s_{x2} & 0 & -n_x^2 s_{x2} & m_x & 0 & \mathrm{sgn}(m_x) & 0 & c_{x1} & c_{x12} \\ 0 & m_x c_{x2} & 0 & m_x & n_x & 0 & n_x^2 s_{x2} & 0 & 0 & n_x & 0 & \mathrm{sgn}(n_x) & 0 & c_{x12} \end{array}\right] \tag{21}$$

$$\begin{aligned} \boldsymbol{p}_x(t_*) = [&a_{11}\eta''_{x1}(t_*) \quad a\eta''_{x1}(t_*) \quad a\eta''_{x2}(t_*) \quad a_{22}\eta''_{x1}(t_*) \quad a_{22}\eta''_{x2}(t_*) \\ &a\eta'_{x1}(t_*)\eta'_{x2}(t_*) \quad a\eta''_{x1}(t_*) \quad a\eta''_{x1}(t_*) \quad d_1\eta'_{x1}(t_*) \quad d_2\eta'_{x2}(t_*) \\ &\rho_1\mathrm{sgn}(\eta'_{x1}(t_*)) \quad \rho_2\mathrm{sgn}(\eta'_{x2}(t_*)) \quad g_1 \quad g_2]^{\mathrm{T}} \end{aligned} \tag{22}$$

Note that the matrix $N_x(t_*)$ is known because they are composed of the only command parameters but the vector $\boldsymbol{p}(t_*)$ is unknown because it contains the uncertain dynamics parameters. The vector $\boldsymbol{p}(t_*)$, by substituting eqs.(15) and (17) into eq.(22), can be transformed into

$$\boldsymbol{p}_x(t_*) = \begin{bmatrix} \alpha_{x1}^2(t_1) a_{11}\ddot{\eta}(t_1) - \alpha_{x1}^3(t_1)\beta_{x1}(t_1) a_{11}\dot{\eta}(t_1) \\ \alpha_{x1}^2(t_1) a\ddot{\eta}(t_1) - \alpha_{x1}^3(t_1)\beta_{x1}(t_1) a\dot{\eta}(t_1) \\ \alpha_{x2}^2(t_2) a\ddot{\eta}(t_2) - \alpha_{x2}^3(t_2)\beta_{x2}(t_2) a\dot{\eta}(t_2) \\ \alpha_{x1}^2(t_1) a_{22}\ddot{\eta}(t_1) - \alpha_{x1}^3(t_1)\beta_{x1}(t_1) a_{22}\dot{\eta}(t_1) \\ \alpha_{x2}^2(t_2) a_{22}\ddot{\eta}(t_2) - \alpha_{x2}^3(t_2)\beta_{x2}(t_2) a_{22}\dot{\eta}(t_2) \\ \alpha_{x1}(t_1)\alpha_{x2}(t_2) a\dot{\eta}(t_1)\dot{\eta}(t_2) \\ \alpha_{x1}^2(t_1) a\dot{\eta}^2(t_1) \\ \alpha_{x2}^2(t_2) a\dot{\eta}^2(t_2) \\ \alpha_{x1}(t_1) d_1\dot{\eta}(t_1) \\ \alpha_{x2}(t_2) d_2\dot{\eta}(t_2) \\ \mathrm{sgn}(\alpha_{x1}(t_1))\rho_1\mathrm{sgn}(\dot{\eta}(t_1)) \\ \mathrm{sgn}(\alpha_{x2}(t_2))\rho_2\mathrm{sgn}(\dot{\eta}(t_2)) \\ g_1 \\ g_2 \end{bmatrix} \tag{23}$$

The expression (23) means that the unknown vector can be described on the basis of the original motion primitive $\eta(t)$ defined by eq.(5) instead of the time-scale transformed motion primitives ($\eta_{x1}(t_*)$ and $\eta_{x2}(t_*)$). Hence, if the unknown vector $\boldsymbol{p}_x(t_*)$ is derived from the dataset of basis-motion torque based on the time-scale of the original motion primitive, the desired feedforward torque $\boldsymbol{\tau}_x$ can be formed.

Now, for adequately given m_j, n_j, q_{j01}, q_{j02}, and $r_j(t)$ $(j = a, \ldots, e)$, five motions

$$\begin{cases} (a): q_{a1}(r_a(t)) = m_a\eta_a(r_a(t)) + q_{a01}, & q_{a2}(r_a(t)) = n_a\eta_b(r_a(t)) + q_{a02} \\ (b): q_{b1}(r_b(t)) = m_b\eta_b(r_b(t)) + q_{b01}, & q_{b2}(r_b(t)) = n_b\eta_b(r_b(t)) + q_{b02} \\ (c): q_{c1}(r_c(t)) = m_c\eta_c(r_c(t)) + q_{c01}, & q_{c2}(r_c(t)) = n_c\eta_c(r_c(t)) + q_{c02} \\ (d): q_{d1}(r_d(t)) = m_d\eta_d(r_d(t)) + q_{d01}, & q_{d2}(r_d(t)) = n_d\eta_d(r_d(t)) + q_{d02} \\ (e): q_{e1}(r_e(t)) = m_e\eta_e(r_e(t)) + q_{e01}, & q_{e2}(r_e(t)) = n_e\eta_e(r_e(t)) + q_{e02} \end{cases} \tag{24}$$

are chosen as basis motions. The linear time-scale functions $(r_j(t) = \gamma_j t\ (j = a, \ldots, e)$ for positive constants $\gamma_j)$ are chosen, and the time-scale transformed motion primitives $\eta_j(r_j(t))$ $(j = a, \ldots, e)$ are defined in the same manner as $\eta_{xi}(r_{xi}(t_i))$. Note that the time-scale functions of both joints are defined so as to be coincident in selection of basis motions. Then, all of the feedforward torque for realizing the basis motions are obtained by the ILC. The obtained basis-motion feedforward torque should satisfy the form of eq.(19) by replacing the subscript x in eq.(19) with $a, \ldots e$, respectively. By referring to the relations in eq.(15), the expressions can be written by

$$\begin{cases} \tau_{j1} = m_j\alpha_j^2 a_{11}\ddot{\eta} + (2m_j + n_j)\alpha_j^2 c_{j2} a\ddot{\eta} \\ \qquad + n_j\alpha_j^2 a_{22}\ddot{\eta} - (2m_j n_j + n_j^2)\alpha_j^2 s_{j2} a\dot{\eta}^2 \\ \qquad + m_j\alpha_j d_1\dot{\eta} + \mathrm{sgn}(m_j\alpha_j)\rho_1\mathrm{sgn}(\dot{\eta}) \\ \tau_{j2} = m_j\alpha_j^2 c_{j2} a\ddot{\eta} + (m_j + n_j)\alpha_j^2 a_{22}\ddot{\eta} \\ \qquad + m_j^2\alpha_j^2 s_{j2} a\dot{\eta}^2 + n_j\alpha_j d_2\dot{\eta} + \mathrm{sgn}(n_j\alpha_j)\rho_2\mathrm{sgn}(\dot{\eta}) \end{cases} \tag{25}$$

Note that the terms including β_j $(j = a, \ldots, e)$ disappear in these expressions (*i.e.*, $\beta_j = 0$ $(j = a, \ldots, e)$) because the linear time-scale functions are chosen. Hence, each basis-motion feedforward torque in eq.(25) can be rewritten in the form of the linear equation of elements of $\boldsymbol{p}(t)$:

$$\boldsymbol{\tau}_j = N_j(t)\boldsymbol{p}(t) \tag{26}$$

where

$$N_j(t) = \left[\begin{array}{cccccccccc} m_j\alpha_j^2 & (2m_j + n_j)\alpha_j^2 c_{j2} & n_j\alpha_j^2 & -(2m_j n_j + n_j^2)\alpha_j^2 s_{j2} & m_j\alpha_j & 0 & \mathrm{sgn}(m_j\alpha_j) & 0 & c_{j1} & c_{j12} \\ 0 & m_j\alpha_j^2 c_{j2} & (m_j + n_j)\alpha_j^2 & m_j^2\alpha_j^2 s_{j2} & 0 & n_j\alpha_j & 0 & \mathrm{sgn}(n_j\alpha_j) & 0 & c_{j12} \end{array}\right] \tag{27}$$

$$\boldsymbol{p}(t) = \begin{bmatrix} a_{11}\ddot{\eta} & a\ddot{\eta} & a_{22}\ddot{\eta} & a\dot{\eta}^2 & d_1\dot{\eta} & d_2\dot{\eta} & \rho_1\mathrm{sgn}(\dot{\eta}) & \rho_2\mathrm{sgn}(\dot{\eta}) & g_1 & g_2 \end{bmatrix}^{\mathrm{T}} \tag{28}$$

Thus, over the time duration $t \in [0, T]$, all of the basis-motion torque can be described in the form

$$\boldsymbol{\tau}_{BM}(t) = B(t)\boldsymbol{p}(t) \tag{29}$$

where

$$\boldsymbol{\tau}_{BM}(t) = \begin{bmatrix} \boldsymbol{\tau}_a(t) \\ \boldsymbol{\tau}_b(t) \\ \boldsymbol{\tau}_c(t) \\ \boldsymbol{\tau}_d(t) \\ \boldsymbol{\tau}_e(t) \end{bmatrix}, \quad B(t) = \begin{bmatrix} N_a(t) \\ N_b(t) \\ N_c(t) \\ N_d(t) \\ N_e(t) \end{bmatrix}, \tag{30}$$

and $N_a(t), \ldots, N_e(t) \in \Re^{2\times 10}$ are defined in eq.(27), respectively. In eq.(29), the vector $\boldsymbol{p}(t)$ is unknown because the elements of $\boldsymbol{p}(t)$ include the unknown dynamics parameters. If the basis motions are adequately chosen so that the matrix $B(t)$ does not degenerate over $[0, T]$, then the inverse of $B(t)$ can be derived. Multiplying eq.(29) by the inverse of $B(t)$ from the left-hand, we obtain

$$\boldsymbol{p}(t) = B^{-1}(t)\boldsymbol{\tau}_{BM}(t) \tag{31}$$

Thus, the unknown vector $\boldsymbol{p}(t)$ can be derived from the basis-motion torque. Note that the matrix $B(t)$ is known because it is composed of the only command parameters for the basis motions. Furthermore, when the elements of the derived vector $\boldsymbol{p}(t)$ are described by $\boldsymbol{p}(t) = (p_1(t), \ldots, p_{10}(t))$, the unknown vector $\boldsymbol{p}_x(t_*)$ in eq.(23) can be also derived from the result of eq.(31):

TABLE I

PARAMETER SETTINGS FOR THE BASIS MOTIONS

(i)	$r_i(t)$	m_i	n_i	q_{i01} [deg]	q_{i02} [deg]
Motion (a)	t	0.8	1.1	0.0	5.0
Motion (b)	t	0.8	0.8	20.0	25.0
Motion (c)	t	1.0	1.0	5.0	-15.0
Motion (d)	$0.8t$	0.7	0.7	-30.0	20.0
Motion (e)	$0.9t$	0.8	0.8	15.0	15.0

$$\begin{aligned} \boldsymbol{p}_x(t_*) &= \begin{bmatrix} \alpha_{x1}^2 p_1(t_1) - \alpha_{x1}^3\beta_{x1}\int_0^{t_1} p_1(\xi)\mathrm{d}\xi \\ \alpha_{x1}^2 p_2(t_1) - \alpha_{x1}^3\beta_{x1}\int_0^{t_1} p_2(\xi)\mathrm{d}\xi \\ \alpha_{x2}^2 p_2(t_2) - \alpha_{x2}^3\beta_{x2}\int_0^{t_2} p_2(\xi)\mathrm{d}\xi \\ \alpha_{x1}^2 p_3(t_1) - \alpha_{x1}^3\beta_{x1}\int_0^{t_1} p_3(\xi)\mathrm{d}\xi \\ \alpha_{x2}^2 p_3(t_2) - \alpha_{x2}^3\beta_{x2}\int_0^{t_2} p_3(\xi)\mathrm{d}\xi \\ \alpha_{x1}\alpha_{x2}\sqrt{p_4(t_1)p_4(t_2)} \\ \alpha_{x1}^2 p_4(t_1) \\ \alpha_{x2}^2 p_4(t_2) \\ \alpha_{x1} p_5(t_1) \\ \alpha_{x2} p_6(t_2) \\ \mathrm{sgn}(\alpha_{x1}) p_7(t_1) \\ \mathrm{sgn}(\alpha_{x2}) p_8(t_2) \\ p_9(t_1) \\ p_{10}(t_2) \end{bmatrix} \\ &= \boldsymbol{f}(\boldsymbol{p}(t_1), \boldsymbol{p}(t_2)) \end{aligned} \tag{32}$$

where the following relations were used:

$$\begin{cases} a_{11}\dot{\eta}(t_1) = \displaystyle\int_0^{t_1} a_{11}\ddot{\eta}(\xi)\mathrm{d}\xi \\ a\dot{\eta}(t_1)\dot{\eta}(t_2) = \sqrt{a\dot{\eta}^2(t_1)\cdot a\dot{\eta}^2(t_2)} \end{cases} \tag{33}$$

The bottom relation in eq.(33) is the key in the improved BMC which allows generation of a motion with different velocity profiles between joints. Thus, the unknown vector $\boldsymbol{p}_x(t_*)$ in eq.(20) was derived on the basis of the basis-motion torque. Then, by substituting eqs.(17), (31) and (32) into eq.(20), the desired feedforward torque can be obtained as follows:

$$\boldsymbol{\tau}_x(t_*) = N_x(t_*)\boldsymbol{f}(B^{-1}(t_1)\boldsymbol{\tau}_{BM}, B^{-1}(t_2)\boldsymbol{\tau}_{BM}) \tag{34}$$

Note that N_x is the known matrix composed of the desired-motion parameters and $B(t)$ is the known matrix composed of the basis-motion parameters.

Consequently, it is concluded that if the basis motions are chosen adequately so that $B(t)$ does not degenerate over the time duration $t \in [0, T]$ then the feedforward torque for achieving the desired motion can be derived from the basis-motion dataset pairs (torque and joint trajectories) and the desired-motion joint trajectory as shown in eq.(34).

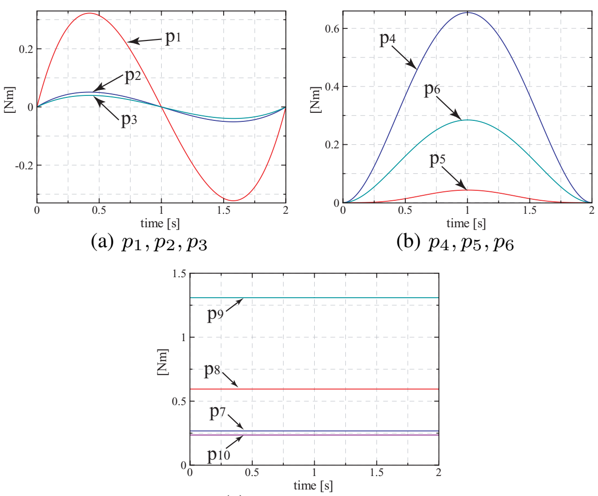
IV. NUMERICAL SIMULATIONS

In order to confirm the effectiveness of enhanced BMC, we conducted numerical simulations. Motions of a two-joint robot arm in gravity as shown in Fig.3 were considered. The motion primitive of eq.(5) was set with $\phi = 80$[deg] and $T = 2.0$[s], and the five basis motions of eq.(24) were determined by the parameters in TABLE I. Firstly, under this condition, all of the feedforward torque for achieving the basis motions were obtained in advance by the ILC. Secondly, applying the

TABLE II

PARAMETER SETTINGS FOR THE DESIRED MOTIONS

	$r_{x1}(t_1)$ / $r_{x2}(t_2)$	m / n	q_{x01} / q_{x02}
Case 1	$0.275t_1^3 - 0.475t_1^2 + 1.1t_1$	0.8	10.0 [deg]
	$1.25t_2$	1.2	-10.0 [deg]
Case 2	$1.25t_1$	0.8	10.0 [deg]
	$0.275t_2^3 - 1.175t_2^2 + 2.5t_2$	1.2	-10.0 [deg]
Case 3	$0.125t_1^3 - 0.125t_1^2 + 0.5t_1$	0.8	10.0 [deg]
	$0.125t_2^3 - 0.625t_2^2 + 1.5t_2$	1.2	-10.0 [deg]
Case 4	$0.20t_1^3 - 0.35t_1^2 + 1.0t_1$	-0.9	60.0 [deg]
	$0.20t_2^3 - 0.85t_2^2 + 2.0t_2$	-1.1	60.0 [deg]
Case 5	$0.20t_1^3 - 0.35t_1^2 + 1.0t_1$	-0.5	40.0 [deg]
	$0.20t_2^3 - 0.85t_2^2 + 2.0t_2$	-0.6	30.0 [deg]



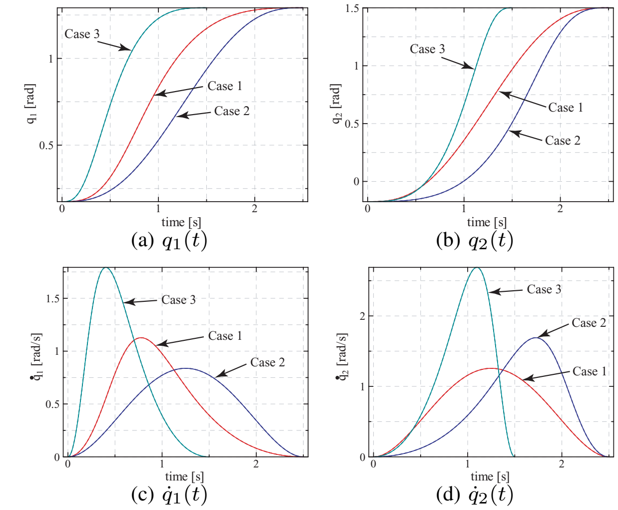
(a) p_1, p_2, p_3 (b) p_4, p_5, p_6 (c) p_7, p_8, p_9, p_{10}

Fig. 6. The vector $\boldsymbol{p}(t)$ derived from the basis-motion torque

enhanced BMC, we attempted to generate five test motions shown in TABLE II. The desired feedforward torque was derived from eq.(34) off-line, and the derived torque was applied to the input $\boldsymbol{u}$ in eq.(1). Then, the solution trajectories (the joint trajectories) of eq.(1) were obtained on the base of the Runge-Kutta scheme. Figure 6 depicts the vector $\boldsymbol{p}(t)$ derived by the enhanced BMC. The figure demonstrates that $\boldsymbol{p}(t)$ was derived adequately without any degeneration of the matrix $B(t)$ over the time interval $t \in [0, T]$.

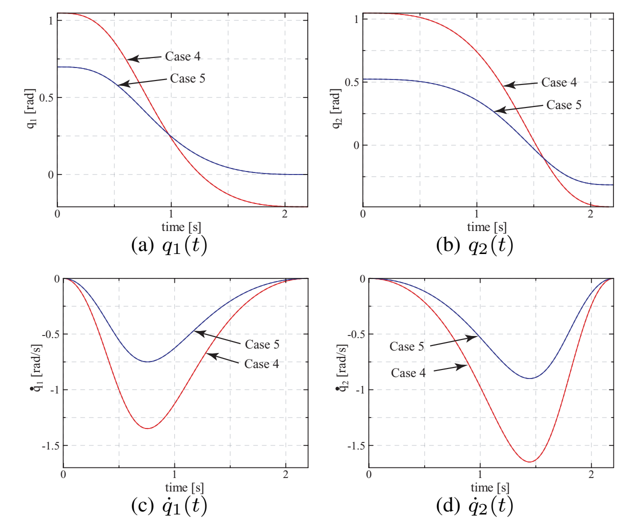
Figure 7 depicts the transient responses of joint angles and angular velocities derived from eq.(1) in Cases 1, 2, and 3 where the motion-scale parameters (m_x, n_x) were fixed but the different time-scale functions ($r_{x1}(t_1)$ and $r_{x2}(t_2)$) were chosen. The terminal time in Cases 1 and 2 is $T_x = 2.5$[s] and that in Case 3 is $T_x = 1.5$[s]. As shown in Fig.7, the trajectories of both angles and angular velocities are completely coincident with the corresponding desired trajectories even in the case of the different time-scale functions.

Figure 8 depicts the joint-angle profiles and the joint-angular-velocity profiles in Cases 4 and 5 where the time-scale function was fixed but the different motion-scale parameters and the different initial poses were set ($T_x = 1.7$[s] in both cases). In the cases, the negative motion-scale



(a) $q_1(t)$ (b) $q_2(t)$ (c) $\dot{q}_1(t)$ (d) $\dot{q}_2(t)$

Fig. 7. Simulation results in Cases 1, 2, and 3 (in the case of the different time-scale functions)



(a) $q_1(t)$ (b) $q_2(t)$ (c) $\dot{q}_1(t)$ (d) $\dot{q}_2(t)$

Fig. 8. Simulation results in Cases 4 and 5 (in the case of the different motion-scale parameters (m_x and n_x) and the different initial position (q_{x01} and q_{x02}))

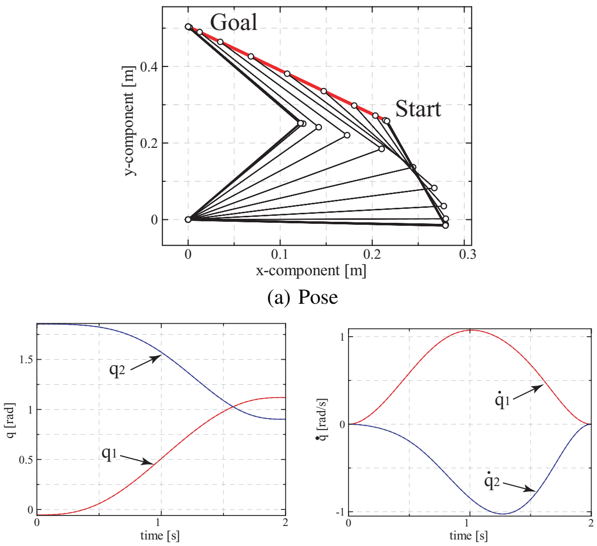
parameters were set, that is, the motion directions of desired tasks are opposite to those of basis motions. Regardless of the conditions, the derived feedforward torque realized the desired motions.

The simulation results demonstrate that the enhanced BMC achieves the desired motions even in the case of different velocity profiles between joints. The generable motions of BMC approach are increased dramatically. The enhanced BMC also allows generation of a motion such that a robot tracks a desired endpoint trajectory given in task space. Figure 9 depicts transient responses of arm poses, joint angles, and angular velocities when the robot tracked a straight endpoint trajectory given by

$$\boldsymbol{x}_d(t) = \xi(t)(\boldsymbol{x}_g - \boldsymbol{x}_s) + \boldsymbol{x}_s \tag{35}$$

where $\boldsymbol{x}_g$ and $\boldsymbol{x}_s \in \Re^2$ are constant vectors and

$$\xi(t) = \left[6\left(\frac{t}{T}\right)^5 - 15\left(\frac{t}{T}\right)^4 + 10\left(\frac{t}{T}\right)^3 \right] \tag{36}$$



(a) Pose

(b) $\boldsymbol{q}(t)$ (c) $\dot{\boldsymbol{q}}(t)$

Fig. 9. Simulation results in the case that the desired trajectory is given in task space

The parameters in eqs.(35) and (36) were set with $T = 2.0[\mathrm{s}]$, $\boldsymbol{x}_s = [0.216[\mathrm{m}], 0.257[\mathrm{m}]]^{\mathrm{T}}$, and $\boldsymbol{x}_g = [0.000[\mathrm{m}], 0.504[\mathrm{m}]]^{\mathrm{T}}$. Firstly, the desired endpoint trajectory was transformed into the joint trajectory by solving the inverse kinematics. Next, the time-scale variables α_{x1}, α_{x2}, β_{x1}, and β_{x2} were calculated from the desired joint trajectory by referring to eq.(15). Then, the desired feedforward torque was derived from eq.(34), and the motion in the case was applied. The derived motion was completely coincident with the desired motion.

V. APPLICABLE SYSTEMS AND SITUATIONS

The effectiveness of the enhanced BMC was confirmed in the only case of the two-joint robot arm under the gravity. Furthermore, the proposition was given for the special case. However, the class of applicable systems and situations of the enhanced BMC is not restricted to the special case because the algorism of the enhanced BMC does not depend on the specific robot. Even if spatial motions of a six-joint robot arm are considered, the enhanced BMC can be applied to the case in the same manner by assuming an adequate dynamics structure for the robot and by choosing adequate basis motions so that $B(t)$ in eq.(29) does not degenerate over a time duration. The number of basis motions depends on the number of unknown elements in the vector $\boldsymbol{p}_x(t_*)$ in eq.(20). In the case of Fig.3, since the number of unknown elements in $\boldsymbol{p}_x(t_*)$ is ten, five basis motions (ten sets of torque data) are required at least. From the same sense, the enhanced BMC allows generation of desired motions even in different situations like in water if an adequate dynamics structure is assumed.

Thus, the proposition in section 2 can be generalized naturally as follows:

Proposition — In the case of motions of a multi-joint robot under the assumptions in section 2, for given ϕ and T, a motion primitive is defined by eq.(5), and some basis motions are chosen adequately based on the motion primitive. Feedforward torque for the basis motions are obtained by the ILC, and some pairs of joint trajectories and time-series toque data $((\boldsymbol{q}_a(t), \boldsymbol{\tau}_a(t)), \ldots, (\boldsymbol{q}_j(t), \boldsymbol{\tau}_j(t)))$ are prepared. Then, for given a desired motion like eq.(18) with motion command parameters, the corresponding desired feedforward torque can be obtained from arithmetic operations of the dataset pairs.

VI. CONCLUSIONS

The class of generable torque of BMC was extended so that it could generate motions with different velocity profiles among joints, like motions with straight endpoint trajectories. Furthermore, the class of applicable systems and situations was discussed. If a dynamics structure is assumed adequately and adequate basis motions are chosen, the enhanced BMC can be applied to various systems or situations.

The enhanced BMC does not require priori information on dynamics parameters. It requires only the assumption of dynamics structure differently from the CTM. It may be considered that the assumption incurs a disadvantage. However, the assumption of dynamics structure contributes to save of the number of datasets to be stored. It makes implementation of the enhanced BMC into an actual robot easy. The experimental verification was not carried out but the effectiveness is presumable from the previous experimental results [8], [9].

The class of generable torque of the enhanced BMC is still restricted. That is, it cannot generate a motion such that the endpoint draws a circle in work space. How to achieve such a motion is discussed in future work.

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