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Passivity-based Model Predictive Control for Mobile Robot Navigation Planning in Rough Terrains

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Abstract— **This paper presents a novel navigation and motion planning algorithm for mobile vehicles in rough terrains. The main purpose of the algorithm is to generate feasible trajectories while selecting smoother paths, in the sense of level of roughness, toward the goal position. The purpose is achieved by adapting the passivity-based model predictive control optimization setup (PB/MPC), recently proposed for flat terrains, to the case of an outdoor irregular terrain. The passivity-based concept is used to enhance MPC in order to stabilize the goal position guaranteeing the task completion. The framework which is obtained can exploit any vehicle model in order to carefully take into account the vehicle dynamics and terrain structure as well as the wheel-terrain interaction. The inherited property of the MPC optimization allows to impose any additional constraint into the PB/MPC navigation, such as those needed to prevent vehicle rollover and unnecessary sideslip. The cost function representing the level of roughness along a candidate path is used to select the appropriate terrain areas toward the goal position. The results have been verified by several simulation examples.**

I. INTRODUCTION

The popularity of the research of the unmanned ground vehicles has been increased recently due to their usefulness in different operation environments. Planetary explorations, search and rescue missions in hazard areas, surveillance, humanitarian de-mining, as well as agriculture applications such as pruning vine and fruit trees, represent possible fields of using autonomous vehicles in natural environments. Differently from the case of indoor mobile robotics where exclusively flat terrains are considered, the outdoor robotics deals with all possible natural terrains. The unstructured environment and the terrain roughness including dynamic obstacles and poorly traversable terrains pose a challenging problem for the autonomy of the vehicle.

A nice overview of motion planning has been presented in [1] and [2]. The main focus of the early research stage was finding collision-free paths. In [3] the potential field approach for real-time obstacle avoidance was introduced while the concept of navigation functions was illustrated in [4] and [5]. The research on motion planning evolved by adding the capability of taking into account the vehicle motion dynamics constraints within the well known dynamic window approach [6], [7]. This subject was extended to the high-speed navigation of a mobile robot in [8] by the global dynamic window approach, as the generalization of the dynamic window approach. A combination of the dynamic window approach with other methods yielded to some improvements in long-term real-world applications [9]. Dubowski and Iagnemma extended the dynamic window approach to rough terrains introducing the vehicle curvature-velocity space. In this space the stability constraints of the vehicle, for instance expressed by limit values of the roll-over and side slip indexes, can be easily described. The given algorithm was also suitable for high speed vehicles and appropriate for real-time implementation [10]–[12].

Rapidly-exploring random trees (RRT) is a type of probabilistic planners originally developed to cope with differential constraints by LaValle and Kufffner [13]. In [14] the authors introduced quasi-PRM and lattice roadmap (LRM) algorithms. LRM was extended in [15] to allow the state lattice to represent the differential constraints of the mobile vehicle. Inverse trajectory generation was used in [16] and [17] to navigate UAV and UGV, respectively.

The passivity-based nonlinear model predictive control (PB/MPC) has been introduced in [18], where the authors presented the connection of this approach to optimal control. A mobile vehicle navigation framework based on the passivity theory concept combined with nonlinear model predictive control has been recently proposed in [19], where the PB/MPC has been used to stabilize the goal position with the help of the navigation function. Namely, the vehicle model was shaped according to the energy-shaping technique using the navigation function constructed for the given terrain configuration. Then, this new virtual model was used to generate feasible trajectories by satisfying vehicle differential constraints, the limits on the vehicle inputs, as well as safety constraints. Moreover, the passivity-based constraint, inherently included in the PB/MPC optimization setup, enhanced the navigation framework by guaranteeing the task completion. This framework was proposed for flat terrains where the estimated terminal cost-to-go value, required in the MPC optimization, was created by the navigation function.

This paper extends the PB/MPC navigation scheme from flat to rough terrains. The advantage of this framework to easily adapt a more complex vehicle model that accounts for terrain structure as well as wheel-terrain interaction is exploited for mobile vehicle navigation in rough terrains. Such model generates more appropriate trajectories in rough terrains, comparing to those generated by models made for flat terrains, where the effects imposed by the terrain irregular structure are intended to be eliminated afterward, within the control loop. According to the MPC optimization, any additional constraint can be imposed into the PB/MPC navigation, such as those related to vehicle stability, preventing

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from vehicle rollover and unnecessary sideslip. In order to use the PB/MPC navigation scheme for rough terrains as an online approach, the appropriate prediction of the cost-to go (roughness-to go), that is the estimate of the roughness yet to be traversed toward the goal position, is also proposed. To summarize, the obtained algorithm has the following properties: 1. guaranteed task completion, 2. easy adaptable to any vehicle model, 3. easy adaptable to any additional constraints, 4. capability of avoiding obstacles and selecting less rough terrain areas toward the goal position, and 5. an online optimization based on estimation of the roughness-to go value.

II. The PB/MPC navigation planning

The PB/MPC navigation framework is given by the following setup (1-8):

$$J(\mathbf{u}) = \int_{t_0}^{t_0+T} \gamma(\mathbf{x},\, \mathbf{u})dt + \Gamma(t_0+T), \tag{1}$$

$$V(\mathbf{x}) = kNF(x_{cg}, y_{cg}) + \frac{1}{2}v^2, \tag{2}$$

$$\frac{d}{dt}\mathbf{x} = f(\mathbf{x}) + g(\mathbf{x})\mathbf{u} \tag{3}$$

$$\mathbf{y} = h(\mathbf{x}) = [\frac{\partial V}{\partial x} g(\mathbf{x})]^T, \tag{4}$$

$$\mathbf{u}^T(t)\mathbf{y}(t) < -\mathbf{y}^T(t)\phi(t) \tag{5}$$

$$\tau : [0,1] \to C_{free}, \tau(0) = \mathbf{q}(t_0), \tau(1) = \mathbf{q}(t_0+T) \tag{6}$$

$$v(t_0+T) = 0 \tag{7}$$

$$\begin{array}{c} \cos\angle(\nabla NF, \mathbf{e}_{\dot{r}})|_{t=t_0+T_1} < 0 \\ \cos\angle(\nabla NF, \mathbf{e}_{\dot{r}})|_{t=t_0+T} < 0 \end{array} \tag{8}$$

The task of this optimization is to find the input u of the vehicle (traction force and steering angle momentum) along the optimization time horizon $t \in (t_0, t_0+T)$, that is over all potential candidate paths, by minimizing the cost function $J(\mathbf{u})$ given in (1). The integrand $\gamma(\mathbf{x},\mathbf{u})$ is selected depending on what is locally required to minimize. The terminal part $\Gamma(t_0+T)$ represents the estimation of the cost-to-go from the end of the horizon to the goal position in the sense of the selected measure. Similar to [20], where the authors developed a navigation algorithm for a unicycle mobile vehicle in flat terrains based on model predictive control combined with control Lyapunov function, the value of the energy storage function at the end of the optimization horizon, $J = V(\mathbf{x}(t_0+T))$, was selected in the given navigation framework. The energy storage function is selected as in (2) and it includes a virtual potential term constructed by the navigation function of the given terrain and a kinetic term, where (x_{cg}, y_{cg}) and v are the current coordinates and velocity of the vehicle. Eq. (3) represents the virtual model obtained by shaping the energy of the real vehicle dynamics by the navigation function, where $\mathbf{x}$ are the new states. The choice of the output (4) forces the system to be a passive one with respect to the radially unbounded and continuously differentiable storage function V, and is based on the passivity control theory concept. Constraint (5), where ϕ injects a damping to the model, makes the goal position an asymptotically stable point.

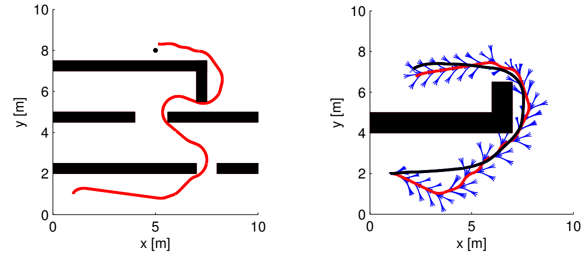

Fig. 1. Left: Generated path based on the limited set of motion primitives. Right: The path presented with black color and no lattices was obtained by the GA algorithm while the other one by the optimization with a few motion primitives.

Eq. (6) constraints the optimization on the collision-free configurations, where τ is the map from the initial to the final vehicle configuration $\mathbf{q}$, into the collision free space C_{free}. Constraint (7) gives the guarantee that there exists such control $\mathbf{u}$ that can stop the vehicle at the end of the horizon satisfying the collision-free constraint (6). If this constraint holds then any state space point $\mathbf{x}(T_1)$ preserves safe policy, where $T_1 < T$. Not allowing the vehicle to reach zero velocity, the optimization is repeated each T_1 for the horizon T. By the choice $J = V(\mathbf{x}(t_0+T))$, condition (7) implies $J = kNF(\mathbf{r}(t_0+T))$, where $\mathbf{r} = (x_{cg}, y_{cg})$ is the current position vector of the vehicle. Such minimization task ensures the shortest possible path to the goal, since the selected navigation function represents the shortest path to the goal for each vehicle position.

Conditions given in (8), where $\cos\angle(\nabla NF, \mathbf{e}_{\dot{r}})$ is the current angle between gradient of the navigation function and current vehicle velocity direction, are the terminal conditions which keep the vehicle oriented toward the decrease of the navigation function at the end of each PB/MPC optimization cycle. This condition helps the optimization to find a feasible solution.

The optimization problem (1-8) can be solved by control parametrization within the given horizon where the problem became a nonlinear programming optimization (see e.g., [17]). Another approach is based on a priori defined motion primitives [21] that has widely been used in mobile vehicle navigation, was also used in [15]. In [19], the GA algorithm was implemented for local optimization, where chromosomes consisted of the potential values of the vehicle accelerations and steering angles. Fig. 1 illustrates the approach based on the limited set of maneuvers and the GA approach.

III. The PB/MPC cost function for rough terrains

A. Decrease of navigation function

The main task of the PB/MPC navigation scheme (1-8), used in [19], is to select such a control u giving the minimal value of the energy function, that is $J = V(\mathbf{x}(t_0+T))$ at the end of the optimization horizon, and if condition (7) holds, this means to obtain the minimal value of the navigation function, $NF(\mathbf{r}(t_0+T))$. Such optimization policy certainly implies the shortest path to the goal under the selected local optimization technique. However, choosing the shortest path to the goal position may be a rather strict constraint especially when the vehicle moves in unknown rough terrains.

In order to ensure the decrease of the navigation function along the selected paths if different cost function is used, the additional constraint (9) has been imposed. Since the optimization is performed within time T, while the control action u is applied only over T_1, both conditions of (9) need to be included.

$$NF(\mathbf{r}(t_0+T)) < NF(\mathbf{r}(t_0+T_1)) < NF(\mathbf{r}(t_0)) \quad (9)$$

B. Local measure of roughness

Let us explain some possible cost functions that can describe the level of roughness of the selected path and can be used within the PB/MPC navigation planning. The first candidate to describe the hardness of a path traversability is certainly the function that penalizes high roll and pitch values along the path. One such function is used in [15] and is given by:

$$\gamma(\mathbf{x}, \mathbf{u}) = \int_{t_0}^{t_0+T} (1+\alpha(\varphi^2+\theta^2))dt, \quad (10)$$

where φ, θ are the roll and the pitch angles along the candidate path. Coefficient α represents the tradeoff between the minimum-time and minimum slope-dwell solutions. If $\alpha = 0$, then the solution gives the fastest path along the horizon.

The second candidate was proposed in [22] and is given by:

$$\gamma(\mathbf{x}, \mathbf{u}) = \int_{t_0}^{t_0+T} \frac{dt}{v_{max}(\mathbf{r})}. \quad (11)$$

This function describes the roughness level in terms of the high mobility of the vehicle, where $v_{max}(\mathbf{r})$ is the maximal value of the vehicle velocity at each point of a path that still do not cause sideslip and rollover of the vehicle. This function is more descriptive when it is important to increase the vehicle mobility. It favors those paths that allow high speeds while preserving vehicle stability constraints.

Other possibilities for the estimation of the hardness of traversability of a candidate path are given in [23]–[25], where the authors introduced a traversability index as a parameter that described the roughness of the terrain.

For demonstration purposes, the local measure of the roughness is expressed by the relative height of the terrain, describing its deviation from flatness. This approximation is done for all candidate paths within the optimization horizon, and does not decrease the generality of the approach since any roughness function mentioned above can be used instead.

C. Roughness-to-go value

The vehicle optimizes the level of roughness toward the goal position in order to select smother paths. In order to optimize the residual roughness, the cost-to-go terminal part $\Gamma(t_0+T)$ represents roughness-to-go value, is added to the local measure of the roughness, in accordance with (1). If it is possible to estimate the optimal roughness-to-go value in case the terrain cost map is given with respect to the vehicle constraints, then the PB/MPC navigation policy would be an optimal one. The computation of the optimal path toward the goal position for each MPC horizon according to the given terrain cost map is rather expensive and cannot be implemented during the navigation. In the optimal control of nonlinear systems, hence in the MPC, the estimation of the optimal cost-to-go value is often impossible to find and, therefore, some rough estimations of this value are needed. For this purpose, the estimated value of the roughness-to-go function is calculated with a conservative assumption that the vehicle goes straight from the end position of the optimization horizon, $x(t_0+T)$, toward the goal position.

IV. Virtual model of the vehicle with shaped energy

A. Model of the vehicle in rough terrain

The main challenge to derive the vehicle model acting in rough terrain is to include the terrain uncertainties. Both tire compliance and suspension compliance should also be modeled (see e.g., [26]–[28]).

The vehicle model driven with traction force input u_p and steering input u_s is given by the state space form:

$$\dot{x} = f(x) + g(x)\begin{pmatrix} u_p \\ u_s \end{pmatrix}, \quad (12)$$

where

$$\dot{x} = (\dot{x}_{cg} \quad \dot{y}_{cg} \quad \dot{v} \quad \dot{\psi} \quad \ddot{\psi} \quad \dot{\beta} \quad \ddot{\varphi} \quad \dot{\delta})^T$$

$$f(x) = (vf_{e_x} \quad vf_{e_y} \quad f_v \quad \dot{\psi} \quad f_\psi \quad f_\beta \quad f_\varphi \quad f_\delta)^T$$

$$g(x) = \begin{pmatrix} 0 & 0 & g_{v_1} & 0 & 0 & g_{\beta_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_{\delta_2} \end{pmatrix}^T$$

ψ, δ, φ and β are current vehicle orientation with respect to the given reference frame, steering angle, body roll angle, and angle of velocity direction with respect to vehicle reference frame, respectively.

The nonholonomic constraints are

$$f_{e_x} = \cos\beta\cos\psi - \sin\beta\sin\psi, \quad f_{e_y} = \cos\beta\sin\psi + \sin\beta\cos\psi$$

f_v and g_{v_1} in (12) are obtained using the vehicle longitudinal dynamics motion equation:

$$m\dot{v} = F_x\cos\beta + F_y\sin\beta, \quad (13)$$

where m is total vehicle mass, F_x and F_y are forces acting along the x and y directions of the vehicle internal reference frame, respectively, given by:

$$F_x = [-2sin\delta F_{s.f} + u_p], \quad F_y = [2cos\delta F_{s.f} + 2F_{s.r} + \textstyle\sum_{i=1}^{4} T_i],$$

$F_{s.f}$ and $F_{s.r}$ being lateral forces of the front and rear wheels. These forces can be approximated using the stiffness coefficients, C_f and C_r, as $F_f = C_f\alpha_f$ and $F_r = C_r\alpha_r$, α_f and α_r being the slip angles of the front and rear tyres approximated by:

$$\alpha_f = \delta - \arctan\frac{v\sin\beta + L_f\dot{\psi}}{v\cos\beta}, \quad \alpha_r = -\arctan\frac{v\sin\beta - L_r\dot{\psi}}{v\cos\beta},$$

where L_f and L_r are the distances of the front and rear wheels from the vehicle center of mass, respectively.

T_i represents the terrain disturbance force acting at each wheel $i = 1..4$. Assuming the terrain elevation is a continuous and differentiable function $z(x,y)$, T_i is given by:

$$T_i = N_i\left(-\sin\psi \frac{\partial z}{\partial x_0} + \cos\psi \frac{\partial z}{\partial y_0}\right),$$

where N_i is the normal contact force at wheel i while $\frac{\partial z}{\partial x_0}$ and $\frac{\partial z}{\partial y_0}$ are gradients calculated in the vehicle body frame.

f_β and g_{β_1} are obtained using the momentum equation:

$$mv(\dot\beta + \dot\psi) = -F_x \sin\beta + F_y \cos\beta + m_s h\ddot\varphi, \tag{14}$$

m_s being the mass of the chassis and h the height of the chassis center of mass.

$\ddot\varphi$ can be extracted from the suspension compliance model equation:

$$I_{xx}\ddot\varphi = F_y h + M_{roll} + M_s, \tag{15}$$

where I_{xx} is the roll moment inertia of the chassis, $M_{roll} = m_s gh\varphi$ is the moment caused by the inclination of the chassis center of mass and M_s the suspension moment on sloped terrain which can be given as:

$$M_s = -k_f(\varphi - \varphi_f) - k_r(\varphi - \varphi_r) - b_f(\dot\varphi - \dot\varphi_f) - b_r(\dot\varphi - \dot\varphi_r),$$

k_f, k_r being the stiffness and b_f, b_r the damping rates of the respective axles. φ_f and φ_r are the roll disturbances caused by the terrain. By the assumption that wheels do not lose contact with the terrain, these disturbances are given by:

$$\varphi_{f,r} = \frac{z_{f,r+1} - z_{f,r}}{y_w}, \quad \dot z_{f,r} = \frac{\partial z}{\partial x_0} v\cos(\psi + \beta) + \frac{\partial z}{\partial y_0} v\sin(\psi + \beta)$$

$z_{f,r}$ are the positions of front and rear wheels respectively, and y_w is the vehicle width.

f_ψ is obtained using the momentum equation:

$$I_{zz}\ddot\psi = 2F_{s.f}L_f\cos\delta - 2F_{s.r}L_r + \sum_{i=1}^{4} T_i L_i, \tag{16}$$

I_{zz} being the yaw moment of inertia, L_i the longitudinal position of each wheel with respect to the vehicle center of mass.

The last raw of (12) represents the steering dynamics of the vehicle from which f_δ and g_{δ_2} can be extracted.

B. Energy-shaping

The main task of this section is to make a virtual model that is passive with globally asymptotically stable equilibrium point in which information on the goal position is included. More precisely, this task could be considered by making the subsystem described by triple state of interest $\begin{pmatrix} x_{cg} & y_{cg} & v \end{pmatrix}_e = \begin{pmatrix} x^* & y^* & 0 \end{pmatrix}$, where (x^*, y^*) is the goal position, to be globally asymptotically stable. The position coordinates could be transformed with $e_x = x_{cg} - x^*$, $e_y = y_{cg} - y^*$, transforming the desired equilibrium point into the zero-state, that is $\begin{pmatrix} e_x & e_y & v \end{pmatrix}_e = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$.

Now, the model of the system given in (12) becomes:

$$\dot e = f(e) + g(e)\begin{pmatrix} u_p \\ u_s \end{pmatrix}, \tag{17}$$

where

$$\dot e = (\dot e_x \quad \dot e_y \quad \dot v \quad \dot\psi \quad \ddot\psi \quad \dot\beta \quad \ddot\varphi \quad \dot\delta)^T, \quad f(e) = f(x), g(e) = g(x).$$

The energy of the given system could be shaped with the traction force input given in the following way:

$$u_p = \frac{1}{g_{v_1}}(-f_v - k\nabla NF(\mathbf{r})\mathbf{e}_{\dot r}) + v_p, \tag{18}$$

where

$$k\nabla NF(\mathbf{r})\mathbf{e}_{\dot r} = k\|\nabla NF(\mathbf{r})\|\cos\angle(\nabla NF(\mathbf{r}), \mathbf{e}_{\dot r}) \tag{19}$$

is the scaled inner product of the gradient of navigation function $NF(\mathbf{r})$ with a unit vector of current direction represented by the vector $\dot{\mathbf{r}}$. This term favors those directions toward the decrease of navigation function $NF(\mathbf{r})$, hence toward the goal position. For instance, if the vehicle goes in the direction of the steepest descent of navigation function, (19) will have a minimum possible value thus providing the maximum value in (18). If the vehicle climbs the surface of the navigation function, the component (19) will have positive values, hence decreasing the speed and stopping the vehicle. v_p is new traction force control input of the system.

The virtual model obtained by energy-shaping technique applied to the model (17) is:

$$\dot e = \tilde f(e) + \tilde g(e)\begin{pmatrix} u_p \\ u_s \end{pmatrix}, \tag{20}$$

where $\tilde g(e) = g(e)$ and

$$\tilde f(e) = (vf_{e_x} \quad vf_{e_y} \quad -k\nabla NF(\mathbf{r})\mathbf{e}_{\dot r} \quad \dot\psi \quad f_\psi \quad \tilde f_\beta \quad f_\varphi \quad f_\delta)^T$$

Note that

$$\dot v = -k\nabla NF(\mathbf{r})\mathbf{e}_{\dot r} f_r + g_{v_1} v_p \tag{21}$$

and the function f_β was changed into $\tilde f_\beta$ since $g_{\beta_1} \neq 0$.

In [19] authors have shown that for the general case of the vehicle energy shaped model, the subsystem that contains triple state of interest x_{ss} has the zero-state equilibrium point, that is $x_{ss_e} = (e_x \quad e_y \quad v)_e^T = (0 \quad 0 \quad 0)^T$. For the purpose of clarity, the proof is recalled here.

Assuming $\dot e_x = 0$, $\dot e_y = 0$ and $\dot v = 0$, namely that there is no movement in both directions of the reference frame, the vehicle velocity is equal to zero, $v = 0$. From $\dot v = 0$, using (21) with input $v_p \equiv 0$, it follows $\nabla NF(\mathbf{r})\mathbf{e}_{\dot r} = 0$. One possible solution of the latter equation, $cos(\nabla NF(\mathbf{r})\mathbf{e}_{\dot r})) = 0$, implies that the first condition in (8) is not satisfied since this equality also holds at the end of the operating time horizon T_1. This means that this equality is true only for the second possible solution, $\mathbf{r} = (x^* \; y^*)$, that is $e_x = 0$ and $e_y = 0$, since the unique minimum of the navigation function $NF(\mathbf{r})$ is at this point.

C. Passivity-based stability

1) Output of the system: The virtual model will become passive when the output is selected according to (4), that is:

$$\mathbf{y}^T = \frac{\partial V}{\partial x} g(x) = \left(\frac{\partial V}{\partial \mathbf{e}_x} \quad \frac{\partial V}{\partial \mathbf{e}_y} \quad \frac{\partial V}{\partial v} \quad \mathbf{0}\right) g(x) = (g_{v_1} v \quad 0) \tag{22}$$

This means that despite the fact that there are two inputs, the output of interest with respect to the given storage function is

$$y = g_{v_1} v = \frac{1}{m} v. \tag{23}$$

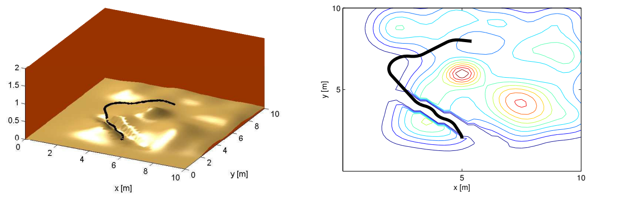

Fig. 2. Vehicle follows flat terrain on the left-side toward the goal position

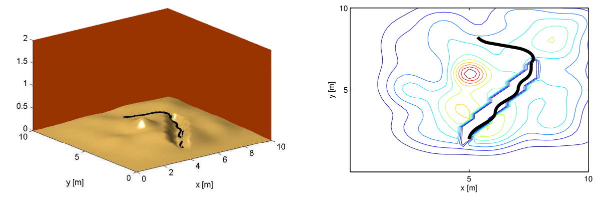

Fig. 3. Vehicle follows flat terrain section on the right-side toward the goal position

2) *Zero-state observability:* In [19] it has been shown that, for the general case of the vehicle with energy-shaped model, the subsystem that contains the triple state of interest x_{ss} was zero-state observable. For the purpose of clarity, this proof discussion is recalled here.

The subsystem of interest is:

$$\begin{pmatrix}\dot{e}_x\\ \dot{e}_y\\ \dot{v}\end{pmatrix} = \begin{pmatrix} v f_{e_x}\\ v f_{e_y}\\ -k\nabla NF(\mathbf{r})\mathbf{e}_{\dot{r}}\end{pmatrix} + \begin{pmatrix}0 & 0\\ 0 & 0\\ g_{v1} & 0\end{pmatrix}\begin{pmatrix}v_p\\ u_s\end{pmatrix} \qquad (24)$$

ZSO conditions $y \equiv 0$ and $u \equiv 0$ imply $v \equiv 0$ if y_1 is selected according to (23). This means $\dot{e}_x = 0$ and $\dot{e}_y = 0$, as well as $\dot{v} = 0$. Using (21), it follows $\nabla NF(\mathbf{r})\mathbf{e}_{\dot{r}} = 0$. Similar to the discussion given in Subsection B, the latter equation implies $\mathbf{r} = (x^*\ y^*)$, that is $e_x = 0$ and $e_y = 0$, so that $x_{ss} = 0$, namely the subsystem (24) is zero-state observable.

3) *Damping injection:* Since all conditions of Theorem 1 are satisfied, the new traction force input v_p could be selected in the form $v_p = -\phi(y)$, where ϕ is any locally Lipschitz function such that $\phi(0) = 0$ and $y^T\phi(y) > 0$ for all $y \neq 0$. One possible choice of damping injection using function $\phi(y)$ is (see e.g., [29]):

$$v_p = -\varepsilon\frac{1}{g_{v_1}}\frac{2}{\pi}\arctan(k_v v) = -\varepsilon m\frac{2}{\pi}\arctan(k_v v), \qquad (25)$$

where ε and k_v are positive constants that should be selected.

In order to obtain stability, with the assumption on the vehicle velocity $v \geq 0$, we can write:

$$v_p \leq -\varepsilon m\frac{2}{\pi}\arctan(k_v v). \qquad (26)$$

This choice of v_p satisfies (5) making the equilibrium x_{ss_e} to be a globally asymptotically stable point.

This claim could be easily checked using the time derivative of the energy storage function V along trajectories of the system (20) with the input v_p that satisfies (26).

With the help of (21), we have:

$$\dot{V} = \frac{\partial V}{\partial x}\dot{x} = \left(\frac{\partial V}{\partial \mathbf{e}_x}\quad \frac{\partial V}{\partial \mathbf{e}_y}\quad \frac{\partial V}{\partial v}\quad \mathbf{0}\right)\dot{x} = g_{v_1} v v_p, \qquad (27)$$

and, using (26), the condition on the derivative of the energy storage function along trajectories of the closed loop system is obtained

$$\dot{V} = g_{v_1} v v_p \leq -\varepsilon v\frac{2}{\pi}\arctan(k_v v). \qquad (28)$$

Hence, $\dot{V}$ is negative semidefinite and $\dot{V} = 0$ if and only if $v = 0$. By zero-state observability, $y \equiv 0$ and $u \equiv 0$ implies $x_{ss} = 0$. Therefore, by the invariance principle, the origin of the subsystem that contains triple states of interest x_{ss} is globally asymptotically stable.

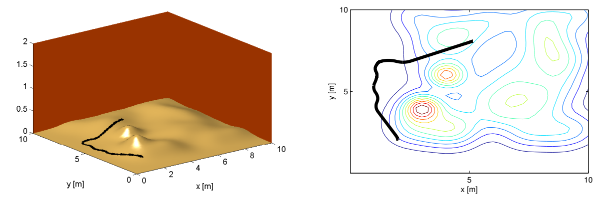

Fig. 4. Vehicle avoids rough terrain sections toward the goal position

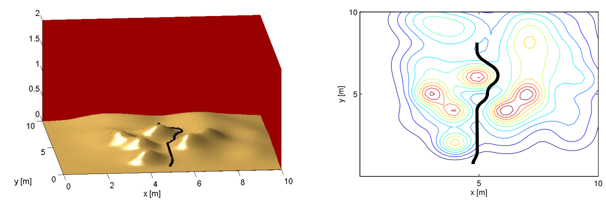

Fig. 5. Vehicle avoids highly rough terrain sections toward the goal position

V. Simulation

In all given simulations, we assume that the terrain is perfectly known. In the case there are unknown terrains sections toward the goal position, these sections are assumed to be completely flat when the roughness-to-go is estimated. However, this problem of unperfect terrain information is unavoidable and such driving paradigm is coherent with the natural human driving policy.

The obtained results show a desirable behavior of the vehicle, meaning that the vehicle was capable to select more traversable terrain sections. All presented examples are illustrated by two subfigures, the left one that represents the generated path in rough terrain, and the right one which depicts the contour plot representing the cost field of the level of roughness. Fig. 2 and 3 show the capability of the vehicle to follow flat parts of the terrain toward the goal position. In Fig. 4 and 5, two different rough terrains are given where the vehicle avoided more difficult rough areas while approaching the goal position.

The presented results show that even if a limited set of motion primitives is used, a desirable vehicle behavior is obtained. In the case when there is no obstacle but only rough terrain, a quadratic function can be used to form a navigation function having a unique minimum at the goal position. However, if this is not the case, any form of navigation function can be used instead [1].

VI. Conclusion

A novel navigation and motion planning algorithm for mobile vehicles in rough terrains has been proposed. In this algorithm, the MPC navigation paradigm is enhanced with the passivity control concept to stabilize the goal position. This gives the important property of the algorithm of guaranteeing the task completion.

The PB/MPC navigation and motion planner can be used with any vehicle model suitable to carefully describe the vehicle behavior for any terrain configuration as well as wheel-terrain interaction, in order to generate feasible trajectories. Such motion policy is more general and reliable comparing to the one where trajectories for rough terrains are generated as for flat terrains, where the effects imposed by the terrain are intended to be eliminated afterward, within the control loop.

The inherited property of the MPC optimization allows one to impose any additional constraint into the PB/MPC navigation such as the constraint on the vehicle stability that can be described by rollover and sideslip angles.

The cost function that estimates the roughness-to-go value, that is the hardness of the residual path traversability, helps the vehicle to select smother areas toward the goal position.

The proposed PB/MPC navigation planning in unknown terrains seems to be natural and consistent with the safe driving policy adopted by humans in such terrains, where the cost function represents the roughness-to-go value. In addition, the MPC paradigm is suitable for on-line use, unlike the off-line approaches used by sample-based algorithms.

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