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Multi-robot Navigation with Limited Communication - Deterministic vs Game-Theoretic Networks

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Abstract— **This paper presents a novel approach to the navigation of dynamically communicating robots via a bidirectional interaction model between the robot network and the continuous states. First, the robot dynamics is formulated as being dependent on the communication network where two robots - if in communication - can access each other's position and goal information. Next, three alternative strategies for establishing the communication network depending on the robots' states are presented: deterministic, game-theoretic and mixed approaches. In the first approach, the network is defined deterministically based on the robots' states and the communication range. The game-theoretic network formation is based on utilizing the conflict between the communication gain and cost. The mixed approach integrates features from deterministic and game-theoretic approaches. An extensive statistical study investigates comparative performance characteristics for exploration, zone and procession type goals.**

I. Introduction

Many multi-robot applications require that all the robots navigate simultaneously to their goal positions without any collisions along the way. Most previous work has assumed that each robot has complete information about the other robots at all times – possibly over a communication channel. However, this may not be possible since in general the communication process is resource constrained. This paper presents a novel approach to the navigation of multiple robots under limited communication. The contribution of the paper is that it proposes three alternative strategies – deterministic, game-theoretic and mixed - that may be employed in setting up the underlying network using a model of bidirectional interaction between continuous states and the network graph.

Three areas in the literature are related: robotic navigation, hybrid systems and network games. Multi-robot navigation problem has been addressed in two different approaches. In the computational geometry based approaches, the kinematic planning is separated from the dynamic control stage. Planning is achieved in general either in a centralized or decoupled manner [21], [14], [24]. Alternatively, feedback based approaches use various forms of artificial potential functions with the aim of establishing a formal analysis framework [10], [13], [11] - assuming access to complete information regarding all the robots. Sensor uncertainty has been considered in [2] as the discrepancy between the robots' real and and measured positions may lead to jerky movements or even collisions. The impact of time-varying fading communication links on the performance of a mobile network is studied in [18]. However, these work have not considered the hybrid nature of the underlying navigation problem. The models proposed for hybrid systems depend to a great extent on the type of system requirements [7], [26]. The relation between graphs and control has been uncovered [22], [19], [16]. Embedded graph transition systems have been proposed as a grammatical approach to modeling and controlling the switching of a system's network topology, continuous controllers and discrete modes [15] which has then been employed in multi-robot deployment and coordination in [23]. Here, a set of well-defined guarded rules define the manner in which the network graph evolves over time [15]. The definition of these rules needs to be expanded if the agents have a separate payoff decision-making mechanism with respect to the graph evolution. On the other hand, game theory has been used to model collaboration models in automatic networks. One approach is based on cooperative game theory, where cooperation is based on coalition formation [20]. Most theoretical models use a two stage structure [1], [17], [5] where approaches differ in the manner these two steps are accomplished. First, the players decide whether or not to join a coalition followed by choosing their behavior depending on the coalition structure in the second stage. However, these models do not take the dynamics of the players into account. Hence, the interplay between continuous dynamics, evolving robot network and the communication strategy remains relatively less studied.

Consider a collection of $p \in \mathbb{Z}^+$ cylinder-shaped robots arbitrarily placed within a disk-like workspace that need to move to their a priori specified target locations without any collisions along the way. We assume that each robot is capable of determining its own position relative to its target[1]. Furthermore, at each instance, it communicates with its neighboring robots as determined by the network graph. Once in communication, it can access the other's position and distance to goal information which are then used to update its the state dynamics accordingly. This communication is very useful as it can adjust its movement accordingly in order to avoid collisions or not to block the other robots from getting to their goal positions. In this paper, we consider three alternative approaches to communication network formation– deterministic, game-theoretic and mixed – based on a model of bidirectional interaction between continuous states and the network graph. The deterministic approach is based on maintaining a required level of signal-to-noise (SNR) ratio

[1]Here, we assume that this information is not noisy, also the extension to noisy case has been studied previously [2].

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between each pair of communicating robots. However such an approach may be employed if each robot is capable of detecting the received signal values from all the other robots at all times. In case this is not possible, each robot can use a game-theoretic approach based on pairwise games which requires no communication or synchronization for selecting the robot pairs. The robot pairs use the fundamental tradeoff between attaining a high SNR and cost of communication in order to adjust the communication network. In the mixed model, each robot employs a hybrid approach for the dynamic network formation. The outline of the paper is as follows. Multi-robot networks and dynamics are defined in Section II. The next three sections describe each approach – deterministic, game-theoretic and mixed networks respectively. Extensive simulation results are discussed in Section VI. The paper concludes with a brief summary including future work.

II. ROBOTS DYNAMICS

Let $\mathcal{P} = \{1, \ldots, p\}$ be the set of robots all residing in a two-dimensional workspace of radius ρ_0. Each robot $i \in P$ is associated with the radius $\rho_i \in \mathbb{R}$, a state $b_i : \mathbb{R}^{\geq 0} \to \mathbb{R}^2$ and a goal position $h_i \in \mathbb{R}^2$. The state of the robots $b(t) \in \mathbb{R}^{2p}$ is defined as $b(t) = \sum_{i \in P} b_i(t) \otimes e_i$ and the goal of the robots $h \in \mathbb{R}^{2p}$ is defined as $h = \sum_{i \in P} h_i \otimes e_i$ where $e_i \in \mathbb{R}^p$ are the unit vectors in $\mathbb{R}^p$. Let $\delta_{ij} = \parallel b_i - b_j \parallel$ denote the robots' pairwise relative distance and let $\beta_{ij} = \delta_{ij}^2 - \rho_{ij}^2$ where $\rho_{ij} = \rho_i + \rho_j$. Since robots cannot overlap each other, it is required that

$$\forall i, j \in \mathcal{P} \qquad \beta_{ij} \geq 0 \tag{1}$$

Furthermore, let $\beta_{0i} = \rho_{0i} - \parallel b_i \parallel^2$ where $\rho_{0i} = \rho_0 - \rho_i$. Each robot should also stay in workspace that is bounded by radius ρ_0 which means that:

$$\forall i \in P \qquad \beta_{0i} \geq 0 \tag{2}$$

The free robot configuration space $F \subset \mathbb{R}^{2p}$ satisfies Eq.1 and Eq.2.

A. State Dependent Network Graph

The communication among the robots is determined by a state-dependent graph map $g : F \to G$. Here $G = \{g' | g' \subseteq g^P\}$ is the set of all possible graphs on P and g^P is the complete graph. The graph map is defined as $g(b) = (P, E(b))$ where $E(b)$ denotes the set of edges as defined by a communication matrix $A(b) = [a_{ij}(b)]$. The corresponding $E(b)$ is defined as:

$$E(b) \quad = \quad \{ij \mid a_{ij} = a_{ji} = 1\} \tag{3}$$

If $a_{ij} = 1$, this means that robot i wants to communicate with robot j. If both robots want to communicate, a communication link is setup. An edge $ij \in E(b)$ exists iff a communication link between robots i and j is established.

B. Network Graph Dependent State

The underlying graph on the other hand induces a set of coupled gradient systems on F that define the state of each robot. First note that each graph defines for each robot i, the set of its immediate neighbors $\mathcal{N}_i(g) = \{j \in P | ij \in E(b)\}$[2] with which it is in direct communication. Robot pairs that are in communication with each other access other's position and goal information. This information designates the state dynamics as:

$$\dot{b}_i = -D_{b_i}\varphi_i(b, g) \tag{4}$$

The function $\varphi_i : F \times G \to [0, 1]$ is constructed as follows: i.) The goal position for robots $\{i\} \cup \mathcal{N}_i(g)$ are encoded; ii.) The obstacle robots $\mathcal{N}_i(g)$ are to be avoided. In this manner, the state dynamics of each robot aims to coordinate with those of its neighboring robots. The formulation of φ_i is a modified version of that presented in [11]:

$$\varphi_i(b, g) = \sigma_d \circ \sigma \circ \widehat{\varphi}_i(b, g) \tag{5}$$

The function $\widehat{\varphi}_i : F \times G \to [0, \infty)$ is defined as:

$$\widehat{\varphi}_i(b, g) = \frac{\gamma_i(b, g)^{k_i}}{\beta_i(b, g)} \tag{6}$$

where $k_i \in \mathbb{Z}^+$ is the relative weighting parameter. The function $\widehat{\varphi}_i$ is made admissible via composing it with $\sigma : [0, \infty) \to [0, 1]$ defined as by $\sigma = \frac{x}{x+1}$. In order to make the goal a non-degenerate critical point, further composition with a sharpening function $\sigma_d : [0, 1] \to [0, 1]$ defined as $\sigma_d(x) = x^{1/k}$ is used. The formulation of $\widehat{\varphi}_i$ is dependent on the communication network since the numerator term γ_i and the denominator term β_i both depend on the neighborhood set $\mathcal{N}_i(g)$. The function $\gamma_i : F \times G \to [0, \infty)$ encodes its distance to the goal as well as the corresponding information communicated by its neighboring set $\mathcal{N}_i(g)$ as:

$$\gamma_i(b, g) = (b_i - h_i)^T(b_i - h_i) + \sum_{j \in \mathcal{N}_i(g)} (b_j - h_j)^T(b_j - h_j) \tag{7}$$

The denominator $\beta_i : F \times G \to [0, \infty)$ encodes the distance from freespace boundary based on its own state information as well as that communicated to it via its neighborhood set $\mathcal{N}_i(g)$.

$$\beta_i(b, g) = \beta_{0i} \prod_{j \in \mathcal{N}_i(g)} \beta_{ij}\beta_{0j} \tag{8}$$

For example, consider a scenario initially as shown in Fig. 1(left) where only robots 1 and 2 are communicating. Hence, while the state dynamics of robots 1 and 2 are coupled, the state dynamics of robot 3 is completely independent of them. As the robots navigate, their states change which can potentially change the network graph. If robots 1 and 3 approach each other and the communication network is changed as seen in Fig. 1(right), the state dynamics of robots 1 and 3 change and start taking each other into account.

[2]As g is an undirected graph, $j \in \mathcal{N}_i(g) \leftrightarrow i \in \mathcal{N}_j(g)$.

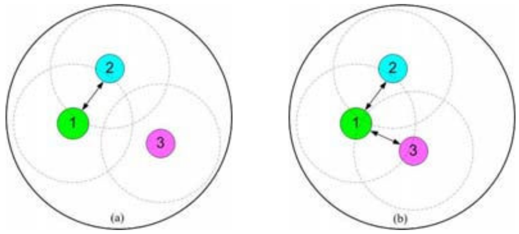

Fig. 1. A sample 3-robot navigation scenario – as robots 1 and 3 get close to each other, the network graph changes accordingly.

C. Communication Network Formation Models

The formation of the graph map g and hence the communication matrix is dependent on the environment and the communication network formation model. Consider an environment having environmental noise with variance σ_n^2. Suppose a scheduling mechanism like TDMA or FDMA is used in order to ensure that there is no interference caused by simultaneous communication of different robots in the network. Assume that each robot has a fixed amount transmitter power P_T allocated for communication. Three alternative approaches to communication network formation are proposed: 1) Deterministic; 2) Game-Theoretic and 3) Mixed. In the sequel, each will be explained in detail.

III. Deterministic Network

In deterministic communication, it is assumed that each robot has a set of detectors that measure the received signal levels from the other robots and hence determine the quality of link for each pairwise connection. It allocates its transmission power in a static manner to the other robots depending on the quality of links established. In general, as the quality of the link will be associated with whether received signal to noise ratio is above a given threshold τ, this restriction yields that $\forall j \in \mathcal{N}_i(g)$, $\delta_{ij} < \rho_c$ as shown in the following Lemma.

Lemma 1: Consider fixed P_T. SNR $> \tau$ iff $\delta_{ij} < \rho_c$ where $\rho_c = \sqrt{\frac{P_T}{(p-1)10^{0.1\tau}\sigma_n^2}}$.

Proof: A good quality link requires that $SNR > \tau$. Since

$$SNR = 10log(\frac{P_T}{(p-1)\,\delta_{ij}^2\sigma_n^2}) \tag{9}$$

$$\tau < 10log(\frac{P_T}{(p-1)\,\delta_{ij}^2\sigma_n^2}) \rightarrow 10^{0.1\tau} < \frac{P_T}{(p-1)\,\delta_{ij}^2\sigma_n^2}$$

which implies that

$$\delta_{ij} < \sqrt{\frac{P_T}{(p-1)\,10^{0.1\tau}\sigma_n^2}} \equiv \rho_c \quad \square \tag{10}$$

Hence the graph map g is defined by the communication matrix $A(b)$ that is symmetric with binary elements.

$$a_{ij}(b) = \begin{cases} 1 & \delta_{ij} \le \rho_c \text{ and } i \neq j \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Let it be noted that the magnitude of the communication range ρ_c will affect the connectivity of the underlying graph g. If the communication range is on the order of workspace coverage – namely $\rho_c \cong \rho_0$, then underlying graph will be a fully connected graph $g = g^p$. On the other hand, if the communication range is very small $\rho_c << \rho_0$, then the underlying graph will on average have a low connectivity. Furthermore, $|\mathcal{N}_i(g)| < p-1$ means that some of the transmission power remain unused.

IV. Game-Theoretic Communication

As the distance between a pair of communicating robots increases, the power usage efficiency decreases. Hence, there is a trade-off between attaining high SNR and power usage efficiency. An alternative approach to communication network formation to use a game-theoretic model based on a payoff function that takes this trade-off into consideration. In this approach, given (g, b), each robot has the policy of trying to allocate its transmission power uniformly only among its $\mathcal{N}_i(g)$ neighbors as $\frac{P_T}{|\mathcal{N}_i(g)|}$. As the network is changing over time, this means that power allocation per communication link is also dynamic. Furthermore, in contrast to deterministic network formation, such an approach does not require any detectors for the received signal levels.

A. Payoff Function

The pairwise payoff function $v_{ij} : G \times F \rightarrow \mathbb{R}$ is defined as follows:

$$v_{ij}(g, b) = \frac{\varsigma_i}{1 + e^{-a_i(\frac{P_T l(\delta_{ij})}{|\mathcal{N}_i(g)|N_0} - \tau')}} - \frac{P_T}{|\mathcal{N}_i(g)|\, l(\delta_{ij})} \tag{12}$$

where $\tau' = 10^{0.1\tau}$, l denotes the loss factor due to diffusion and absorption in the environment and is taken as $l(\delta_{ij}) = 1/\delta_{ij}^2$ and $N_o = 2\sigma_n^2$ denotes the noise spectral density. This function represents the degree of satisfaction of the robots to the link quality, as well as a cost function to measure the cost incurred. Generally, the quality of link is known to depend on SNR, so we let the first term to be an increasing function of SNR with the additional to property that its value must be equal to 0 in case $SNR = 0$. This means that a robot is more and more satisfied with the service as the quality improves. We use a sigmoid function for this purpose [25]. However, it should be noted that our scheme can be applicable to many other functions of similar nature. Secondly, the cost for the robot is defined as a function of power and distance. As power is itself a valuable commodity, the specific cost function should reflect the expense of establishing a link with a particular robot. In particular, as their pairwise distance gets larger, so does the cost of this communication link. The total payoff function that encodes this trade-off is formulated as

$$v_i(g, b) = \sum_{j \in \mathcal{N}_i(g)} v_{ij}(b) \tag{13}$$

The goal is to maximize the total payoff defined as such by adjusting the communication network. Hence each robot will try to maximize its own total payoff.

B. Pairwise Game

Each individual robot plays a pairwise game with the rest of the robots. Let p_{ij} be the probability that it chooses to play with robot j. Consequently, it is associated with a set of decisions $x_i(t) = (x_{i,1}(t), \ldots, x_{i,i-1}(t), x_{i,i+1}(t), \ldots, x_{i,p}(t))$ where $x_{i,j}(t) \in \{0, 1\}$ as:

$$x_{i,j}(t) = \begin{cases} 1 & \text{with probability } p_{ij} \\ 0 & \text{with probability } 1 - p_{ij} \end{cases} \quad (14)$$

If $x_{i,j}(t) = 1$, robot i wants to play a game with j. In case a robot wants to play with more than one robot, it chooses one player j with uniform probability. This dynamic process requires no communication or synchronization for selecting the robot pairs. Once this decision is made, the pair ij is selected to play a pairwise game. Note that the order is not important – the pair ij being selected is equivalent to the pair ji being selected. Let $\mathcal{C}(t)$ be the set of robot pairs that are selected.

C. Game Graph Update

In the game-theoretic network formation, the graph g is updated based on a game-theoretic approach. Given that $ij \in \mathcal{C}$ play the game, if the edge ij is already in the robot network, the decision is to deactivate it or not while otherwise, the decision is to activate it or not – all based on the payoff function. Robots act individually and decide to activate a link if it makes each robot at least as well off and one better off and remove a link if its deletion makes either robot better off. The pair ij then updates the communication matrix $A(b(t))$ as follows:

$$a_{ij}(b) = \begin{cases} 0 & \text{if } ij \in \mathcal{C}(t) \text{ and} \\ & v_i(g - ij, b) > v_i(g, b) \\ 1 & \text{if } ij \in \mathcal{C}(t) \text{ and} \\ & v_i(g + ij, b) > v_i(g, b) \\ a_{ij}(b) & \text{otherwise} \end{cases} \quad (15)$$

$$a_{ji}(b) = \begin{cases} 0 & \text{if } ij \in \mathcal{C}(t) \text{ and} \\ & v_j(g - ij, b) > v_j(g, b) \\ 1 & \text{if } ij \in \mathcal{C}(t) \text{ and} \\ & v_j(g + ij, b) > v_j(g, b) \\ a_{ji}(b) & \text{otherwise} \end{cases} \quad (16)$$

Hence, the communication matrix changes depending on the pairwise games that are being played. All matrix entries where there are no corresponding games being played remain unchanged. As a result of game between i and j, an existing communication link between them is removed iff $v_i(g - ij) < v_i(g)$ or $v_j(g - ij) < v_j(g)$ while a new communication link is established iff $v_i(g + ij) < v_i(g)$ and $v_j(g + ij) < v_j(g)$.

V. Mixed Strategy

A third approach is to use a mixed communication strategy that combines features from deterministic and game-theoretic communication. First, the neighbor set is partitioned into two disjoint sets depending on whether each neighboring robot is outside the ρ_c range or not. Hence, Letting $\mathcal{N}_i'(g) \equiv \{j \in \mathcal{N}_i(g) \mid \delta_{ij} > \rho_c\}$ and $\mathcal{N}_i''(g) \equiv \{j \in \mathcal{N}_i(g) \mid \delta_{ij} \leq \rho_c\}$, then $\mathcal{N}_i(g) = \mathcal{N}_i'(g) \cup \mathcal{N}_i''(g)$. It then devises its communication strategy accordingly. For each robot in $\mathcal{N}_i''(g)$, $\frac{P_T}{p-1}$ is allocated. Letting $n_i = |\mathcal{N}_i''(g)|$ be the number of such robots, the total amount of power allocated for this communication is $n_i \frac{P_T}{p-1}$.

Secondly, for robots outside this range and hence in $\mathcal{N}_i'(g)$, it employs a modified version of game-theoretic strategy for setting up links. First, each robot is associated with a set of decisions with only the robots in $\mathcal{P} - \mathcal{N}_i''(g)$ in regards to playing a game or not. Let each decision set be denoted by $x_i(t) = \{x_{i,k}(t) \mid k \in \mathcal{P} - \mathcal{N}_i''(g)\}$ where $x_{i,j}(t) \in \{0, 1\}$ is defined as in Eq. 14. Again, in case that a robot chooses to play with more than one robot, then the robot chooses one player $x_i = j$ with uniform probability. Once this decision is made, the pair ij is selected to play a pairwise game. It allocates the remaining amount of power $P_T' \equiv P_T - n_i \frac{P_T}{p-1}$ uniformly among these robots. The pairwise payoff function $w_{ij} : G \times F \to \mathbb{R}$ is defined as follows:

$$w_{ij}(g, b) = \frac{\varsigma_i}{1 + e^{-a_i\left(\frac{P_T' l(\delta_{ij})}{|\mathcal{N}_i'(g)| N_0} - \tau'\right)}} - \frac{P_T'}{|\mathcal{N}_i'(g)|\, l(\delta_{ij})} \quad (17)$$

The total payoff function that encodes this trade-off is formulated as:

$$w_i(g, b) = \sum_{j \in \mathcal{N}_i'(g)} w_{ij}(b) \quad (18)$$

After this, the decision making proceeds as in purely game-theoretic communication. Recalling that $\mathcal{C}(t)$ is the set of robot pairs that are selected, if the edge ij is already in the robot network, the decision is to deactivate it or not while otherwise, the decision is to activate it or not – again all based on the payoff function. The graph map g is defined by the communication matrix $A(b)$ as:

$$a_{ij}(b) = \begin{cases} 1 & \delta_{ij} \leq \rho_c \text{ and } i \neq j \\ 0 & \text{if } \delta_{ij} > \rho_c \text{ and } ij \in \mathcal{C}(t) \text{ and} \\ & w_i(g_t - ij, b) > w_i(g_t, b) \\ 1 & \text{if } \delta_{ij} > \rho_c \text{ and } ij \in \mathcal{C}(t) \text{ and} \\ & w_i(g_t + ij, b) > w_i(g_t, b) \\ a_{ij}(b(t)) & \text{otherwise} \end{cases} \quad (19)$$

VI. Simulations

This section presents results from a set of simulations that are conducted with 20 robots all having radii 25cm. As explained, the task of all the robots is to navigate simultaneously to their a priori specified goal positions in a workspace of radius 15m. We consider three different kind of goals as shown in Fig. 2 which differ with respect to the distance between two vertically or horizontally adjacent robot goal positions defined as ρ_d. In exploration type goals (ET), the robots are required to navigate to goal positions that cover the given workspace where ρ_d is large. Here, $\rho_d = 5$m. In procession type goals (PT), the robots are required to position themselves in a cortege arrangement and hence ρ_d is relatively small. Here, $\rho_d = 1$m. Finally, in the zone type goals (ZT), ρ_d is somewhere in between those of the two other type goals. Here $\rho_d = 2.5$m.

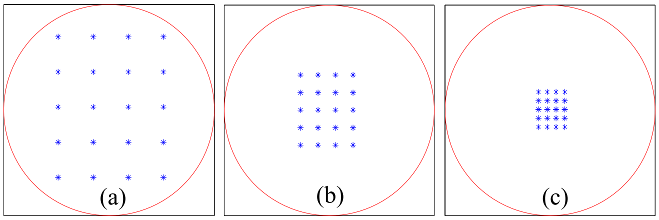

Fig. 2. Sample configurations for different goal types: a) Exploration type (ET), b) Zone type (ZT), c) Procession type (PT).

We study the nature of the resulting navigation behavior under limited communication. Each robot has transmitter power $P_T = 10$ dBm allocated for wireless communication. The environmental noise is considered in levels varying from low, medium and high as given by $\sigma_n^2 \in \{5\times10^{-5}, 15\times10^{-5}, 5\times10^{-4}\}$. In deterministic networks, with $\tau = 10$, these in turn correspond to three different communication ranges ρ_c calculated using Eq.10:

$$\rho_c = \begin{cases} 3.2m & \text{if} \quad \sigma_n^2 = 5\times10^{-5} \\ 1.9m & \text{if} \quad \sigma_n^2 = 15\times10^{-5} \\ 1m & \text{if} \quad \sigma_n^2 = 5\times10^{-4} \end{cases} \tag{20}$$

Hence, for deterministic networks, as the environmental noise level increases, the communication range ρ_c decreases. The parameters ς_i and a_i in the pairwise payoff function are set to 10 and 1, respectively. For a zone type goal with $\sigma_n^2 = 15\times10^{-5}$, a sampled time evolution of the navigation task with a game-theoretic network is shown in Fig. 3. In each graph, the star marks indicate the goal positions. The first graph shows the robots' initial positions and the network topology at the first iteration of the game. The next graphs show the progress of the tasks along with the corresponding networks. The last graph shows the final network when the task is completed.

For each network and goal types and noise level, 100 simulations are performed with the robots starting at random initial positions. In the game-theoretic networks, at the start of all the simulations, the robots are initialized to be fully disconnected. We study three performance statistics: Task completion percentage, average connectivity and task completion efficiency.

The task completion percentage is defined to be the percentage of navigation tasks that have been successfully completed in finite time. Hence, failure is any case where navigation of all the robots to their target locations is not achieved. The performance in task completion is presented in Table-I where it is observed that successful navigation can occur despite restricted communication ranges and hence partial connectivity. For $\sigma_n^2 = 5\times10^{-5}$, for both deterministic and game-theoretic networks, task completion percentage is better in procession type goals whereas for higher noise levels, task completion is better in exploration type goals. Mixed networks exhibit similar performance, regardless of the noise level. As link activation in the game-theoretic approach is performed in a probabilistic manner, collisions become likely. The mixed strategy, which copes

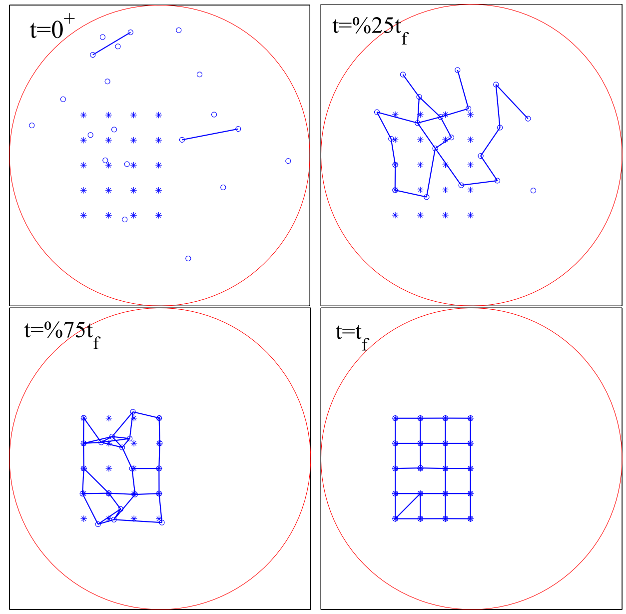

Fig. 3. A sampled time evolution in zone type goal navigation with a game-theoretic network.

with this deficiency using deterministic approach, has the best performance with respect to task completion percentage.

TABLE I
TASK COMPLETION PERCENTAGE

σ_n^2	5×10^{-5}			15×10^{-5}			5×10^{-4}		
Strategy	ET	ZT	PT	ET	ZT	PT	ET	ZT	PT
Deterministic	90	86	96	99	87	87	99	96	42
Game-theoretic	91	96	99	97	90	93	100	98	72
Mixed	100	100	99	99	100	99	100	99	100

The average connectivity $\bar{D}$ is defined as the time average of the average normalized degree of the robot network defined as:

$$\bar{D} = \frac{1}{t_f}\frac{100}{p}\int_0^{t_f}\sum_{i\in\mathcal{P}}\frac{1}{p-1}\left|\mathcal{N}_i(g(b(t)))\right| dt$$

where t_f denotes the task completion time. The closer is this value to 100, higher is the connectivity. Table II shows average degree percentage of the resulting robot networks. As expected, exploration type goals have inherently lower connectivity as compared to procession type goals. Furthermore, in all, the connectivity decreases as noise level increases.

TABLE II
AVERAGE CONNECTIVITY $\bar{D}$

σ_n^2	5×10^{-5}			15×10^{-5}			5×10^{-4}		
Strategy	ET	ZT	PT	ET	ZT	PT	ET	ZT	PT
Deterministic	3.3	13.6	32.4	0.8	2.7	16	0.4	0.7	2
Game-theoretic	15.4	23.3	33	8.2	14	21.7	3	7.1	13.3
Mixed	16	26.1	38.8	8.3	14.6	24.5	3	7.2	13.7

Task completion efficiency (tce) is measured by normalized path length which is the average of the total distance traveled by the robots from their initial positions to their goal locations normalized by the Euclidean distance between their initial and goal positions as:

$$tce = \frac{1}{p} \sum_{i \in \mathcal{P}} \frac{\int_0^{t_f} \left\| \dot{b}_i(t) \right\| dt}{\| b_i(0) - h_i \|}$$

$b(0)$ denotes the initial position of the robots. Note that a smaller *tce* value indicates greater overall efficiency in task completion. Task completion efficiency results for the different type of goals are presented in Table III. In the deterministic network, interestingly, for all task types, task completion efficiency increases as the communication range becomes smaller with the effect being more observable for the exploration type goals. This can possibly be attributed to the fact that as the robots become more myopic, they take less of other robots' states and goals into consideration and hence navigate more efficiently. Of course, as task completion percentages deteriorate simultaneously, this means while tasks are less likely to be completed, if completed, their realization efficiency is higher. On the other hand, for game theoretic and mixed networks, task efficiency remains roughly the same with respect to task types and noise levels.

TABLE III

TASK COMPLETION EFFICIENCY – *tce*

σ_n^2	5×10^{-5}			15×10^{-5}			5×10^{-4}		
Strategy	ET	ZT	PT	ET	ZT	PT	ET	ZT	PT
Deterministic	3.66	3.12	3.18	1.58	2.52	2.67	1.4	1.52	1.94
Game-theoretic	2.75	2.94	2.99	3.22	2.74	2.73	2.67	2.5	2.9
Mixed	2.78	2.92	3.06	3.24	2.77	2.79	2.77	2.52	2.37

VII. CONCLUSION

This paper considers the problem of navigation of dynamically communicating robots where there is a bidirectional interaction between the robot network and continuous states. First, the robot dynamics is formulated as being dependent on the communication network where two robots - if in communication - can access each other's position and goal information. Next, three alternative strategies for establishing the communication network depending on the robots' states are presented: deterministic, game-theoretic and mixed approaches. In the first approach, the network is defined deterministically based on the robots' states and the communication range. The game-theoretic approach is based on utilizing the conflict between the communication gain and cost. The third approach is a mixture of the deterministic and game-theoretic approaches. Our extensive simulation results indicate that for goal positions including exploration, zone and procession types, employing a mixed network formation yields best task completion percentages. As part of ongoing research, we are considering different type of game-theoretic networks.

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REFERENCES

[1] Aumann,R.J. and J.H. Dreze, "Cooperative games with coalition structure", *Int. J. Game Theory*, no.3, pp.217-237, 1974.

[2] Bayram, H., E. Ertuzun and H.I. Bozma, "Reactive Rearrangement of Parts under Sensor Inaccuracy: Particle Filter Approach", *Proc. of IEEE Int. Conf. Robot. Autom.*, pp.2029-2034, 2006.

[3] Franceschetti, M. and R. Meester,*Random Networks for Communication*,Cambridge Univ. Press,2007.

[4] Hart, S. and A. Mas-Colell, "A simple adaptive procedure leading to correlated equilibrium", *Econometrica*, 68(5), pp.1127-1150, 2000.

[5] Jiang, T. and J.S.Baras, "Fundamental tradeoffs and constrained coalitional games in automatic wireless networks", *Proc. of 5th Int. Symp. on Modeling and Opt. in Mobile, Ad Hoc & Wireless Networks*, 2007.

[6] Jiang, T. and J.S. Baras, "Coalition formation through learning in automatic networks", *Proc. of Int. Conf. Game Theory for Networks*, pp.10-16, 2009.

[7] Henzinger,T. A. ,"The theory of hybrid automata" in Verification of Digital and Hybrid Systems (M.K. Inan, R.P. Kurshan, eds.), 1790, pp.265-292, Springer-Verlag 2000.

[8] Jackson, M.O. and A. Wolinsky, "A strategic model of social and economic networks", *J. Econ. Theory*, 77, pp. 44-74, 1996.

[9] Karagöz, C. S. "A Game-Theoretic Approach to Objects' Moving Problem With Mobile Robots", Ph.D. Thesis, Elec. Electr. Eng., Bogazici Univ., 2001.

[10] Warren, C. "Multiple Robot Path Coordination Using Artificial Potential Functions", *Proc. of the IEEE Int. Conf. Robot. Autom.*, pp. 500-505, 1990.

[11] Karagöz, C. S., H. I. Bozma and D. E. Koditschek, "Coordinated Navigation of Multiple Independent Disk-Shaped Robots", Tech. Rep., CSE-TR-486-04, The Univ. of Michigan, Comp. Sci. Eng. Div., Dept. of Elec.Eng. & Comp. Sci., 2004.

[12] Karl, H. and A. Willig, *Protocols and Architectures for Wireless Sensor Networks*, Wiley, 2005.

[13] Koditschek, D. E., "Task Encoding: Toward a Scientific Paradigm for Robot Planning and Control", *Robot. Auton. Syst.*, 9, pp. 5-39, 1992.

[14] LaValle, S.M. and S. Hutchinson, "Optimal Motion Planning for Multiple Robots Having Independent Goals",*IEEE T. Robot. Autom.*, 14(6), pp.912-925, 1998.

[15] McNew, J.M. and E. Klavins, "Locally interacting hybrid systems with embedded graph grammers", *Proc. of 45th IEEE Conf. on Decision & Control*, pp.6080-6087, 2006.

[16] Mesbahi, M., "On State-dependent dynamic graphs and their controllability properties", *IEEE T Automat Contr*, 50(3), pp. 387 - 392, 2005.

[17] Michiardi, P. and R. Molva, "Analysis of coalition formation and cooperation strategies in mobile ad hoc networks", *Ad Hoc Networks*, pp. 193-219, 2005.

[18] Mostofi, Y., "Communication-Aware Motion Planning in Fading Environments", *Proc. IEEE Int. Conf. Robot. Autom.*, pp. 3170-3174, 2008.

[19] Muhammad,A. and M. Egerstedt, "Connectivity graphs as models of local interactions", *Appl. Math. Comput.*, 168(1), pp. 243-269, 2005.

[20] Myerson, R., "Graphs and cooperation in games", *Math. Oper. Res.*, 2(3), pp. 225-229, 1977.

[21] Ramanathab, G. and V.S. Alagar, "Algorithmic Motion Planning in Robotics: Coordinated Motion of Several Disks Amidst Polygonal Obstacles", *Proc. IEEE Int. Conf. Robot. Autom.*, pp. 514-522, 1985.

[22] Saber,R. and R. Murray, "Agreement problems in networks with directed graphs and switching topology", *Proc. IEEE Conf. Decision & Control*, 2003.

[23] Smith, B., A. Howard, J.M. McNew, J. Wang and M. Egerstedt, "Multi-robot deployment and coordination with Embedded Graph Grammers", *Auton. Robot*, 26, pp. 76-98, 2009.

[24] Siméon, T., S. Leroy and J. Laumond, "Path Coordination for Multiple Mobile Robots: A Resolution-Complete Algorithm", *IEEE T. Robot. Autom.*, 18(1), pp. 42-49, 2002.

[25] Xiao, M. , N.B. Shroff and E.K.P. Chong, "A Utility-Based Power-Control Scheme in Wireless Cellular Systems", *IEEE T. Networking*, 11(2), pp.210-221, 2003.

[26] Wisniewski, R., "Towards Modelling of Hybrid Systems", *Proc. of the 45th IEEE Conf. Decision & Control*, pp. 911-916, 2006.