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Path following of a vehicle-trailer system in presence of sliding: Application to automatic guidance of a towed agricultural implement

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***Abstract*— This paper addresses the problem of sliding parameter estimation and lateral control of an off-road vehicle-trailer system. The aim is to accurately guide the position of the trailer with respect to a planned trajectory, whatever ground conditions and trajectory shape. Relevant sliding parameter estimation is first proposed, based on the kinematic model of the system extended with side slip angles. Then, a vehicle steering control algorithm is presented to move away the vehicle from the reference trajectory in order for the trailer to achieve accurate path tracking. Reported experiments demonstrate the capabilities of the proposed algorithms.**

I. INTRODUCTION

For many years, researchers and manufacturers have widely pointed out the benefits of developing automatic guidance systems for agricultural vehicles, in particular to improve field efficiency while releasing human operator from monotonous and dangerous operations. Auto-steering systems are becoming common place (*e.g. Agco AutoGuide, Agrocom E-drive, Autofarm AutoSteer, Case IH AccuGuide, John-Deere AutoTrac, New-Holland IntelliSteer*) and focus on accurately guide the vehicle along parallel tracks in the field. However, more advanced functionalities are today required, in particular accurate guidance of towed implements. In fact, these implements are attached at one hitch point on a drawbar at the rear of the tractor, and various effects can force them to shift away from the vehicle's path (*e.g. ground conditions, uneven load, curve, slope*) leading to unsatisfactory implement position and inaccurate agricultural work, as highlighted in figure 1. The trend toward increasingly long and heavy implements increases moreover this problem.

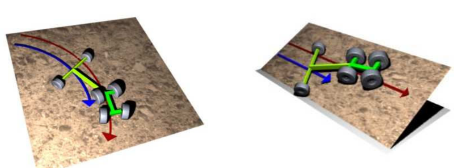

Fig. 1. Behaviour of a passive towed implement in curve and slope

Very few approaches have been proposed to counteract such a behaviour. They can be classified into two categories.

The first one uses active solutions, either with some additional steering disks mounted on the back of the implement acting as a rudder to keep the implement on the tractor's track, or with active drawbars, see for example [1], [4] and the commercial guidance systems named *AutoFarm AFTracker*, *Sunco Acura Trak* and *Sukup Slide Guide*. Such solutions are however quite expensive, as they require large actuation forces to shift the implement, and are fitted to a limited class of implement.

The second category is interested in solutions moving away the vehicle from the reference path in order to keep the implement on this objective path. Such approaches are attractive because any towed implement can be considered, even those presenting no active steering capability. However, the control of a tractor-implement combination is a challenging problem, all the more in off-road conditions. In fact, numerous dynamic phenomena, often disregarded in classical control design, may lead to large path tracking errors or unexpected oscillations. In particular, as pointed out in [14], if the control algorithms are designed from pure rolling without sliding assumptions, the accuracy of path following may be seriously damaged, especially in curves and in slippery slopes.

The solutions proposed in the literature to control vehicle-trailer systems have often neglected sliding phenomena, see for example [7], [10], [13]. Among the few approaches accounting for sliding, [3] is interested in accurate maneuvers of a vehicle-trailer system in headland, but the position of the trailer is not specifically controlled. [2] and [5] aim at controlling a towed agricultural implement using GPS antennas on both the tractor and the implement. However, a vehicle dynamic model is considered, and if the parameter values of such model upon which the control algorithm is based are not well-estimated, the overall performances may then be decreased. These parameters are often difficult to obtain through preliminary experimental identification, and their on-line adaptation, in order to reflect changing operating conditions (varying loads, ground conditions, implement used, tire configurations) is even more challenging. [6] and the commercial system named *John-Deere iGuide* have proposed to correct a constant implement drift, using also GPS antennas on both the tractor and the implement. No detail with respect to vehicle modeling and control algorithms has been supplied. Such guidance systems are nevertheless quite expensive and only efficient once the tractor-implement combination is in steady-state conditions.

In this paper, we propose first an alternative approach to describe and estimate relevant sliding parameters of a vehicle-trailer system, relying on observer theory. Theses parameters are next used to feed an adaptive control algorithm ensuring that the trailer achieves accurate path following. Capabilities of the proposed algorithms are finally investigated through full-scale experiments.

II. Kinematic model extended with sliding parameters

Dynamic models have numerous parameters (masses, wheel-ground contact conditions, cornering stiffnesses, etc) most often badly known and very difficult to derive through experimental identification as they are depending on the interaction with the environment. Online identification of soil properties is in particular quite hard to perform, all the more in off-road conditions. In previous work [8], a classical kinematic model extended with sliding parameters has been proposed to describe a two-wheel steering vehicle moving off-road. This work is here extended, considering an additional trailer hitched up at some distance from the center of the vehicle rear axle (i.e. the general 1-trailer system). Consequently, each two front and rear wheels of the vehicle and the two wheels of the trailer are first considered equivalent to three virtual wheels located at mid-distance between the actual ones, as depicted in figure 2. Then, in order to account for sliding phenomena, three additional parameters - homogeneous with side slip angles in a dynamic model - are added to the classical representation. These three angles, denoted respectively β_F, β_R and β_T, represent the difference between the theoretical direction of the linear velocity vector at wheel centers, described by the wheel plane, and their actual direction. These angles are assumed to be entirely representative of the sliding influence on the dynamics of the vehicle-trailer system. The notations used in this paper are listed below and depicted in figure 2.

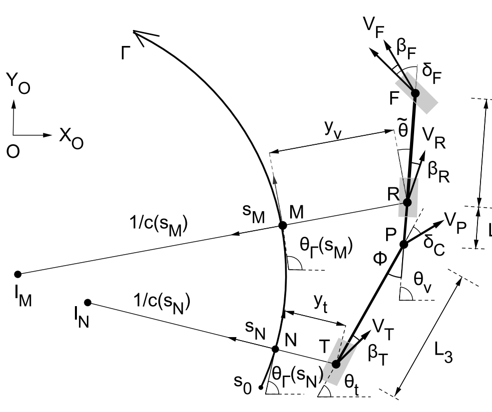

Fig. 2. Parameters of the vehicle-trailer system

- F, R and T are respectively the centers of the vehicle front and rear virtual wheels, and the center of the trailer virtual wheel. P is the hitch point.
- L_1 and L_3 are the vehicle and trailer wheelbases. L_2 is the vehicle tow-hitch.
- θ_v and θ_t are the orientation of the vehicle and trailer centerlines with respect to an absolute frame $[O, X_O, Y_O)$.
- δ_F is the front steering angle. It constitutes the first control variable.
- V_R is the vehicle linear velocity at point R, and constitutes the second control variable.
- β_F, β_R and β_T are respectively the vehicle front and rear side slip angles, and the trailer side slip angle.
- M and N are the points on the reference path Γ which are respectively the closest to R and T.
- s_M and s_N are the curvilinear abscissas of points M and N along Γ.
- $c(s_M)$ and $c(s_N)$ are the curvatures of path Γ at points M and N.
- $\theta_\Gamma(s_M)$ and $\theta_\Gamma(s_N)$ are the orientations of the tangent to Γ at points M and N with respect to the absolute frame $[O, X_O, Y_O)$.
- $\tilde{\theta} = \theta_v - \theta_\Gamma(s_M)$ is the vehicle angular deviation with respect to Γ.
- $\tilde{\theta}_t = \theta_t - \theta_\Gamma(s_N)$ is the trailer angular deviation with respect to Γ.
- y_v and y_t are respectively the vehicle and trailer lateral deviations at points R and T with respect to Γ.
- ϕ is the vehicle-trailer angle.
- δ_C is the angle between the trailer centerline and the velocity vector orientation at point P.

The kinematic model of the vehicle-trailer combination extended with the sliding parameters can be established with respect to the path Γ:

$$\begin{cases} \dot{s}_M &= V_R \frac{\cos(\tilde{\theta}-\beta_R)}{1-c(s_M)\, y_v} \\ \dot{y}_v &= V_R \sin(\tilde{\theta}-\beta_R) \\ \dot{\tilde{\theta}} &= V_R \left[\lambda_1 \cos\beta_R - \lambda_2\right] \\ \dot{\phi} &= -\frac{V_R}{L_1 L_3}\left[\lambda_3 \tan(\delta_F - \beta_F) + \lambda_4\right] \end{cases} \quad (1)$$

with:

$$\lambda_1 = \frac{\tan(\delta_F-\beta_F)+\tan\beta_R}{L_1} \;;\; \lambda_2 = \frac{c(s_M)\cos(\tilde{\theta}-\beta_R)}{1-c(s_M)\, y_v}$$
$$\lambda_3 = L_3\cos\beta_R + \frac{cos\beta_R}{cos\beta_T} L_2\cos(\phi-\beta_T)$$
$$\lambda_4 = L_3\sin\beta_R + \frac{cos\beta_R}{cos\beta_T}\left[L_1\sin(\phi-\beta_T) + (L_1+L_2)\cos(\phi-\beta_T)\tan\beta_R\right]$$

The first three equations have been established in [8]. The last equation is obtained by using the equiprojectivity property of velocity vectors and then $\dot{\phi} = \dot{\theta}_t - \dot{\theta}_v$.

Model (1) accurately describes the motion of the vehicle-trailer system in presence of sliding as soon as the three additional parameters β_F, β_R and β_T are known. Therefore, the estimation of these three variables appears to be of crucial importance.

III. Sliding parameter estimation

As pointed out for example in [12], the direct measurement of side slip angles appears to be hardly feasible at a reasonable cost. It then appears necessary to apply indirect estimation. Nevertheless, the variability of the soil conditions encountered in an agricultural environment does not permit to apply classical observer theory, as used in on-road context.

A first solution consists in reporting the numerical derivation of the measured lateral and angular deviations and the one of the vehicle-trailer angle into the last three equations in model (1). A system of three equations with three unknowns is then obtained, and sliding parameters can therefore be directly calculated. However, the numerical derivation of measured signals may lead to a noisy sliding estimation. To improve it, an alternative solution consists in developing an observer. It is here proposed to generalize the observer designed in previous work [8] for a single vehicle, to the vehicle-trailer system addressed in this paper. Such an observer is based on the duality between observation and control: β_F, β_R and β_T are considered as control variables to be designed in order to ensure the convergence of the extended model outputs $X_{obs} = (y_v, \tilde{\theta}, \phi)_{obs}$ with the measured variables $X_{mes} = (y_v, \tilde{\theta}, \phi)_{mes}$, as depicted in the scheme in figure 3.

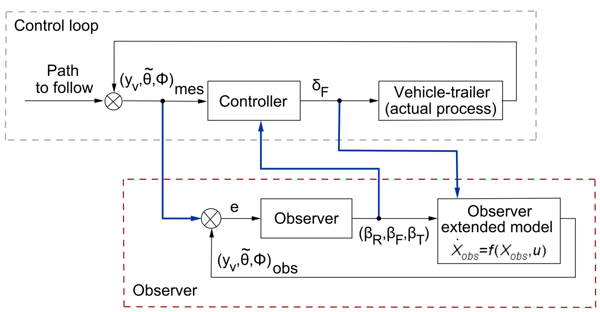

Fig. 3. Observer principle

More precisely, the extended model (1) can be rewritten, without curvilinear abscissa evolution, as the non-linear state representation (2), with δ_F considered as a known parameter, and $u = (u_1, u_2, u_3) = -(\beta_R, \beta_F, \beta_T)$ the variables to be controlled:

$$\dot{X}_{obs} = f(X_{obs}, \delta_F, u) \tag{2}$$

As side slip angles do not exceed few degrees in practice, this state equation can be linearized with respect to the control vector u in the vicinity of zero (i.e. no sliding). This leads to:

$$\dot{X}_{obs} \quad = \quad f(X_{obs}, \delta_F, 0) + B(X_{obs}, \delta_F)u \tag{3}$$

with B denoting the derivative of f with respect to u evaluated at $u = (0, 0, 0)$.

$$B(X_{obs}, \delta) = \begin{bmatrix} V_R \cos\tilde{\theta}_{obs} & 0 & 0 \\ b_1 - \frac{V_R}{L_1} & b_2 & 0 \\ \frac{V_R}{L_1} + \frac{V_R \cos\phi_{obs}}{L_3} + b_3 & -b_2 + b_4 & b_5 \end{bmatrix} \tag{4}$$

with:

$b_1 = \frac{V_R c(s_M) \sin\tilde{\theta}_{obs}}{1 - c(s_M) y_{v\,obs}}$; $b_2 = \frac{V_R}{L_1 \cos^2 \delta_F}$

$b_3 = \frac{V_R L_2 \cos\phi_{obs}}{L_1 L_3}$; $b_4 = -\frac{V_R L_2 \cos\phi_{obs}}{L_1 L_3 \cos^2 \delta_F}$

$b_5 = -\frac{V_R}{L_3}(\cos\phi_{obs} - \frac{L_2 \sin\phi_{obs} \tan\delta_F}{L_1})$

The matrix B is invertible when $\tilde{\theta}_{obs} \neq \frac{\pi}{2}[\pi]$, $V_R \neq 0$, and $b_5 \neq 0$, i.e. $\frac{L_2}{L_1} \tan\phi_{obs} \tan\delta_F \neq 1$. The two first conditions are always met in practical path following applications and the last one is also satisfied, in view of experimental system features $L_1 = 1.2m$, $L_2 = 0.46m$, $\delta_{Fmax} = 25°$ and $\phi_{max} = 65°$.

Relying on exact linearization of equation (3), the convergence of the observation error $e = X_{obs} - X_{mes}$ can be obtained by imposing:

$$u \quad = \quad B(X_{obs}, \delta_F)^{-1} \left\{ G.e - f(X_{obs}, \delta_F, 0) + \dot{X}_{mes} \right\} \tag{5}$$

as it leads to the error dynamic $\dot{e} = G \cdot e$.

G has to be chosen as an Hurwitz matrix and constitutes the observer gain, defining the settling times for the observed states. As a result, the convergence of the observed state with the measured one ensures that u is a relevant estimation of side slip angles $(\beta_R, \beta_F, \beta_T)$, which can then be injected into the control loop, see figure 3. Since all the variables of model (1) are known, control law design can be considered.

IV. Control Law Design

The control objective is to guarantee that the trailer accurately follows the reference path, i.e. that y_t converges with 0, see figure 2. To meet this objective, control design has been divided into three steps: first, the trailer is considered as an independent virtual vehicle, with a virtual front steering wheel located in P. A control law is then designed for the virtual front steering angle δ_C in order that y_t converges with 0. Next, the vehicle-trailer angle ϕ_{ref} that would lead to such a value for δ_C is inferred. Finally, the vehicle front steering angle δ_F is designed in order to impose that the actual vehicle-trailer angle ϕ converges with ϕ_{ref}.

A. Trailer as a virtual vehicle

The trailer is first considered as an independent virtual vehicle, with a fixed rear-wheel located at point T and a virtual front steering wheel located at the hitch point P. The aim is then to calculate the direction δ_C of the linear velocity vector $\vec{V}_P$ that would ensure the convergence of this virtual vehicle to the reference path Γ. A model for this virtual vehicle can easily be inferred from the first three equations in model (1):

$$\begin{cases} \dot{s}_N & = & V_T \frac{\cos(\tilde{\theta}_t - \beta_T)}{1 - c(s_N)\, y_t} \\ \dot{y}_t & = & V_T \sin(\tilde{\theta}_t - \beta_T) \\ \dot{\tilde{\theta}}_t & = & V_T \left[\bar{\lambda}_1 \cos\beta_T - \bar{\lambda}_2\right] \end{cases} \tag{6}$$

with: $\bar{\lambda}_1 = \frac{\tan\delta_C + \tan\beta_T}{L_3}$; $\bar{\lambda}_2 = \frac{c(s_N)\cos(\tilde{\theta}_t - \beta_T)}{1 - c(s_N)\, y_t}$

Model (6) can then be converted in an exact way into linear equations, see [9] and [11], according to the following state and control transformations:

$$\begin{aligned} &[s_N, y_t, \tilde{\theta}_t] \rightarrow [a_1, a_2, a_3] = \\ &\qquad [s_N, y_t, (1 - c(s_N)\, y_t) \tan(\tilde{\theta}_t - \beta_T)] \\ &[V_T, \delta_C] \rightarrow [m_1, m_2] = [\tfrac{V_T \cos(\tilde{\theta}_t - \beta_T)}{1 - c(s_N)\, y_t}, \tfrac{da_3}{dt}] \end{aligned} \tag{7}$$

Finally, if derivatives are expressed with respect to the curvilinear abscissa, the following chained form is obtained:

$$\begin{cases} a_2' &= \frac{da_2}{da_1} &= a_3 \\ a_3' &= \frac{da_3}{da_1} &= m_3 = \frac{m_2}{m_1} \end{cases} \qquad (8)$$

Since chained form (8) is a linear model, a natural expression for the virtual control law m_3 is:

$$m_3 = -K_d a_3 - K_p a_2 \qquad (K_p, K_d) \in \Re^{+2} \qquad (9)$$

since it leads to:

$$a_2'' + K_d a_2' + K_p a_2 = 0 \qquad (10)$$

which implies that both a_2 and a_3 converge with zero, i.e. $y_t \to 0$ and $\tilde{\theta}_t \to \beta_T$. The gains (K_d, K_p) impose a settling distance. The inversion of control transformations provides then the expected direction δ_C of the linear velocity $\vec{V}_P$:

$$\delta_C = \arctan\left\{\frac{L_3}{\cos\beta_T}\left(\frac{c(s_N)\cos\tilde{\theta}_2}{\alpha} + \frac{A\cos^3\tilde{\theta}_2}{\alpha^2}\right) - \tan\beta_T\right\} \qquad (11)$$

$$\text{with: } \begin{cases} \tilde{\theta}_2 &= \tilde{\theta}_t - \beta_T \\ A &= -K_p\, y_t - K_d\, \alpha \tan\tilde{\theta}_2 + c(s_N)\alpha \tan^2\tilde{\theta}_2 \\ \alpha &= 1 - c(s_N) y_t \end{cases}$$

B. Reference vehicle-trailer angle ϕ_{ref}

The next step consists in computing the vehicle-trailer angle ϕ_{ref} leading to such a velocity vector $\vec{V}_P$ at P. For that, the steady state of the vehicle-trailer system, i.e when the instantaneous center of rotation (ICR) of the trailer coincides with the vehicle ICR see figure 4, is considered. The ICR may be at infinity in the case of a straight line motion.

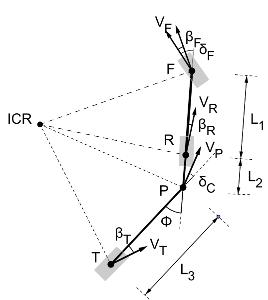

Fig. 4. Coincidence of the instantaneous centers of rotation

The relation (12) between ϕ_{ref} and δ_C in this steady state configuration can easily be inferred from basic geometrical considerations. This relation is identical, whether the ICR is at infinity or not.

$$\phi_{ref} = \delta_c + \beta_R + \arcsin\frac{L_2\cos\beta_R\sin(\delta_c + \beta_T)}{L_3\cos\beta_T} \qquad (12)$$

C. Front-wheel steering control law

Finally, the vehicle-trailer angle ϕ is stabilized on ϕ_{ref} relying on the fourth equation in model (1): the error dynamic $\dot{\phi} = K_R(\phi_{ref} - \phi)$ (with $K_R > 0$) can be imposed with the following front-wheel steering control law:

$$\delta_F = \beta_F + \arctan\frac{-\frac{L_1 L_3 K_R(\phi_{ref}-\phi)}{V_r} - \lambda_4}{\lambda_3} \qquad (13)$$

The stability of the whole non-linear control strategy, composed of control laws (11) and (13), can be checked using Lyapunov theory. Consider Lyapunov function candidate $V = \frac{1}{2}(K_p y_t^2 + (\alpha\tan\tilde{\theta}_2)^2 + \epsilon^2)$ with $\epsilon = \phi_{ref} - \phi$. The derivative of the positive function V with respect to curvilinear abscissa leads to $\frac{dV}{ds} = -K_d\alpha^2\tan^2\tilde{\theta}_2 - K_R\epsilon^2$ which is always negative. The stability of the control strategy is then ensured.

V. EXPERIMENTAL RESULTS

In this section, the capabilities of the proposed algorithms are investigated using the vehicle-trailer system depicted in figure 5. The vehicle is an all-terrain mobile robot whose weight is 650kg, wheelbase $L_1 = 1.2m$, and tow-hitch $L_2 = 0.46m$. The only exteroceptive sensor is an RTK-GPS receiver, whose antenna has been located straight up above the center R of the rear axle. The GPS antenna is usually fixed at this point on farm vehicles, in order to maximize satellites visibility and to avoid antenna displacement when the implement is changed. The RTK-GPS receiver supplies an absolute position accurate to within 2cm, at a 10Hz sampling frequency. The trailer wheelbase is $L_3 = 2.34m$ and the vehicle-trailer angle is measured using a low cost potentiometer at the hitch point. This solution is less expensive than installing another localization system on the implement (e.g. one or several GPS antennas), and supplies however an accurate information (resolution of $0.35°$ using a 10-bit Analog-to-Digital Converter). The control period is $0.1s$.

Fig. 5. Experimental vehicle-trailer system

A. Curved path following

A reference path is first recorded with the GPS receiver during a manual run on a wet and irregular ground, see figure 6. The high curvature of the two successive circles will enable to clearly separate the trailer's and vehicle's trajectories and to highlight sliding phenomena.

As a first step, to point out that the trailer does not follow the vehicle's track on such a reference path, path following is performed at $1.4m/s$ with respect to the vehicle lateral and angular deviations $(y_v, \tilde{\theta})$, i.e. the trailer is ignored in that case. For that, the control law detailed in [9] has been used. This control law takes into account the vehicle side slip

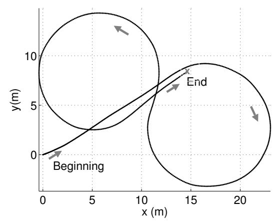

Fig. 6. Reference path

angles (β_F, β_R): it is equivalent to control law (11), applied to the actual vehicle instead of the trailer.

It can be seen in figure 7 that the vehicle follows satisfactorily the reference path, whereas the trailer is shifted inside the two successive circles. The lateral deviation of the vehicle recorded at point R and the lateral deviation of the trailer recorded at point T, with respect to the reference path, are reported in figure 8. They are plotted according to the curvilinear abscissa. The vehicle starts at about $18cm$ from the path to be followed. Then, it reaches the planned path and maintains an overall lateral error within $\pm 10cm$ (except at the beginning of the first circle, at curvilinear abscissa $17m$, due to a fast curvature variation and actuator delays). In contrast, the trailer is shifted for roughly $50cm$ during the first circle and for more than $30cm$ during the second one.

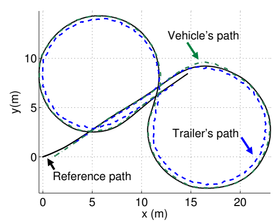

Fig. 7. Path following with respect to the vehicle

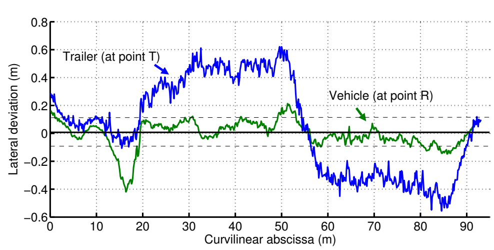

Fig. 8. Lateral deviations when the point R is controlled

In the second test, the side slip angle observer (5) and control law (13) have been used, in order to explicitly control the position of the trailer with respect to the reference path. The results are depicted in figures 9, 10 and 11.

It can be seen in figure 9 that the vehicle has moved outside the two successives circles (for roughly $40cm$, as shown in figure 10), enabling an accurate path tracking for the trailer. Figure 10 allows to appreciate the capabilities of the proposed control algorithm: the trailer starts at about $30cm$ from the path to be followed. Then, it reaches the planned path and maintains a satisfactory overall lateral error within $\pm 10cm$. The small overshoots (about $20cm$) at each variation in the curvature of Γ (curvilinear abscissas $18m$, $52m$, $57m$ and $90m$) are mainly due to actuator delays at such transient phases. The introduction of some anticipation will be investigated in future development with predictive actions. The front steering angle measurement is reported in figure 11. The values are quite smooth, with variations within $\pm 10°$.

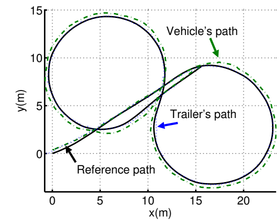

Fig. 9. Path following with respect to the trailer

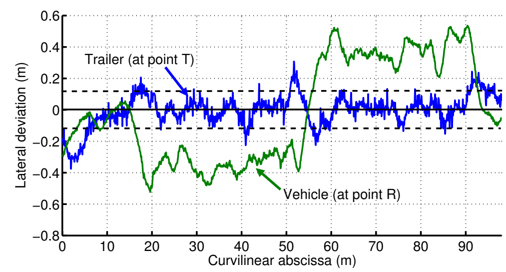

Fig. 10. Lateral deviations when the point T is controlled

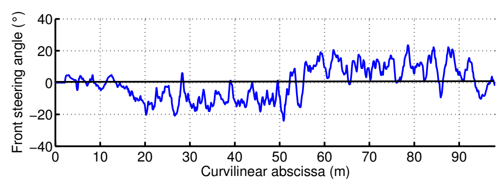

Fig. 11. Front steering angle

Finally, the same control algorithm has been used in the third test, but sliding phenomena have been disregarded, i.e. $(\beta_F, \beta_R, \beta_T)$ were set to $(0, 0, 0)$. The result is reported in figure 12.

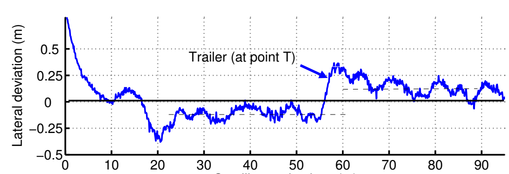

Fig. 12. Without taking into account side slip angles

It can be observed that the accuracy of path following for the trailer is seriously damaged, with an error about $15cm$ during each circular part of Γ. This result shows on one hand the necessity to explicitly take into account sliding phenomena into the control algorithms, and on the other hand that observer (5) provides relevant side slip angle values, since they enable a more accurate control of the vehicle-trailer system, as depicted in figure 10.

B. Straight line following in sloping fields

A heavy water drum ($60L$) has been added at the extremity of the trailer, see figure 5, in order to obtain significant sliding phenomena in the sloping field. A straight line is then recorded during a manual run on an irregular and side sloped ground varying from 0% to 25%. Path following is then achieved with side slip angle observer (5) and control algorithm (13). In figure 13 are reported the three side slip angles. As the slope increases progressively, the side slip angles also distinctly raise up to $5°$ for β_F, $3°$ for β_R, and more than $10°$ for β_T.

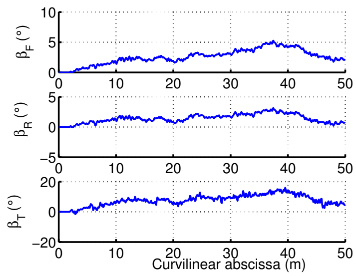

Fig. 13. Side slip angles $(\beta_F, \beta_R, \beta_T)$

It can be seen in figure 14 that the trailer follows satisfactorily the reference path despite the variation in the slope, with an overall error within $\pm 10cm$. Moreover, it can be noticed that the vehicle moves away from the reference trajectory Γ (about $10cm$) in order to keep the trailer correctly on Γ.

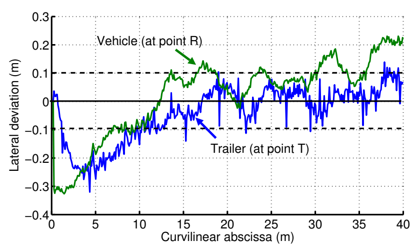

Fig. 14. Lateral deviations on a side sloped field

VI. Conclusion and Future Work

This paper addresses the problem of accurate path following of a vehicle-trailer system in presence of sliding, with application to automatic guidance of a towed agricultural implement. The objective is to control the position of the trailer's center with respect to a planned trajectory. An extended kinematic model accounting for sliding effects via three side slip angles has first been considered. An observer has then been developed in order to obtain relevant estimations of the side slip angles. Next, a control law has been designed according to three steps: the trailer is first considered as an independent virtual vehicle and a control law calculates the direction of the linear velocity vector at the hitch point that would ensure the convergence of this virtual vehicle to the planned trajectory. A reference vehicle-trailer angle leading to such a velocity vector is then inferred. Finally, a control law is designed to stabilize the actual vehicle-trailer angle on the reference vehicle-trailer angle. Promising results, based on full scale experiments, have been presented: the proposed algorithms succeed in stabilizing the trailer with a satisfactory overall lateral error within $\pm 10cm$ during path following of circles and on a side sloped ground.

To go further, the capabilities of the proposed algorithms could be improved according to two directions: on one hand small overshoots of about $20cm$ have been observed transiently when the curvature of the reference path is fast varying. Model predictive control techniques could be investigated to counteract such a behaviour. On the other hand, the extension of this work to control the vehicle's velocity in order that the trailer maintains a constant velocity, in particular in curve, would be of practical interest. In fact, a towed agricultural implement able to work at a constant velocity whatever ground conditions and trajectory shape is significant in precision agriculture and constitutes another challenging problem.

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