

An Adaptive Switching Behavior between Levy and Brownian Random Search in a Mobile Robot Based on Biological Fluctuation

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Abstract – Biological creatures, some of them are very simple, seem to perform efficient search strategy. Recent researches show that noise in their internal mechanism may have an important role to manage this behavior. This paper focuses on realizing a simple, noise utilizing, mathematical framework that enables a mobile robot to perform random search adaptively and efficiently under changing target density. Our approach is to model and implement bacterial movement based on a recent perspective of noise utilizing mechanism in living beings: biological fluctuation. As a result, the robot will adaptively switch its random search pattern between Levy walk and Brownian walk to increase the search efficiency in a patchy environment where the target density naturally alternates.

I. INTRODUCTION

SEARCH is an essential function of mobile robots, with many potential applications like cleaning, harvesting, or search and rescue. The general aim is to maximize the chance of finding the targets under certain constraints, such as the time elapsed or travelled distance. Hence, the search efficiency can be observed by comparing number of targets found to those constraints. With apriori information like the targets distribution, it is possible to calculate certain optimal strategy [1][2]. Without it, the robot can only do random search. Furthermore, the environment can change, so the search must be adaptive. While there are some researches on random search by mobile robots [3][4], none seem to focus on how to properly adapt the statistical properties of the search.

In nature, actually many predators do random search as they have to make foraging, searching for foods, decisions with little, if any, knowledge of present resource distribution and availability [5]. Interestingly, various creatures, some of them are very simple, seem to show efficient random search with proper statistical properties [5][9-10][12-14]. In relation with this, the question of what is the most efficient statistical strategy to optimize a random search has been addressed in [6]. It is shown that for sparse targets (i.e low target density), the efficiency, defined as number of targets found divided by the traveled distance, is maximized when the flight lengths follows power law distribution with a heavy tail: a Levy walk. However, Brownian walk, a

common random walk where the distribution of the flight lengths, the moving lengths between subsequent changes of direction, is not heavy tailed, is actually not a null model should be improved. While Levy walk is more efficient for a search with sparse, smaller and slower targets, the opposite conditions generally favor Brownian walk [7].

Recently, it is found that animal random search strategies can be subject to noise in the form of a presumably internally generated variability in animal's choice of movement. These phenomena are shown to exist in creatures as simple as zooplankton [9], or fruit-fly [10].

The aim of our research is to realize a simple mathematical framework that enables a mobile robot to perform random search adaptively and efficiently under changing target density. We are interested on the role of noise in biological creatures and focus on the simplest one: bacteria. Our approach is to model and implement bacterial movement to mobile robot, based on a recent perspective of noise-utilizing mechanism called the biological fluctuation [8][11]. The searching behavior is implemented in a patchy environment, where the target density naturally alternates. As a result, the robot will adaptively switch between Levy and Brownian walk pattern, able to increase the search efficiency compared to either one.

The organization of the paper is as follow. First, we will explain the fundamentals about Levy and Brownian walk along with examples occur in biological creatures. Then, we will focus on bacteria. After that, we will explain biological fluctuation and how an adaptive random search can be realized based on it. Simulation experiments are performed to confirm the validity of our approach. We will explain the experiment setup and condition, before discussing the result. At the end, the conclusion and future work will be explained.

II. RANDOM SEARCH IN BIOLOGICAL CREATURES

A. Levy and Brownian Walk

The term Levy flight is used to describe a specialized random walk in which the move steps are drawn from a probability distribution with a power-law tail [5][6]:

$$p(l) \approx l^{-\mu} \quad (1)$$

with $1 < \mu < 3$, and l is the flight length. It means that rare but extremely long flight lengths can happen in the random walk trajectory. Without tail truncation, sums of those flight lengths converge to a Levy stable distribution. For $\mu \geq 3$, there is no heavy tail in the distribution and the sums of the flight lengths converge to a Gaussian distribution due to the Central Limit Theorem, thus we recover Brownian walk.

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The case of $\mu \leq 1$ does not correspond to distributions that can be normalized.

The trajectory of a Levy flight comprises of walk clusters' of short flight lengths with longer reorientation jumps between them, repeated across all scales, with the resultant scale-invariant clusters creating trajectories with fractal patterns. To be exact, a technically correct term is actually Levy walk: essentially means Levy flight with time cost that depends on the flight lengths. Levy walk leads to anomalous diffusion, meaning that the mean squared displacement from the starting point increases faster than linearly with time t , while Brownian walk is a normal diffusion where the increase is linear. Mathematically, in order to create probability distribution whose tail follows (1), one can surely sample an approximation of Levy stable distribution [16]. However, it is not the only way. In [7], a transformation method is used to generate power law random variables from uniform distribution. In [17], it is shown that fractional Brownian motion, a generalized form of Brownian motion with correlation time, can also cause a similar distribution.

In [10][12][13], it is shown that trajectory of animals like fruit flies, marine predators, and spider monkeys fit a Levy walk pattern. More interestingly, zooplankton switches from Brownian to Levy walk trajectory as the resource availability becomes sparse [14]. The underlying mechanism on how a Levy walk trajectory is generated in those creatures is considered as an interesting topic. For example, in spider monkeys, memory about profitable target locations that keeps them to move forward for long periods seems to be the explanation [13].

As will be explained further, we concentrate on Levy walk trajectory found in bacteria, whose key mechanism is long correlation time [15]. It means that the mechanism can be said as a family of fractional Brownian motion. However, here we do not differentiate the term based on the underlying process and focus on the trajectory. We simply use the term "Levy walk" to describe a trajectory pattern with heavy tailed power law distribution, and "Brownian walk" for trajectory pattern without such heavy tail. It must also be noticed that the trajectory are not generated by a Levy process. Therefore, like common assumption about the Levy walk trajectory found in animals, the flight length distribution has a large, but finite variance, or in essence, truncated [5]. It means after an extreme long period, the sum of the flight lengths will actually converge to a Gaussian distribution as well, causing the random search pattern to become a Brownian walk.

B. Bacterial Levy Walk and The Role of Noise

In bacteria, such as *Escherichia coli*, the motion can be characterized as a sequence of smooth - swimming runs, punctuated by intermittent tumbles that effectively randomize the direction of the next run [18]. The switching probability between the two modes is dictated by measurement of attractant chemical gradient in the environment, obtained from comparison of current and past concentration. When the bacterium perceives conditions to be worsening, the tendency to tumble is enhanced and vice versa. As a result, when the bacterium runs up a gradient of attractant, it will do chemotaxis, a biased random walk

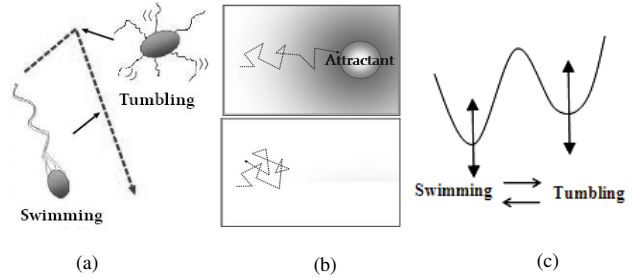


Fig. 1 Bacterial movement: (a) the two basic modes (b) a biased random walk with attractant, and random walk without any (c) the energy barrier model of switching probability between the two modes without attractant

toward the source. However, in the absence of this attractant, the bacterium will simply do random walk (Fig 1.b).

Bacterial movement attracts a lot of attentions in mobile robot researches in the context of realizing chemotaxis behavior, a simple biased random walk mechanism for searching gradient sources [19][20]. However, here we focus on target density, and concentrate on the random walk part.

In recent researches, unlike the conventional expectation that the swimming mode duration of *Escherichia coli* follows Poisson-like distribution in absence of gradient, a power law distribution is found [21]. A possible cause has been explained. It can be modeled that the switching probability between swimming and tumbling mode is an exponential function of conceptual energy barrier [22] (Fig. 1.c), whose level keeps changing due to Gaussian fluctuation of certain protein inside the bacteria. In [15], it is shown that power law switching between the swimming and tumbling mode, a Levy walk trajectory, can occur if this protein fluctuates with a long correlation time.

III. ADAPTIVE SWITCHING BETWEEN LEVY AND BROWNIAN WALK BASED ON BIOLOGICAL FLUCTUATION

A. The Principle

Recent researches show that certain noise utilizing mechanism called "biological fluctuation", or "Yuragi" in Japanese language, plays important role in various stages from molecules to brains in life sciences [8]. The mechanism is also found in bacteria adaptation to environmental changes by altering their gene expression when they are lack of certain nutrient. Based on this behavior, in [11] a simple model to explain the biological fluctuation was built. Here, the gene expression is modelled to be controlled by a dynamical system with some attractors. The model is also called "the attractor selection model" and represented by Langevin equation as:

$$\dot{x}(t) = -\nabla U(x(t))A(t) + \varepsilon(t) \quad (2a)$$

$$= f(x(t))A(t) + \varepsilon(t) \quad (2b)$$

where $x(t)$ and $f(x(t))$ are the state and the dynamics of the model at time t , with $f(x(t))$ can be designed to have some attractors in potential $U(x(t))$. $\varepsilon(t)$ is the noise term. $A(t)$ is a variable called "activity" which indicates the fitness of the state to the environment. From the equation, $f(x(t))A(t)$

becomes dominant when the activity is large, and the state transition approaches deterministic. When the activity is small, $\varepsilon(t)$ becomes dominant, and the state transition becomes more stochastic. The activity is therefore designed to be large when the state is suited to the environment and vice versa.

While the framework offers many possible designs, in this paper we focus on seeing whether the robot can adaptively adjust its random search property based on simple findings from the environment. To be exact, we would like to see whether the robot can adaptively switch between Levy walk pattern, an efficient strategy for sparsely placed, low density targets, and Brownian walk pattern, expected to be a favorable strategy for higher target density inside the patch. The realized searching behavior is explained as follow.

B. The Realized Searching Behavior

In order to implement the ‘‘Yuragi’’ equation in (2) for realizing bacterial based searching behavior, the first step is to properly choose the state of the attractor selection model. As in bacteria the probability of switching between the two modes is an exponential function of energy barrier whose level keeps changing due to fluctuation of certain chemical protein [15], a natural choice for the state of attractor selection model explained in (2) is a variable representing this chemical fluctuation, here simply called $z(t)$. Furthermore, to model the switching probability, one can draw a probabilistic state machine shown in Fig. 2 (center).

Here, ‘‘ P ’’ is the tumbling probability (i.e. switching from swimming to tumbling mode). The swimming mode is defined as moving forward with a certain distance, while tumbling means changing direction randomly. In the current paper, the swimming probability (i.e. switching from tumbling to swimming mode) is simply considered as 1. It is the simplest case which causes the robot to stop turning at the next time step. This suits our purpose, as in this paper we concentrate on the relationship between the search efficiency and the flight length distribution. For other focuses, for example to investigate the efficiency when it is assumed that it takes time to understand the environment [23], it may be necessary to change the consideration. However, such focus is outside the scope of this paper. In bacteria, the effect of the chemical fluctuation to the tumbling probability and its counterpart, the swimming

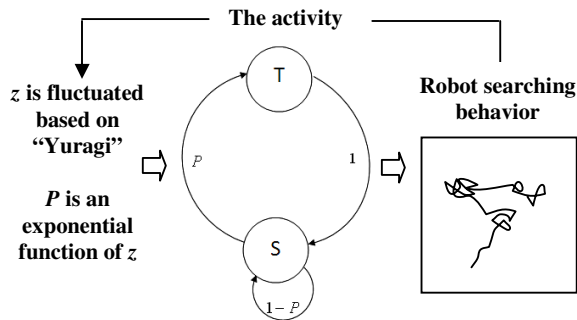


Fig.2 The principle of searching behavior based on biological fluctuation (‘‘Yuragi’’) model of bacterial movement.

bacterial motor [15]. As can be seen, for our purpose we concentrate on the fluctuation effect to the tumbling probability and how the flight length distribution can be controlled by changing the fluctuation correlation time.

Furthermore, to investigate whether adaptive switching between Levy and Brownian walk pattern can emerge, we design the dynamic of $z(t)$ following a simple unimodal potential function $U(z(t))$ shown below:

$$U(z(t)) = (z(t) - h)^2 \quad (3)$$

causing the dynamics of $z(t)$ shown in (4):

$$\dot{z}(t) = -\frac{dU(z(t))}{dz} A(t) + \varepsilon(t) \quad (4a)$$

$$= -2(z(t) - h)A(t) + \varepsilon(t) \quad (4b)$$

while, modeled from real bacteria, P at time t is assumed as an exponential function of $z(t)$:

$$P(t) = \exp(-z(t)) \quad (5)$$

The first term in (4) represents slow adaptation toward a preferred value of h , which corresponds to the attractor. The noise term, $\varepsilon(t)$ is zero mean Gaussian white noise, represents the stochastic driving force of the Gaussian internal protein fluctuation in bacteria. The activity $A(t)$ changes the shape of the potential $U(z(t))$ and correlation time of state $z(t)$, as the key mechanism of the approach. Following bacteria behavior, a Levy walk pattern in the robot trajectory supposes to happen when $z(t)$ fluctuates with a long term correlation. On the other hand, shorter correlation time supposes to realize a less correlated random walk, with the sum of those flight lengths converge to Gaussian distribution, i.e. a Brownian walk.

From (4a), it can be seen that small value of $A(t)$ will cause $U(z(t))A(t)$, potential $U(z(t))$ multiplied by $A(t)$, to be flat and variable $z(t)$ supposes to fluctuate with long correlation time. Therefore, Levy walk pattern in the robot trajectory with certain power law exponent in (1): $1 < \mu < 3$, supposes to be realized. When the activity $A(t)$ has a large value, the shape of $U(z(t))A(t)$ supposes to be sharp and variable $z(t)$ supposes to fluctuate with short correlation time. Therefore, Brownian walk pattern supposes to be realized, with the exponent μ equal or larger than 3.

In realizing adaptive searching behavior, the activity is defined as a function of sensory input. When no targets are found, the activity should be low such that the shape of $U(z(t))A(t)$ is adequately flat and Levy walk is performed as a default random search. However, once some targets are found, the activity should be high, and the shape of the $U(z(t))A(t)$ should become sharp, supposedly reduces the correlation time, causing the Levy walk to switch to a Brownian walk pattern. Here, the activity function can be summarized in equation (6) to (8) with $0 < C < 1$, while k_F is a constant with a large value in comparison to A_{min} . Anytime one or more targets are found at time t , $F(t)$ will be triggered to 1. It can therefore be said that $F(t)$ is a step function whose input is the finding of targets. The definition of

finding the target will be explained in the next section. By employing such function, the correlation time will be reduced once a target is found and gradually increases to the original value, if no more targets are found. The overall principle is shown in Fig.2.

$$A(t) = \begin{cases} A_{\min}, & \text{if } \alpha(t) \leq A_{\min} \\ \alpha(t), & \text{if } \alpha(t) > A_{\min} \end{cases} \quad (6)$$

$$\alpha(t) = C\alpha(t-1) + k_f F(t) \quad (7)$$

$$F(t) = \begin{cases} 1, & \text{if one or more targets are} \\ & \text{found at time } t \\ 0, & \text{if no targets are found at} \\ & \text{time } t \end{cases} \quad (8)$$

IV. SIMULATION EXPERIMENTS

A. Experiment Setup and Conditions

In order to verify our assumption that adaptive behavior will occur, a simulation experiment was conducted. Here, the simulation settings and the experiment conditions will be explained.

Fig.3 shows the screenshot of the simulation from two different scales. Fig. 3 (a) shows the screenshot of the whole area. The size of the area is 1000x1000 [units]. The targets inside the patch are not shown for clarity. The starting position of the robot is from the center of the screen. Here we use periodic boundary condition, a common approach used in observing random search performance [7], which means once the robot passes the simulation boundary, it will reappear from the other end. This will present an experiment result that is unaffected by different boundaries of the area, and, with long enough simulation time, where the robot starts the search. For the swimming mode, the length of moving forward is defined as 1 [unit], while the random turning angle in the tumbling mode is set to be uniformly distributed from 0 to 360 [deg]. The simulated time is 10000 [s]. To obtain a new value of $P(t)$, (4) is discretized with time sampling 0.1 [s].

The position of the attractor is chosen as $h=0.7$, corresponds to $P(t) \approx 0.5$ according to (5). In order to let $P(t)$

adequately fluctuates between 0 and 1 as a function of $z(t)$, the value of $z(t)$ is limited between 0 and 5, corresponds to those values of $P(t)$. The size of noise $\varepsilon(t)$ in (4) is 0.5, defined by the standard deviation.

Fig. 3 (b) shows the zoomed in condition when the robot approaches some targets, shown by the crosses. One or more targets are considered to be found, and disappear at the next time step, if their position is inside the robot sensing diameter, d_s , representing a limited sensing capability of the robot. The Levy walk has been shown to be better when the targets are sparse, that is the target site has a low target density. In other word, the average distances among the targets are much larger than this diameter. However, Brownian walk is shown to be favorable when the target density is higher. If the sensing diameter is infinite, then the robot does not have to do any search as it will automatically find all of the targets regardless where the robot is. For further study about the comparison, one can refer to [5][6][7]. Our aim here is to investigate whether the robot can adaptively choose the more proper random search in a patchy target setting, i.e. the target density varies.

In creating the patchy target setting, we make sure that the patches are sparsely placed while each patch has denser target density. One can easily see that if the patches are not sparsely placed then it cannot be called patchy environment as the overall target density will become high, in which Brownian walk is expected to be more favorable. On the other hand, if each patch only contains a few targets then it cannot be called patchy environment either, as the overall target density will become low in which Levy walk is the better strategy [7]. Therefore, we deploy 10 circular shape patches with a small radius of 10 [units] in the 1000x1000 [units] search area. To make sure that each patch is dense, 100 targets are deployed inside each of them. The robot sensing diameter d_s is 2 [units]. The simulation screenshot showing the patches, the targets inside them, and d_s can be seen from Fig. 3. While investigating various kind of target distribution is a part of our future works, here we simply make sure that the target density is changing and set the targets inside the patches and the center of each patch in the whole area to be uniformly distributed.

To measure the performance of the search, we observe the search efficiency, defined as the number of targets found divided by total distance travelled. Such criteria is related with the energy efficiency, as moving forward generally takes more energy than changing direction randomly, therefore also used widely in animal random search literatures [5][6][7].

At first, we want to confirm whether at certain values of the activity, the robot will show Levy and Brownian walk pattern. Therefore, we choose a certain range of the activity value and observe the effect to the shape of $U(z(t))A(t)$, to the correlation time of state $z(t)$, and the realized random search pattern performing by the robot.

As the second step, we want to investigate whether the robot can adaptively switch between Levy and Brownian walk pattern and whether it is beneficial. By choosing a certain minimum value of the activity A_{\min} supposedly causes

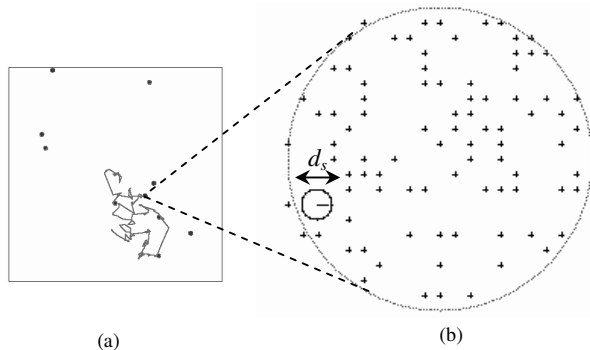


Fig.3 The simulation screenshot: (a) from a usual scale, and (b) the zoomed-in condition when the robot enters a patch of targets

the robot to do a Levy walk, and implement the activity rule explained in (7) to (8), it is assumed that the adaptive behavior can be realized. The performance of the adaptive search is then compared with Levy and Brownian walk alone.

For every experiment, we perform 20 trials. We also use the commonly known t-test if it is necessary to confirm a statistical significance of a data comparison.

B. Experiment Results

Based on the mentioned conditions, the experiment results can be explained as follow.

1) *Confirmation of Levy and Brownian walk pattern.* At first, we observe fluctuation of $z(t)$, its correlation time and the resulting value of μ with a range of the activity $A(t)$ values. To determine whether the realized searching behavior follows a Levy or Brownian walk pattern, the most common approach is to plot the log-log histogram of the flight's frequency $N(l)$ versus the lengths l . The frequency is normalized (i.e. divided by the histogram bin width and total frequency), while the bins are increased logarithmically. Because the minimum value of l is one, here we use bin breaks of 1,2,4, and so on. Power law statistic is indicated if a straight line fits the plotted data. This method is called "LBN" (logarithmic binning with normalization), recommended in [24], and actually the slope of the fitted line will be equal to minus of the power law exponent, μ , in (1). However, in calculating this value of μ , we use a more recent, accurate, method based on maximum likelihood explained in [25] shown in (9), where L is the whole data set of the flight lengths. The equation, derived by maximum likelihood method, can be used because the minimum value of the flight length is 1. A Levy walk trajectory will be shown if $1 < \mu < 3$.

$$\mu = 1 + \frac{1}{\text{mean}(\log(L))} \quad (9)$$

In order to observe the correlation time of state $z(t)$, we plot the autocorrelation function, defined as $R(z(t))$, and calculate the correlation time, t_{r_z} . We use the common definition of correlation time, that is the time when the autocorrelation value of the state is already at a factor of $1/e$ down from its maximum value at $t=0$ [26].

Fig. 6 (a), (b) and (c) show the examples of the realized shape of potential $U(z(t)).A(t)$, the resulting example of fluctuation of $z(t)$ along with the autocorrelation graph with shown position of t_{r_z} in the first 25 [s]. It can be seen that with small value of the activity, the potential $U(z(t)).A(t)$ is flat and let $z(t)$ fluctuate with large correlation time t_{r_z} . On the other hand, with large enough $A(t)$, $z(t)$ fluctuates around the position of attractor $h=0.7$ with a small correlation time t_{r_z} .

Fig. 7 shows the corresponding trajectory for each value of the activity shown in Fig. 6, with the resulting exponent μ that indicates whether the robot performs Levy or Brownian walk. It can be observed that due to some long flight lengths in between the intensive searches, the Levy walk pattern explores a wider area compare to the Brownian walk.

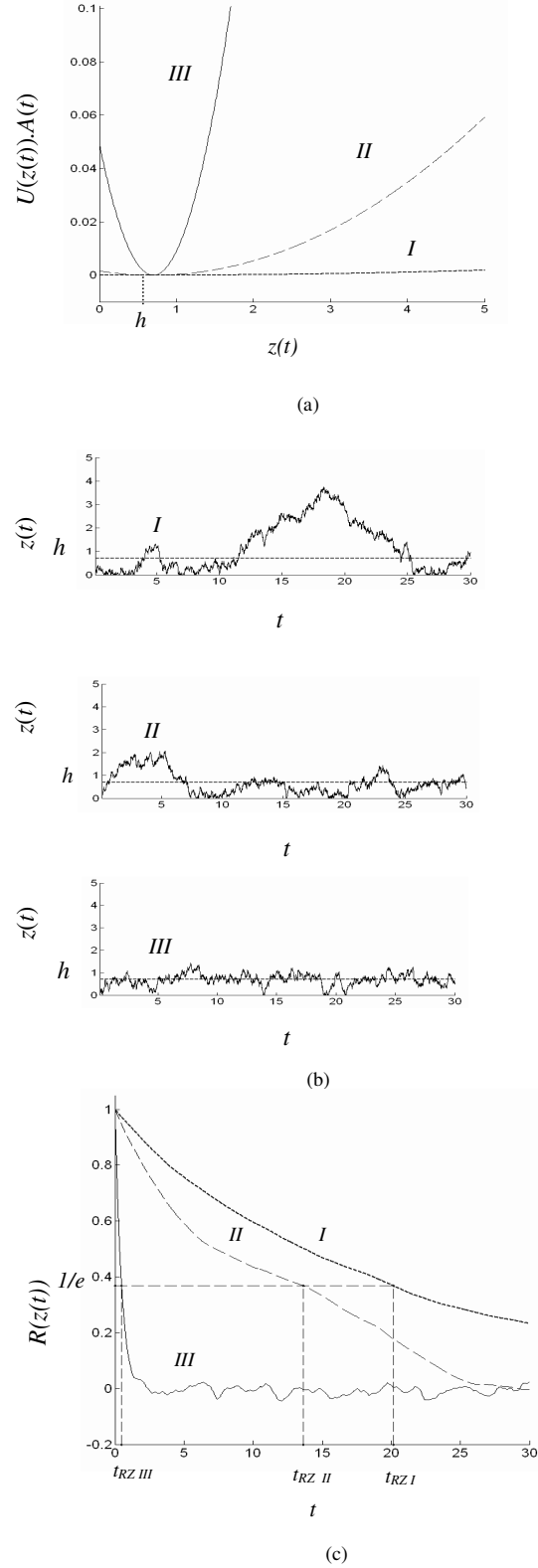


Fig.6 (a) The shape of potential $U(z(t)).A(t)$ (b) the resulting fluctuation of $z(t)$ around the attractor at $h=0.7$ (c) the autocorrelation function $R(z(t))$ with the correlation time t_{r_z} I, II, III corresponds to $A(t) = 10^{-4}, 10^{-2.5}, 10^{-1}$ consecutively

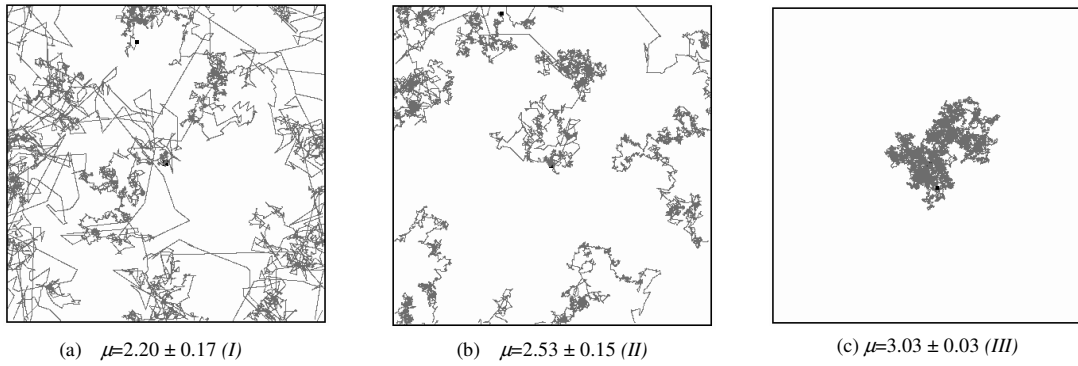


Fig.7 Trajectory examples for the activity values shown in Fig. 6 with the average calculated value of exponent μ

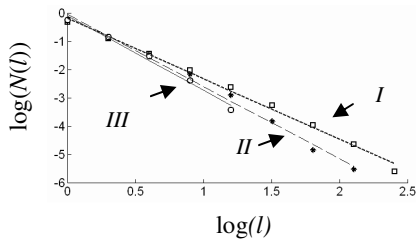


Fig.8 Log-log histogram and the approximated fitted line of normalized flight lengths frequency $N(l)$ versus the flight lengths l for each activity value shown in Fig. 6

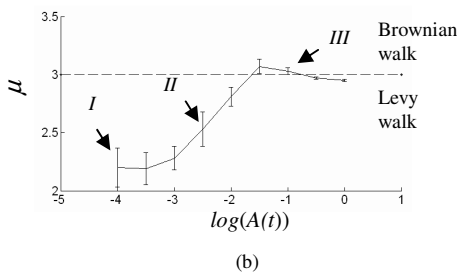
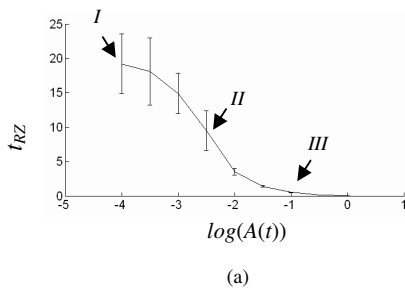


Fig. 9 (a) correlation time t_{RZ} and (b) exponent μ versus the activity $A(t)$. For clarity, the activity is plotted in a log scale

Furthermore, Fig. 8 shows examples of the log-log graph of flight lengths frequency $N(l)$ versus the flight lengths l and an approximated straight line that indicates power law statistic.

Fig. 9 explains the behavior of the random search in a more thorough way. The figure shows the relationship between log of the activity $A(t)$ versus the correlation time

t_{RZ} and exponent μ . The vertical bars show the standard deviation. It can be seen clearly that when the activity has a small value, causing a correlation time, the robot will do a Levy walk pattern. When the activity gets larger, the correlation time is reduced and the random search will have stronger tendency to a Brownian walk. When the activity equals to 1, the value of μ already about equals to the condition if $z(t)$ is simply kept constant at h .

2) *Confirmation of Adaptive Behavior and Its Benefit.* The adaptive behavior means the change of the value of the activity will cause the robot to immediately switch to Brownian walk once one or more targets are found and gradually switch back to Levy walk when number of targets found decreases.

To be exact, we implement the activity rule in (6) to (8) with $A_{min}=10^{-4}$ whose properties indicated by the number I in Fig. 6 to 9. The constant k_F is set to 10^{-1} such that when some targets are found and $F(t)$ in (8) equals to 1, the robot will immediately switch to a Brownian walk whose properties indicated by the number III in Fig. 6 to 9. When no more targets are found, the robot will gradually switch back to Levy walk with the activity $A(t)$ equals to 10^{-4} . The value of constant C in (7) is 0.9.

The trajectory comparison between Levy, Brownian walk and the adaptive search is shown in Fig. 10. It can be seen that due to the occasional long flight lengths, as variable $z(t)$ sometimes fluctuates near a high value as shown in Fig. 6.b (top), the Levy walk pattern finds more patches than the Brownian walk. This causes higher search efficiency even that the total traveled distance is also a little bit higher. However, it is interesting to notice that while Brownian walk finds less number of patches, the ratio between targets found and visited patches are actually higher, as the search is more intensive when a patch is found. This indicates that switching between the two random search behaviors might be beneficial. It is confirmed in table I, which shows the mean and standard deviation of the efficiency, along with other criteria. It can be seen that performing the adaptive behavior is better than either Levy or Brownian walk alone. The statistical significance of this result has been confirmed by using t-test.

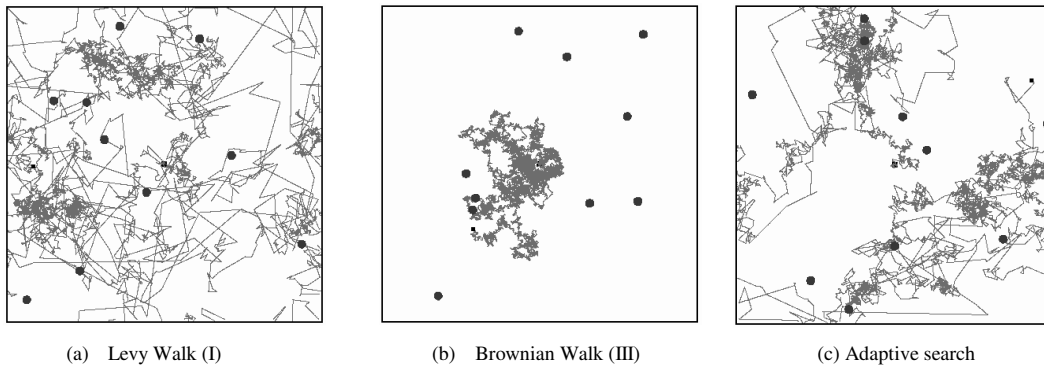


Fig.10 Trajectory examples for Levy Walk (I), Brownian Walk (III) and the adaptive search with some patches

TABLE I. PERFORMANCE OF DIFFERENT SEARCHING BEHAVIORS

Searching behavior	Search Efficiency	Targets found	Travelled distance	Visited patches (out of 10)
Levy Walk (I)	1.54 ± 0.73	$(1.32 \pm 0.65) \times 10^2$	$(8.48 \pm 0.26) \times 10^4$	5.25 ± 1.80
Brownian Walk (III)	0.86 ± 0.82	$(0.57 \pm 0.55) \times 10^2$	$(6.67 \pm 0.02) \times 10^4$	1.45 ± 1.15
Adaptive search	2.78 ± 1.10	$(2.23 \pm 0.83) \times 10^2$	$(8.08 \pm 0.27) \times 10^4$	4.75 ± 1.16

TABLE II. AVERAGE ACTIVITY VALUE OF DIFFERENT SEARCHING BEHAVIORS

Searching behavior	Inside the patches	Outside the patches
Levy Walk (I)	10^{-4}	10^{-4}
Brownian Walk (III)	10^{-1}	10^{-1}
Adaptive search	$(1.14 \pm 0.27) \times 10^{-1}$	$(5.03 \pm 1.35) \times 10^{-4}$

From the trajectories in Fig. 10, it can be seen that, unlike the Levy walk which does not react when a target is found, in the adaptive search, the intensive searches are concentrated near the patches. This behavior can be explained by comparing the average activity value of Levy and the adaptive search shown in Table 2. It can be seen that for the adaptive search, the average activity inside the patch, in a log scale, corresponds to the exponent μ for a Brownian walk pattern shown in Fig. 9, confirming the switching behavior. However, outside the patches, the value of the activity is also slightly higher as it takes sometimes for the robot to gradually switch back to Levy walk after it does not find any targets for certain period. As a result, the exponent μ outside the patch will have a slightly higher value, according to Fig. 9 (b).

Parameter C in (7) decides how strong the tendency to switch to Brownian walk and how fast the robot switches back to Levy walk (see appendix). Too small value of C is meaningless, as it will not be able to increase the value of the activity large enough to make the robot adequately switch to Brownian walk inside the patches. On the other hand, too large value of C , may cause a longer time to switch back to the Levy walk pattern. For the used patchy environment, $C=0.9$ is shown to be the best value.

It is also easy to imagine that under extreme conditions of a very high density (i.e. there are abundant numbers of patches) or a very low density (i.e. there are only a few

targets inside patches) the robot would do Brownian and Levy walk consecutively, the expected favorable strategy for those conditions. This is caused by the value of the activity that most of the time will likely be high for the first extreme condition, or never increase significantly for the second one.

V. CONCLUSION AND FUTURE WORK

In this paper, we have presented a simple, noise-utilizing, mathematical framework that models and implements bacterial movement to a mobile robot based on biological fluctuation. Based on the framework, we have shown that an adaptive random search behavior in a patchy environment can be realized by adaptively switching between Levy and Brownian walk pattern, shown to be able to increase the efficiency of the search as compared to do just either one.

Indeed, biological creatures are inspirational resources to realize a mechanism not yet utilized in human engineered system. In principle, here we show that by exploiting some stochastic natures of the simplest creature, using a very simple model, adaptive and efficient searching behavior can emerge by simple interaction with relatively complex environment.

For a future work, we plan to try the framework in a more complex environment, starting with a more complex target distribution. In relation with this, it is interesting to notice that the diffusion speed of a Brownian walk can be different although similarly linearly proportional to t [27]. As have been shown in this paper, using single attractor is enough to make adaptive switching between Levy and Brownian walk pattern. However, we have preliminary results showing that we can make the robot tend to choose a more suitable attractor, corresponds to a more suitable diffusion speed, inside patches with different sizes and target densities. We plan to further explore this behavior. It is also interesting to combine certain learning algorithm with the activity function.

APPENDIX A: AVERAGE ACTIVITY VALUE OF THE ADAPTIVE SEARCH WITH DIFFERENT PARAMETERS

C	Inside the patches	Outside the patches
0.99	$(5.52 \pm 1.30) \times 10^{-1}$	$(78.73 \pm 32.31) \times 10^{-4}$
0.90	$(1.14 \pm 0.27) \times 10^{-1}$	$(5.03 \pm 1.35) \times 10^{-4}$
0.50	$(0.50 \pm 0.16) \times 10^{-1}$	$(1.17 \pm 0.08) \times 10^{-4}$

APPENDIX B: PERFORMANCE OF THE ADAPTIVE SEARCH WITH DIFFERENT PARAMETERS

C	Search Efficiency	Targets found
0.99	2.33 ± 1.07	$(1.81 \pm 0.82) \times 10^2$
0.90	2.78 ± 1.10	$(2.23 \pm 0.83) \times 10^2$
0.50	1.69 ± 0.63	$(1.41 \pm 0.51) \times 10^2$

C	Traveled distance	Visited patches (out of 10)
0.99	$(7.81 \pm 0.15) \times 10^4$	4.05 ± 1.54
0.90	$(8.08 \pm 0.27) \times 10^4$	4.75 ± 1.16
0.50	$(8.42 \pm 0.27) \times 10^4$	4.90 ± 1.33

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