Scan-Based SLAM with Trajectory Correction in Underwater Environments

Antoni Burguera, Gabriel Oliver and Yolanda González

Abstract—This paper presents an approach to perform Simultaneous Localization and Mapping (SLAM) in underwater environments using a Mechanically Scanned Imaging Sonar (MSIS) not relying on the existence of features in the environment. The proposal has to deal with the particularities of the MSIS in order to obtain range scans while correcting the motion induced distortions. The SLAM algorithm manages the relative poses between the gathered scans, thus making it possible to correct the whole Autonomous Underwater Vehicle (AUV) trajectories involved in the loop closures. Additionally, the loop closures can be delayed if needed.

The experiments are based on real data obtained by an AUV endowed with an MSIS, a Doppler Velocity Log (DVL) and a Motion Reference Unit (MRU). Also, GPS data is available as a ground truth. The results show the quality of our approach by comparing it to GPS and to other previously existing algorithms.

I. INTRODUCTION

Nearly all advanced mobile robotic tasks require some knowledge of the robot location in the environment. For example, those tasks involving the robot to reach a specific target require knowledge about the current robot pose in order to plan a path to the goal. Also, exploration tasks require some estimate of the robot location in order to decide whether a specific region has been already visited by the robot or not. The problem of computing the robot pose is known as the mobile robot localization problem or localization for short.

The most successful approach to estimate the robot pose is the Simultaneous Localization and Mapping (SLAM) [1], [2]. Although a wide variety of SLAM algorithms exist, the most common SLAM implementations are based on feature maps composed of straight lines and corners. This approach has proved to be robust and accurate in structured terrestrial [3] and underwater [4], [5] environments. However, the use of feature maps reduces the scenarios where the robot can be used. This problem is especially important in underwater scenarios where man made, structured, environments are uncommon.

Some studies propose the use of raw range data in SLAM not assuming any type of feature in the environment [6]. A recent study [7] has shown the feasibility of SLAM using raw range information in underwater environments using an Autonomous Underwater Vehicle (AUV) endowed with a Mechanically Scanned Imaging Sonar (MSIS).

When used to perform SLAM based on raw range readings, an MSIS has two important problems. Firstly, this sensor does not provide range scans but acoustic images. Hence, the sensor information has to be processed before being used in this context. A simple yet effective method to perform such process has been described by [5]. Secondly, the scan time of an MSIS is not negligible. For example, in our particular configuration, the sensor needs more than 13 seconds to gather a 360° scan. As a consequence, it can not be assumed that the robot remains static while the scan is being obtained. Some considerations regarding this issue using terrestrial Polaroid sensors are provided in [8]. Moreover, a recent study by [9] shows the feasibility of underwater localization based on matching MSIS scans.

This paper proposes a SLAM approach based on scans gathered by an MSIS sensor. The problem is summarized in section II. In order to obtain range information from the MSIS acoustic profiles, a beam segmentation process is proposed in section III. Section IV states our proposal to build the scans taking into account the motion-induced distortion. The proposed scan-based SLAM is described in section V. This description focuses on an observation function that explicitly takes into account the whole set of robot motions involved in a loop. It also describes the measurement model, which is based on scan matching. The experimental results are provided in section VI. The results, which are based on real data gathered by an AUV, show the benefits of the presented approach by comparing it with other previously existing methods. Finally, section VII concludes the paper.

II. PROBLEM STATEMENT

The experiments conducted in this paper have been performed using the sensor data gathered by the Ictineu AUV. This AUV was designed and developed at the University of Girona (see [5] for more details). Among other sensors, the AUV is endowed with a Doppler Velocity Log (DVL) which measures the velocities of the unit with respect to bottom and water, a Motion Reference Unit (MRU) that provides absolute attitude data by means of compass and inclinometers, and an MSIS.

The MSIS obtains 360° scans of the environment by rotating a sonar beam through 200 angular steps in about 13.8 seconds. At each angular position, the sensor provides a set of 500 values named bins. These values represent a 50 m long echo intensity profile with a resolution of 10 cm. Each of these sets of 500 bins will be referred to as a beam.
By accumulating this information, an acoustic image of the environment can be obtained.

As stated previously, different problems arise when using an MSIS to perform scan-based localization. In order to solve them several processes are necessary. Our proposal is summarized in Figure 1. First, range information is extracted from each MSIS measurement by means of the beam segmentation. Also, DVL and MRU readings are fused by means of an Extended Kalman Filter (EKF) to obtain dead reckoning estimates, as described by [5]. Both the obtained range information and the dead reckoning estimates are stored in two buffers, called readings history and transformations history respectively. When the MSIS has obtained a 360° view of the environment, the information in these buffers is used by the scan building process to compensate the robot motion and build the scan $S_k$. Also, the scan builder computes the motion $x_k$ between the previously gathered scan and the current one. For the particular case of the first obtained scan, $x_1$ denotes the robot motion from the start position to the first scan itself. The scans are stored in the so called scan history and the motions are used to augment the SLAM state vector. Afterwards, the new scan $S_k$ is matched against the scans in the scan history to perform the SLAM state update in an Iterated Extended Kalman Filter (IEKF) framework. This state update exploits the relations between consecutively gathered scans, similarly to the top level in the Hierarchical SLAM approach [10]. Finally, the robot pose with respect to a fixed coordinate frame, $x^p_k$, is computed by the pose extraction process. The rest of the paper is devoted to describing the abovementioned processes.

### III. BEAM SEGMENTATION

Our goal is to obtain range scans from the beams as they are provided by the MSIS. Accordingly, the beam segmentation is in charge of computing the distance from the sensor to the largest obstacle in the beam. In some cases, this distance corresponds to the bin with the largest intensity value. However, in some other, very frequent, situations, the distance can not be computed in such way. To deal with these situations, the following procedure is proposed. Firstly, when the MSIS provides a new beam, the first bins are discarded as they correspond to echoes produced by the sonar casing. Afterwards, the beam segmentation process obtains the corresponding range measurement by means of the following three steps:

**Thresholding**: An echo intensity threshold is dynamically selected as follows. Firstly, the histogram of echo intensities corresponding to the beam under analysis is computed and smoothed. Afterwards, the threshold is located at the largest echo intensity value that locally minimizes the smoothed histogram. In this way, the threshold separates two clearly defined areas in the echo intensity space. Finally, those bins whose intensity is below the threshold are discarded. Figure 2-a exemplifies the thresholding step.

**Erosion**: The remaining bins are eroded. That means that those bins that, after the thresholding, do not have another bin in their immediate neighborhood, are removed. The purpose of this step is to remove spurious bins.

**Selection**: At this point, it is usual that a single cluster of bins remains. The bin with the largest echo intensity value is selected, and the distance corresponding to this bin represents the range value for the beam under analysis. Let the point corresponding to this range be named the range reading.

The results of selecting the maximum intensity bin and those of applying the method proposed in this paper are exemplified in Figures 2-b and 2-c respectively. It is clear that our approach is able to obtain a much more accurate range scan than a simple maximum intensity selection.

### IV. SCAN BUILDING

The MSIS data cannot be treated as a synchronous snapshot of the world. Instead, the sonar data is actually acquired whilst the AUV is moving. Thus, the robot motions during the sonar data acquisition have to be taken into account in order to correct the induced distortion. The scan building process epitomizes this idea.
The range readings provided by the beam segmentation constitute the range information used to build the scans. Our proposal is to model each measurement in a scan by a normal distribution. In that way, the scans not only hold information about the place where an obstacle has been detected but also about the uncertainty in this detection.

Let \( r_t = N(\hat{r}_t, \sigma^2_t) \) denote a measurement obtained at time \( t \) in form of a random Gaussian variable (RGV). Let this measurement be represented with respect to a coordinate frame centered on the MSIS and having the \( x \) axis aligned with the beam acoustic axis at time \( t \). In this case, the mean vector has the form \( \hat{r}_t = [\rho_t, 0]^T \), where \( \rho_t \) denotes the range reading provided by the beam segmentation at time \( t \). The obtention of the covariance matrix \( \sigma^2_t \) is out of the scope of this paper. Some details regarding this issue are available in [8].

Let \( z_t \) represent the measurement \( r_t \) with respect to the robot coordinate frame. It is straightforward to obtain \( z_t \) from \( r_t \) and the MSIS beam angle at time \( t \). For the sake of simplicity, henceforth the \( z_t \) will be referred to as the sonar reading.

The sonar readings have to be stored in the so called readings history so that they can be easily accessed by the scan building process. The readings history at time \( t \) contains the most recent \( N \) sonar readings gathered until time \( t \). It is defined as follows:

\[
RH_t = \{z_{t-N+1}, ..., z_{t-2}, z_{t-1}, z_t\} \tag{1}
\]

The value of \( N \) has to be decided so that \( RH_t \) can store one full 360° scan. In our particular sensor configuration a full MSIS scan is composed of 200 beams. Thus, \( N \) is set to 200.

Let \( \bar{x}_t \) denote the robot motion from time step \( t-1 \) to time step \( t \). This robot motion is modeled as a RGV and is provided by the dead reckoning EKF. Similarly to the readings history, let the transformations history be defined as a history of the most recent \( N \) robot motions. That is,

\[
TH_t = \{\bar{x}_{t-N+1}, ..., \bar{x}_{t-2}, \bar{x}_{t-1}, \bar{x}_t\} \tag{2}
\]

As the AUV is moving while acquiring the scan, each reading in \( RH_t \) may have been obtained at a different robot pose. The goal of the scan building process is to represent each reading in one scan with respect to a common coordinate frame.

Let us denote by \( z_{i,j} \) the measurement \( z_i \in RH_t \) represented with respect to the robot pose at time \( j \), where \( t - N + 1 \leq i \leq t \) and \( t - N + 1 \leq j \leq t \), being \( t \) the current time step. \( z_{i,j} \) can be computed as follows:

\[
z_{i,j} = \begin{cases} z_i, & i = j \\ \bar{x}_{j+1} \oplus \bar{x}_{j+2} \oplus ... \oplus \bar{x}_i \oplus z_i, & j < i \\ (\ominus \bar{x}_j) \oplus (\ominus \bar{x}_{j-1}) \oplus ... \oplus (\ominus \bar{x}_{i+1}) \oplus z_i, & i < j \end{cases} \tag{3}
\]

where the operators \( \oplus \) and \( \ominus \) denote the compounding and inversion operations commonly used in the context stochastic mapping.

The robot motions involved in the Equation 3 are those in \( TH_t \). Hence, by means of this equation, each reading in \( RH_t \) can be represented with respect to any coordinate frame referenced in \( TH_t \) while taking into account the robot motion. Next, it has to be decided which coordinate frame choose to build a scan. The chosen coordinate frame corresponds to the central position of the trajectory followed by the robot when collecting the readings involved in the scan. The central position has been chosen for two main reasons. On the one hand, because of the similarity to the scans generated by a laser range finder. On the other hand, in order to reduce the maximum uncertainty of each reading with respect to the reference frame. Thus, every time the MSIS performs a 360° scan, \( S_k \) is built as follows:

\[
S_k = \{z_{i,t}, \forall i, t - N + 1 \leq i \leq t\} \tag{4}
\]

where \( t_c \) corresponds to the time step at which the robot was at the central position of the trajectory followed while the MSIS acquired the scan.

Figure 3-a illustrates the result of the scan builder by the raw range data before and after the scan building. Additionally, 3-b shows the acoustic image corresponding to the corrected scan overlayed to a satellite view to show the effects of the correction.

The RGV \( x_k = N(\bar{x}_k, P_{xk}) \), which denotes the relative pose of the coordinate frame of the current scan \( S_k \) with respect to the coordinate frame of the previous scan \( S_{k-1} \), can be easily computed from the dead reckoning estimates. As stated previously, the value of \( x_1 \) (i.e. no previous scan is available) denotes the relative position of the first scan frame with respect to the robot’s starting pose. In the context of this paper, \( \bar{x}_k \) is a vector of the form \([x, y, \theta]\) denoting displacement and rotation.

Every time a new scan \( S_k \) and the associated robot motion \( x_k \) are available a new SLAM step can be executed.

V. SCAN-BASED SLAM

Our scan-based approach is based on EKF concepts. The state vector contains the relative positions between consecutively gathered scans. That is, the mean \( \bar{X}_k \) of the state vector \( X_k \) after gathering the k-th scan is

\[
\bar{X}_k = [\bar{x}_1, \bar{x}_2, ..., \bar{x}_k]^T \tag{5}
\]

![Fig. 3. (a) Range data before and after the scan building process. (b) Acoustic image of one MSIS scan after the scan building process.](image-url)
and the state vector covariance is denoted by $P_k$.

The robot pose with respect to an earth-fixed coordinate frame could be included in the state vector. However, for the sake of simplicity, we have decided to build a state vector containing only the map. As the map stores relative poses between consecutively obtained scans, the pose of the most recent scan with respect to the robot’s starting pose is $\hat{x}_1 \oplus \hat{x}_2 \oplus \ldots \oplus \hat{x}_k$, where the $\hat{x}_k$ come from the state vector. Additionally, all the information needed to compute the current robot pose with respect to the last scan is available in the transformations history. Using this data the current robot pose with respect to the first robot pose, $x^W_1$, can be easily computed by the pose extraction process (see Figure 1).

A. Prediction and state augmentation

As stated previously, the state vector holds information about the relative scan poses. Under the assumption of a static environment, the EKF prediction step does not change the state vector. That is, $X^-_k = X^+_{k-1}$.

After the execution of the scan building process, both the scan $S_k$ and the relative motion $x_k$ are available. The scan is stored in the scans history $SH_k$ so that

$$SH_k = \{S_1, S_2, \ldots, S_k\} \quad (6)$$

The relative motion $x_k = N(\hat{x}_k, P_{xk})$ is used to augment the state. The mean of the augmented state vector is $\hat{X}_k = [X_{k-1}, \hat{x}_k]^T$. At this point, the relative motion is independent to the previous ones. Thus, the covariance is constructed as follows:

$$P_k = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\vdots & \vdots & \vdots \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix} \quad (7)$$

B. Data association

The measurement update step is performed at the scan level. Thus, the first step is to determine which ones of the stored scans sufficiently overlap with the most recent scan. Our proposal is to take this decision based on a proximity criteria. Although our implementation uses euclidean distance, other distances could be used.

The displacement and rotation from a scan $S_i \in SH_k$, with $i < k$, to $S_k$ can be estimated from the state vector as follows:

$$\delta(i, X_k) = \hat{x}_{i+1} \oplus \hat{x}_{i+2} \oplus \ldots \oplus \hat{x}_k \quad (8)$$

Thus, the euclidean distance $d(i, k)$ from the coordinate frame of $S_i$ to the one of $S_k$ is the norm of the $[x, y]$ components of $\delta(i, X_k)$. The proposal of this paper is, similarly to [7], to select as possible associations those scans in $SH_k$ that are at an euclidean distance below a certain threshold $\gamma$. Let the associated scans set $AS_k$ be defined as the set of possible associations as follows:

$$AS_k = \{i | d(i, k) < \gamma\} \equiv \{a_1, a_2, \ldots, aM\} \quad (9)$$

C. The measurement model

In order to measure the displacement and rotation between each of the associated scans, a scan matching technique is used. This paper proposes the use of the sonar probabilistic Iterative Correspondence (spIC) because it has shown to be a reliable, stand-alone, scan matching algorithm both using terrestrial ultrasonic range finders [8] and underwater MSIS [11]. Moreover, the spIC has also been successfully applied to underwater SLAM [7]. The explanation of the spIC algorithm is out of the scope of this paper. The reader is directed to the aforementioned studies to have a full description of the algorithm.

The spIC is executed for each $S_i$ with $i \in AS_k$ to estimate the displacement and rotation between $S_i$ and $S_k$. These scan matching estimates constitute the measurements and will be denoted by $z_{spIC, i}$.

The observation function $h_i$ is in charge of predicting the spIC measurement corresponding to $S_i$ from the state vector $X_k^-$. In other words, the observation function estimates the displacement and rotation from $S_i$ to $S_k$ using the state vector. Thus, $h_i$ is as follows:

$$h_i(X^-_k) = \hat{x}_{i+1} \oplus \hat{x}_{i+2} \oplus \ldots \oplus \hat{x}_k \quad (10)$$

The computation time required to compute this function can be reduced if one takes into account that it is similar to Equation 8 and, thus, the results obtained when selecting the associated scans can be used here. As this observation function explicitly takes into account the whole chain of motions involved in each loop closure, the proposed approach is able to correct the whole robot trajectories involved in loops.

The observation matrix $H_i$ is

$$H_i = \frac{\partial h_i}{\partial X_k^-} \quad \begin{bmatrix}
\frac{\partial h_i}{\partial x_{1}} & \frac{\partial h_i}{\partial x_{2}} & \ldots & \frac{\partial h_i}{\partial x_{k}}
\end{bmatrix} \quad (11)$$

where $x_i$ denote the state vector components. It is straightforward to see that

$$H_i = \begin{bmatrix}
000 & \frac{\partial h_i}{\partial x_{i+1}} & \frac{\partial h_i}{\partial x_{i+2}} & \ldots & \frac{\partial h_i}{\partial x_{k}}
000 & \frac{\partial h_i}{\partial x_{i+1}} & \frac{\partial h_i}{\partial x_{i+2}} & \ldots & \frac{\partial h_i}{\partial x_{k}}
\end{bmatrix} \quad (12)$$

By applying the chain rule, the non-zero terms of this Equation are as follows:

$$\frac{\partial h_i}{\partial x_j} = \begin{bmatrix}
\frac{\partial h_i}{\partial x_{i+1} \oplus x_{i+2} \oplus \ldots \oplus x_j}
\end{bmatrix}_{x_k^-} \quad (13)$$
According to [10] this can be computed as follows:

\[
\frac{\partial h_i}{\partial x_j} \bigg|_{X_k} = J_1 \otimes \{g_j, \oplus g_j \otimes h_i\} |_{X_k} \cdot J_2 \oplus \{g_j \odot x_j, x_j\} |_{X_k}
\]

(14)

where \(J_1\) and \(J_2\) are the Jacobians of the composition of transformations [12] and

\[
g_j = x_{i+1} \oplus x_{i+1} \odot \ldots \odot x_j
\]

(15)

At this point, the measurements \(z_{spIC,i}\) coming from the scan matching and the observation function \(h_i\), as well as the observation matrix \(H_i\) are available for all \(i \in AS_k\). Let us build the measurement vector \(z_{spIC}\) considering all the associated scans as follows:

\[
z_{spIC} = \begin{bmatrix}
z_{spIC,a1} \\
z_{spIC,a2} \\
\vdots \\
z_{spIC,aM}
\end{bmatrix}
\]

(16)

where \(a_1, a_2, \ldots, a_M\) denote the items in \(AS_k\) (see Equation 9). The observation function \(h\) and the observation matrix \(H\) considering all the associated scans are

\[
h(X_k) = \begin{bmatrix}
h_{a1} \\
h_{a2} \\
\vdots \\
h_{aM}
\end{bmatrix}, \quad H = \begin{bmatrix}
H_{a1} \\
H_{a2} \\
\vdots \\
H_{aM}
\end{bmatrix}
\]

(17)

D. The update step

By means of \(z_{spIC}\), \(h\) and \(H\) the EKF-SLAM update step can be performed. However, the effects of the linearizations in the observation model may be problematic especially when closing large loops. In order to alleviate this problem, our proposal is not to use an EKF but an IEKF [13]. Roughly speaking, the IEKF consists on iterating an EKF and relinearizing the system at each iteration until convergence is achieved.

At the \(j-th\) iteration of the IEKF, the mean of the state vector is as follows:

\[
\hat{X}_{j+1} = \hat{X}_j + P_j H_j^T (z_{spIC} - h(X_0)) - P_j P_0^{-1} (\hat{X}_j - \hat{X}_0)
\]

(18)

where \(H_j\) denotes the observation matrix \(H\) evaluated at \(\hat{X}_j\) (i.e. each \(H_i\) in Equation 11 is evaluated at the value of the state vector in the previous IEKF iteration). The terms \(X_0\) and \(P_0\) denote the state vector and its covariance before starting the IEKF and \(P_{spIC}\) is a block diagonal matrix containing the scan matching covariances corresponding to the items in \(z_{spIC}\). The state vector covariance is updated by the IEKF by the following expression:

\[
P_j = P_0 - P_0 H_j^T (H_j P_0 H_j^T + P_{spIC})^{-1} H_j P_0
\]

(19)

When the IEKF achieves convergence, the obtained state vector constitutes the \(X^*_k\).

It is important to emphasize that this step only updates the items in the state vector involved in the detected loops. Thus, the matrix \(H\) in Equation 17 could be reduced by removing all the zero valued columns on the left side of the matrix and then updating only the part of the state vector involved in all the detected loops.

Moreover, the presented update step makes it possible to store different loops when they are detected and close them later simultaneously, not necessarily at each SLAM step [10]. Thanks to this, the loop closure can be delayed if the computational resources are not available at a certain time step. Also, the overall computational cost is reduced because, prior to the loop closing, the newly gathered scans are independent and those parts of the covariance matrix related to the new scans are block diagonal.

VI. EXPERIMENTAL RESULTS

The experimental data used to validate our underwater SLAM approach was obtained by [5] in an abandoned marina situated near St. Pere Pescador in the Costa Brava (Spain). A satellite view of this environment is available in [14]. The Ictineu AUV was teleoperated along a 600m trajectory at an average speed of 0.2m/s. The trajectory includes a small loop as well as a 200m long straight path. The gathered data included measurements from the DVL, the MRU and the MSIS. Additionally, a buoy with a GPS was attached to the robot in order to obtain the ground truth.

Figure 4-a shows the trajectories provided by dead reckoning (DVL+MRU) and the GPS. Also, the sonar readings are plotted according to the dead reckoning trajectory for visual inspection. The problems of dead reckoning can be appreciated. For example, the entrance to the canal is misaligned (i.e. the loop is not closed) due to the drift error.

The results provided by the scan based SLAM are shown in Figure 4-b together with the ground truth provided by the GPS. It can be observed how the SLAM trajectory is very similar to the ground truth. The loop at the entrance to the canal has been detected and closed. Also, no significant differences between the GPS and the SLAM positions appear at the end of the trajectory, after more than 200 meters without revisiting known areas. The ellipses shown in Figure 4-b correspond to the \(2\sigma\) bounds for some scans positions with respect to the first robot pose after the whole SLAM process. It can be observed that the \(2\sigma\) bounds are smaller on the left side of the image where different loop closures are performed.

The absolute errors corresponding to dead reckoning and SLAM are shown in Figure 4-c. The represented error corresponds to the distance from the estimated pose, both for dead reckoning and SLAM, to the GPS ground truth.

Finally, the proposed scan based SLAM approach has been compared to three scan matching approaches. On the one hand, the underwater sonar Iterative Closest Point (usICP) [11], which is an improvement of the well known and widely used ICP algorithm [15] to be used underwater with a MSIS. On the other hand, the presented approach has been compared to the MSIS probabilistic Iterative Correspondence
The results are summarized in Table I in terms of the mean and the standard deviation of the error as well as the maximum error. The improvements with respect to all the tested methods are significant, both in terms of maximum error and in terms of mean and standard deviation of the error.

VII. CONCLUSION

This paper presents an approach to perform SLAM in underwater environments using an MSIS not relying on the existence of features in the environment. The proposal deals with the particularities of the MSIS in order to obtain range scans while correcting the motion induced distortions. The SLAM algorithm manages the relative poses between the gathered scans, thus making it possible to correct the whole AUV trajectories involved in the loop closures. Additionally, the loop closure can be delayed if needed.

The experiments are based on real data obtained by an AUV endowed with an MSIS, a DVL and a MRU. Also, GPS data is available as a ground truth. The results show the quality of our approach by comparing it to GPS and to other previously existing algorithms.

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REFERENCES