The 2010 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 18-22, 2010, Taipei, Taiwan

Human Hand Motion Recognition Using Empirical Copula

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Abstract— **Programming by Demonstration (PbD) enables robotic hands to learn human manipulation skills through storing motion primitives and recognizing motion types. In this paper, Empirical Copula is introduced to recognize dynamic human hand motions for the first time using the proposed motion template and matching algorithm. The huge computational cost of Empirical Copula is alleviated by the proposed re-sampling processing. The experiments with human hand motions including grasps and in-hand manipulations demonstrate Empirical Copula outperforms the Time Clustering (TC) method, Gaussian Mixture Models (GMMs) and Hidden Markov Models (HMMs) in terms of recognition rate. In addition, Empirical Copula is also proved to be able to recognize different motions from different subjects.**

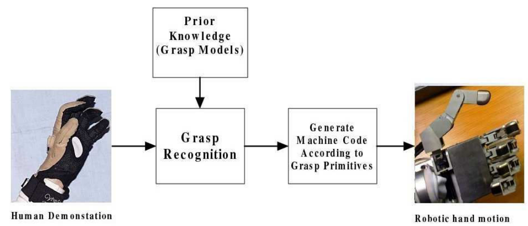

Fig. 1. Programming by Demonstration in the context of grasping tasks

I. Introduction

As one of the most distinguished features which differ from other animals, the human hand is attracting more and more research interests for building human-like robotic hand with not only similar dexterous manipulation, which can be of great help in mobile robots, industries and healthcare. Simulating human hand's mechanism has enabled anthropomorphic robotic hands develop fast in the past decades, such as the JPL/Standford Hand[1], the Belgrad/USC hand[2], the Utah/MIT Hand[3], the Cog Hand[4], the Robonout hand[5] and the ShadowHand[6]. Especially, the ACT hand [7] has not only the same kinematics but also the similar anatomical structure with the human hand, providing a good start for the new generation of anatomical robotic hands. However anatomically correct robotic hand is still a far way to go due to the lack of appropriate sensory systems, unsolved human-robot interaction (HRI) problems, mysterious neuroscience issues, etc.

Though artificial hand may perform stronger and faster motions than the human hand, the high dimensionality and reliable safety make it challenging to program and manipulate human-like robotic hands for dexterous manipulations as human does. The need for hands that can adapt to a variety of grasps and augment the arm's manipulative capacity with fine position and force control is addressed[8], [9]. To solve this problem, Programming by Demonstration (PbD in figure 1) was introduced for complex robotic applications in HRI domain such as grasping and dexterous manipulation [10].

Motion capturing and modeling had been intensively studied such as [11], [12], while motion recognition is still an open problem due to the ambiguity of dynamic grasps though it had been investigated in the past two decades. Various methods have been used or created for the human hand motion recognition. Neural network approach demonstrated its powerful grasp recognition in Glove-Talk project [13], support vector machine was fused to recognize hand gestures [14], and the finite sate machine was also proposed to track and recognize hand gestures with interaction of video games [15]. Natal *et al* [16] and Bedregal *et al* [17] developed fuzzy rule-based methods for the recognition of hand gestures, however the methods are highly dependent on a detailed previous analysis of the recognized gesture features with manual transfer involved. Note that Heumer *et al* [18] attempted to provide a performance criteria for hand grasp classification including 28 algorithms provided by Weka data mining software package. However these algorithms usually require more parameters such as finger angular speed instead of position trajectories only, they are also paid less attention to the context of intelligent robotics, for instance real-time recognition requirement. Wu *et al* [19] proposed a signature descriptor based on elaborated invariants and used a non-linear inter-signature matching algorithm for signature's trajectory recognition, which can solve the problems of occlusion and differences of observing viewpoints, viewing distance and speed, but it has no capability to deal with the continuous motions.

State of the art in grasp recognition can be represented by methods based on Gaussian Mixture Models (GMMs) [20] and Hidden Markov Models (HMMs) [21], [22]. For instance, Calinon and Billard [20] applied GMMs on robot learning of human gestures. On the other hand, Bernardin *et al* [22] presented a method of using HMMs to recognize continuously executed sequences of grasping gestures. Ju *et al* [23] compared Time Clustering (TC) with HMMs and GMMs on the recognition rate of 13 types of different grasps. Gaussian Model is confined to normal distributed clusters, so that more Gaussian components are needed for approximating the data with curve manifolds[24]. The reason that these two statistical methods are popular is that they are very rich

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in mathematical structure and hence their theoretical basis can be adopted for a wide range of applications.

Empirical Copulas were introduced and first studied by Deheuvels in 1979 [25], which can be used to study the interrelations of marginal variables with unknown underlying distributions. Dempster *et al.* [26] constructed an Empirical Copula for collateralized debt obligation tranche pricing and achieved better performance than the dominant base correlation approach in pricing non-standard tranches. Ma and Sun [27] proposed a Chow-Liu like method based on a dependence measure via Empirical Copula to estimate maximum spanning product copula with only bivariate dependence relations while Morettin *et al.* [28] proposed wavelet estimators based on Empirical Copula which can be used for independent, identically distributed time series data. Copula has has been applied widely to areas of finance [29] such as option pricing and portfolio value-at-risk to deal with the skewness. However, few research has been done applying Copula to machine learning field.

In this paper, the principle of PbD is adopted. We introduce another statistical method, Empirical Copula, into the field of recognizing human hand motions for the first time. The results of experiments on the grasp dataset and in-hand manipulation dataset are compared with those of GMMs, HMMs and TC, to demonstrate its recognition efficiency. Datasets of these motions are captured by Cyberglove. This paper is organized as follows: Section II describes the theoretical foundation of Copula and Empirical Copula; Section III proposes the recognition method using Empirical Copula; SectionIV demonstrates the experiment results and compares them with GMMs, HMMs and TC methods. Finally the paper is concluded with conclusions and future work.

II. Dependence Structure Estimation via Empirical Copula

As a general way of formulating a multivariate distribution, copula can be used to study various general types of dependence between variables. Other ways of formulating multivariate distributions include conceptually-based approaches in which the real-world meaning of the variables is used to imply what types of relationships might occur. In contrast, the approach via copulas might be considered as being raw, but it does allow much more general types of dependencies to be included than would usually be invoked by a conceptual approach. The measures of dependence via copula include Kendall's tau, Spearman's rho, Gini's gamma and Blomgvist Beta, whose relationships have been analyzed in [30]. Spearman's rho is considered in this paper. In this section, we revisit the theoretical foundation of copula and Empirical Copula, then introduce the theorem of calculating Spearman's rho using bivariate Empirical Copula.

A. Copula

A n-dimensional copula is defined as a multivariate joint distribution on the n-dimensional unit cube $[0,1]^n$ such that every marginal distribution is uniform on the interval $[0,1]$.

Definition II-A.1. *A n-dimensional copula is a function C from I^n to I with the following properties [30]:*

1 *For every u_i in I,*

$$C(0,\cdots,u_i,\cdots,0)=0,(u_i=0,i=1,\cdots,n) \tag{1}$$

and

$$C(1,\cdots,u_i,\cdots,1)=1,(u_i=1); \tag{2}$$

2 *C is grounded and N-increasing, ie., for each $B=x_{i=1}^n[x_i,y_i]\subseteq[0,1]^n$*

$$V_c(B)=\sum_{z\in x_{i=1}^n\{x_i,y_i\}}(-1)^{N(z)}C(z)\geq 0 \tag{3}$$

where the $N(z)=card\{k\,|z_k=x_k\}$. $V_c(B)$ is the so called C-volume of B.

Sklar's Theorem [31] is central to the theory of copula and underlies most applications of the copula. It elucidates the role that copula plays in the relationship between multivariate distribution functions and their univariate margins.

Sklar's Theorem II-A.1. *Let H be a joint distribution function with margins $F_i(i=1,2,\cdots,n)$. Then there exists a copula C such that for all x_i in $\bar{R}$,*

$$H(x_1,\cdots,x_n)=C(F_1(x_1),\cdots,F_n(x_n)) \tag{4}$$

where C is a n-dimensional copula, F_i are marginal distribution function of x_i.

If $F_i(i=1,\cdots,n)$ are continuous, C is unique. If C is a n-dimensional copula and $F_i(i=1,\cdots,n)$ are distribution functions, then the function H defined by equation 4 is a joint distribution function with margins $F_i(i=1,\cdots,n)$. More details can be seen in [30], [32].

B. Empirical Copula and Dependence Estimation

The Empirical Copula is a characterization of the dependence function between variables based on observational data using order statistics theory and it can reproduce any pattern found in the observed data. If the marginal distributions are normalized, the Empirical Copula is the empirical distribution function for the joint distribution. Because only bivariate Empirical Copula will be employed in this paper (see section III), details of bivariate Empirical Copula is given as follows.

Definition II-B.1. *Let $\{(x_k,y_k)\}_{k=1}^n$ denote a sample of size n from a continuous bivariate distribution. The Empirical Copula is the function C_n given by*

$$C_n(\tfrac{i}{n},\tfrac{j}{n})=\frac{Num((x,y)\big|x\leq x_{(i)},y\leq y_{(j)})}{n} \tag{5}$$

where $x_{(i)}$ and $y_{(j)}$, $1\leq i,j\leq n$, denote order statistics from the sample [30].

The Empirical Copula frequency c_n is given by

$$c_n(\tfrac{i}{n},\tfrac{j}{n})= \begin{cases} \frac{1}{n}, & \textit{if } (x_{(i)},y_{(j)}) \textit{ is an element of the sample} \\ 0, & \textit{otherwise} \end{cases} \tag{6}$$

Note that C_n and c_n are related via

$$C_n(\frac{i}{n}, \frac{j}{n}) = \sum_{p=1}^{i}\sum_{q=1}^{j} c_n(\frac{p}{n}, \frac{q}{n}) \tag{7}$$

Theorem II-B.1. *Let C_n and c_n denote, respectively, the Empirical Copula and the Empirical Copula frequency function for the sample $\{(x_k, y_k)_{k=1}^n$. If ρ denotes the sample version of Spearman's rho [33], [34], then*

$$\rho = \frac{12}{n^2-1}\sum_{i=1}^{n}\sum_{j=1}^{n}[C_n(\frac{i}{n}\cdot\frac{j}{n}) - \frac{i}{n}\cdot\frac{j}{n}] \tag{8}$$

Spearman's rho is used to measure two variables' association [30]. According to the definition and theorem, we can estimate one-to-one correlations between variables using Empirical Copula based on Spearman's rho.

III. Motion Recognition using Empirical Copula

A. Pre-processing

Supposing the number of objects is n and number of attributes is m, for $m << n$, according to the equations 5 and 8, the time complexity of Spearman's rho is $O(n^3)$. If number of samples is huge, computational time would be too long to get the result, which will be proved by the experiments in Section IV. In order to alleviate the burden of huge computational cost, re-sampling processing is adapted for fewer data points. One intuitional and fast method of re-sampling is to take the samples at equal interval on the original datasets, e.g., in figure 2, the number of sample points is set to be 6 on one grasp motion data. Before re-sampling, low-Pass filter is applied to remove high frequency factor and uncertainty noise in order to achieve smooth signals.

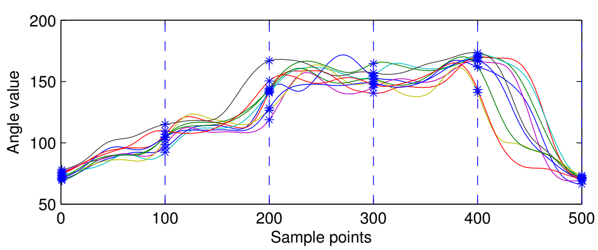

Fig. 2. Re-sample the data at equal interval

B. One-to-one correlation and motion template

Supposing there are m variables which could be joint angles or articulation positions in every motion, C_m^2 is the total number of the one-to-one correlations. Let ρ_{ij} be the Spearman's rho between ith and jth variables, and the motion template is defined as the matrix $\mathbf{P}$ of Spearman's rhos:

$$\mathbf{P} = \begin{pmatrix} \rho_{11} & \cdots & \rho_{1m} \\ \vdots & \ddots & \vdots \\ \rho_{m1} & \cdots & \rho_{mm} \end{pmatrix}$$

where $\rho_{ij} = \rho_{ji}$ when $i \neq j$ and $\rho_{ij} = 1$ if $i = j$. Given s observations for one motion, the template is trained by taking the average of all Spearman's rho matrices.

$$\widehat{P} = \frac{\sum_{i=1}^{s} w_i \mathbf{P}_i}{\sum_{i=1}^{s} w_i} \tag{9}$$

where $\mathbf{w} = [w_1, \cdots, w_s]$ is a weight vector used to store the relative difference of each observation in the estimated template, so that more valid observation may carry larger weight than those with more uncertainties, which may be caused by noise, capturing devices, softwares and the environment. Figure 3 shows an example of the motion template representing the one-to-one correlations among the finger angles when grasping a big ball.

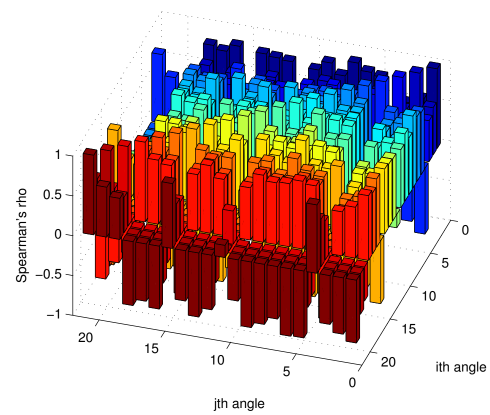

Fig. 3. An example of motion template (grasping a big ball)

The matrix $\mathbf{P}$ effectively aggregates the dependence relations of m variables into just one $m \times m$ matrix, a highly reduced dimensionality of feature space. The relation matrix is naturally uniformed that the matrix is not dependent on differently sampled trials associated with specific speeds. This makes direct comparisons of relation matrices with differently sampled data feasible and computationally efficient.

C. Motion recognition

Motion recognition is straightforward with the proposed template. It is achieved by finding the best match between an observed motion template and pre-trained motion templates. The proposed algorithm is applied on an observed motion to generate its motion template $\mathbf{U} = \{\varrho_{ij}|i, j = 1, \cdots, m\}$. Its dissimilarity with the pre-trained template is achieved by

$$D_t = \|\mathbf{U} - \mathbf{P}\|_t = \left(\sum_{i=1}^{m}\sum_{j=1}^{m}|\rho_{ij} - \varrho_{ij}|^t\right)^{\frac{1}{t}} \tag{10}$$

D_t is t-norm distance; $t \geq 1$ and is a real number; usually we take $t \in \{1, 2, \infty\}$ that D_1 is the taxicab norm, D_2 is the Euclidean norm and D_∞ is the maximum norm. The derived D_t norm infers the dissimilarity between the observed motion and the trained motions. The threshold of the template **P** is defined as

$$th_{\mathbf{P}} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m}|\rho_{ij}|}{\alpha}; \tag{11}$$

where $\alpha \geq 1$, which indicates the threshold is $100/\alpha$ percent of the whole absolute value of the template. Different datasets may have different α values. In this paper, we set $\alpha = 10$, then the threshold of the template in fig 3 would be 28.46. The matching criterion of the motion recognition is that if $D_t \leq th_{\mathbf{P}}$, the observed motion is recognized as belonging to the trained motion.

IV. Experiment

Two datasets are captured by Cyberglove from different subjects, including grasp and in-hand manipulation data. 21 hand angles are recorded every time and their positions are shown in figure 4.

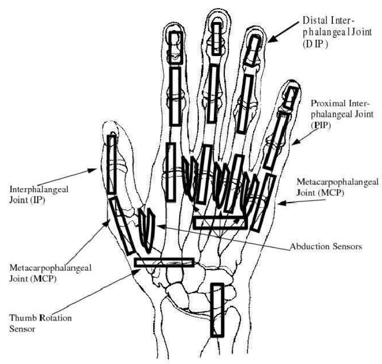

Fig. 4. Positions of the recorded hand angles

A. Datasets

According to the human hand grasp taxonomy [35], hand grasps are classified into 6 atomic different types, for instance, power grasps, precision grasps, and circular/prismatic grasps, etc. We selected 13 different grasp motions as shown in Fig. 5 to test the algorithm recognition ability. Note that every type of grasp is repeated 10 times for training and testing and lasts about 3 seconds.

Fig. 5. Selected grasping tasks

The other dataset is for in-hand manipulation, which is much more complex than simple grasp motion, and associated with the most complex human motor skills. It's the ability to change the position/orientation or adjust an object within one hand. Ten types of manipulations are recorded also by Cyberglove 10 times per type as listed in table I, and a few examples are shown in figure 6.

TABLE I

10 Types of In-Hand Manipulation

1	Open a mobile phone and then close it.
2	Screw to open a small bottle using only thumb, index finger and middle finger.
3	Pick up a coin and move it from the fingertip to the palm
4	Remove the pencil from back to front for writing, as shown in figure 6: (a) pencil walking
5	Pick up a pencil and simply rotate to write, as show in figure 6:(c) simple rotation
6	Pick up a pencil and complexly rotate to write, as show in figure 6:(d) complex rotation
7	Screw to open a big bottle using all five fingers
8	Roll a small cylinder
9	Pick up a scissor and cut paper
10	Pencil flips, as shown in figure 6:(b) pencil flips

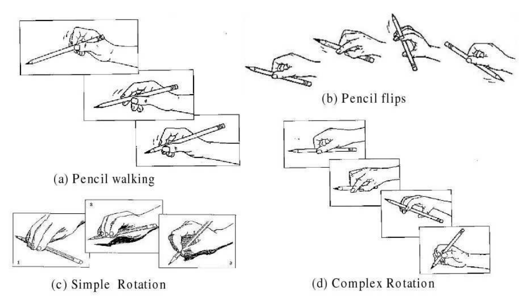

Fig. 6. Four examples of pencil in-hand manipulation

B. One person

We evaluate the proposed method by the experimental set-up including 13 types of hand grasping motions and 10 types of in-hand manipulations from one subject as mentioned, in which the whole datasets are divided into ten equal parts, one tenth of which are used for training the models/templates and the rest for testing the algorithms. In our experiments, it is assumed that there are all types of motion templates which are corresponding to all possible testing motion samples, which ensures the existence of local minimums for recognition.

These recognition results of Empirical Copula is compared with those of GMMs, HMMs and TC from our previous paper [23] for full understanding of its performances. Figure 7 presents the recognition rate of Empirical Copula using one tenth training data along different number of re-sampled points. It can be seen that, when more than 5 re-sampled points from one tenth training data are considered, the recognition rate reaches more than 85.5% and even 90%. It's the highest recognition rate of these four methods using only one tenth of dataset for training, while 0% for HMMs, 55.56% for GMMs and 85.47% for TC [23].

On the other hand, the similar result can be seen in figure 8 and paper [23] (72% for TC, 57% for GMM and 0%

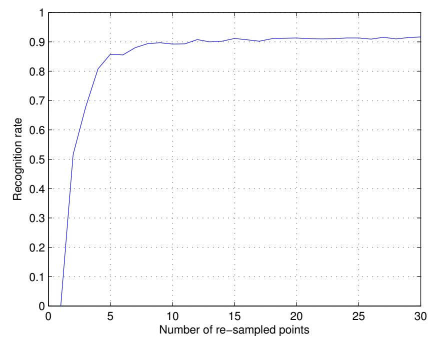

Fig. 7. Recognition rate of Empirical Copula using grasps data from the same subject with different number of re-sampled points from one tenth training data.

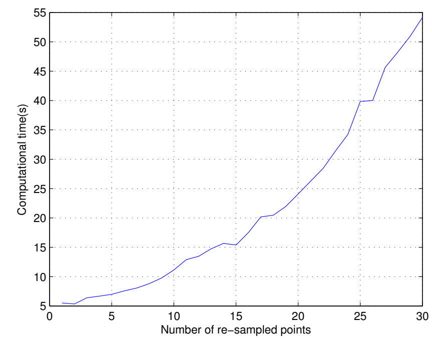

Fig. 9. Computational time of Empirical Copula algorithm with re-sampled points using in-hand manipulation data with different number of training data

for HMM) for the in-hand manipulation data. The reason that recognition rates of these four methods for in-hand manipulation data rise less slowly in general than those for grasps data maybe that the in-hand manipulation datasets are more complex and more non-linear than grasp data and their dependence structures are more distinctive. Therefore, more training data or more re-sampled points are needed for in-hand manipulation operations.

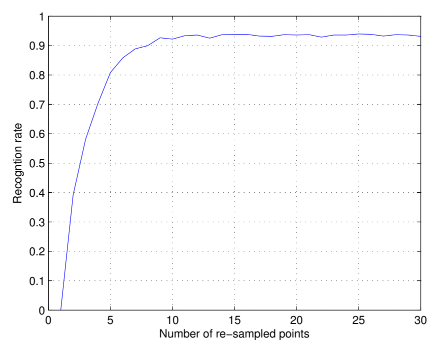

Fig. 8. Recognition rate of Empirical Copula using in-hand manipulation data from the same subject with different number of re-sampled points from one tenth training data.

The Empirical Copula algorithm saves the computational time using fewer data points for modeling by re-sampling training data at equal interval, so the fewer re-sampled points are considered, the less time is needed for modeling. Figure 9 presents the used time of Empirical Copula algorithm with the re-sampling processing using in-hand manipulation data. If the training data is not re-sampled and one tenth of each dataset is used for training, the computational time used by modeling for in-hand manipulation is 1236 seconds, which is more than 80 times of the cost time by Empirical Copula with 15 re-sampled points. It proves the re-sampling processing efficiently reduces the cost time of Empirical Copula and makes Empirical Copula more practical.

C. *Different persons*

Even for the same motion, different subjects perform grasps/manipulations in different ways subject to the different personal habits, manners and speeds. These personal differences ask for a more effective recognition algorithm which can manage not only different motions but also different subjects. In order to test the algorithm's robustness to different person, four-subject datasets have been collected, including two men and two women with different heights. Each motion is repeated 10 times by each subject. One tenth of the data is used for training, while others are for testing. Results are shown in figure 10 for grasp and figure 11 for in-hand manipulation, where recognition rates can reach 80% for grasp and 84% for in-hand manipulation. They are a little lower compared to figure 7 and 8, but they show that Empirical Copula still gets more that 80% recognition rates and it is capable of identifying different motions from different subjects.

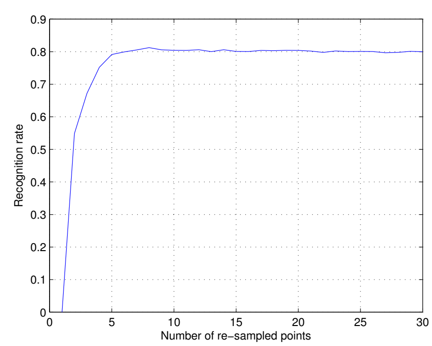

Fig. 10. Recognition rate of Empirical Copula using grasp data from different subjects with different number of re-sampled points from one tenth training data.

V. Concluding Remarks

In this paper, Empirical Copula is introduced to be one of recognition methods for dynamic human hand motion for the first time. Because of the ability of Empirical Copula to analyze dependence relations between variables, we use the structure of the dependence relations among the finger angles as the motion template. This template frees the limitation of the motion speed when comparing different motions, so that recognition process is achieved by finding the dissimilarity

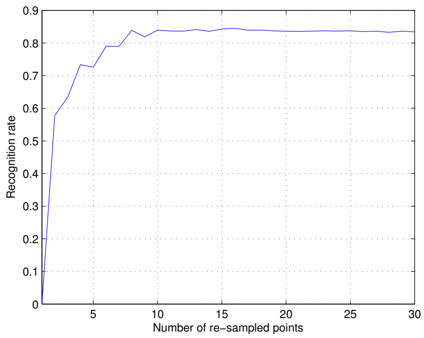

Fig. 11. Recognition rate of Empirical Copula using in-hand manipulation data from different subjects with different number of re-sampled points from one tenth training data.

between an observed motion template and pre-trained motion template. The experiments with one subject convince us that the recognition method using Empirical Copula outperforms other three methods because of its higher recognition rate with only one tenth data as the training data; experiments with different subjects prove that it is capable of identifying different motions from different subjects. Based on those, it can be regarded as one of the most efficient recognition methods. In addition, the proposed re-sampling pre-processing overcomes the main drawback of Empirical Copula, huge computational cost, and makes it more practical, while maintaining high recognition rates. Future work is mainly focused on how to find the proper number of re-sampled points for different datasets, how to set the value of α and qualitative descriptions [36], [37], [38], [39], [40].

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