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Robust Nonlinear Real-time Control Strategy to Stabilize a PVTOL Aircraft in Crosswind

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***Abstract*— A robust control strategy to stabilize a PVTOL aircraft in the presence of crosswind is proposed in this paper. The approach makes use of Robust Control Lyapunov Functions (RCLF) and Sontag's universal stabilizing feedback. A nonlinear dynamic model of the aircraft taking account the crosswind has been developed. Likewise, a robust nonlinear control strategy is proposed to stabilize the PVTOL aircraft using RCLF, and we have employed the Riccati equation's parameters to compute and tune it in real-time. To validate the proposed control strategy, various simulations have been carried out. The controller has been also applied in real-time to a PVTOL prototype undergoing crosswinds. The experimental results show the good performance of the control algorithm.**

I. INTRODUCTION

The problem of controlling the Planar Vertical Take-Off and Landing aircraft (PVTOL) has been the object of study for many researchers worldwide. It is due to the fact that the PVTOL's dynamics keep the main features of a real aircraft but with a minimal number of states and inputs. Several control strategies have been described in the literature to solve the control problem for PVTOL aircraft. The PVTOL aircraft represents the most simple model of a plane and also the longitudinal model of an helicopter. This makes it a suitable test-bed for researchers, teachers and students working on flying vehicles. Indeed, the PVTOL aircraft is non-minimum phase system since the linearized system possesses an unstable zero dynamics that comes from the coupling between the roll moment and the lateral acceleration of the aircraft [3].

In [3], an approximate input-to-output linearization method is proposed to achieve the bounded tracking and asymptotic stability for the V/STOL (Vertical/ Short Take Off and Landing) aircraft. Fantoni *et al.* [7] introduced a control algorithm for the PVTOL aircraft based on the forwarding technique. This approach has allowed the design of a Lyapunov function insuring asymptotic stability. In [1], [4], [8], [9], [10] control strategies taking into account (arbitrary) bounded inputs have been developed, by using embedded saturations functions. Some of them have permitted to obtain global asymptotic stability of the origin in closed-loop.

Wood et al. [11] have introduced an extension of approaches found in [6], [5], with an optimal state feedback, for the case where the aerodynamical forces cannot be neglected. Recently, a nonlinear control scheme using a feedback law that casts the system into a cascade structure and proved its global stability has been proposed in [13]. Global stabilization was also achieved by Ye *et al.* [12] through a saturated control technique by previously transforming the PVTOL dynamics into a chain of integrators with nonlinear perturbations.

Nevertheless, only a few studies about robust control of this aircraft in the presence of wind can be found in the literature. The main goal of this paper is to present a robust control strategy to stabilize the PVTOL aircraft in crosswind using Robust Control Lyapunov Functions (RCLF) and Sontag's universal stabilizing feedback. Significant advances in nonlinear control systems, using the Control Lyapunov Functions (CLF) and RCLF, had been presented in the last decades, for example, it can be cited [16], [18], [21], [17]. However, only a relative few experimental results have been reported [15], [19] and [20], because of difficulties associated either with solving the Hamilton Jacobi Isaacs partial differential equations or with proposing a CLF or a RCLF to guarantee good performance for applications (it is not obvious). Moreover, we consider the effort of illustrating the applicability of Control Lyapunov Function to practical problems is very important. In our case, the idea is to chose a simple RCLF. Additionally, we use the equation's Riccati parameters to tune the controller and thus obtain a good performance in the practical system. We then demonstrate the effectiveness of the controller not only by simulations but also by experimental evaluation on a PVTOL prototype aircraft.

The outline of this paper is as follows: the nonlinear model of the PVTOL aircraft, in presence of wind, is introduced in Section II. The methodology for designing the robust nonlinear controller using the inverse optimality approach is developed in Section III. Section IV contains the simulation results of the performance of the proposed control laws in presence of wind. Furthermore, a validation of the proposed control algorithm in real-time experiments, is shown in Section V. Finally, the conclusions are discussed in Section VI.

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II. DYNAMICAL MODEL OF THE AIRCRAFT IN PRESENCE OF CROSSWIND

The PVTOL aircraft is an underactuated system, since it possesses two inputs, u_1, u_2, and three degrees of freedom, (x,y,ϕ), and that it moves on a plane. The PVTOL aircraft is composed of two independent motors which produce a force and a moment on the vehicle. The main thrust, u_1, is the sum of each motor thrust. The roll moment, u_2, is obtained by the difference of motors angular velocities. In real conditions, the aircraft is generally exposed to crosswind. If the PVTOL is affected by a crosswind, the aircraft will be pushed over or rolled away from the wind. Consequently, this leads to include additional forces acting over each rotor, see Figure 1. These forces are due to the airflow generated by the lateral wind. It means that, the magnitude of these forces, is a function of the incoming lateral airflow coming from the wind, see Figure 2.

The induced wind speed in a propeller is defined as $V = \left(\frac{f}{2\rho A}\right)^{\frac{1}{2}}$, where f is the thrust generated by the propeller, ρ is the air density and A is the propeller area [14], [22]. The thrust, $f_{k_T} = f_k + f_{w_k}$, $k = 1,2$, could be expressed as (see Figure 2)

$$f_{k_T} = 2\rho A\hat{V}V_p \tag{1}$$

where V_p is the induced wind speed in the propeller and $\hat{V}$ is the total induced wind speed by the rotor and lateral wind. Moreover, $\hat{V} = \left[(V_w\cos\alpha + V_p)^2 + (V_w\sin\alpha)^2\right]^{\frac{1}{2}}$, where α is the angle between the rotor axis and the lateral wind axis, with $\phi = 0°$ and a wind coming from the right in the x-axis, $\alpha = 90°$, see Figure 2. It is important to notice that, without lateral wind, $V_w = 0$, then this gives $\hat{V} = V_p$, $f_{w_k} = 0$, and (1) becomes $f_{k_T} = f_k = 2\rho AV_p^2$; $\quad \forall\ k = 1,2$, which represents the classical equation of induced wind speed in a propeller.

The dynamical model of the PVTOL aircraft, in presence of crosswind, can be obtained from Figure 1 and using

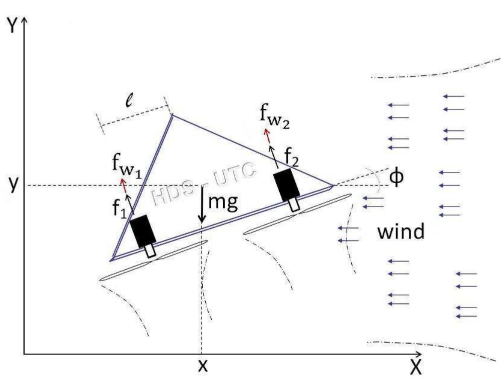

Fig. 1. The PVTOL aircraft in presence of crosswind.

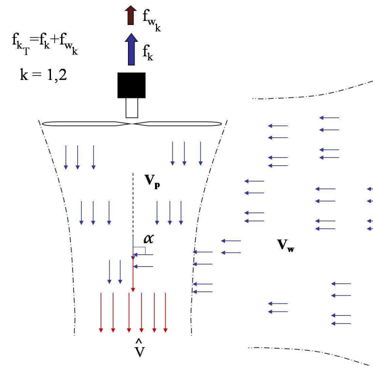

Fig. 2. Analysis of the rotor with crosswind.

Newton - Euler's approach,

$$\begin{aligned}
m\ddot{x} &= -(f_{1_T} + f_{2_T})\sin(\phi) + \varepsilon(f_{1_T} - f_{2_T})l\cos(\phi)\\
m\ddot{y} &= (f_{1_T} + f_{2_T})\cos(\phi) + \varepsilon(f_{1_T} - f_{2_T})l\sin(\phi) - mg\\
I\ddot{\phi} &= (f_{1_T} - f_{2_T})l
\end{aligned}$$

where x,y denote the horizontal and the vertical position of the aircraft's center of mass, ϕ is the roll angle of the aircraft made with the horizon, m is the total mass of the aircraft, g is the gravitational acceleration, l is the distance between the rotor and the aircraft's center of mass and I is the moment of inertia. $f_{1_T} = f_1 + f_{w_1}$ and $f_{2_T} = f_2 + f_{w_2}$ are the total forces produced by the thrust of the motors f_1 and f_2 and the forces due to the wind, f_{w_1} and f_{w_2}, in each motor. The parameter ε is a small coefficient which characterizes the coupling between the new rolling moment (the normal rolling moment with crosswind) and the lateral acceleration of the aircraft. This coefficient, ε, is very small $\varepsilon << 1$, not always well-known, and also neglected. Thus, neglecting ε and normalizing the mass, the moment of inertia and the gravity, the simplified model is

$$\ddot{x} = -u_1\sin(\phi) - w_1\sin(\phi) \tag{2a}$$

$$\ddot{y} = u_1\cos(\phi) + w_1\cos(\phi) - 1 \tag{2b}$$

$$\ddot{\phi} = u_2 + w_2 \tag{2c}$$

where $u_1 = (f_1 + f_2)$ and $u_2 = (f_1 - f_2)l$ are the main thrust and the roll moment respectively, and $w_1 = (f_{w_1} + f_{w_2})$ and $w_2 = (f_{w_1} - f_{w_2})l$ represents the disturbances due to the crosswind. Notice that, if there is no wind, that is, $f_{w_k} = 0$, consequently, $w_k = 0$, $\forall\ k = 1,2$, and the above reduces to the classical simplified dynamic model of the PVTOL aircraft [3].

III. CONTROL STRATEGY

The goal of this section is to propose a control strategy, using Robust Control Lyapunov Functions (RCLF), to stabilize the PVTOL in presence of wind. In order to apply this approach, it is more convenient to have the equilibrium point of the system at the origin. From (2) it can be observed that this is not the case, hence, the following change of variables is proposed: $u_1 \cong v_1 + 1$. Therefore, (2) yields

$$\ddot{x} = -(v_1+1)\sin(\theta) - \sin(\theta)w_1 \tag{3a}$$
$$\ddot{y} = (v_1+1)\cos(\theta) + \cos(\theta)w_1 - 1 \tag{3b}$$
$$\ddot{\theta} = u_2 + w_2 \tag{3c}$$

The previous system can be also written as

$$\dot{\bar{\mathbf{x}}} = \begin{bmatrix} x_2 \\ -\sin(x_5) \\ x_4 \\ \cos(x_5)-1 \\ x_6 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\sin(x_5) & 0 \\ 0 & 0 \\ \cos(x_5) & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{u} + \begin{bmatrix} 0 & 0 \\ -\sin(x_5) & 0 \\ 0 & 0 \\ \cos(x_5) & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{w}$$

or

$$\dot{\bar{\mathbf{x}}} = \mathbf{f_0}(\bar{\mathbf{x}}) + \mathbf{f_1}(\bar{\mathbf{x}})\mathbf{u} + \mathbf{f_2}(\bar{\mathbf{x}})\mathbf{w} \tag{4}$$

with $\bar{\mathbf{x}} = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T = [x\ \dot{x}\ y\ \dot{y}\ \theta\ \dot{\theta}]^T$, $\mathbf{u} = [v_1\ u_2]^T$, $\mathbf{w} = [w_1\ w_2]^T$.

Notice that, the system (4) has the form $\dot{\bar{\mathbf{x}}} = \mathbf{f}(\bar{\mathbf{x}}, \mathbf{u}, \mathbf{w})$, where $\bar{\mathbf{x}} \in \mathscr{X}$ is the state variable, $\mathbf{u} \in \mathscr{U}$ is the control input, $\mathbf{w} \in \mathscr{W}$ is the disturbance in a convex space. We are interested in proposing a control strategy to reject disturbances in the system, and we found from the literature that, Freeman and Kokotovic have introduced a control approach to compute control laws to reject disturbances using RCLF in a nonlinear system [2]. The control strategies obtained allow the rejection of perturbations in the system. This method can be summarized as follows (for more details, see [2], [16]):

Let

$$\dot{\bar{\mathbf{x}}} = \mathbf{f_0}(\bar{\mathbf{x}}) + \mathbf{f_1}(\bar{\mathbf{x}})\mathbf{u} + \mathbf{f_2}(\bar{\mathbf{x}})\mathbf{w} \tag{5}$$

be a non linear system in presence of disturbances, with some continuous functions $\mathbf{f_0}, \mathbf{f_1}, \mathbf{f_2}$, and V a RCLF satisfying the following assumptions

$$D(\bar{\mathbf{x}}, \mathbf{u}) := \max_{\mathbf{w} \in \mathscr{W}(\bar{\mathbf{x}})} \left[L_f V(\bar{\mathbf{x}}, \mathbf{u}, \mathbf{w}) + \alpha_v(\bar{\mathbf{x}})\right] \tag{6}$$
$$K(\bar{\mathbf{x}}) := \left\{\mathbf{u} \in \mathscr{U}(\bar{\mathbf{x}}) : D(\bar{\mathbf{x}}, \mathbf{u}) < 0\right\} \tag{7}$$

where $D : \mathscr{X} \times \mathscr{U} \longrightarrow \mathbb{R}$ and $K : \mathscr{X} \rightsquigarrow \mathscr{U}$. Then, the control strategy that stabilizes the system (5) is

$$\mathbf{u}(\bar{\mathbf{x}}) = \begin{cases} -\frac{\psi_0(\bar{\mathbf{x}})\psi_1(\bar{\mathbf{x}})}{\psi_1^T(\bar{\mathbf{x}})\psi_1(\bar{\mathbf{x}})} & \text{when} \quad \psi_0(\bar{\mathbf{x}}) > 0 \\ 0 & \text{when} \quad \psi_0(\bar{\mathbf{x}}) \leq 0 \end{cases} \tag{8}$$

where

$$\psi_0(\bar{\mathbf{x}}) := \nabla V(\bar{\mathbf{x}}) \cdot \mathbf{f_0}(\bar{\mathbf{x}}) + \|\nabla V(\bar{\mathbf{x}}) \cdot \mathbf{f_2}(\bar{\mathbf{x}})\| + \alpha_v(\bar{\mathbf{x}})$$
$$\psi_1(\bar{\mathbf{x}}) := [\nabla V(\bar{\mathbf{x}}) \cdot \mathbf{f_1}(\bar{\mathbf{x}})]^T$$

and

$$D(\bar{\mathbf{x}}, \mathbf{u}) = \psi_0(\bar{\mathbf{x}}) + \psi_1^T(\bar{\mathbf{x}})\mathbf{u}$$
$$K(\bar{\mathbf{x}}) = \left\{\mathbf{u} \in \mathscr{U} : \psi_0(\bar{\mathbf{x}}) + \psi_1^T(\bar{\mathbf{x}})\mathbf{u} < 0\right\}$$

where $\alpha_v(\bar{\mathbf{x}}) > 0$.

Observe that, (8) depends on α_v through the ψ_0–function. Note also that, there is no division by zero because the set $K(\bar{\mathbf{x}})$ is nonempty.

Remark: The function α_v represents the desired negativity of the Lyapunov derivative, and it can be adjusted to achieve a tradeoff between the control effort and the rate of convergence of the state to zero. □

The previous method was used to propose a control strategy to stabilize the system (4). Define $V(\bar{\mathbf{x}}) = \frac{1}{2}\bar{\mathbf{x}}^T P \bar{\mathbf{x}}$, where $P_{6\times6}$ is a symmetric and positive definite matrix. Therefore, $\nabla V(\bar{x}) = \begin{bmatrix}\nabla_{V_1} & \nabla_{V_2} & \nabla_{V_3} & \nabla_{V_4} & \nabla_{V_5} & \nabla_{V_6}\end{bmatrix}^T$, with $\nabla_{V_i} = \bar{\mathbf{x}}^T P_i$, where P_i is the i-th row of the matrix P.

To calculate $\psi_0(\bar{\mathbf{x}})$ and $\psi_1(\bar{\mathbf{x}})$, every term of these equations needs to be performed. Thus,

$$\nabla V(\bar{\mathbf{x}}) \cdot \mathbf{f_0}(\bar{\mathbf{x}}) = \nabla_{V_1}x_2 - \nabla_{V_2}\sin(x_5) + \nabla_{V_3}x_4 + \nabla_{V_4}\cos(x_5) + \nabla_{V_5}x_6$$

and

$$\nabla^T V(\bar{\mathbf{x}})\mathbf{f_2}(\bar{\mathbf{x}}) = \nabla V(\bar{\mathbf{x}}) \cdot \mathbf{f_2}(\bar{\mathbf{x}}) = \begin{bmatrix} -\nabla_{V_2}\sin(x_5) + \nabla_{V_4}\cos(x_5) \\ \nabla_{V_6} \end{bmatrix}$$

hence,

$$\|\nabla V(\bar{\mathbf{x}}) \cdot \mathbf{f_2}(\bar{\mathbf{x}})\| = \sqrt{\nabla_{V_6}^2 + \left[-\nabla_{V_2}\sin(x_5) + \nabla_{V_4}\cos(x_5)\right]^2}.$$

Consequently,

$$\psi_0(\bar{\mathbf{x}}) = \nabla_{V_1}x_2 - \nabla_{V_2}\sin(x_5) + \nabla_{V_3}x_4 + \nabla_{V_4}\cos(x_5) + \sqrt{\nabla_{V_6}^2 + \left[-\nabla_{V_2}\sin(x_5) + \nabla_{V_4}\cos(x_5)\right]^2} + \nabla_{V_5}x_6 + \alpha_v(\bar{\mathbf{x}})$$

Define, $\alpha_v(\bar{\mathbf{x}}) = \bar{\mathbf{x}}^T M \bar{\mathbf{x}}$, where $M_{6\times6}$ is a diagonal positive definite matrix, such that,

$$\alpha_v(\bar{\mathbf{x}}) = x_1^2 M_{11} + x_2^2 M_{22} + x_3^2 M_{33} + x_4^2 M_{44} + x_5^2 M_{55} + x_6^2 M_{66}$$

On the other hand, ψ_1 is given by

$$\psi_1(\bar{\mathbf{x}}) = \begin{bmatrix} -\nabla_{V_2}\sin(x_5) + \nabla_{V_4}\cos(x_5) \\ \nabla_{V_6} \end{bmatrix}$$

Moreover, $\mathbf{f_1} = \mathbf{f_2}$. Finally, the robust control law, $\mathbf{u}$, has the form

$$v_1(\bar{\mathbf{x}}) = \begin{cases} -\frac{\psi_0(\bar{\mathbf{x}})\left[-\nabla_{V_2}\sin(x_5) + \nabla_{V_4}\cos(x_5)\right]}{\left[-\nabla_{V_2}\sin(x_5) + \nabla_{V_4}\cos(x_5)\right]^2 + \nabla_{V_6}^2} & , \psi_0(\bar{\mathbf{x}}) > 0 \\ 0 & , \psi_0 \leq 0 \end{cases} \tag{9}$$

and

$$u_2(\bar{\mathbf{x}}) = \begin{cases} -\frac{\psi_0(\bar{\mathbf{x}})\nabla_{V_6}}{\left[-\nabla_{V_2}\sin(x_5) + \nabla_{V_4}\cos(x_5)\right]^2 + \nabla_{V_6}^2} & , \psi_0(\bar{\mathbf{x}}) > 0 \\ 0 & , \psi_0 \leq 0 \end{cases} \tag{10}$$

Remark: The disturbance on the system is the crosswind. In order to satisfy that the disturbance is in a convex space, we suppose that the crosswind can be approximated as a Sum of Squares (SOS) [23]. With this assumption the disturbances can be consider belonging on a convex space.

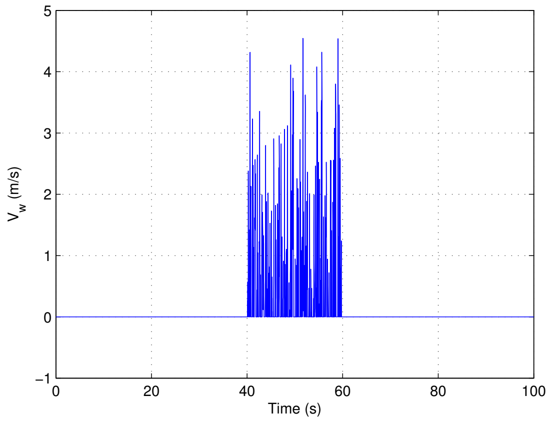

Fig. 3. Signal disturbance applied to the PVTOL.

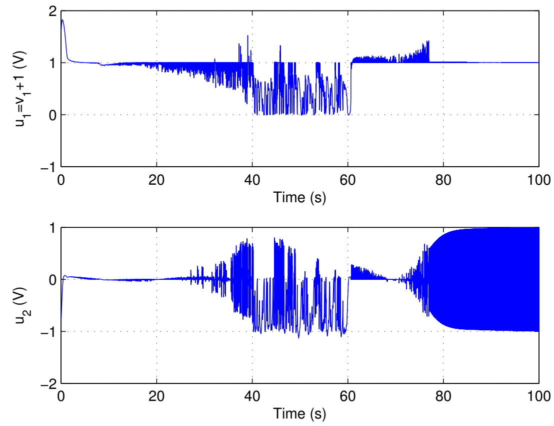

Fig. 5. Control inputs u_1 and u_2

IV. SIMULATION RESULTS

To validate the performance of the proposed control law, (9) and (10), in closed-loop, some simulations are carried out. The objective is to stabilize the PVTOL system in a desired position, even in presence of disturbances, i.e, $x \rightarrow x_d = 20$m, $y \rightarrow y_d = 15$m and $\phi \rightarrow \phi_d = 0$ rad.

To represent in simulation the disturbances due to the crosswind, a band-limited white noise has been used (see Fig. 3). In order to execute the control law, an appropriate matrix P, was found by linearizing the system (3) and by solving the algebraic Riccati equation. Hence, the obtained matrix P, is given by

$$P = \begin{bmatrix} 449 & 958 & 0 & 0 & -1168 & -707 \\ 958 & 3132 & 0 & 0 & -4538 & -3175 \\ 0 & 0 & 676 & 2236 & 0 & 0 \\ 0 & 0 & 2236 & 15120 & 0 & 0 \\ -1168 & -4538 & 0 & 0 & 8015 & 6773 \\ -707 & -3175 & 0 & 0 & 6773 & 8260 \end{bmatrix}$$

On the other hand, the matrix M was chosen such that, the tradeoff between the control effort and the rate of convergence was suitable to be applied in real time. The selected value is: $M = 1 \times 10^{-7}\ I_{6\times 6}$.

Figure 4 shows the performance of the system when applying the control strategies. Observe that, the control algorithms perform well even in presence of disturbances. In this figure, the solid line represents the system response and the dashed line the desired value. The control input responses, u_1 and u_2, are shown in Figure 5.

V. EXPERIMENTAL RESULTS

In this section, real-time experimental results when applying the control strategy to the PVTOL aircraft are described. The experimental platform is composed of a vehicle with two rotors moving on a sloping plan, a vision sensor to compute its position (x, y) and orientation (ϕ), and the Matlab XPC target system where the control laws are implemented (ground station), see Figure 6.

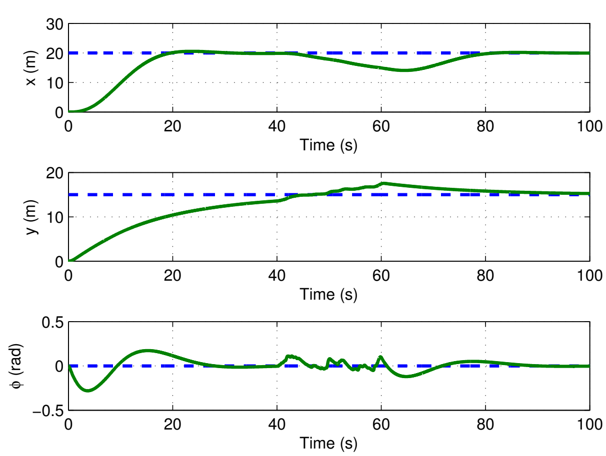

Fig. 4. System responses, x, y and ϕ, in closed-loop system when applying the controllers.

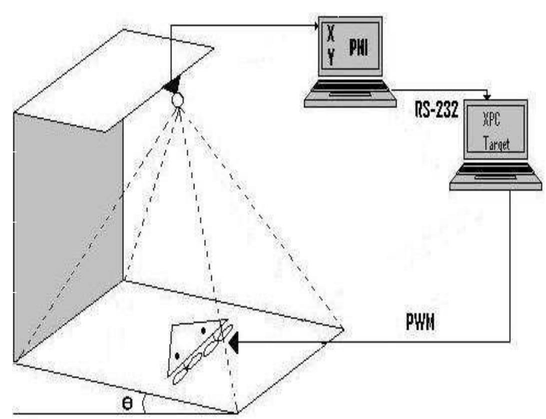

Fig. 6. Scheme of the real-time platform

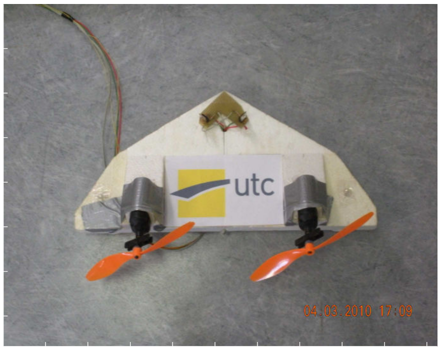

Fig. 7. The PVTOL prototype.

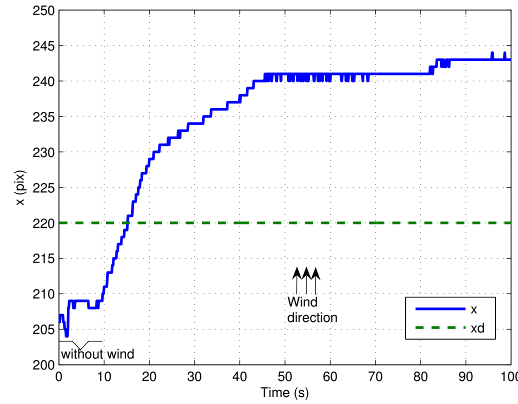

Fig. 9. x-position of the PVTOL aircraft

The vision sensor, composed of a fixed webcam, is located in a parallel plan above the PVTOL's workspace plan. The PVTOL is equipped with two powerful LEDs to help its localization with the camera. A photo of the prototype is shown in Figure 7. Measurements of the position and orientation are given in pixels (1 pix ≈ 0.55 cm), and they are sent to the ground station using XPC Target and the RS-232 serial communication, to compute the control inputs, see Figure 6. The control inputs are then sent to the PVTOL motors through Advantech PCL-726 output cards. The advantage of this platform is the great case of the implementation for control algorithms by using Simulink from Matlab. Indeed, the simulation files used can be used in the XPC Target's application for real-time experiments.

The control objective is to stabilize the aircraft in a desired position in presence of crosswind. For y–position a desired trajectory is proposed, while for x–position a constant desired value is chosen. For ϕ_d, the desired value is, at first, equal to zero but the angle will change in the opposite direction to the wind if a lateral displacement is measured. The idea is to apply a crosswind to the aircraft when it is moving to the desired values. The crosswind, in our experience, is generated by a fan and focused in the x–axis to perturb the (x,ϕ)–subsystem. The control algorithms, (9) and (10), were implemented using the simulation parameters, and only the states velocities were tuned in order to improve the performance of the control laws. Indeed, due to the fact that the vision sensor gives only position and orientation, the velocities were approximated using the 'derivative' Simulink block. This fact generates some errors when the velocity is estimated by the processor, because small variations in the position, give sometimes, big velocity values. Additionally, the calculated control inputs were scaled to improve the behavior of the system in practice.

Figures 8-12 show the experimental results when applying the control strategy to the aircraft in presence of wind. In these figures, the solid lines represent the system response and the dashed lines the desired values. Figure 8 shows the y-displacement following a small trajectory. The goal is, that, the aircraft reaches to the desired altitude, $y_{d_{max}} = 150$ pix (≈ 27.5 cm).

In Fig. 9, the x-displacement can be observed. In this experience $x_0 = 205$ pix and $x_d = 220$ pix. The wind comes from the right side with a velocity of 4m/s. Note in figure that, after 10 sec the crosswind was activated and then, the PVTOL moves in the x-axis. Notice also that, this displacement is slow and the control law reacts changing

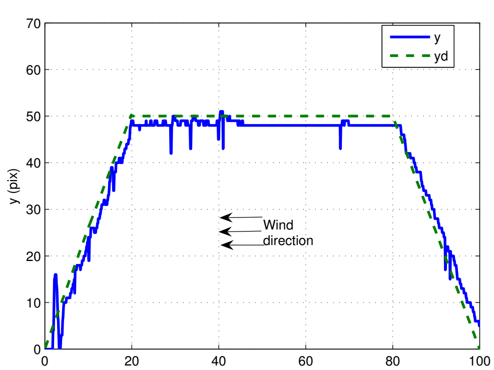

Fig. 8. y-position of the PVTOL aircraft

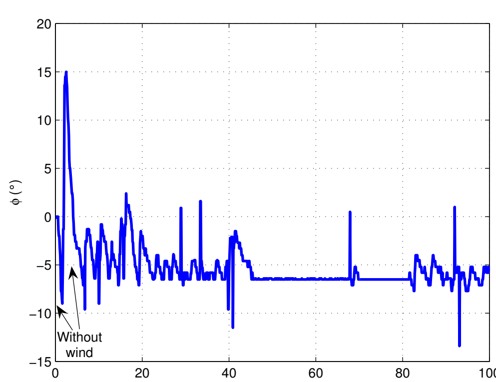

Fig. 10. ϕ-angle of the PVTOL aircraft

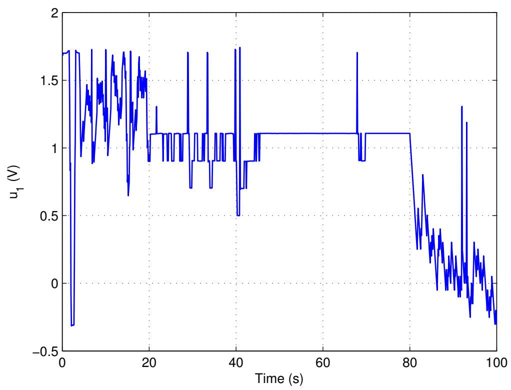

Fig. 11. Control input u_1

the roll angle (from 0° to -5°) to avoid this displacement, see Figure 10. In this figure, the performance of the ϕ angle is illustrated.

From these figures it can be observed that, although, there is a small displacement in x, the proposed control strategy has a good performance and the PVTOL remains stable with low-cost inputs, see Figures 11 and 12.

VI. CONCLUSIONS

In this paper, we have used RCLF to propose a control strategy to stabilize the PVTOL aircraft in presence of crosswind. The use of this approach is not new, but we have proved that, this method could be used in UAV control area to reject disturbances like crosswind. A new nonlinear model of the aircraft was obtained taking into account the disturbances produced by the lateral wind. The obtained algorithms were tested in simulations and also in real time. The results have demonstrated the good performance of the control laws in closed-loop system, even in presence of disturbances.

The future work is to extend these results to an rotorcraft moving in 3D and to apply the strategies, in real time, in an embedded control system.

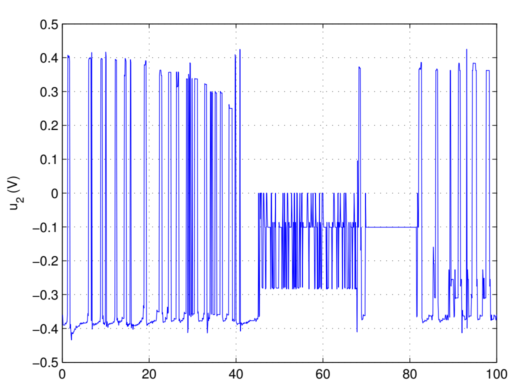

Fig. 12. Control input u_2

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