The 2010 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 18-22, 2010, Taipei, Taiwan

# A distributed Transferable Belief Model for collaborative topological map-building in Multi-Robot Systems

Cristina Carletti, Maurizio Di Rocco, Andrea Gasparri and Giovanni Ulivi

*Abstract*— **In this paper the problem of multi-robot collaborative topological map-building is addressed. In this framework, a team of robots is supposed to move in an indoor office-like environment. Each robot, after building a local map by using infrared range-finders, achieves a topological representation of the environment by extracting the most significant features via the Hough transform and comparing them with a set of predefined environmental patterns. The local view of each robot which is significantly constrained by its limited sensing capabilities is then strengthened by a collaborative aggregation schema based on the Transferable Belief Model (TBM). In this way, a better representation of the environment is achieved by each robot with a minimal exchange of information. A preliminary experimental validation carried out by exploiting data collected from a self-made team of robots is proposed.**

## I. INTRODUCTION

Map-building addresses the problem of acquiring spatial models of physical environments by mobile robots [1]. The map-building problem is generally considered as one of the most important problems in the pursuit of building truly autonomous mobile robots. Two different approaches for modeling an indoor environment have been proposed: the metric approach and the topological approach [2]. Metric maps capture the geometric properties of the environment, whereas topological maps describe the connectivity of different places. These approaches exhibit orthogonal strengths and weaknesses [3]. On the one hand, topological maps are computationally efficient, easy to maintain even in large scale environments while metric maps suffer from their enormous space and time complexity. On the other hand, metric maps provide a very detailed description of the environment while topological maps offer a limited representation of the surrounding world.

The majority of the approaches available in the literature deals with the simultaneous localization and mapping problem (SLAM) consisting in both building the map of the environment and localizing the robot that is moving within it [4]. In [5] an approach to build a topological map based on the concept of Voronoi random fields is introduced. The idea is to extract a Voronoi graph from an occupancy grid map generated with a laser range-finder, and then represent each point on the Voronoi graph as a node of a conditional random field. The resulting Voronoi random field estimates the label of each node, integrating features from both the map and the Voronoi topology. Several works have been proposed for the SLAM problem in a multi-robot scenario as well. In [6] a platoon of four robots performs on-line the map building with a particle filter algorithm. In [7] an alternative approach for less effective sensors, e.g., sonar range-finders, is proposed. The idea is to build a grid-map representation of the environment modeling uncertainty by means of the fuzzy theory. In [8], an improvement of this work is presented. In particular, the grid-maps are exploited to extract a knowledge of the surrounding environment along which the robot travels. The uncertainties are managed through the Possibility Theory [9].

In this work a collaborative topological map-building approach for a team of robots moving in an indoor office-like environment is proposed. Each robot, after building a local map by infrared range-finders, builds a set of hypotheses about the topological nature of the surrounding environment by comparing the features extracted using the Hough transform with a set of predefined environmental patterns. The local view of each robot which is significantly constrained by its limited sensing capabilities is then strengthened by a collaborative aggregation schema based on the Transferable Belief Model. In this way, a better representation of the environment is achieved by each robot by means of a minimal exchange of information.

The rest of the paper is organized as follows. In Section II the problem setting is described. In Section III the robotic hardware platform SAETTA developed at the Robotics Lab of the University of "Roma Tre" is detailed. In Section IV the topological multi-robot map-building process is explained. In Section V an experimental validation of the proposed collaborative map-building technique by exploiting the SAETTA multi-robot system is proposed. In Section VI conclusions are drawn and future work is discussed.

## II. PROBLEM SETTING

In the proposed framework, a team of robots which explores an unknown office-like environment is considered. Robots are equipped with a sensorial system composed of an array of infrared range-finders along with an analog compass which allows the team to share a common heading direction. Therefore, a wireless channel is available for communication purposes.

The team of robots is assumed to move in a rigid formation. Indeed, this can be achieved by exploiting one of the several control laws available in literature, e.g., [10], [11]. In addition, robots are assumed to be aware of the sensing

C. Carletti is with the Dept. of Computer Science, Management and Automation, University "Politecnica delle Marche", Via Brecce Bianche, Ancona, 60131 Italy. Email: carletti@diiga.unipvm.it

M. DiRocco, A. Gasparri and G. Ulivi are with the Dept. of Computer Science and Automation, University of "Roma Tre", Roma, 00146 Italy. Email: {dirocco, gasparri, ulivi}@dia.uniroma3.it

Authors are listed in alphabetical order.

occlusions due to the other robots. Note that, this is not a strong limitation as robots are assumed to move in a rigid formation. In particular, for each couple of robots an angular section with respect to their line of sight is considered as occluded.This information can be taken into account when building the set of hypotheses to describe the surrounding environment. Figure 1 depicts the adopted geometrical model of occlusion for a team of three robots.

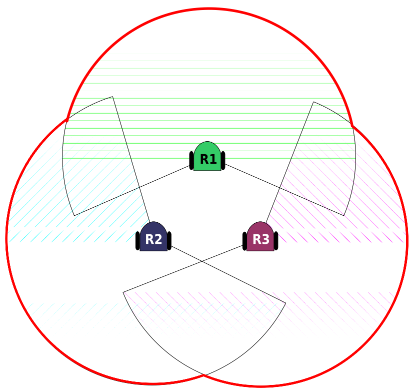


Fig. 1. Geometric model of occlusion for the multi-robot system.

The office-like environment in which the team of robots moves is approximated by the union of a set of environmental patterns detailed in Subsection IV-A. In particular, the following patterns are taken into account in this framework: L-turn, corridor, dead-end, T-junction, and crossing.

The objective of this work is to develop a collaborative technique to let the team of robots achieve a proper topological description of the surrounding environment. The key idea is to provide an effective collaborative framework to make up for the limited sensorial capabilities of each single robot.

## III. The SAETTA Robot Design

The SAETTA robotic hardware platform developed at the Robotics Lab of the University of "Roma TRE" is a low-cost robot. It features a complete sensorial system, a very accurate traction in indoor environment, and a ZigBee transceiver for multi-robot applications. The platform has been reproduced into 12 units.

The SAETTA architecture can be conceptually decomposed into a two-tiers architecture. The first tier is constituted by the interaction between low level components, such as traction and sensorial system, while the other is in charge of executing high level tasks. This conceptual division has an immediate correspondence in the hardware realization: each tier is realized on an electronic board and equipped by a CPU unit. The low level is managed by a Programmable Interface Controller (PIC) while the high level is a Linux embedded board (FOX) from Acme Systems. The first board, which manages strictly time constrained tasks, has a control cycle (25ms) shorter than the other one (200ms) that is supported, on the other hand, by a more powerful CPU to implement higher level tasks. The inter-board communication is realized

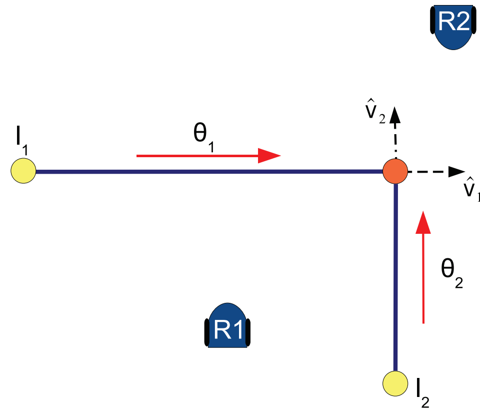


Fig. 2. Corner detection. Robot R1 detects a convex corner while robot R2 detects a concave one.

by exploiting a RS232 channel. At the moment, a gyroscope, a magnetometer, an accelerometer and 5 infrared (IR) sensors are present.

Regarding the traction system, a very convenient choice has been the use of stepper motors instead of the more common d.c. motors. They offer several advantages, first of all the absence of tachometers or encoders and, as a consequence, of the circuitry associated with the transducer. Moreover, a stepper motor requires low supply voltages as it has a very low back EMF. Further details can be found in [12].

## IV. Topological Multi-Robot Map-Building

In the following the collaborative topological multi-robot map-building technique is described. First, the patterns adopted to represent the environment are described in Subsection IV-A. Then, the feature extraction process to build an actual view of the surrounding environment is explained in Subsection IV-B. Successively, the topological map building is explained in Subsection IV-C. Finally, the collaborative approach based on the TBM conjunctive rule to improve the local view of each robot is described in Subsection IV-D.

### A. Environmental Patterns

In order to have a meaningful representation of an office-like area, environmental patterns are introduced. Indeed, these models constitute, in our context, a valid approximation into which classify more complex environments. In the proposed scenario, maps are composed of a combination of the following elements: *L-turn* $(L)$, *corridor* $(O)$, *dead-end* $(D)$, *T-junction* $(T)$, *crossing* $(X)$.

In order to derive a mathematical description of these patterns some preliminary concepts must be introduced.

Let us first introduce the set $\mathcal{A} = \{a_1, \ldots, a_k\}$ of *atomic elements* as the set of basic features that a robot can detect. In this work, walls (W) and corners (C) are considered. An atomic element $a_i$ is described by means of a simple parametrization with respect to the reference frame of the detecting robot. In detail, a wall is described by the angular coefficient $\theta$ of the detected segment, while a corner is represented by the angular coefficient $\theta_1$ and $\theta_2$ of the oriented segments connecting the two end-points to the vertex. Note that, two different kinds of corners are considered: convex and concave. According to the situation shown in Figure 2, a corner is said to be convex if the robot $r$ belongs to the third quadrant of the reference frame $\hat{O}(\hat{v}_1, \hat{v}_2)$ attached to the vertex $v$, concave otherwise.

Let us now introduce the concept of *relational features*. A relational feature $f_i$ is a couple of atomic elements for which a set of geometric relationships hold. From a mathematical perspective, a relational feature $f_i$ can be defined as follows:

$$f_i = \{a_j, a_k\} \;:\; \mathcal{R}(a_j, a_k) = \text{true}$$

where $\mathcal{R}(\cdot) \;:\; \mathcal{F} \times \mathcal{F} \longrightarrow \{\text{false, true}\}$ is a boolean map describing a given set of geometric relationships. Note that, among all the possible relationships also the identity map, i.e., $\mathcal{R}(a_i, a_i) = \text{true}$, is considered. Indeed, this allows to define a relational feature even if only a single atomic element is recognized. The idea of relational feature turns out to be very useful as it allows to provide a graph-like representation of an environmental pattern and therefore a simple decomposition of it.

It is now possible to provide a formal characterization of the set $\mathcal{P} = \{p_1, \ldots p_n\}$ of environmental patterns. In particular, an environmental pattern $p_q$ can be described by a set $F_q = \{f_1, \ldots, f_h\}$ of relational features. Note that, in order to be a valid representation, the set $F_q$ must satisfy the following property:

$$F_q \nsubseteq F_k \,\wedge\, F_q \nsupseteq F_k,\, \forall p_k \in \mathcal{P}\backslash\{p_q\}, \tag{1}$$

which guarantees the pattern $p_q$ to be *fully* described.

This formalization leads, as previously mentioned, to an intuitive graph-like representation of an environmental pattern where links represent the geometric relationships existing among each couples of atomic elements. Therefore, according to this alternative representation, an environmental pattern can be also thought as a fully connected graph of atomic features. Figure 3 gives a graphical overview of such a representation for the set of environmental patterns adopted in this paper. In particular, a corridor can be viewed like a two *wall-nodes* graph connected by an edge for which the relationship of equal orientation holds. A T-junction graph is instead represented by two *concave-corner-nodes* and a *wall-node*. The *corner-nodes* share a common bearing for a couple of segments while the other one has a phase displacement of $\pi$. In addition, each of them shares a common bearing with *wall-node*. Note that, in order to avoid a wrong association between atomic elements, the relationship of parallelism is constrained by a minimal distance between the considered segments. A similar description can be easily derived for the remaining patterns.

### B. Local Topological Features Extraction

Each robot while moving builds a local map of the surrounding environment by exploiting infrared range-finders. The expressiveness of the obtained map is highly constrained by the limited amount of data which can be collected at each time, i.e., only an array of 5 infrared sensors arranged over the 180° with respect to the heading direction of the robot is available. Figure 4 depicts an example of local map built exploiting the raw data collected by the robot while rotating over 360°. In particular, it can be noticed the effect of the noise affecting the measurements due to the intrinsic characteristics of the sensor. The local map is used to extract some features of interest by means of a Hough transform [13].

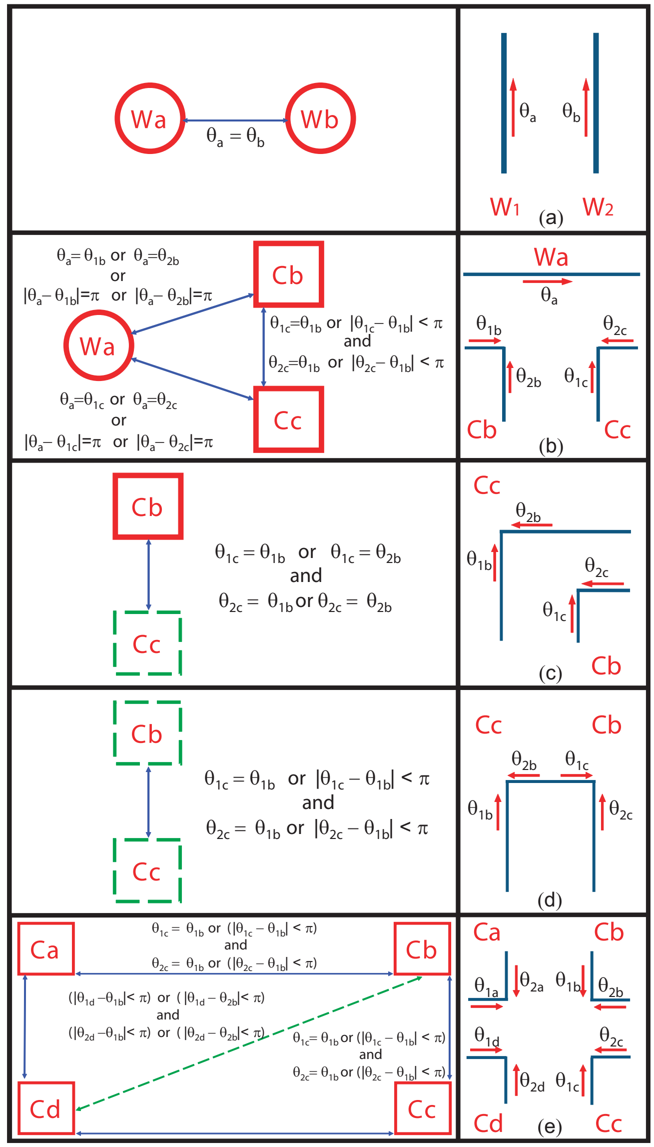


Fig. 3. Graph-based representation of an environmental pattern. Nodes represent atomic elements linked to each other by *relational edges*. Dashed squares are convex corners, solid squares are concave corners and solid circles are walls. a) corridor b) T-junction c) L-turn d) dead-end e) crossing.

### C. Local Topological Map Building

In order to achieve the topological description of the environment, the framework of the "Theory of Evidence" introduced by Shafer in [14] is exploited. It gives an effective mathematical model for the representation of uncertainty.

Fig. 4. Example of local map built by letting the robot rotate over $360°$.

Hence, it turns out to be very suitable in a robotics context when dealing with noisy measurements coming from low-cost sensors. In this framework, an environmental pattern represents the proposition of a set $\Omega$ called *frame of discernment*, while the set of all propositions of interest corresponds to the elements of the power-set $\Gamma$. In addition, it can be defined a function $m : \Gamma \rightarrow 1$ called Basic Belief Assignment (BBA) which associates to each element $\gamma \in \Gamma$ a belief mass. This mass $m(\gamma)$ describes the proportion of all relevant and available evidence that supports the claim that the actual "state" belongs to $\gamma$ but to no particular subset of it. This framework suits very well the multi-robot topological mapping problem. In fact, elements of the power-set can be used to model the subset of patterns which fits the limited set of features that can be extracted from the partial view of a single robot. For example, if a corner is detected by a robot during the feature extraction process the set given by the union of all the environmental patterns which contain a corner is considered. Note that, due to the limited sensing capability, the whole surrounding environment will be hardly recognized by a single robot. For this reason an aggregation among the knowledge acquired by the team must be introduced. In this ways, ambiguities can be reduced and the set of plausible patterns can be restricted. Conditions under which the correct pattern can be detected will be discussed in Theorem 1.

As far as the construction of the set of masses $\mathcal{M} = \{m(\gamma_1), \ldots, m(\gamma_{|\Gamma|})\}$ is concerned, let us assume each robot can extract from its local map a set of relational features $s = \{f_1, \ldots, f_d\}$ every $T$ seconds. In addition, let us define $\alpha_i = p_i \cap s$, with $\alpha \in [0, 1]$, as the "similarity" between the set of features $s$ and an environmental pattern $p_i$. At this point, the set of masses, which must sum to one by definition, is computed as follows:

- a fraction $\eta$, is assigned to the union $A$ of all patterns compatible with the features acquired that is:

$$m(A) = \eta, \quad A = \bigcup_{p_i \cap s \neq \emptyset} p_i \tag{2}$$

- the remaining $1 - \eta$ of the mass is assigned proportionally (w.r.t. to the similarity) to each subset $B_j$ having cardinality minus one respect to $A$, except for the atomic patterns:

$$\begin{aligned} m(B_j) &= (1-\eta) \cdot \chi_j, \\ &\forall \quad B = \bigcup_{p_i \cap s \neq \emptyset} p_i \quad s.t. \quad |B| = |A| - 1 > 1 \end{aligned} \tag{3}$$

with

$$\chi_j = \frac{\sum_{p_k \in B_j} \alpha_k}{\sum_{\substack{B_j \in A \\ |B_j| = |A| - 1}} \sum_{p_k \in B_j} \alpha_k}.$$

Note that, the condition $|A| \geq 2$ is required in order to have an assignment to be self-consistent, i.e., a combination of a set $\mathcal{M}$ with itself should not introduce contradiction.

### *D. Topological Maps Aggregation*

The Transferable Belief Model (TBM) introduced by Smets [15] is exploited for the aggregation of the topological description of the environment built by each robot. TBM allows to combine evidence from different sources and arrive at a degree of belief that takes into account all the available evidences.

For a couple or robots $(i, j)$, the TBM conjunction rule can be defined as follows:

$$(m_i \otimes m_j)(\gamma_a) = \sum_{\substack{\gamma_b, \gamma_c \\ \gamma_b \cap \gamma_c = \gamma_a}} m_i(\gamma_b) m_j(\gamma_c). \tag{4}$$

From a computational perspective, the aggregation can be performed in two different ways. A simple approach is to let robots broadcast the acquired topological description and then each robot performs locally the aggregation. A more refined approach is to devise a local interaction rule which provides the same result as the previous one avoiding the overhead of the broadcast [16]. The first approach is simpler and reasonable for a team of few robots, while the second approach is more suitable for a large team of robots.

Note that the proposed framework turns out to be exact in the ideal case of measurements without noise. The following theorem provides a mathematical characterization of this correctness.

***Theorem 1:*** Let us consider an environmental pattern $p_q \in \mathcal{P}$ described by a set $F_q = \{f_1, \ldots, f_h\}$ of relational features. Let us assume each robot to build a set of masses according to the rules given in eq. (2) and eq. (3). Finally, let us assume the set of masses to be aggregated according to the combination rule given in eq. (4). A sufficient condition for the recognition of the environmental pattern $p_q$ is that :

$$\bigcup_{i}^{N} s_i = F_q,$$

where $N$ is the number of robots and $s_i$ is the set of relational features computed by the $i$-th robot.

*Proof:* In order to prove the theorem, let us consider without any lack of generality a partition over $F_q$ such that every subset of $F_q$ is correctly identified by one robot. Let $A_i$ be the union of all the patterns compatible with the observed subset of features related to the $i$-th robot. Now, let us consider two robots $j$ and $k$ performing an aggregation of their masses $\mathcal{M}_j$ and $\mathcal{M}_k$ according to eq. (4). The elements $m(\gamma)$ of the resulting set $\mathcal{M}_{j,k}$ are obtained as follows:

$$m(\gamma) = \begin{cases} m_j(A_j) \cdot m_k(A_k) & \text{if} \quad \gamma = A_j \cap A_k \\ \geq 0 & \text{if} \quad \gamma \cap (A_j \cap A_k) \neq \emptyset \\ 0 & \text{if} \quad \text{otherwise} \end{cases}$$

Therefore, only the elements $\{\gamma_l\}$ of the power-set $\Gamma$ representing the subset of environmental patterns supported by the intersection of the two sets of relational features $A_j$ and $A_k$ are assigned with a mass greater than 0, while the remaining content of information accumulates in the mass of the element $\gamma = \emptyset$ emphasizing the contradiction between sources. At this point, by iterating this aggregation process, the obtained set of masses $\mathcal{M}_{j,k}$ will be necessarily aggregated with the set $\mathcal{M}_r$ provided by one of the other robots. Now, by recalling from eq. (1) that an environmental pattern is fully described by the union of its own relational features and by assuming measurements to be perfect, the following holds:

$$\bigcap_{i}^{N} A_i = p_q.$$

Therefore, after all the set of masses are aggregated, only the element $\gamma = p_q$ will have a mass greater than 0. Hence, the correct environmental pattern can be identified. Furthermore, the mass associated to $\gamma = \emptyset$ can be thought as the amount of contradiction due to the initial occlusions experienced by each robot. ■

Although a proper mathematical characterization has been derived under the assumption of ideal measurements availability, in the real world data coming from sensors is always affected by noise. For this reason a simple but effective workaround to deal with the eventuality that a robot might perform an incorrect feature extraction is proposed. The idea is to assign a fraction $\zeta$ to the *universe set*, e.g. $m(\mathcal{P}) = \zeta$. This trick allows to partially recover from a bad feature extraction process. In this way also non common hypotheses can survive. A viable solution, making the assumption that a sort of index measuring the quality of the feature extraction is available, is to assign $\zeta$ proportionally to the goodness of the feature extraction process. Finally, the remaining part after assigning $\eta$ and $\zeta$ could be assigned as explained before.

### *E. Computational Complexity and Implementation Details*

From a computational perspective the proposed collaborative technique for topological map building cannot be implemented as it is. Indeed, a few tricks are required to let the technique be computationally affordable for a team of low-cost robots.

Strictly speaking, the major bottleneck is related to the construction of the power-set $\Gamma$. Indeed, given a frame of discernment $\Omega$ with cardinality $|\Omega|$ the related power-set will have a cardinality equals to $2^{|\Omega|}$, letting the formalization become intractable very quickly. In order to overcome this limitation, the problem formulation has been equivalently split in two parts. In detail, while the original formalization considers a frame of discernment where profiles are oriented, in the algorithmic solution profiles are not oriented and the discrimination is made in two steps: first the profiles which are plausible according to the set of information coming from the team are discriminated, successively the correct one is identified by means of an agreement over the orientation. Indeed, this two-steps procedure allows to significantly reduce the cardinality of the set $\Omega$ and therefore to keep the overall complexity of the proposed technique affordable.

## V. Experimental Results

In this section, an experimental validation of the proposed topological map-building technique is proposed. Experiments have been carried out by exploiting two robots SAETTA having a complementary 180° field of view (see Figure 5 ). The experiments encompass all the environmental patterns described in Section IV-A. In particular, three different scenarios have been considered.

The motion of a robot is regulated by a very simple rule: if an obstacle is sensed on the heading direction, or if a large discontinuity is detected by the lateral sensors, a 360° rotation is performed by each robot. Note that, the rotation maneuver is required only to make up for the limited sensing resolution of the array of infrared range-finders. Indeed, this could be avoided if a sensor with a more granular resolution were available, e.g., a laser range-finder.

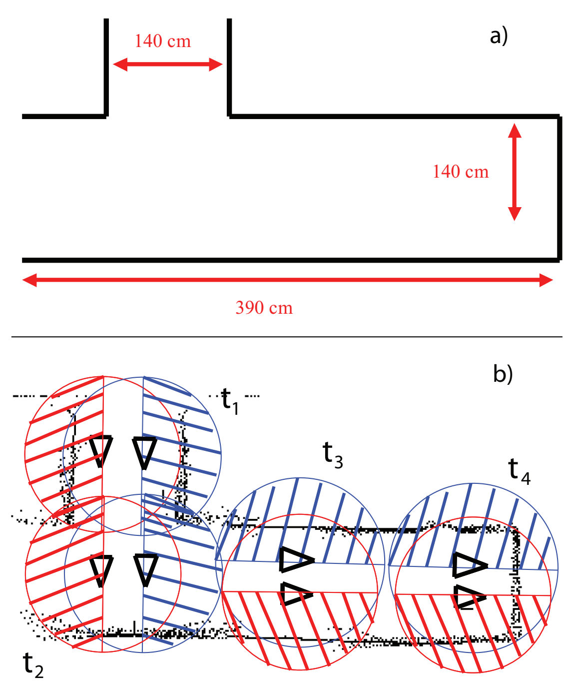


Fig. 5. First scenario. a) scenario of the first experiment: the environment can be partitioned into a sequence constituted by a corridor (upper part), a T-junction, another corridor and a dead end b) the environment reconstructed by IR sensors is visible in the background. Triangles represents robot poses over time, while semi circles show the parts to be monitored by each robot

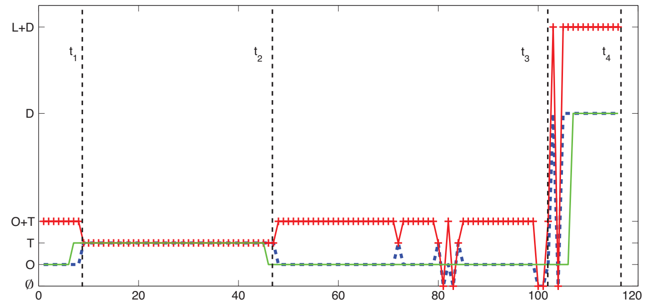


Fig. 6. Estimation process over time for the first experiment: solid (green) line represents the ground-truth, cross-dashed (red) line represents the output of the sensor fusion using the TBM conjunctive rule, dashed (blu) line represents the output of the algorithm using feature orientation information

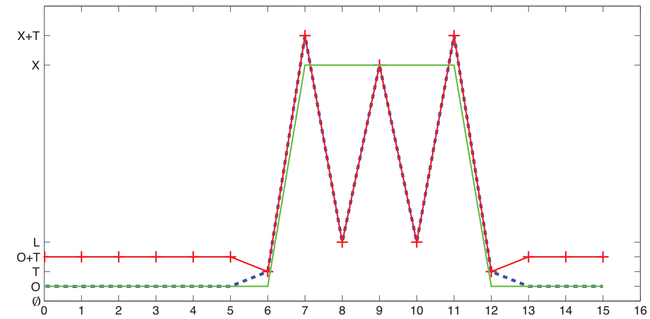


Fig. 7. Estimation process over time for the second experiment: solid (green) line represents the ground-truth, cross-dashed (red) line represents the output of the sensor fusion using the TBM, dashed (blu) line represents the output of the algorithm using feature orientation information

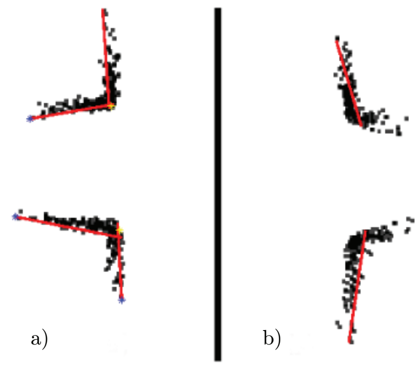


Fig. 8. Feature extraction. a) a robot correctly detect a double angle b) a robot fails to recognize the double angle estimating a meaningless dashed line

The first scenario, with the related robots paths, is depicted in Figure 5. The sequence of mass aggregation performed by the two robots is shown in Figure 6. In particular, the solid (green) line represents the ground-truth, while the cross-dashed (red) line represents the output of the coarse aggregation involving a set of non oriented environmental patterns, and the dashed (blu) line describes the resulting (oriented) patterns obtained after the two-step aggregation procedure is performed.

From the beginning until time $t_1$, each robot detects a wall and, therefore the element of the power-set supporting both a corridor and a T-junction will be set with a mass greater than $0$. Now, since the algorithmic solution described in Subsection IV-E is considered, the aggregation (first-step) cannot solve the ambiguous situation, due to the lack of knowledge about the orientation. However, by performing the second step a conflict regarding the orientation of the T-junction arises and therefore the corridor is taken as the correct pattern. Successively, at time $t_2$ the formation moves into a T-junction. In this case both robots detect a corner along with a wall, and therefore no doubt concerning the correct pattern remains after the masses aggregation (first-step) is performed. Furthermore, as a large discontinuity is detected, the two robots perform a full rotation as explained above. After that, till time $t_3$ the two robots remain within a corridor performing the same aggregation as discussed for the first time interval. Note that, due to the noise affecting the measurements, a couple of times a convex corner is erroneously detected by one of the two robots, making the two observations contradictory. As a result, a high value for the $m(\emptyset)$ is obtained by the two robots. After time $t_3$, the two robots approach a dead-end and also in this case a full rotation is performed. Apart from a few erroneous features extraction both robots detect a convex corner which supports both the L-turn and the dead-end. As for the corridor, the lack of information about the orientation does not allow to disambiguate the proper profile after the first step. Nevertheless, the dead-end is properly recognized after the second step is performed.

The second scenario, which considers two intersecting corridors, is shown in Figure 7. After a correct initial estimation of the corridor, the couple of robots repeatedly fails to estimate the crossing. Indeed, this is due to the wrong feature extraction performed by one of the two robots. In particular, three erroneous detections are performed. On the first case, one of the two robots correctly identifies two corners while the other one simply recognizes a segment. This leads to the wrong (but plausible according to the data) detection of a T-junction. On the second case as shown in Figure 8, one of the two robots correctly identifies two corners while the other one recognizes two roughly parallel segments which do not respect the minimum distance constraint discussed in Subsection IV-A. Now, since no feature is available for the faulty robot the entire mass is assigned to the union of all the hypotheses, i.e., $m(\mathcal{P}) = 1$. As a result, the two profiles crossing and T-junction, supported by the detection of the other robot, cannot be disambiguated. Finally, on the third case an L-turn is detected. This can be explained by the fact that one of the two robots erroneously recognizes a convex corner instead of a concave one. As a consequence, by combining the patterns supported by the correct detection of a concave corner (by one robot), with the patterns supported by the wrong detection of a convex corner (by the other robot), the only plausible pattern turns out to be the L-turn (Figure 10). Differently, the crossing is properly recognized when both robots recognize a couple of concave corners each. Note that, this situation could be partially recovered by assigning a mass $m(\mathcal{P}) > 0$ to the element representing

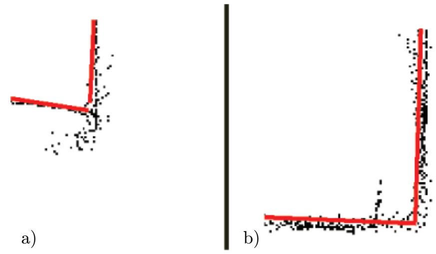


Fig. 9. Corner identification. a) the robot correctly identify a concave corner b) the robot correctly identify a convex corner

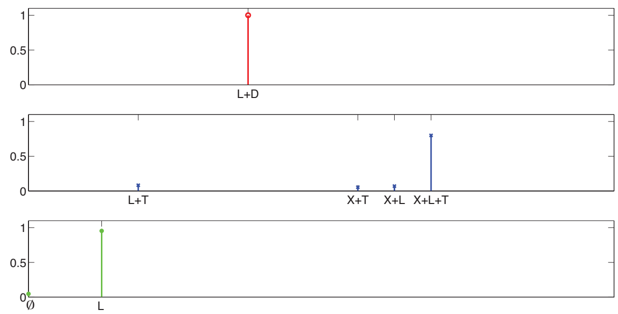


Fig. 10. Mass aggregation for a couple of robots under the assumption of perfect measures. a) mass assignment for the first robot b) mass assignment for the second robot c) result

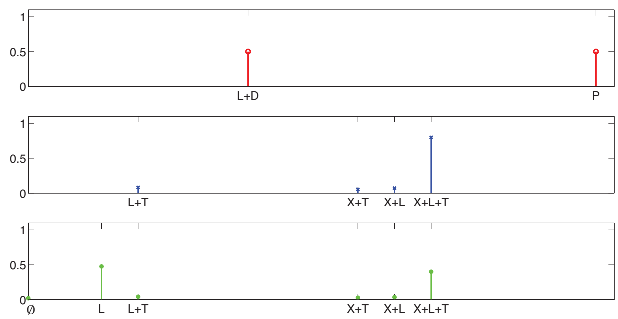


Fig. 11. Mass aggregation for a couple of robots under the assumption of noisy measurements. a) mass assignment for the first robot b) mass assignment for the second robot c) result

the whole set of environmental patterns. In addition, if an index of quality about the feature extraction process were available, the value of $m(\mathcal{P})$ could even be accurately tuned. Obviously, the higher the observation reliability is the lower the value of the mass would be. For sake of clarity, let us now consider a numerical example describing this situation. Figure 10 shows the result of the aggregation in the case of ideal measurements, while Figure 11 shows the result of the aggregation if the suggested workaround is taken into account. In the first case, the result of the aggregation does not allow to detect the correct pattern even if further aggregations are considered, while in the second case this would be possible as the correct pattern is still considered plausible.

The last experiment involves the detection of the L-turn. Also in this case the orientation of the relational feature allows to properly estimate the surrounding environment as can be seen in Figure 9 with no contradiction.

## VI. Conclusions

In this paper the collaborative topological map-building problem for multi-robot systems has been addressed. In the proposed framework, a team of robots is supposed to move in an indoor office-like environment. Each robot, after building a local map by infrared range-finders, compares the surrounding area to a set of (pre-defined) environmental patterns to achieve a local topological description of the environment. The local view of each robot, which is highly constrained by the low sensorial and computational capability of the robotic platform, is then strengthened by a collaborative aggregation schema based on the Transferable Belief Model. Several interesting challenges still remain for future work. From a theoretical standpoint, a more detailed mathematical investigation of the graph-like modeling to describe the environmental patterns might be of interest. From an experimental point of view, a validation of the proposed technique with more complex environments along with an investigation of the effectiveness with respect to different sensorial systems might be of interest.

## References

[1] S. Thrun, "Robotic mapping: a survey," *Exploring artificial intelligence in the new millennium*, pp. 1–35, 2003.

[2] N. Tomatis, I. Nourbakhsh, and R. Siegwart, "Hybrid simultaneous localization and map building: a natural integration of topological and metric," *Robotics and Autonomous Systems*, vol. 44, no. 1, 2003.

[3] S. Thrun and A. Buecken, "Integrating grid-based and topological maps for mobile robot navigation," in *AAAI 1996*, 1996.

[4] H. Durrant-Whyte and T. Bailey, "Simultaneous localization and mapping: part i," *IEEE RAS*, vol. 13, no. 2, pp. 99–110, June 2006.

[5] D. Fox, F. S., and P. H., "Voronoi random fields: extracting topological structure of indoor environments via place labeling," in *Proc. of the International Joint Conference on Artificial Intelligence*, 2007.

[6] A. Howard, G. Sukhatme, and M. Mataric, "Multirobot simultaneous localization and mapping using manifold representations," *Proceedings of the IEEE*, vol. 94, no. 7, pp. 1360–1369, July 2006.

[7] G. Oriolo, G. Ulivi, and M. Vendittelli, "Real-time map building and navigation for autonomous robots in unknown environments," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, vol. 28, no. 3, pp. 316–333, Jun 1998.

[8] S. Panzieri, D. Petroselli, and G. Ulivi, "Topological localization on indoor sonar based fuzzy maps," in *Intelligent Autonomous System 6*. IOS Press, 2000.

[9] D. Dubois and H. Prade, "Possibility theory, probability theory and multiple-valued logics: A clarification," *Annals of Mathematics and Artificial Intelligence*, vol. 32, pp. 35–66, 2001.

[10] S. Yadlapalli, S. Darbha, and K. Rajagopal, "Information flow and its relation to stability of the motion of vehicles in a rigid formation," *IEEE TAC*, vol. 51, no. 8, pp. 1315 –1319, 2006.

[11] L. Krick, M. Broucke, and B. Francis, "Stabilisation of infinitesimally rigid formations of multi-robot networks," *International Journal of Control*, vol. 82, no. 3, pp. 423 – 439, 2009.

[12] M. D. Rocco and G. Ulivi, "An efficient implementation of a particle filter for localization using compass data," in *IAV '10*, 2010.

[13] P. Hough, "Method and means for recognizing complex patterns." U. S. Patent 3, 069 654, December, 18 1962.

[14] G. Shafer, *A Mathematical Theory of Evidence*. Princeton: Princeton University Press, 1976.

[15] P. Smets, "The combination of evidence in the transferable belief model," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 12, no. 5, pp. 447–458, 1990.

[16] M. D. Rocco, F. Flavio, A. Gasparri, and S. Panzieri, "A networked transferable belief model approach for distributed data aggregation - static version," in *Submitted to the 49th IEEE Conference on Decision and Control, CDC10*, 2010.