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An Approach to Posture Control of Free-Falling Twin Bodies Using Differential Flatness

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Abstract— **Often, electronics and packages must be prevented from damages resulting from awkward falls. The goal of this paper is to explore how actuators and control can be used to reorient bodies during free fall. It is well known that motion of a free-falling body or a set of interconnected bodies is characterized by the principle of conservation of angular momentum. The governing angular momentum equations are nonholonomic, i.e., are non-integrable rate equations.**

In this paper, we assume that the falling system consists of twin bodies interconnected by a hinge joint. One of the twin bodies is designated as the primary body which needs to be reoriented during fall. The secondary body is connected to the primary body by a hinge joint, similar to a protective cover on a cell phone. The relative angle between the twin bodies is actively controlled. In addition, two rotors are mounted on the primary body with axes orthogonal to the axis of the hinge between the twin bodies. The goal of this study is to use the framework of differential flatness to compute trajectories for the hinge joint and the two rotors so that the twin bodies achieve prespecified orientations at the end of the free fall.

I. Introduction

Thousands of dollars are lost each year in accidents when electronic components fall on the ground. To prevent these damages, manufacturers use a variety of innovative packing materials that provide soft cushion during landing. We believe that packaging industries and electronic designs can benefit by innovative methods using which the posture of falling bodies can be actively modulated by onboard actuators.

It is well known that a falling cat is capable of reorienting its body with its flexible musculoskeletal system and the tail. A cat always lands on the ground on four limbs. In the past, robot cats have been investigated by researchers, including Kawamura et al [1]. A robot cat designed by them consisted of a front and a rear body. The rear body was connected to the front body by a flexible backbone, which was actuated to initiate twisting motion during fall. Yamafuji et al also employed a flexible backbone for posture control of a mobile robot with articulated twin legs [2], [3].

Chen and Sreenath studied modeling and controllability of a coupled rigid body by assuming zero angular momentum condition [4]. Based on Ritz approximations, Chris Fernandes et al developed a simple algorithm for near optimal nonholonomic motion planning of a falling cat [5]. Weng and Nishimura investigated the twisting motion of a falling robot cat with two torque inputs around its waist by applying iterative error learning [6].

Based on Lagrange-Euler formulation, Yang et al studied the landing posture control of a 5-DOF twin-body system, which is a simplified model of a cellular phone. Two controllers, a PD controller and an input-output linearization with computed torques, were applied to achieve the desired final landing posture [7]. Yang et al also conducted research on the landing posture control of a generalized twin-body system [8],[9]. In these studies, the degrees-of-freedom of the system is larger than the number of independent inputs. As a result, instead of controlling the orientation of the falling body, they control the relative orientation between the bodies. However, since their system is not fully feedback linearizable, it is not possible to guarantee that the system can reach a final point in the state space via a continuous trajectory.

Apart from research on free falling objects, there are studies which employ the principle of conservation of angular momentum during motion. For example, Guo et al presented a prototype model of an underwater fish-like micro robot [10], which possess a body posture adjuster to facilitate the swimming motion. Agrawal et al have investigated point-to-point maneuvers of free-floating space robots using the theory of differential flatness [11].

This study proposes a novel approach to posture control of a system containing twin free-falling bodies. The system is designed to be differentially flat through proper inertia distribution within the system. This allows the free falling twin-body to attain a pre-determined landing posture. The paper is organized as follows. The model of twin free-falling bodies is described in Section II. The underlying structure of the governing equations is highlighted in this section. Differential flatness of the falling twin bodies for the chosen inertia distribution is proved in Section III. The trajectory planning and optimization are illustrated in Section IV. In order to remove the effects of initial error, a PD controller is designed and implemented in Section V. Finally, conclusions and discussions are drawn at the end of the paper.

II. Model of a Free-Falling Twin Body

A. Geometry of the System

A schematic of the system is shown in Fig. 1. The primary body is numbered as body 0 and the basis vectors $(\hat{\mathbf{x}}_0, \hat{\mathbf{y}}_0, \hat{\mathbf{z}}_0)$ are attached to it rigidly. These basis vectors constitute the frame $\mathcal{F}_0$. This frame coincides with the inertial frame in the zero configuration. Two rotors, labeled as bodies 1 and 2, are mounted on body 0 with axes along $\hat{\mathbf{x}}_0$ and $\hat{\mathbf{z}}_0$, respectively. It is assumed that the center of mass of body 1 is on $\hat{\mathbf{x}}_0$ and the center of mass of body 2 is on $\hat{\mathbf{z}}_0$. The secondary body, rotor 3, is labeled as body 3 and is hinged to body 0 along the axis $\hat{\mathbf{y}}_0$. It is assumed that the center of mass of body 3 lies on $\hat{\mathbf{y}}_0$.

The orientation of body 0 in the inertial frame is described by a body-fixed rotation sequence 1-2-3 with angles ψ, θ, and ϕ. The relative orientations of rotors 1 and 2 along their respective axes are denoted by θ_1 and θ_2, while the relative orientation of body 3 with respect to 0 is labeled as θ_3. Given the specific body-fixed rotation sequence, the angular velocity of the body 0 is given by the following equation:

$$^{\mathcal{F}_0}\omega_0 = \begin{bmatrix} \omega_{01} \\ \omega_{02} \\ \omega_{03} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & \sin\phi & 0 \\ -\cos\theta\sin\phi & \cos\phi & 0 \\ \sin\theta & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} \tag{1}$$

The inertial angular velocity of bodies 1, 2, 3 have the following expressions in $\mathcal{F}_0$

$$\begin{aligned} ^{\mathcal{F}_0}\omega_1 &= ^{\mathcal{F}_0}\omega_0 + \dot{\theta}_1\hat{\mathbf{x}}_0 \\ ^{\mathcal{F}_0}\omega_2 &= ^{\mathcal{F}_0}\omega_0 + \dot{\theta}_2\hat{\mathbf{z}}_0 \\ ^{\mathcal{F}_0}\omega_3 &= ^{\mathcal{F}_0}\omega_0 + \dot{\theta}_3\hat{\mathbf{y}}_0 \end{aligned} \tag{2}$$

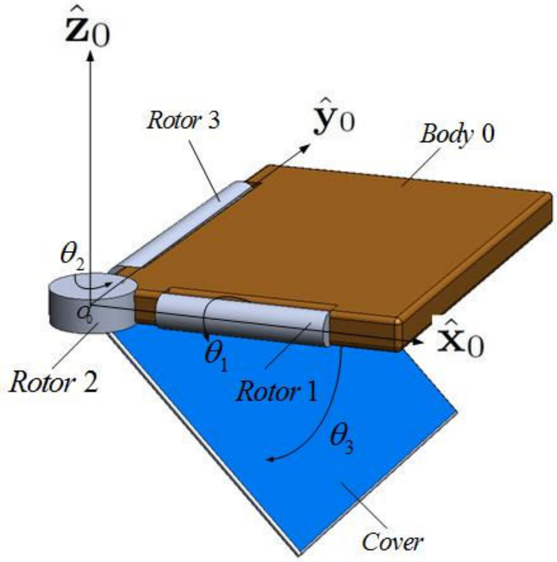

Fig. 1. A schematic of the twin free-falling bodies.

B. Angular Momentum Equations

The center of mass of the system is denoted as C_*. With the given assumptions of center of mass for bodies 0-3, the center of mass C_* is a fixed point on body 0. The total angular momentum $\mathbf{H}_{C_*}$ for the system, consisting of 0, 1, 2, and 3, about C_* can be written as

$$\mathbf{H}_{C_*} = \sum_{i=0}^{3} \mathbf{I}_i \cdot \omega_i + \mathbf{r}_{C_*C_i} \times m_i \frac{d}{dt}\mathbf{r}_{C_*C_i}, \tag{3}$$

where $\mathbf{I}_i$ is the inertia dyadic of body i, ω_i is the angular velocity, and $\frac{d}{dt}\mathbf{r}_{C_*C_i}$ is the velocity of the center of mass of body i. In general, a vector $\mathbf{r}_{C_*C_i} = \alpha_{i1}\hat{\mathbf{x}}_0 + \alpha_{i2}\hat{\mathbf{y}}_0 + \alpha_{i3}\hat{\mathbf{z}}_0$, where α_{i1}, α_{i2}, α_{i3} are constants specific to a body i. Hence,

$$\frac{d}{dt}\mathbf{r}_{C_*C_i} = ^{\mathcal{F}_0}\omega_0 \times (\alpha_{i1}\hat{\mathbf{x}}_0 + \alpha_{i2}\hat{\mathbf{y}}_0 + \alpha_{i3}\hat{\mathbf{z}}_0) \tag{4}$$

In the absence of external forces and moments, the angular momentum of the system around the system center of mass C_* remains inertially fixed. For example, if the system was released from rest, $\mathbf{H}_{C*} = 0$. On expressing $\mathbf{H}_{C_*}$ in the coordinate frame $\mathcal{F}_0$ and equating to zero, one obtains the following form of the governing angular momentum equations

$$A \begin{bmatrix} \omega_{01} \\ \omega_{02} \\ \omega_{03} \end{bmatrix} + B \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = 0, \tag{5}$$

where A and B are constant matrices. On solving for $(\omega_{01}, \omega_{02}, \omega_{03})^T$ from Eq. (5) and substituting in Eq. (1), the governing equations of motion can be expressed in the following form

$$\begin{aligned} \dot{\psi} &= \bar{f}_{11}(\theta,\phi)\dot{\theta}_1 + \bar{f}_{12}(\theta,\phi)\dot{\theta}_2 + \bar{f}_{13}(\theta,\phi)\dot{\theta}_3 \\ \dot{\theta} &= \bar{f}_{21}(\theta,\phi)\dot{\theta}_1 + \bar{f}_{22}(\theta,\phi)\dot{\theta}_2 + \bar{f}_{23}(\theta,\phi)\dot{\theta}_3 \\ \dot{\phi} &= \bar{f}_{31}(\theta,\phi)\dot{\theta}_1 + \bar{f}_{32}(\theta,\phi)\dot{\theta}_2 + \bar{f}_{33}(\theta,\phi)\dot{\theta}_3 \end{aligned} \tag{6}$$

It is important to point out that the form of Eq. (6) is possible if and only if the following two conditions are met: (i) the matrix A is invertible, and (ii) the determinant of matrix in Eq. (1) is nonzero, i.e., $\cos\theta \neq 0$ or $\theta \neq \pi/2$ or $3\pi/2$.

C. Problem Statement

The initial orientation of the twin body is described by the four variables $\psi(t_0)$, $\theta(t_0)$, $\phi(t_0)$, and $\theta_3(t_0)$ and a final orientation $\psi(t_f)$, $\theta(t_f)$, $\phi(t_f)$, and $\theta_3(t_f)$ before hitting the ground at time t_f is prescribed. The control inputs are the two rotor speeds $\dot{\theta}_1$, $\dot{\theta}_2$ and the hinge speed $\dot{\theta}_3$. The statement of the problem is to choose trajectories of $\theta_1(t)$, $\theta_2(t)$, $\theta_3(t)$ such that the four orientation variables $\psi(t)$, $\theta(t)$, $\phi(t)$, and $\theta_3(t)$ transfer from their initial values at t_0 to the given values at t_f.

A few quick notes about this problem are: (i) the governing equations are under-actuated, i.e., the three speeds $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\theta}_3$ are required to manipulate four variables $\psi(t)$, $\theta(t)$, $\phi(t)$, and $\theta_3(t)$, (ii) The governing equations (6) are non-holonomic, i.e., are non-integrable, (iii) The relative angle of the twin body θ_3 does not explicitly appear in the governing equations (6) since it was assumed that the center of mass of twin body 3 lies on the axis $\hat{\mathbf{y}}_0$. It will be shown later that this condition is critical to solving the problem via the differential flatness theory, (iv) The falling duration $t_f - t_0$ is characterized by the initial distance of the system center of mass C_* from the ground, since this point has free-fall. It is possible that the actual falling time may be smaller than $t_f - t_0$ as it will be dictated by the actual shape and size of the twin body.

III. Structure of Angular Momentum Equations

Eq. (6) can be rewritten in the following form

$$\begin{aligned}
\dot{\psi} &= f_{11}(\theta,\phi)u_1 + f_{12}(\theta,\phi)u_2 + f_{13}(\theta,\phi)u_3 \\
\dot{\theta} &= u_1 \\
\dot{\phi} &= u_2 \\
\dot{\theta}_3 &= u_3,
\end{aligned} \tag{7}$$

on making the following subsititution $\bar{f}_{21}(\theta,\phi)\dot{\theta}_1 + \bar{f}_{22}(\theta,\phi)\dot{\theta}_2 + \bar{f}_{23}(\theta,\phi)\dot{\theta}_3 = u_1$, $\bar{f}_{31}(\theta,\phi)\dot{\theta}_1 + \bar{f}_{32}(\theta,\phi)\dot{\theta}_2 + \bar{f}_{33}(\theta,\phi)\dot{\theta}_3 = u_2$ and $\dot{\theta}_3 = u_3$. The functions $f_{11}(\theta,\phi)$, $f_{12}(\theta,\phi)$, $f_{13}(\theta,\phi)$ are a results of the above input transformation. A condition that needs to be satisfied in order to carry out this input transformation is $\bar{f}_{21}\bar{f}_{32} - \bar{f}_{31}\bar{f}_{22} \neq 0$.

Some salient features of these governing equations are: $f_{ij}(\theta,\phi)$ are independent of the base angle ψ and the momentum wheel rotation angles θ_1 and θ_2. The intuitive justifications for these are: (i) the first coordinate ψ is a cyclic coordinate, hence it does not appear in the equation, (ii) Due to symmetry of the two rotors along the axis, the angular momentum equations remain unaffected by the angular position of these wheels, (iii) The same argument holds for the variable θ_3.

A. Differential Flatness

A driftless system with 4 states and 3 inputs is differentially flat if and only if it is controllable. For the system given by Eqs. (7), the controllability condition formally is to check if the dimension of

$$< X_1, X_2, X_3, \{[X_i, X_j], i,j = 1,\ldots,3\} > \tag{8}$$

is equal to 4, where $[X_i, X_j]$ denotes Lie bracket of two vector fields. The vector fields X_1, X_2, X_3 are

$$X_1 = \begin{pmatrix} f_{11}(\theta,\phi) \\ 1 \\ 0 \\ 0 \end{pmatrix},\ X_2 = \begin{pmatrix} f_{12}(\theta,\phi) \\ 0 \\ 1 \\ 0 \end{pmatrix},$$
$$X_3 = \begin{pmatrix} f_{13}(\theta,\phi) \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Due to the special structure of the vector fields, it is differentially flat if and only if there exists i, j such that

$$\frac{\partial f_{1j}(\theta,\phi)}{\partial x_i} - \frac{\partial f_{1i}(\theta,\phi)}{\partial x_j} \neq 0, \tag{9}$$

where $(x_1, ..., x_4) = (\psi, \theta, \phi, \theta_3)$. This is a straightforward verification.

B. Choice of Flat Outputs and Diffeomorphism

Assuming that the system is controllable, a way to obtain the flat outputs is to consider a 'one prolongation' of the two inputs u_1 and u_2. It should be noted that differential flat outputs are not unique. The prolonged system has the form

$$\begin{aligned}
\dot{\psi} &= f_{11}(\theta,\phi)u_1 + f_{12}(\theta,\phi)u_2 + f_{13}(\theta,\phi)u_3 \\
\dot{\theta} &= u_1 \\
\dot{\phi} &= u_2 \\
\dot{\theta}_3 &= u_3 \\
\dot{u}_1 &= w_1 \\
\dot{u}_2 &= w_2
\end{aligned} \tag{10}$$

This new system with 6 states and 3 inputs w_1, w_2, and u_3 should be static feedback linearizable. A sufficient condition to show this property is to choose three flat outputs y_1, y_2, y_3 whose relative degrees together add to the order of this prolonged system, in this case 6. A choice of linearizing outputs for the system (10), possessing this property, is

$$\begin{aligned}
&y_1 = \theta,\ y_2 = \phi, \\
&y_3 = \psi - f_{13}(\theta,\phi)\theta_3.
\end{aligned} \tag{11}$$

Differentiation of these outputs with respect to time gives

$$\dot{y}_1 = u_1, \quad \dot{y}_2 = u_2, \quad \ddot{y}_1 = w_1, \quad \ddot{y}_2 = w_2 \tag{12}$$

$$\begin{aligned}
\dot{y}_3 &= \dot{\psi} - \dot{f}_{13}(\theta,\phi)\theta_3 - f_{13}(\theta,\phi)\dot{\theta}_3 \\
&= f_{11}(\theta,\phi)u_1 + f_{12}(\theta,\phi)u_2 - \dot{f}_{13}(\theta,\phi)\theta_3 \\
&= [f_{11} - \frac{\partial f_{13}(\theta,\phi)}{\partial\theta}\theta_3]u_1 + \\
&\quad [f_{12} - \frac{\partial f_{13}(\theta,\phi)}{\partial\phi}\theta_3]u_2
\end{aligned} \tag{13}$$

Hence, $\dot{y}_3$ can be differentiated once more before the inputs w_1, w_2, u_3 appear in its expression. On differentiating $\dot{y}_3$ once more, the three outputs can be written as follows:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \lambda_1 & \lambda_2 & \lambda_3] \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \lambda_4 \end{bmatrix} \tag{14}$$

where

$$\begin{aligned}
\lambda_1 &= f_{11} - \tfrac{\partial f_{13}}{\partial\theta}\theta_3 \\
\lambda_2 &= f_{12} - \tfrac{\partial f_{13}}{\partial\phi}\theta_3 \\
\lambda_3 &= -[\tfrac{\partial f_{13}}{\partial\theta}u_1 + \tfrac{\partial f_{13}}{\partial\phi}u_2] \\
\lambda_4 &= \dot{f}_{11}u_1 + \dot{f}_{12}u_2 - \theta_3[\tfrac{d}{dt}\{\tfrac{\partial f_{13}}{\partial\theta}\}u_1 + \tfrac{d}{dt}\{\tfrac{\partial f_{13}}{\partial\phi}\}u_2]
\end{aligned}$$

The relative degree of all three outputs is 2. In order to achieve an input transformation, the $(3,3)$ element of the decoulping matrix should be nonzero, i.e., $\frac{\partial f_{13}}{\partial\theta}u_1 + \frac{\partial f_{13}}{\partial\phi}u_2 \neq 0$ at every point on the trajectory. The three outputs together have the relative degree of 6, the order of the system (10). Hence, y_1, y_2, and y_3 provide the three outputs that will statically feedback linearize the system (10). The static feedback linearized system is described by three double integrator plants.

TABLE I
INITIAL AND FINAL PHYSICAL STATES VALUES IN DEGREE/SEC OR DEGREE

Variable	Initial	Final
t	0 sec	0.8 sec
$(\psi, \theta, \phi, \theta_3)$	$(30, 20, 20, 5)$	$(0, 0, 0, 10)$
$(\dot{\psi}, \dot{\theta}, \dot{\phi}, \dot{\theta}_3)$	$(*, 0.0573, 0.0573, 0.0573)$	$(*, 0.0573, 0.0573, 0.0573)$
$(\ddot{\psi}, \ddot{\theta}, \ddot{\phi}, \ddot{\theta}_3)$	$(*, 0, 0, 0)$	$(*, 0, 0, 0)$

The diffeomoprphism has the following expressions in terms of outputs and their derivatives.

$$\theta = y_1, \quad \phi = y_2$$

$$\theta_3 = -\frac{\dot{y}_3 - f_{11}(y_1, y_2)\dot{y}_1 - f_{12}(y_1, y_2)\dot{y}_2}{[\frac{\partial f_{13}(y_1,y_2)}{\partial y_1}\dot{y}_1 + \frac{\partial f_{13}(y_1,y_2)}{\partial y_2}\dot{y}_2]} \qquad (15)$$

$$\psi = y_3 + f_{13}(y_1, y_2)\theta_3(y_1, y_2, \dot{y}_1, \dot{y}_2, \dot{y}_3) \qquad (16)$$

This diffeomorphism is valid if the denominator of Eqn. (15) is different from 0. This is consistent with the nonzero condition of Eq. (9).

IV. TRAJECTORY PLANNING AND OPTIMIZATION

Given the initial orientation of the twin body $\psi(t_0)$, $\theta(t_0)$, $\phi(t_0)$, $\theta_3(t_0)$, the final orientation $\psi(t_f)$, $\theta(t_f)$, $\phi(t_f)$, $\theta_3(t_f)$ is selected so that the free-falling twin bodies can land on the ground safely. Trajectory planning is performed to provide continuous motion between the two points before hitting the ground at time t_f.

The method outlined in Section III is illustrated using the model shown in Fig. 1 with the design assumptions. The bodies 1, 2 and 3 are cylinders. We modify the mass distribution of body 3 to place its center of mass oon the axis $\hat{\mathbf{y}}_0$. The mass and geometric parameters are: body 0 length 0.32m, width 0.22m, and height 0.012m. The lengths of rotors 1, 2 and 3 are 0.12m, 0.012m and 0.22m respectively. The radii of rotors 1 and 3 are 0.006m and for rotor 2 is 0.06m. The mass of body 0 is 0.8kg, while the masses of rotors 1, 2 and 3 are 0.2kg, 0.1kg and 0.3kg respectively. These parameters and the inertia matrices were selected based on a typical laptop.

The initial and final values of the planned state variables are given in Table I. These were arbitrarily chosen to show that the presented methodology can steer the system from the initial states to the desired final states. Note that the initial and final values marked with a '$*$' , e.g. $\dot{\psi}$ cannot be independently prescribed due to the nonholonomic constraint given in Eq. (6). Therefore they are not listed in Table I. For this specific case, $\dot{\psi}(t_0) = 0.0019$, $\dot{\psi}(t_f) = 0.0052$, $\ddot{\psi}(t_0) = 3.47 \times 10^{-6}$, $\ddot{\psi}(t_f) = 2.49 \times 10^{-6}$.

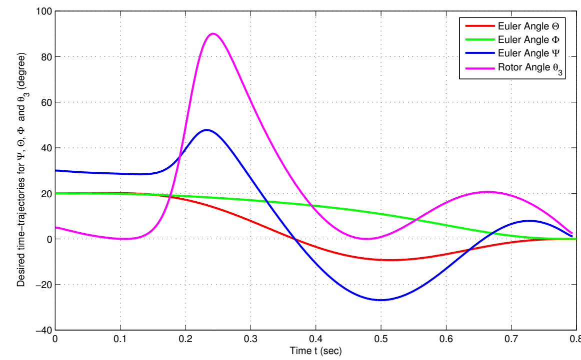

Fig. 2. Desired trajectory of the angles of the twin body between a start and end postures: $\psi(t)$, $\theta(t)$, $\phi(t)$, and $\theta_3(t)$.

The underlying planning method is to use Eq. (11) to transform the given initial and final conditions in the flat-space. Then, smooth polynomials are used to connect these points in the flat output space. Finally, the planned trajectories in the flat output space are then transformed back to the original space via Eqs. (15) and (16). Note that the start and final values of Euler angles as well as angular velocities are selected to be different from zero to avoid singularities. In order to address geometric or saturation constraints, an optimization is performed within the family of admissible trajectories connecting the initial and final points. For example, in the problem posed in Fig. 1, we assume that θ_3 is limited to $(0, \pi/2)$ and all angular speeds are limited to 60rad/sec.

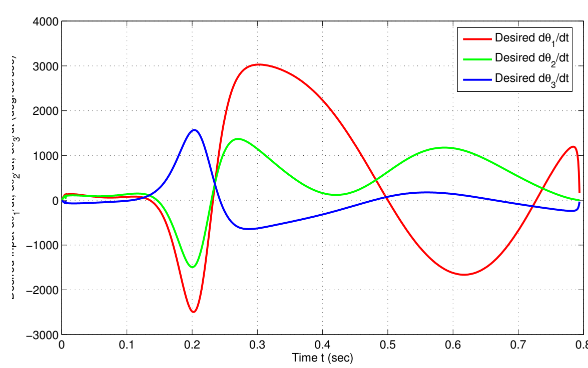

Fig. 3. Desired trajectory of the rotor speed : $\dot{\theta}_1(t)$, $\dot{\theta}_2(t)$ and $\dot{\theta}_3(t)$.

In our example, we choose the admissible class of flat output trajectories as $y_k(t) = p_{k1}(t) + t^3(t - t_f)^3 p_{k2}(t)$, $p_{k1}(t) = \sum_{i=0}^{5} \alpha_{ki} t^i$ and $p_{k2}(t) = \sum_{i=0}^{4} \beta_{ki} t^i$, and $t_f = 0.8$. With this choice, $p_{k1}(t)$ ensures that the flat output trajectory passes through the start and final values at the given end times. $p_{k2}(t)$ is a polynomial with free parameters that can be optimized to satisfy auxiliary constraints. Note that $t^3(t - t_f)^3$ and its two derivatives are zero at start and end times. The free parameters in $p_2(t)$ do not change the specified end conditions of the trajectory. To ensure θ_3

to be within limits, the planning problem was formulated numerically as a nonlinear optimization problem and solved by a SQP-based routine ‘fmincon’ available in MATLAB. Trajectory planning and optimization results are shown in Figs 2-3.

As seen in these simulations, the twin-body system can be steered by these rotors to move from their initial to the prescribed final posture. Additionally, θ_3 satisfies the specified limits.

V. Controller Design

This section focuses on the issue of controller design via differential flatness. Since the three flat outputs y_1, y_2, y_3 satisfy Eq. (15), via an input feedback

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{17}$$

the system becomes three double integrator plants. On defining $e_i = y_{id} - y_i$, $i = 1, 2, 3$, the control laws can be chosen in the form

$$v_i = \ddot{y}_{id} + K_{vi}\dot{e}_i + K_{pi}e_i, \quad i = 1, 2, 3 \tag{18}$$

where the control gains K_{vi} and K_{pi} are chosen to ensure stability. Simulations are shown in Figs. 4-7 for $K_{vi} = 40$ and $K_{pi} = 400$, $i = 1, 2, 3$. These results show that this controller can remove any initial errors successfully.

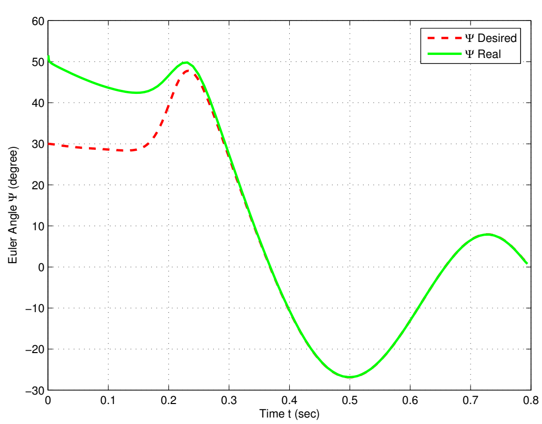

Fig. 4. Response of the controller to initial errors: $\psi(t)$

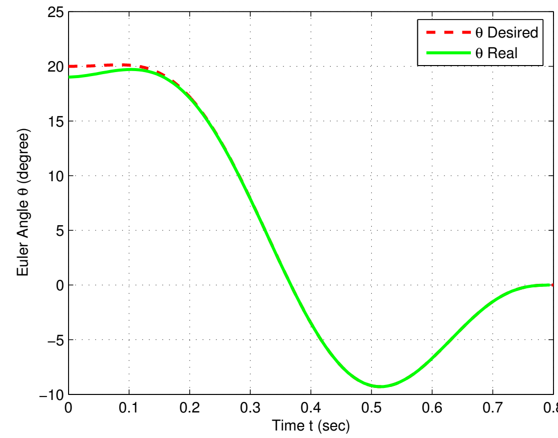

Fig. 5. Response of the controller to initial errors: $\theta(t)$

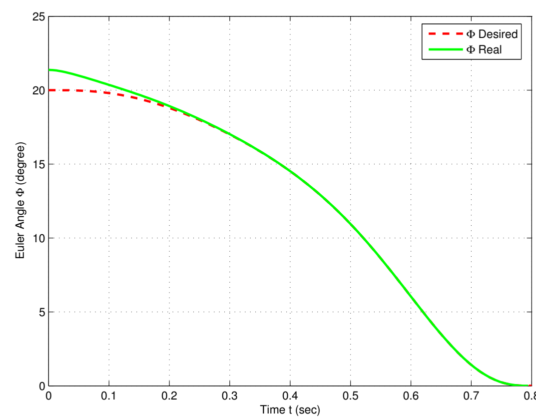

Fig. 6. Response of the controller to initial errors: $\phi(t)$

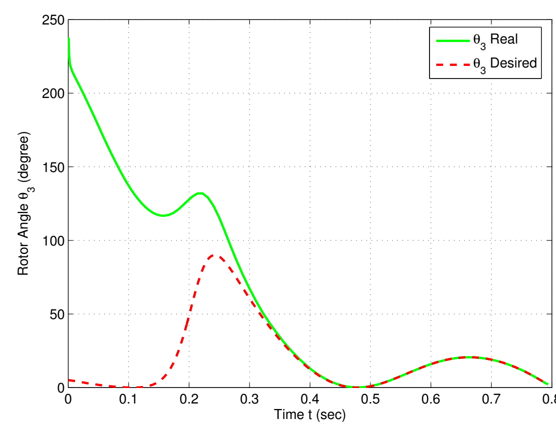

Fig. 7. Response of the controller to initial errors: $\theta_3(t)$

VI. Conclusions

This study examined posture control of a system of free-falling twin bodies using the framework of differential flatness. The motivation is to capture the physics of falling of common electronic devices with the twin body model, such as laptops and cell phones. These systems are described by four degrees-of-freedom to account for full three dimension orientation during falls. We assume that the system possesses three actuators to control the orientation. In this study, a special mass distribution was assumed within the system that allowed the system to be dynamically feedback linearizable, via one prolongation of two inputs. A diffeomorphism between the state space and flat output space was built using governing angular momentum conservation equations. Trajectory planners and controllers were developed to steer the system from an initial posture to a final posture while satisfying auxiliary constraints with initial errors. In the near future, we propose to experimentally verify these results.

REFERENCES

[1] T. Kawamura, K. Yamafuji, and T. Kobayashi, "Study on posture control and soft landing of a free falling robot. (1st report, posture control by turning motion of a cat)", *Transactions of the Japan Society of Mechanical Engineers*, Part C, vol. 57, no. 544, 1991, pp. 3895-3900.

[2] H. Kanai and K. Yamafuji,"Posture change and jumping motion of the controlling twin-leg-type mobile robot". *Transactions of the Japan Society of Mechanical Engineers*, Part C, vol. 57, no. 539, 1991, pp. 2336-2431.

[3] K. Yamafuji, K. Honda, and T. Kobayashi, "Midair Posture Detection and Landing Control of a Robot with Multiarticulated Twin Legs." *Transactions of the Japan Society of Mechanical Engineers*, Part C, vol. 59, no. 565, 1993, pp. 2780-2787.

[4] C. K. Chen and N. Sreenath, "Control of Coupled Spatial Two-Body System with Nonholonomic Constraints".*Proceedings of the IEEE Conference on Decision and Control*, vol. 2, 1993, pp. 949-954.

[5] C. Fernandes, L. Gurvits, and Z. Li, "Near-optimal Nonholonomic Motion Planning for a System of Coupled Rigid Bodies", *IEEE Transactions on Automatic Control*, vol. 39, no. 3, pp. 450-463, March 1994.

[6] Z. Weng and H. Nishimura, "Final-state Control of A Two-link Cat Robot by Feedforward Torque Inputs", *Proceedings of International Workshop on Advanced Motion Control (AMC)*, 2000, pp. 264-269.

[7] Y. Yang, C. Yang, P. Chao and C. Sung, "Landing Posture Control of a Generalized 5-RDOF Twin-body System via Methods of Input-output Linearization and PD Control", *Proceedings of the 46th IEEE Conference on Decision and Control*, pp. 1681-1686, New Orleans, LA, USA, Dec. 12-14, 2007.

[8] Y. Yang, P. Chao, and C. Sung, "Landing Posture Control via Trajectory Planning for a Generalized Twin-Body System", *Proceedings of the 4th International Conference on Autonomous Robots and Agents*, pp. 97-102, Wellington, New Zealand, Feb 10-12, 2009.

[9] Y. Yang, P. Chao, and C. Sung, "Landing Posture Control for a Generalized Twin-Body System Using Methods of Input-Output Linearization and Computed Torque", *IEEE/ASME Transactions on Mechatronics*, vol. 14, no. 3 pp. 326-336, June 2009.

[10] S. Guo, T. Fukuda, and K. Asaka, "A New Type of Fish-Like Underwater Microrobot", *IEEE/ASME Transactions on Mechatronics*, vol. 8, no. 1, pp. 136-141, March 2003.

[11] S. K. Agrawal, K. Pathak, J. Franch, R. Lampariello and G. Hirzinger, "A Differentially Flat Open-Chain Space Robot with Arbitrarily Oriented Joint Axes and Two Momentum Wheels at the Base",*IEEE Transactions on Automatic Control*, vol. 54, no. 9, pp. 2185-2191, September, 2009.

[12] Bradford Engineering, "Space Systems and Components Division: Reaction Wheel Unit", http://www.bradford-space.com/pdf/be_datasheet_rwu_sep2006.pdf, 2007.