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# Robot Task Space PID type regulation with prescribed performance guaranties

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***Abstract*—A prescribed performance regulator for the generalized position of the robot arm endpoint in the task space is proposed. The control input which incorporates a transformed error guarantees a prescribed performance regarding the response of the endpoint generalized position error. The use of two different forms of this transformed error will be presented and compared. Mathematical proof of the controller's success in fulfilling the desired goals is given. A simulation of a three degrees of freedom robot is used to confirm the theoretical findings for both cases of the transformed error.**

## I. INTRODUCTION

Most of the literature on robot position regulation deal solely with the stability problem in the sense of asymptotic convergence of the position errors to zero rather than the system performance in the transient phase. Positions errors are usually defined in the joint space but task coordinates have also been considered. A brief treatment and review of the works on the regulation problem in task coordinates can be found in [1]. In all these works a priori guaranties for transient behavior bounds are not given. At best, performance and robustness are guaranteed by the exponential convergence of a Lyapunov metric that is however dependant on the values of the control gains while added disturbances adversely affect asymptotic convergence [2]. Recently prescribed performance controllers have been proposed for robot joint position regulation [3],[4] that were inspired by the recent work on prescribed performance controllers [5],[6] developed for specific classes of non linear systems. As robot tasks are performed via the robot's end effector, it is important to consider error performance in the task space. Hence, in this work we transfer the design and application of prescribed performance regulators from the joint space [3],[4] to the task space.

## II. PROBLEM DESCRIPTION AND PRELIMINARIES

We consider a $n$ degrees of freedom robot with $q \in \Re^n$ the vector of the generalized joint variables. The dynamic model of the robot is given by the following non linear differential equation:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \tag{1}$$

where $H(q) \in \Re^n \times \Re^n$ is the positive definite robot inertia matrix, $C(q,\dot{q})\dot{q} \in \Re^n$ is the vector of Coriolis and centripetal forces, $g(q) \in \Re^n$ is the gravity vector and $u \in \Re^n$ is the vector of applied torques. Notice that

$$\exists\ \lambda_q, \Lambda_q > 0\ : \lambda_q I_n \leq H(q) \leq \Lambda_q I_n\ \forall\ q \in \Re^n.$$

Further notice that $\dot{H}(q) - 2C(q,\dot{q})$ is a skew-symmetric matrix and $g(q) = \frac{\partial U(q)}{\partial q}$ where $U(q)$ denotes the potential energy due to the gravity field. A known property of the robot dynamics is that there exists a positive constant $c_g$ so that the following inequalities simultaneously hold [1]:

$$U(q) - U(q_d) - e_q^T g(q_d) \quad \geq \quad -c_g\|e_q\|^2 \tag{2}$$

$$e_q^T\Big(g(q) - g(q_d)\Big) \quad \geq \quad -c_g\|e_q\|^2 \tag{3}$$

with $q_d \in \Re^n$ the constant desired robot's position vector expressed in the joint space and $e_q = q(t) - q_d$ the position error in the joint space.

We consider the position regulation problem of the robot arm in the task space with prescribed performance guaranties. More specifically, we want to regulate the generalized position of the robot's end effector $X(t) \in \Re^6$ to a desired value $X_d \in \Re^6$. The generalized position is described by the three Cartesian coordinates of the robot's tip position $x(t) \in \Re^3$ and the three rotation angles that minimally parameterize the end effector orientation. The generalized velocity $\dot{X}(t)$ is related to the joint velocity $\dot{q}(t)$ through the Jacobian $J(q)$ as follows : $\dot{X}(t) = J(q)\dot{q}(t)$. We assume that joint positions and velocities are measured and that the Jacobian $J(q)$ is known. Hence, $X(t), \dot{X}(t)$ can be calculated using the robot forward kinematics. We further consider the problem of satisfying specific prescribed performance requirements for the position error $e_X(t) = X(t) - X_d$ regarding both its transient and steady state response.

By prescribed performance guaranties we mean that each component $e_{Xi}(t)$, $i = 1, ..., 6$ of the position error $e_X(t)$, evolves within a predefined region that is bounded by a decaying function of time. The mathematical expression for the prescribed performance is given by the following inequalities for all $i's$ [5]:

$$-M\rho_i(t) < e_{Xi}(t) < \rho_i(t) \quad \forall t \text{ in case } e_{Xi}(0) \geq 0 \tag{4}$$

$$-\rho_i(t) < e_{Xi}(t) < M\rho_i(t) \quad \forall t \text{ in case } e_{Xi}(0) \leq 0 \tag{5}$$

where $0 \leq M \leq 1$ and $\rho_i(t)$ as defined in [5] is a bounded, smooth, strictly positive and decreasing function satisfying $\lim_{t\to\infty} \rho_i(t) = \rho_\infty > 0$. Further, following [5], we define:

$$\varepsilon_{Xi}(t) = T_i\left(\frac{e_{Xi}(t)}{\rho_i(t)}\right) \tag{6}$$

where $\varepsilon_{Xi}(t)$ is the $i'th$ component of a transformation error vector $\varepsilon_X(t) \in \Re^6$ and $T_i(.)$ is a smooth, strictly increasing function defining an onto mapping :

$$\begin{array}{lll} T_i : (-M, 1) \to (-\infty, \infty) & \text{in case} & e_{Xi}(0) \geq 0 \\ T_i : (-1, M) \to (-\infty, \infty) & \text{in case} & e_{Xi}(0) \leq 0 \end{array} \tag{7}$$

Notice that the choice of the mapping in (7) depends solely on the sign of the initial error and either mapping is possible in case the initial error is zero. It has been shown [6] that owing to the properties of the error transformation, the uniform boundedness of the transformation error $\varepsilon_X(t)$ (i.e. $\varepsilon_X(t) \in \mathcal{L}_\infty$) is sufficient to guarantee the satisfaction of the prescribed performance (4),(5) for the position error.

## III. CONTROLLER DESIGN

The design procedure involves the following steps:

***Step 1***: We specify the error performance function $\rho_i(t)$. Without loss of generality the same function $\rho(t)$ is considered for all $i$'s and it is here given by the following exponentially decaying time function:

$$\rho_i(t) = \rho(t) = (\rho_0 - \rho_\infty)\exp(-lt) + \rho_\infty \quad \text{for all } i\text{'s} \tag{8}$$

where $\rho_0, \rho_\infty, l$ are appropriately chosen positive constants.

A graphical presentation of (4) and (5) with $\rho(t)$ given by (8) is shown in Fig.(1). Constant $\rho_0 = \rho(0)$ is selected such that $\rho_0 > \max_i |e_{Xi}(0)|$ so that (4) and (5) are satisfied at $t = 0$. Constant $\rho_\infty$ represents the maximum allowable size of the position errors at steady state. Constant $l$, the decreasing rate of $\rho(t)$, represents a lower bound on the required speed of convergence of $e_{Xi}(t)'s$. Finally, the maximum overshoot is prescribed less than $M\rho_0$ and may even become zero if we set $M = 0$. Thus, by selecting specific values for $M$ and the performance function parameters, we prescribe specific performance bounds for each component of the position error $e_X(t)$.

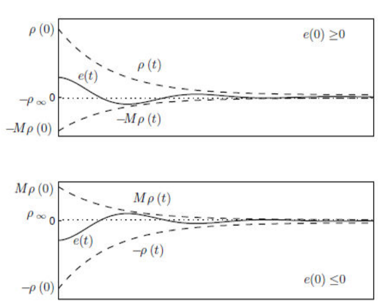


Fig. 1. Performance bounds

***Step 2***: A transformation function that possesses all the required properties is specified. In this work we consider two cases:

$$T_{1i}\left(\tfrac{e_{Xi}(t)}{\rho(t)}\right) = \begin{cases} \ln\left(\frac{M + (e_{Xi}(t)/\rho(t))}{1 - (e_{Xi}(t)/\rho(t))}\right) & e_{Xi}(0) \geq 0 \\ \ln\left(\frac{1 + (e_{Xi}(t)/\rho(t))}{M - (e_{Xi}(t)/\rho(t))}\right) & e_{Xi}(0) \leq 0 \end{cases} \tag{9}$$

$$T_{2i}\left(\tfrac{e_{Xi}(t)}{\rho(t)}\right) = \begin{cases} \ln\left(\frac{M + (e_{Xi}(t)/\rho(t))}{M(1 - (e_{Xi}(t)/\rho(t)))}\right) & e_{Xi}(0) \geq 0 \\ \ln\left(\frac{M(1 + (e_{Xi}(t)/\rho(t)))}{M - (e_{Xi}(t)/\rho(t))}\right) & e_{Xi}(0) \leq 0 \end{cases} \tag{10}$$

They are both based on the natural logarithm but (10) is shifted so that it belongs to a sector $[\kappa, +\infty]$. The two cases are graphically illustrated in Fig.(2) and Fig.(3). Notice that transformation function (10), initially proposed in [4] excludes the choice of a zero value for parameter $M$.

***Step 3***: The following PID-type regulator in the task space is formed where the time argument is dropped from $e_X(t)$, $\varepsilon_X(t)$ and $\dot{X}(t)$ for compactness of presentation:

$$u = J^T(q)(-K_p e_X - K_v \dot{X} - K_I \int_0^t y(\tau) d\tau - k_\epsilon \partial T_X \varepsilon_X) \tag{11}$$

where $\partial T_X = \text{diag}[\partial T_{X1}\ \partial T_{X2}\ ...\ \partial T_{X6}]$ with

$$\partial T_{Xi} \triangleq \frac{\partial T_i}{\partial(e_{Xi}(t)/\rho(t))} \frac{1}{\rho(t)} > 0 \ \ i = 1, ..., 6 \tag{12}$$

and

$$y(t) = \dot{X} + k(t) e_X \tag{13}$$

with

$$k(t) = \left(\frac{-\dot{\rho}(t)}{\rho(t)}\right) + \beta \text{ with } \begin{array}{ll} \beta = 0 & \text{if } T_i(.) = T_{1i} \\ \beta > 0 & \text{if } T_i(.) = T_{2i} \end{array} \tag{14}$$

Finally, $K_p, K_I, K_v \in \Re^n \times \Re^n$ are positive definite diagonal gain matrices while $k_\epsilon$ is a positive control constant.

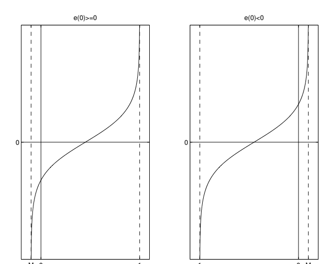


Fig. 2. Function $T_{1i}(.)$

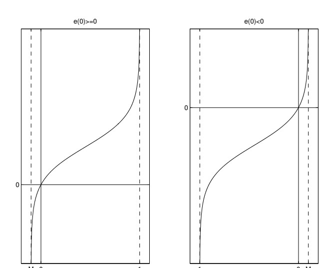


Fig. 3. Function $T_{2i}(.)$

The closed loop system formed by the system (1) and the proposed controller (11), expressed in the task space is as follows:

$$H_X(X)\ddot{X} + C_X(X,\dot{X})\dot{X} + g_X(X) + K_p e_X + K_v\dot{X} + K_I\int_0^t y(\tau)d\tau + k_\epsilon \partial T_X \varepsilon_X = 0 \quad (15)$$

where

$$\begin{aligned} H_X(X) &= (J(q)H(q)^{-1}J(q)^T)^{-1} \\ C_X(X,\dot{X})\dot{X} &= H_X(X)J(q)H(q)^{-1}C(q,\dot{q})\dot{q} - \\ &\quad - H_X(X)\dot{J}(q)\dot{q} \\ g_X(X) &= H_X(X)J(q)H(q)^{-1}g(q) \end{aligned}$$

The task space model is formally valid for a redundant manipulator given a full rank Jacobian although $X$ is no longer a set of generalized system coordinates. Nevertheless redundancy resolution can be performed at either the kinematic level or at the dynamic level exploiting the homogeneous term of the general solution for the joint torques [7].

It is known that matrices $H_X(X)$ and $\dot{H}_X(X) - 2C_X(X,\dot{X})$ share the same properties with matrices $H(q)$ and $\dot{H}(q) - 2C(q,\dot{q})$ respectively. This means

$$\exists\ \lambda_X, \Lambda_X > 0\ : \lambda_X I_n \le H_X(X) \le \Lambda_X I_n\ \forall\ X \in \Re^6$$

and $\dot{H}_X(X) - 2C_X(X,\dot{X})$ is a skew-symmetric matrix. Adding and subtracting $g_X(X_d)$ in (15) yields :

$$H_X(X)\ddot{X} + C_X(X,\dot{X})\dot{X} + g_X(X) - g_X(X_d) + K_p e_X + K_v\dot{X} + K_I z(t) + k_\epsilon \partial T_X \varepsilon_X = 0 \quad (16)$$

where

$$z(t) = \int_0^t y(\tau)d\tau + K_I^{-1} g_X(X_d) \quad (17)$$

Notice that: $\dot{z}(t) = y(t)$. Considering that the derivative of $\varepsilon_X(t)$ is calculated as follows:

$$\dot{\varepsilon}_X(t) = \partial T_X[\dot{e}_X(t) + \left(\frac{-\dot{\rho}(t)}{\rho(t)}\right) e_X(t)] \quad (18)$$

$y(t)$ can be written as $y(t) = \partial T_X^{-1}\dot{\varepsilon_X} + \beta e_X$ using (13) and (18).

The inner product of the closed loop system (16) and $y(t)$ can be written as:

$$\frac{dV}{dt} + W = 0 \quad (19)$$

where

$$V = \frac{1}{2}\dot{X}^T H_X(X)\dot{X} + k(t)e_X^T H_X(X)\dot{X} + \Big(U(q) - U(q_d) - e_q^T g(q_d)\Big) + \frac{1}{2}e_X^T K_p e_X + \frac{1}{2}k(t)e_X^T K_v e_X + \frac{1}{2}k_\epsilon\|\varepsilon_X\|^2 + \frac{1}{2}z^T K_I z \quad (20)$$

and

$$W = -\dot{k}(t)e_X^T H_X(X)\dot{X} + \dot{X}^T[K_v - k(t)H_X(X)]\dot{X} - k(t)e_X^T\dot{H}_X(X)\dot{X} + k(t)e_X^T C_X(X,\dot{X})\dot{X} + k(t)e_X^T\Big(g_X(X) - g_X(X_d)\Big) + k(t)e_X^T K_p e_X - \frac{1}{2}\dot{k}(t)e_X^T K_v e_X + \beta k_\epsilon e_X^T \partial T_X \varepsilon_X \quad (21)$$

We will restrict our analysis to a neighborhood $N_q$ inside which each $q$ corresponds to a unique $X$ and vice versa. This is a neighborhood of $q_d$ (corresponding to $X_d$) that is also assumed to contain the initial joint position $q(0)$. This implies that the forward kinematics mapping $f : q \to X$ in this neighborhood is a continuous vectorial function with a full rank Jacobian $J(q) = \frac{\partial f}{\partial q}$. The satisfaction of the performance bounds for the task space error may allow us to define such a neighborhood by mapping the set $N_X = \{X \in \Re^6 : \underline{X} \le X \le \overline{X}\}$ using the inverse kinematics i.e $N_X \xrightarrow{f^{-1}} N_q$ where $\underline{X}_i$ and $\overline{X}_i$, the $i'th$ components of $\underline{X}$ and $\overline{X}$ respectively, are defined as follows:

$$\text{if } e_{Xi}(0) \ge 0\ :\ \begin{array}{l} \overline{X}_i = \rho_0 + X_{di} \\ \underline{X}_i = -M\rho_0 + X_{di} \end{array}$$

$$\text{if } e_{Xi}(0) \le 0\ :\ \begin{array}{l} \overline{X}_i = M\rho_0 + X_{di} \\ \underline{X}_i = -\rho_0 + X_{di} \end{array}$$

Inside $N_q$, position error $e_q = q - q_d$ corresponds to a unique position error $e_X = X - X_d$ in the task space. An effective way of achieving such correspondence is via the Jacobian $J(q)$. In fact according to the mean-value theorem for vectorial functions [8], there exists $q^* : q < q^* < q_d$ such that:

$$e_X = J(q^*)e_q \quad (22)$$

Having (22) in mind, $V$ can be lower bounded as below:

$$V \ge \frac{1}{4}\dot{X}^T H_X(X)\dot{X} + \frac{1}{4}e_q^T[J(q^*)^T K_p J(q^*) - 4c_g I_n]e_q + \frac{1}{2}k(t)e_X^T[K_v - 2(l+\beta)\Lambda_X I_n]e_X + \frac{1}{2}k_\epsilon\|\varepsilon_X\|^2 + \frac{1}{2}z^T K_I z + \frac{1}{4}e_X^T K_p e_X \quad (23)$$

$V$ is positive definite with respect to $\dot{X}, e_q, e_X, \varepsilon_X, z(t)$ assuming that $K_p, K_v$ are chosen big enough to satisfy the following inequalities:

$$\min_{q^*}\Big(\lambda_{min}(J(q^*)^T K_p J(q^*))\Big) \ \ge\ 4c_g \quad (24)$$

$$\lambda_{min}(K_v) \ \ge\ 2(l+\beta)\Lambda_X \quad (25)$$

where $\lambda_{min}(.)$ denotes the smallest eigenvalue of a matrix. Demanding $K_p$ to be sufficiently high, we ensure that the smallest of all the eigenvalues $\lambda_{min}(J(q^*)^T K_p J(q^*))$, each one corresponding to a different $q^*$, is going to be larger than $4c_g$. Moreover, given that $J(q)$ is full rank inside $N_q$, $J(q^*)^T K_p J(q^*)$ is positive definite. The proof of (23) is given in the Appendix.

Term $[C_X(X,\dot{X})\dot{X} - \dot{H}_X(X)\dot{X}]$ is bounded and quadratic in $\dot{X}$, therefore we can bound it as follows:

$$[C_X(X,\dot{X})\dot{X} - \dot{H}_X(X)\dot{X}] \geq -c_0\|\dot{X}\|^2 \quad (26)$$

for a positive $c_0$. Hence, $W$ can be lower bounded as follows:

$$\begin{aligned} W \geq\ & \dot{X}^T[K_v - (l+\beta)\Lambda_X I_n \\ & - \frac{l^2}{8}\Lambda_X \xi I_n - \Big(l(\rho_0 - \rho_\infty) + \beta\rho_0\Big)c_0 I_n]\dot{X} \\ & + \frac{1}{2}|\dot{k}(t)| e_X^T[K_v - \Lambda_X \xi^{-1} I_n]e_X + \beta k_\epsilon e_X^T \partial T_X \varepsilon_X \\ & + k(t) e_q^T[J(q^*)^T K_p J(q^*) - c_g c_J I_n]e_q \end{aligned} \quad (27)$$

where $\xi$ is a free positive constant and $c_J = \|J(q^*)^T J(q)^{-T}\|$. Analytical proof concerning (27) is given in the Appendix. In the following we distinguish two cases each one employing a different transformation function $T_{1i}(.)$ and $T_{2i}(.)$:

1) if $\varepsilon_{Xi}(t) = T_{1i}\left(\frac{e_{Xi}(t)}{\rho(t)}\right)$, then $\beta = 0$ from (14). Therefore, term $\beta k_\epsilon e_X^T \partial T_X \varepsilon_X(t)$ in (27) no longer exists. If $K_v, K_p$ are chosen sufficiently high to guarantee the following inequalities:

$$\begin{aligned} \lambda_{min}(K_v) \geq\ & [(l+\beta)\Lambda_X + l^2\Lambda_X\frac{\xi}{8} \\ & + c_0\Big(l(\rho_0 - \rho_\infty) + \beta\rho_0\Big)] \end{aligned} \quad (28)$$

$$\lambda_{min}(K_v) \geq \Lambda_X \xi^{-1} \quad (29)$$

$$\min_{q^*}\Big(\lambda_{min}(J(q^*)^T K_p J(q^*))\Big) \geq c_g c_J \quad (30)$$

then $W \geq 0$. Notice that inequality (28) is stricter than (25).

2) if $\varepsilon_{Xi}(t) = T_{2i}\left(\frac{e_{Xi}(t)}{\rho(t)}\right)$, then $\beta > 0$. Since

$$\begin{aligned} |\varepsilon_{Xi}| &> \frac{4}{(M+1)\rho(t)}|e_{Xi}| \\ \partial T_{Xi} &> \frac{4}{(M+1)\rho(t)} \end{aligned} \quad (31)$$

we can lower bound term $\beta k_\epsilon e_X^T \partial T_X \varepsilon_X(t)$ as follows:

$$e_X^T \partial T_X \varepsilon_X(t) \geq c_1\|e_X\|^2 \quad (32)$$

where it is easily proven that: $c_1 = \left(\frac{4}{\rho_0(M+1)}\right)^2$. If $K_v, K_p$ are chosen sufficiently high to guarantee inequalities (28), (29) and (30) then owing to (32), $W \geq 0$.

Since in both the above cases $V$ is positive definite with a non positive derivative $\frac{dV}{dt} = -W$, $V(\dot{X}, e_q, e_X, \varepsilon_X, z(t)) \leq V(0)$ holds and consequently $\dot{X}, e_q, e_X, \varepsilon_X, z(t)$ are bounded. The boundedness of $\varepsilon_X$ implies that $e_X$ is bounded by the performance function according to (4) and (5) as well as that $\partial T_X$ is bounded. From (16) $\ddot{X}$ is also bounded, hence $\dot{X}$ is uniformly continuous.

In case of $T_{1i}(.)$ from (27) since $\lim_{t\to\infty} k(t) = \beta = 0$, $W$ can be lower bounded by a function of $\|\dot{X}\|^2$, thus $\dot{X} \in \mathcal{L}_2$. Using Desoer and Vidyasagar (1975) it is proved that $\dot{X} \to 0$. Moreover in case of $T_{2i}(.)$ since $\lim_{t\to\infty} k(t) = \beta > 0$, $W$ can be lower bounded by a function of $\|\dot{X}\|^2$ and $\|e_X\|^2$ thus $\dot{X}, e_X \in \mathcal{L}_2$. In fact there exist positive constants $\gamma_1$ and $\gamma_2$ such that $W \geq \gamma_1\|\dot{X}\|^2 + \gamma_2\|e_X\|^2$ and hence by integrating $\frac{dV}{dt} \leq -\gamma_1\|\dot{X}\|^2 - \gamma_2\|e_X\|^2$ along the time interval $[0, +\infty]$ we get

$$V(0) - V(\infty) \geq \gamma_1\int_0^\infty \|\dot{X}\|^2 d\tau + \gamma_2\int_0^\infty \|e_X\|^2 d\tau$$

that clearly implies $\dot{X}, e_X \in \mathcal{L}_2$. Since $\dot{X}, e_X$ are uniformly continuous and belong to the $\mathcal{L}_2$ space, using Desoer and Vidyasagar (1975) it is proved, additionally to $\dot{X} \to 0$, that $e_X \to 0$.

This result holds under the assumption that $q(t) \in N_q$ for all $t \geq 0$. Thus we need to establish that given $q(0) \in N_q$ the proposed law (11) does not force $q(t)$ to escape $N_q$ at any time. We shall prove this argument by contradiction, considering only the case of $e_X(0) \geq 0$, since similar analysis holds for the case of $e_X(0) \leq 0$ too. Let us assume that $q(t)$ escapes $N_q$. Thus there exist a time instant $t_1$ at which either $q(t_1) = f^{-1}(\underline{X})$ or $q(t_1) = f^{-1}(\overline{X})$ from which we further conclude that either $X(t_1) = \underline{X}$ or $X(t_1) = \overline{X}$ (position $X1$ in Fig.(4)). Since $e_X(0) \geq 0$ then $X(t_1) = -M\rho_0 + X_d$ or $X(t_1) = \rho_0 + X_d$ (which is the case of $X1$ in Fig.(4)) hence $\forall\ t \in [0, t_1]$, $X(t_1) \leq -M\rho(t) + X_d$ or $X(t_1) \geq \rho(t) + X_d$. Consequently owing to $-M\rho_0 + X_d < X(0) < \rho_0 + X_d$ and the continuity of the solution of the closed loop system $\forall\ t \in [0, t_1]$ we conclude the existence of a time instant $t_2$ satisfying $t_2 < t_1$ for which $q \in N_q \quad \forall\, t \leq t_2$ and either $\lim_{t\to t_2} X(t) = -M\rho(t_2) + X_d$ or $\lim_{t\to t_2} X(t) = \rho(t_2) + X_d$ (position $X2$ in Fig.(4)) hence from (7) $\lim_{t\to t_2} \varepsilon_X = \pm\infty$ which is in direct contradiction with the fact that $\varepsilon_X$ is bounded provided $q \in N_q$. This means that $\forall\ t, q(t) \in N_q$.

*Theorem 1:* The control law (11) applied to the system (1) guarantees the satisfaction of (4) and (5) for all $t > 0$ provided that the controller gains $K_p, K_v$ satisfy conditions (24), (28), (29) and (30) that incorporate minimal information concerning the robot arm model. Furthermore, $\dot{X}$ asymptot-

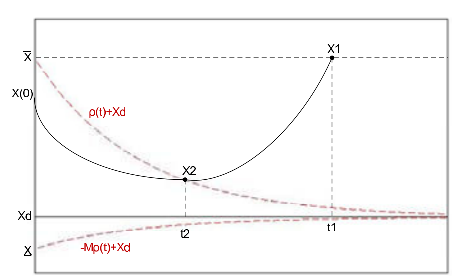


Fig. 4.

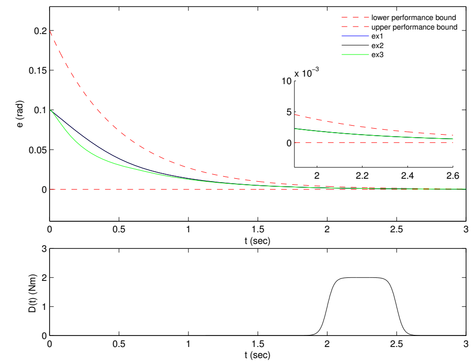


Fig. 5. Position error responses and disturbance input with $T_{1i}(.)$

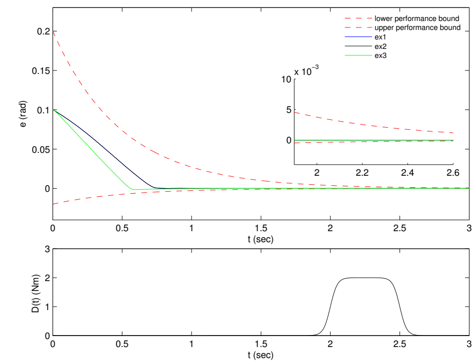


Fig. 6. Position error responses and disturbance input with $T_{2i}(.)$

ically converges to zero and if the error transformation (10) is used then $e_X$ also asymptotically converges to zero .

It is therefore possible to achieve the prescribed performance bounds with (11) under both cases of $T_i(.)$. However a zero overshoot is possible only in case of $T_{1i}(.)$ in the expense of non zero although small steady state error. On the other hand zero steady state error can be achieved when $T_{2i}(.)$ is used in the expense of a small but non zero value of overshoot.

*Remark:* The assumption of a full rank Jacobian is typical of all operational space control systems and is required in order to ensure the positive definiteness of $V$. Notice however that in our work this assumption is easy to satisfy by appropriately setting the desired generalized position and the performance bounds to guarantee that the system trajectories will evolve away from any Jacobian singularities.

## IV. SIMULATION RESULTS

We consider an example of a 3 d.o.f. spatial robotic manipulator with rotational joints with masses $m_1 = m_2 = m_3 = 1$ kg, link lengths $l_2 = l_3 = 0.5$ m and inertias $\mathtt{I}_{z1} = \mathtt{I}_{x2} = \mathtt{I}_{x3} = 4.15 \times 10^{-4}$ kg m$^2$, $\mathtt{I}_{y2} = \mathtt{I}_{z2} = 0.021$ kg m$^2$ and $\mathtt{I}_{y3} = \mathtt{I}_{z3} = 0.0039$ kg m$^2$. The initial robot endpoint position is given by $X(0) = [0.3\ 0.35\ 0.3]^T$ (m). We consider the case of step magnitudes of 0.1 m for each coordinate, hence we set the desired position $X_d$ to $X_d = [0.2\ 0.25\ 0.2]^T$ (m). Regarding the prescribed performance function $\rho(t)$ we choose: $\rho_0 = 0.2 > |e_{Xi}(0)|$ for all i, $\rho_\infty = 10^{-4}$ and $l = 2$. Consequently, the prescribed performance is expressed by the function $\rho(t) = (0.2 - 10^{-4})exp(-2t) + 10^{-4}$. Furthermore we consider the following bounded disturbance torque acting at each joint $(u(t) + D(t))$:

$$D(t) = \tanh(20 \times (t - 2)) - \tanh(20 \times (t - 2.5))$$

Simulations for both cases of transformation functions were performed. For the case of $T_{1i}(.)$ (7), $M = 0$ and control gains are set to the values of $\beta = 0$, $K_p = 170I_3$, $K_v = 120I_3$, $K_I = 10I_3$, $k_\epsilon = 0.8$. For the case of $T_{2i}(.)$ (8), $M = 0.1$ and control gains are set to the values of $\beta = 0.1$, $K_p = 100I_3$, $K_v = 200I_3$, $K_I = 10I_3$, $k_\epsilon = 0.25$.

Simulation results are shown in Fig.(5) - Fig.(9). Notice that in both cases, our controller succeeds in keeping the position errors within the boundaries, defined by the performance function $\rho(t)$ despite the presence of disturbances (Fig.(5) and Fig.(6)). Notice that although disturbances occur at a time where performance bounds are very strict (lower plots in Fig.(5) and Fig.(6)) they do not affect the error response as clearly indicated in the embedded plots. In case of $T_{1i}(.)$, there is an non-zero but small position error at steady state of $2.97 \times 10^{-4}$ magnitude while in case $T_{2i}(.)$ is used, an asymptotical convergence of the position error to zero is achieved. Fig.(7) shows the path of the robot arm endpoint in the three-dimensional space, for both cases of $T_{1i}(.)$ and $T_{2i}(.)$. The embedded subplot shows details of the path in the neighborhood of $X_d$ where the overshoot in the second case as expected by the $M$ value set at 0.1 is clearly depicted as well as the achievement of the desired position in contrast to the first case. Input torques are shown with solid lines in Fig.(8) and Fig.(9) for each case of $T_i(.)$ respectively. Embedded plots show details of early transients. Dashed lines show the input torques in the absence of disturbances for

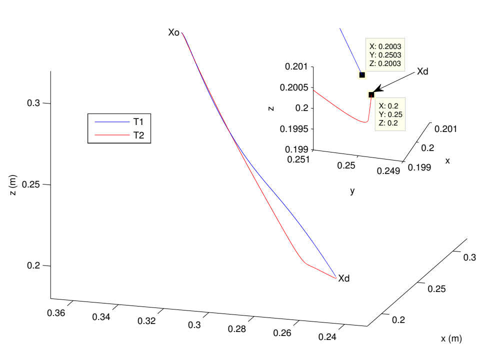


Fig. 7. Robot endpoint movement in task space for each case of $T_i(.)$

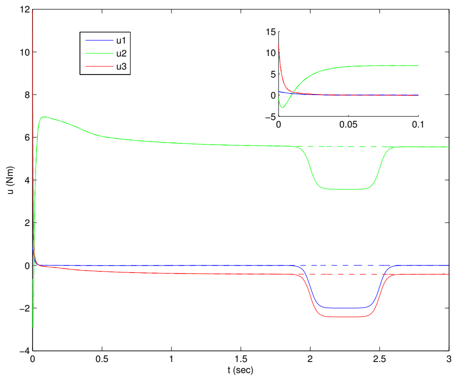


Fig. 8. Input torque responses with $T_{1i}(.)$ in the presence (solid line) and absence (dashed line) of disturbance

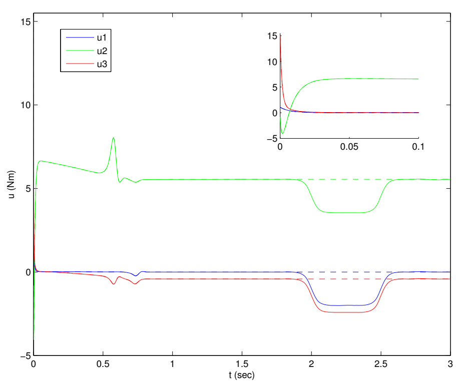


Fig. 9. Input torque responses with $T_{2i}(.)$ in the presence (solid line) and absence (dashed line) of disturbance

comparison purposes. Notice how input torques compensate for disturbances while torque inputs remain at reasonable values in all cases.

## V. CONCLUSIONS

This work proposes a PID type regulator augmented by a transformed position error term that achieves regulation with prescribed performance in the task space. Prescribed performance bounds are constructed by setting a priori values for the maximum overshoot, steady state error and minimum speed of response. Two control variants are analyzed which differ by the transformation function used. It is theoretically proved and demonstrated by the simulation of a three degrees of freedom spatial robot that the control objective is achieved in both control variants and that the transformation which belongs to a sector enables further the asymptotic convergence of the errors to zero.

## VI. APPENDIX

Proof of (23): Notice that the first two terms of $V$ (20) can be written as follows: $\frac{1}{4}\dot{X}^T H_X(X)\dot{X} + \frac{1}{4}(\dot{X} + 2k(t)e_X)^T H_X(X)(\dot{X} + 2k(t)e_X) - k^2(t)e_X^T H_X(X)e_X$. From (2) and using $e_X = J(q^*)e_q$ we can lower bound terms $\Big(U(q) - U(q_d) - e_q^T g(q_d)\Big) + \frac{1}{2}e_X^T K_p e_X$ as follows:

$$\Big(U(q) - U(q_d) - e_q^T g(q_d)\Big) + \frac{1}{2}e_X^T K_p e_X \geq \frac{1}{4}e_q^T[J(q^*)^T K_p J(q^*) - 4c_g I_n]e_q + \frac{1}{4}e_X^T K_p e_X$$

Since $0 < -\frac{\dot{\rho}(t)}{\rho(t)} < l$ we can therefore bound $V$ as shown in (23).

Proof of (27): Notice that the term $-\dot{k}(t)e_X^T H_X(X)\dot{X}$ can be lower bounded as follows: $-\dot{k}(t)e_X^T H_X(X)\dot{X} \geq -|\dot{k}(t)|\Lambda_X\Big(\frac{\|e_X\|^2}{2\xi} + \frac{\xi}{2}\|\dot{X}\|^2\Big)$ for a positive $\xi$. The inverse transformation $T_i^{-1}(\varepsilon_{Xi}) = \frac{e_{Xi}(t)}{\rho(t)}$ is a smooth strictly increasing function with the following properties:

$$\begin{aligned} -M < T_i^{-1}(\varepsilon_{Xi}) < 1 \quad &\text{in case} \quad e_{Xi}(0) \geq 0 \\ -1 < T_i^{-1}(\varepsilon_{Xi}) < M \quad &\text{in case} \quad e_{Xi}(0) \leq 0 \end{aligned} \tag{33}$$

Since $|\dot{\rho}(t)| \leq l(\rho_0 - \rho_\infty)$ and using (26), (33) we can lower bound term $k(t)e_X^T[-\dot{H}_X(X)\dot{X} + C_X(X,\dot{X})\dot{X}]$ as follows : $k(t)e_X^T[-\dot{H}_X(X)\dot{X} + C_X(X,\dot{X})\dot{X}] \geq -\Big(l(\rho_0 - \rho_\infty) + \beta\rho_0\Big)c_0\|\dot{X}\|^2$ for a positive $c_0$. Finally, using (3) we can bound term

$$k(t)e_X^T\Big(g_X(X) - g_X(X_d)\Big) = k(t)e_q^T[J(q^*)^T J(q)^{-T}]\Big(g(q) - g(q_d)\Big)$$

as below:

$$k(t)e_X^T\Big(g_X(X) - g_X(X_d)\Big) \geq -c_g\|J(q^*)^T J(q)^{-T}\|\|e_q\|^2$$

Hence

$$k(t)e_X^T\Big(g_X(X) - g_X(X_d)\Big) + k(t)e_X^T K_p e_X \geq k(t)e_q^T[J(q^*)^T K_p J(q^*) - c_g c_J I_n]e_q$$

Since $|\dot{k}(t)| \leq \frac{l^2}{4}$ we can bound $W$ as shown in (27).

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