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Applications of Potential Fields and Conformal Geometric Algebra for Humanoid Manipulation Maneuvering

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***Abstract*— In this work we show the use of potential fields in conformal geometric algebra to assist the humanoid manipulation planning. Since the computational unit in conformal geometry algebra is the sphere, it appears natural to use the potential fields for computing navigation trajectories avoiding obstacles. Furthermore, by covering obstacles with convex hulls, one can obtain certain shapes which are amenable of being parameterized as spheres, thus this equivalent potential fields can be used so that the robot device can move around the irregular shapes of obstacle without increasing the computation complexity. In this paper we present this theory and show with simulations the effectiveness of the approach.**

***Index Terms*— Conformal Geometry, Kinematics, Grasping, Tracking.**

I. Introduction

For the study of kinematics and motion planning of robot mechanisms different mathematical frameworks have been used like vector calculus, quaternion algebra or linear algebra, being the last one the most used. However, in these mathematical systems it is very complicated to handle the kinematics and dynamics involving geometric primitives like points, lines, planes and spheres. On the other hand, if we want to integrate potential fields for computing suitable trajectories avoiding obstacles, the algebraic complexity is increased if matrix or tensor calculus are still used. In this work, we formulate the kinematics and the motion planning using potential fields in the conformal geometric algebra framework. In conformal geometry algebra [1], [2], the computational unit is the sphere, thus it appears quite natural to use the potential fields [3] for computing navigation trajectories avoiding obstacles. The gradient of the potential fields creates repulsive forces bringing away the robot mechanism from the obstacle. Since this potential field function is formulated with respect to a position vector of the conformal geometric algebra, it can be integrated in the inverse kinematics to find out the appropriate angles of the root arm to get around the obstacles. Interestingly enough, by covering obstacles with convex hulls, we obtain certain shapes which are amenable of being parameterized as spheres [8], therefore they can be used for the inverse kinematics computation in the conformal geometric algebra framework for obstacle avoidance. A similar idea, was used by us to parametrize different shapes of mirrors for omnidirectional catadioptric vision [1]. As a result the parabolic, hyperbolic and elliptic mirror can be represented in a canonical form using a spherical mirror. In this paper, we show how conformal geometric algebra simplifies the algebraic treatment of inverse kinematic with the use of potential fields for planning the robot manipulator maneuvering. Sections two and three presents an outline of geometric algebra and conformal geometric algebra respectively. Section four presents the representation of rigid transformations and direct kinematics in terms of versors. Section five shows the computing of the inverse kinematics of a 5 D.O.F. manipulator using conformal geometric algebra. In section six, we explain the use of potential fields in the conformal geometric algebra framework. Section seven describes the Conformal mapping to represent irregular shapes. Finally section eight is devoted to the conclusions.

II. Geometric Algebra: An Outline

Let G_n denote the geometric algebra of n-dimensions, which is a graded-linear space. As well as vector-addition and scalar multiplication, we have a non-commutative product which is associative and distributive over addition. This is the *geometric* or *Clifford product*.
The inner product of two vectors is the standard *scalar* or *dot* product, which produces a scalar. The outer or wedge product of two vectors is a new quantity which we call a *bivector*. We think of a bivector as an oriented area in the plane containing a and b, which is formed by sweeping a along b.
Thus, $b \wedge a$ will have the opposite orientation making the wedge product anti-commutative. The outer product is immediately generalizable to higher dimensions. For example, $(a \wedge b) \wedge c$, a *trivector*, is interpreted as the oriented volume formed by sweeping the area $a \wedge b$ along vector c. The outer product of k vectors is a k-blade, and such a quantity is said to have *grade* k. A *multivector* (the linear combination of objects of different grades) is a *homogeneous* k-vector if it contains terms of only a single grade k.

In this paper we will specify the geometric algebra G_n of the n dimensional space by $G_{p,q,r}$, where p, q and r stand for the number of basis vectors which square to 1, -1 and 0 respectively and fulfill $n = p + q + r$.

We will use e_i to denote the basis vector i. In geometric algebra $G_{p,q,r}$, the geometric product of two basis vectors is defined as

$$e_i e_j = \begin{cases} 1 & for \quad i=j \in 1,\cdots,p \\ -1 & for \quad i=j \in p+1,\cdots,p+q \\ 0 & for \quad i=j \in p+q+1,\cdots,p+q+r. \\ e_i \wedge e_j & for \quad i \neq j \end{cases}$$

This leads to a basis for the entire algebra:

$$\{1\},\{e_i\},\{e_i \wedge e_j\},\{e_i \wedge e_j \wedge e_k\},\ldots,\{e_1 \wedge e_2 \wedge \ldots \wedge e_n\} \quad (1)$$

Any multivector can be expressed in terms of this basis. The multivectors can be of grade 0 (scalars), grade 1 (vectors), grade 2 (bivectors), grade 3 (trivectors), etc., up to grade n (n-vectors). For example, $G_{4,1,0}$ has the basis

$$\begin{array}{l} \{1\}, \quad \{e_1,...,e_5\}, \quad \{e_{12},e_{13},...,e_{45}\}, \\ \{e_{123},...e_{345}\},\{e_{1234},...,e_{2345}\}, \quad \{e_{12345}=I\} \end{array} \quad (2)$$

where $e_1^2=1$, $e_2^2=1$, $e_3^2=1$, $e_4^2=1$, $e_5^2=-1$. $G_{4,1,0}$ is a five-dimensional geometric algebra with $2^5=32$ multivector blades.

III. Conformal Geometric Algebra

Geometric algebra $G_{4,1}=G_{4,1,0}$ can be used to treat conformal geometry in a very elegant way. To see how this is possible, we follow the same formulation presented in [2] and show how the Euclidean vector space $\mathbb{R}^3$ is represented in $\mathbb{R}^{4,1}$. This space has an orthonormal vector basis given by $\{e_i\}$ and $e_{ij}=e_i \wedge e_j$ are bivectorial bases and a bivector basis e_{23}, e_{31} and e_{12} that corresponds together with 1 to Hamilton's quaternions. The unit Euclidean pseudo-scalar $I_e := e_1 \wedge e_2 \wedge e_3$, a pseudo-scalar $I=I_eE$, and the bivector $E := e_4 \wedge e_5 = e_4 e_5$ are used for computing Euclidean and conformal duals of multivectors. For more about conformal geometric algebra, refer to [1], [2].

A. The point

The vector $x_e \in \mathbb{R}^3$ representing a point after a conformal mapping is rewritten as

$$x_c \quad = \quad x_e + \frac{1}{2}x_e^2 e_\infty + e_0, \quad (3)$$

where the null vectors are the point at infinity $e_\infty = e_4 + e_5$ and the origin point $e_0 = \frac{1}{2}(e_4 - e_5)$ with the properties $e_\infty^2 = e_0^2 = 0$ and $e_\infty \cdot e_0$=1.

B. Spheres and Planes

The equation of a sphere of radius ρ centered at point $p_e \in \mathbb{R}^3$ can be written as $(x_e - p_e)^2 = \rho^2$. Since $x_c \cdot y_c = -\frac{1}{2}(\mathrm{x}_e - \mathrm{y}_e)^2$, where x_e and y_e are the Euclidean components, and $x_c \cdot p_c = -\frac{1}{2}\rho^2$, we can rewrite the formula above in terms of homogeneous coordinates. Since $x_c \cdot e_\infty = -1$, we can factor the expression above to

$$x_c \cdot (p_c - \frac{1}{2}\rho^2 e_\infty) = 0 \quad (4)$$

this equation corresponds to the so called Inner Product Null Space (IPNS) representation. Which finally yields the simplified equation for the sphere as $s = p_c - \frac{1}{2}\rho^2 e_\infty$. Note from this equation that a point is just a sphere with a zero radius. Alternatively, the dual of the sphere is represented as 4-vector $s^* = sI$. The advantage of the dual form is that the sphere can be directly computed from four points as

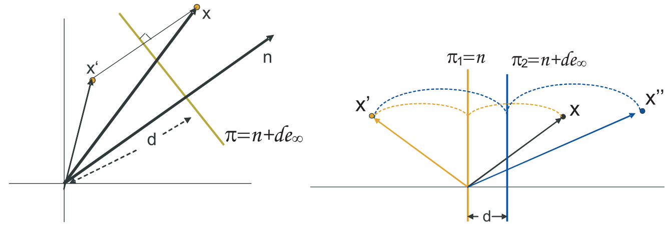


Fig. 1. a) (left column) Reflection of a point x with respect to the plane π. b) (right column) Reflection about parallel planes.

$$s^* = x_{c_1} \wedge x_{c_2} \wedge x_{c_3} \wedge x_{c_4}. \quad (5)$$

If we replace one of these points for the point at infinity, we get the equation of a 3D plane

$$\pi^* = x_{c_1} \wedge x_{c_2} \wedge x_{c_3} \wedge e_\infty. \quad (6)$$

So that π becomes in standard IPNS form

$$\pi \quad = I\pi^* = \quad n + de_\infty \quad (7)$$

Where n is the normal vector and d represents the Hesse distance for the 3D space.

C. Circles and Lines

A circle z can be regarded as the intersection of two spheres s_1 and s_2 as $z = (s_1 \wedge s_2)$ in IPNS. The dual form of the circle can be expressed by three points lying on the circle, namely

$$z^* = x_{c_1} \wedge x_{c_2} \wedge x_{c_3}. \quad (8)$$

Similar to the case of planes, lines can be defined by circles passing through the point at infinity as:

$$L^* = x_{c_1} \wedge x_{c_2} \wedge e_\infty. \quad (9)$$

The standard IPNS form of the line can be expressed as

$$L = \mathbf{n}I_e - e_\infty \mathbf{m} I_e \quad (10)$$

where $\mathbf{n}$ and $\mathbf{m}$ stand for the line orientation and moment respectively. The line in the IPNS standard form is a bivector representing the six Plücker coordinates.

TABLE I
Representation of conformal geometric entities

Entity	IPNS Representation	OPNS Dual representation
Sphere	$s = p - \frac{1}{2}\rho^2 e_\infty$	$s^* = x_1 \wedge x_2 \wedge x_3 \wedge x_4$
Point	$x_c = x_e + \frac{1}{2}x_e^2 e_\infty + e_0$	$x^* = s_1 \wedge s_2 \wedge s_3 \wedge s_4$
Line	$L = \mathbf{n}I_e - e_\infty \mathbf{m} I_e$	$L^* = x_1 \wedge x_2 \wedge e_\infty$
Plane	$\pi = n + de_\infty$	$\pi^* = x_1 \wedge x_2 \wedge x_3 \wedge e_\infty$
Circle	$z = s_1 \wedge s_2$	$z^* = x_1 \wedge x_2 \wedge x_3$
Pair of P.	$P_p = s_1 \wedge s_2 \wedge s_3$	$P_p^* = x_1 \wedge x_2$

IV. Rigid Transformations

We can express rigid transformations in conformal geometry carrying out plane reflections.

1) Reflection: The combination of reflections of conformal geometric entities enables us to form other transformations. The reflection of a point x with respect to the plane π is equal x minus twice the directed distance between the point and plane (see the Figure 1(a)). That is, $ref(x) = x - 2(\pi \cdot x)\pi^{-1}$. We get this expression by using the reflection $ref(x_c) = -\pi x_c \pi^{-1}$ and the property of Clifford product of vectors $2(b \cdot a) = ab + ba$.

For a IPNS geometric entity Q, the reflection with respect to the plane π is given as

$$Q' = \pi Q \pi^{-1} \tag{11}$$

2) Translation: The translation of conformal geometric entities can be done by carrying out two reflections in parallel planes π_1 and π_2 (see Figure 1(b)). That is

$$Q' = \underbrace{(\pi_2\pi_1)}_{T_a} Q \underbrace{(\pi_1^{-1}\pi_2^{-1})}_{\widetilde{T_a}} \tag{12}$$

$$T_a = (n + de_\infty)n = 1 + \frac{1}{2}ae_\infty = e^{\frac{a}{2}e_\infty} \tag{13}$$

With $a = 2dn$.

3) Rotation: The rotation is the product of two reflections at nonparallel planes which pass through the origin, (see Figure 2)

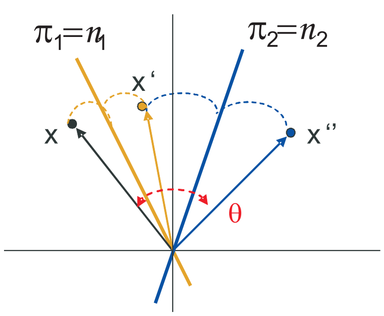


Fig. 2. Reflection about nonparallel planes.

$$Q' = \underbrace{(\pi_2\pi_1)}_{R_\theta} Q \underbrace{(\pi_1^{-1}\pi_2^{-1})}_{\widetilde{R_\theta}} \tag{14}$$

Or computing the conformal product of the normals of the planes.

$$R_\theta = n_2 n_1 = cos(\frac{\theta}{2}) - sin(\frac{\theta}{2})l = e^{-\frac{\theta}{2}l} \tag{15}$$

With $l = n_2 \wedge n_1$, and θ twice the angle between the planes π_2 and π_1. The screw motion *called motor* is related to an arbitrary axis L is $M = TR\widetilde{T}$

$$Q' = \underbrace{(TR\widetilde{T})}_{M_\theta} Q \underbrace{(T\widetilde{R}\widetilde{T})}_{\widetilde{M_\theta}} \tag{16}$$

$$M_\theta = TR\widetilde{T} = cos(\frac{\theta}{2}) - sin(\frac{\theta}{2})L = e^{-\frac{\theta}{2}L} \tag{17}$$

The direct kinematics for serial robot arms is a succession of motors as you can see in [1] and it is valid for points, lines, planes, circles and spheres

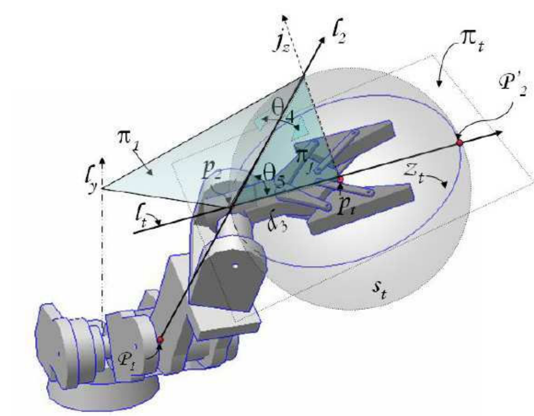
$$Q' = \prod_{i=1}^{n} M_i Q \prod_{i=1}^{n} \widetilde{M}_{n-i+1} \tag{18}$$

.

V. Inverse kinematic using conformal geometric algebra

In this subsection, we will briefly describe the procedure steps to compute the inverse kinematics of a 5 D.O.F robot arm using the conformal geometric algebra framework. We have presented above a similar computation using motor algebra $\mathcal{G}_{3,0,1}$ and the affine plane and next conformal geometric algebra $\mathcal{G}_{4,1}$ [1]. The reader by studying these examples can learn to compute using the geometric algebra and get a better understanding of the potential of this mathematical framework.

Objective: find the joint angles in order to place the robot arm at the point $\boldsymbol{p}_t$ in such a way that the gripper will stay parallel to the plane ϕ_t.



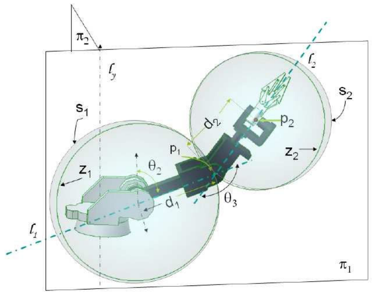
Step 1: Find the position of the point $\boldsymbol{p}_2$

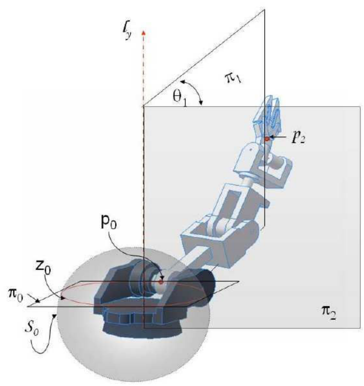
$$\begin{aligned}
\boldsymbol{l}_y^* &= e_2\boldsymbol{E}, \quad \boldsymbol{s}_t = \boldsymbol{p}_t - \frac{1}{2}d_3^2 e_\infty \\
\boldsymbol{z}_t &= \boldsymbol{s}_t \wedge \pi_t, \quad \boldsymbol{j}_z^* = \boldsymbol{z}_t \wedge e_\infty \\
\boldsymbol{l}_d^* &= d(\boldsymbol{p}_t, \boldsymbol{l}_y^*) \wedge \boldsymbol{p}_t \wedge e_\infty, \quad \pi_{jl_d}^* = \boldsymbol{j}_z^* \wedge (\boldsymbol{l}_d^* \boldsymbol{E}) \\
\boldsymbol{P}_{P_2} &= \pi_{jl_d} \wedge \boldsymbol{z}_t, \ \boldsymbol{p}_2 = \frac{\boldsymbol{P}_{P_2}^* + \sqrt{\boldsymbol{P}_{P_2}^* \cdot \boldsymbol{P}_{P_2}^*}}{\boldsymbol{P}_{P_2}^* \cdot e_\infty}.
\end{aligned}$$

where $d(\boldsymbol{p}_t, \boldsymbol{l}_y^*)$ stands for the directed distance between $\boldsymbol{p}_t$ and $\boldsymbol{l}_y^*$.

Step 2: Find the point $\boldsymbol{p}_1$

$$\begin{aligned}
\boldsymbol{s}_1 &= e_0 - \frac{d_1^2}{2}e_\infty, \ \ \boldsymbol{s}_2 = \boldsymbol{p}_2 - \frac{d_2^2}{2}e_\infty \\
\pi_1^* &= \boldsymbol{l}_y^* \wedge \boldsymbol{p}_2 = e_2\boldsymbol{E} \wedge \boldsymbol{p}_2, \ \ \pi_2^* = e_3 I_c \\
\boldsymbol{P}_{P_1} &= \boldsymbol{s}_1 \wedge \boldsymbol{s}_2 \wedge \pi_1, \ \ \boldsymbol{p}_1 = \frac{\boldsymbol{P}_{P_1}^* + \sqrt{\boldsymbol{P}_{P_1}^* \cdot \boldsymbol{P}_{P_1}^*}}{\boldsymbol{P}_{P_1}^* \cdot e_\infty}
\end{aligned}$$





Step 3: Find the point $\boldsymbol{p}_0$

$$
\begin{aligned}
\pi_1^* &= \boldsymbol{l}_y^* \wedge \boldsymbol{p}_2, \;\; \boldsymbol{s}_0 = e_0 - \frac{d_0^2}{2} e_\infty \\
\pi_0 &= e_2 - h e_\infty, \;\; \boldsymbol{z}_0 = \boldsymbol{s}_0 \wedge \pi_0 \\
\boldsymbol{P}_{P_0} &= \boldsymbol{z}_0 \wedge \pi_1, \;\; \boldsymbol{p}_0 = \frac{\boldsymbol{P}_{P_0}^* + \sqrt{\boldsymbol{P}_{P_0}^* \cdot \boldsymbol{P}_{P_0}^*}}{\boldsymbol{P}_{P_0}^* \cdot e_\infty}
\end{aligned}
$$

Step 4: Compute the lines $\boldsymbol{l}_1$, $\boldsymbol{l}_2$ and $\boldsymbol{l}_3$

$$
\boldsymbol{l}_1^* = \boldsymbol{p}_0 \wedge \boldsymbol{p}_1 \wedge e_\infty, \;\; \boldsymbol{l}_2^* = \boldsymbol{p}_1 \wedge \boldsymbol{p}_2 \wedge e_\infty, \;\; \boldsymbol{l}_3^* = \boldsymbol{p}_2 \wedge \boldsymbol{p}_1 \wedge e_\infty \quad (19)
$$

Step 5: Compute the angles

$$
\begin{aligned}
cos(\theta_1) &= \frac{\pi_1^* \cdot \pi_2^*}{|\pi_1^*||\pi_2^*|}, \;\; cos(\theta_2) = \frac{\boldsymbol{l}_1^* \cdot \boldsymbol{l}_y^*}{|\boldsymbol{l}_1^*||\boldsymbol{l}_y^*|} \\
cos(\theta_3) &= \frac{\boldsymbol{l}_1^* \cdot \boldsymbol{l}_3^*}{|\boldsymbol{l}_1^*||\boldsymbol{l}_3^*|}, \;\; cos(\theta_4) = \frac{\pi_1^* \cdot \pi_3^*}{|\pi_1^*||\pi_3^*|} \\
cos(\theta_5) &= \frac{\boldsymbol{l}_3^* \cdot \boldsymbol{l}_2^*}{|\boldsymbol{l}_3^*||\boldsymbol{l}_2^*|}.
\end{aligned}
$$

VI. POTENTIAL FIELDS

There are several strategies in robotics to tackle the problems of obstacle avoidance and path planning for n-links manipulators. The method of potential fields, pioneered by Khatib [3], is a good approach to address these problems. The aim is to generate a field potential with an attractive global minimum at the target and a repulsive local maximum at the obstacle. In others works like [4], [5] it was proposed to use an electrostatic field as an artificial potential function to specify the desired trajectories. These approaches assume knowledge of the location of the obstacle. The principal advantage of a Coulomb potential is the absence of local minima. In [5] the standard Coulomb potential was extended to the n-dimensional Euclidean space for $r > 0$ with

$$
r = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}, \qquad x = (x_1, x_2, x_3) \quad (20)
$$

The generalized harmonic potential of a point charge q is given by

$$
U(r) = \begin{cases} \frac{q}{r^{n-2}}, & n = 1, 3, 4, \cdots \\ q \ln \frac{q}{r}, & n = 2 \end{cases} \quad (21)
$$

The associated electrostatic force field is computed by

$$
E(r) = -gradU(r) \begin{cases} (n-2)\frac{q}{r^{n-1}}, & n = 1, 3, 4, \cdots \\ \frac{q}{r}, & n = 2 \end{cases} \quad (22)
$$

The formulation of (21), it was proposed in [8], [4] to locate a negative unit charge in the goal point and a distributed positive charge in the obstacles. However, the calculations of the field created by a distributed charge is tedious for this reason in [5] it was proposed to locate a positive point charge in each obstacle. To avoid collisions of the robot or end effector whit the obstacles it was proposed in [6] the use of a security circle whit radius R and center in the position of the positive charge q of the obstacle. A key aspect is to construct the potential field such that the gradient lines (trajectories) do not cross the security circle from outside to inside. The *equilibrium point placement method* [6] determines the charge of the obstacle by

$$
q = \frac{R}{R + D} \quad (23)
$$

where R is the radius of the security circle and D is the distance between the two charges.

A. Use of potential fields in conformal geometric algebra

The designed potential field has been developed in a Euclidean space. Now, to solve the inverse kinematics problem of a manipulator arm, we work with conformal points, for this reason we need to transform the conformal points to Euclidean points for the design of the gradient field. Using the *additive split* [1] we can divide a conformal point $x_c \in \mathbb{R}^{n+1,1}$ into its Euclidean and conformal parts by

$$
\begin{aligned}
P_E(x_c) &= (x_c \cdot E)E = \alpha e_0 + \beta e_\infty \in \mathbb{R}^{1,1} \quad (24) \\
P_E^{\perp}(x_c) &= (x_c \wedge E)E = x_e \in \mathbb{R}^n \quad (25) \\
x_c &= P_E(x_c) + P_E^{\perp}(x_c) \quad (26)
\end{aligned}
$$

where $E = e_\infty \wedge e_0$ and α, β are scalars. Now using (25) we can design an unitary gradient field for the Euclidean part of x_c, which is

$$
E(x_e) = E_{obs} - E_{goal} \quad (27)
$$

where E_{obs} is the gradient field in the obstacle for a charge q and E_{goal} is the gradient field for a unitary charge in the target.

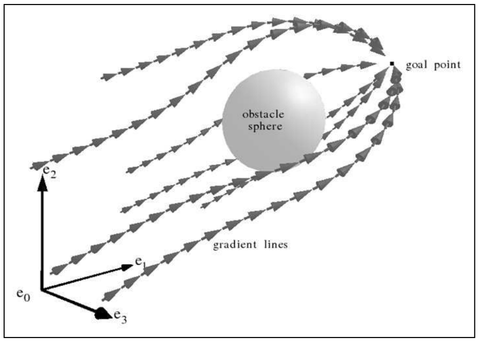


Fig. 3. Gradient field in $\mathbb{R}^3$ for a positive charge q in the sphere center and a negative charge in the goal point.

B. Path generation

To generate the trajectory that the end effector must follow we create a path consisting of points using (27) and the initial position of the end effector. This is an iterative process which evaluates the distance between the end effector and the goal point. This is a description of the method to generate the path for a robotic arm:

1) First, we calculate the distance between the current position of the end effector ($x_c(n) \in \mathbb{R}^{4,1}$) and the position of the goal point ($xgoal_c \in \mathbb{R}^{4,1}$) using $dist[x_c(n), xgoal_c] = \sqrt{-2(x_c(n) \cdot xgoal_c)}$, if it is less or equal to a boundary ε then $x_c(n)$ is the final position of the end effector, else we move to 2.
2) We obtain the Euclidean part($x_e(n)$) of $x_c(n)$ using (25).
3) Then, we compute the following position of the end effector ($x_e(n+1)$) by $x_e(n+1) = x_e(n) + \varepsilon E(x_e(n))$.
4) Using (3) we mapped $x_e(n+1)$ to a conformal point ($x_c(n+1)$).
5) Using the steps of the section V we find the inverse kinematics for $x_c(n+1)$.
6) We update $x_c(n)$ doing $x_c(n) = x_c(n+1)$ then we back to 1.

C. Obstacle Avoidance

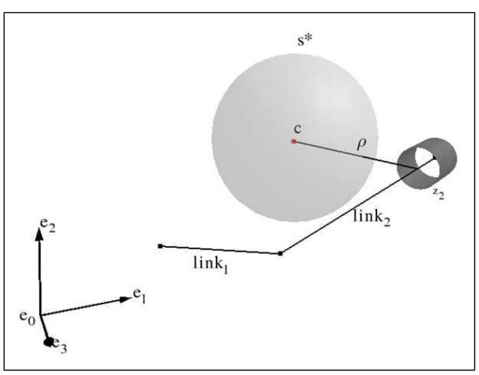


Fig. 4. Two links case for collision avoidance. c is the sphere center and z_2 is a cylinder wich represents the link.

With the generated path we ensure that the end effector does not collide with an obstacle, but we cannot guarantee that the links of the manipulator arm avoid collisions. For this reason we propose a method to avoid collisions using the minimum distances between the obstacle and the links. When we compute the gradient field, we assume knowledge of the position of the positive charge q which is the center of the obstacle sphere (s) and its radius R. The stage of the problem is shown in figure (4). Computing lines like in (19) for each link we can calculate the minimum distance (d) between s and the link by

$$d = \sqrt{\rho^2} - \sqrt{s^2} - r_2 \tag{28}$$

where r_2 is the radius of the cylinder z_2 (see figure(4)), s^2 is the geometric product ss and ρ^2 is computed by

$$\rho^2 = \left[\frac{l^* \cdot \left(s + \frac{1}{2}s^2 e_\infty\right)}{l^* \cdot \left(s + \frac{1}{2}s^2 e_\infty\right) \cdot e_\infty} \right]^2 \tag{29}$$

If the distance is greater or equal than a boundary ς, then the solution of the inverse kinematics calculated on section V is valid for collision avoidance, otherwise we need to find other points to do (19) until the distance condition is satisfied. To find these other points we use a rotor. Suppose that the point p_2 in the step 1 of the section V is not valid for collision avoidance, then we can rotate this initial point with a rotor wich axis of ratation is the axis of the circle z_t shown in the step 1. The angle of rotation is a factor $\Delta\theta$ in this way we can make an iterative process to obtain new points to do (19). The process used, is summarizing by

$$p_2' = R_\theta p_2 \widetilde{R_\theta} \tag{30}$$

where R_θ is

$$R_\theta = e^{-\frac{\theta + \Delta\theta}{2} j_z{}^*} \tag{31}$$

and $j_z{}^*$ is the axis of the circle z_t.

Figure (5) shows a simulation using CLUCalc [7] for a humanoid arm using the proposed method. In the figure we can see the target and the generated path to avoid the obstacle sphere, also we can see a case when a collision can happen and how the rotor works to find other points wich avoid the collision.

VII. Representing Object Shapes in Conformal Geometric Algebra

We would like to represent areas outside of irregular shapes with respect to a canonical representation in conformal geometric algebra, namely the circle. As figure (6) shows, according the Joukowski conformal mapping, exterior areas of ellipse, oriented ellipse and aerofoil shape can be mapped one to one into the exterior area of a circle. Thus, any point (x,y) of $\mathbb{R}^2$ outside of the shape represented using a complex number $q = x + iy$ or multivector $q = x + e_1 e_2 y \in \mathcal{G}_{2,1}$ can be represented in conformal geometric algebra in the following way

$$p_c = p + \frac{1}{2}|p|^2 e_\infty + e_0, \tag{32}$$

where p and q are related for the Joukowski conformal mapping as follows

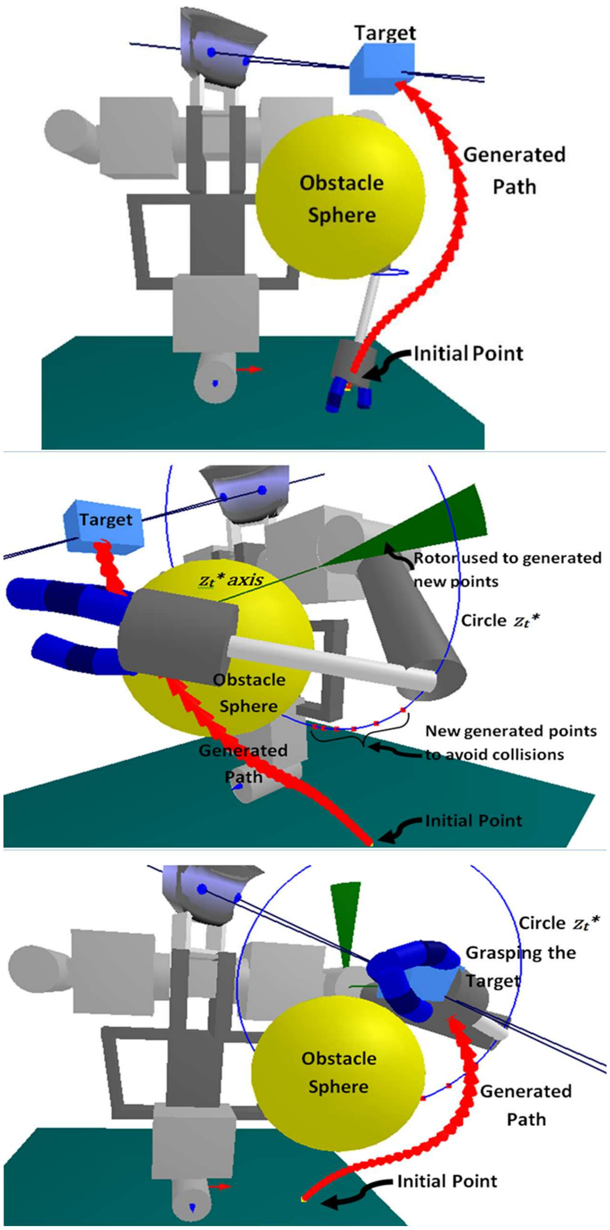


Fig. 5. Colission avoidance of a humanoid arm using potential fields and the proposed method to avoid collision.

Case (a):

$$p = \frac{1}{2}\left[q + \sqrt{q^2 - c^2}\right], \quad q = p + \frac{c^2}{4p}, \quad c^2 = a^2 - b^2 \tag{33}$$

Case (b):

$$p - q_0 = \frac{1}{2}\left[q - q_0 + \sqrt{(q - q_0)^2 - c^2 e^{i2\theta}}\right], \quad q = p + \frac{c^2}{4(p - q_0)} e^{i2\theta} \tag{34}$$

Case (c): (33) applied to a circle whose center is not at the origin.

VIII. Conclusions

This paper has presented a complete mathematical framework for treating real-time obstacle avoidance for manipulators and mobile robots. In the conformal geometric algebra framework, we use potential fields as gradient repulsion functions which helps to compute the inverse kinematics of robot manipulators to avoid obstacles in real time. Since the computing unit of conformal geometric algebra is the sphere, we can easily embed the potential filed in this framework. Our approach takes advantage of two seminal ideas: the Kathib's potential fields and the Wolovich's conformal mappings both for planning and obstacle avoidance. This work generalizes to higher dimensions the issue of complex potential functions and complex mappings. Our future work will use our new formalism to exploit velocity feedback using modern sliding modes techniques for planning and maneuvering.

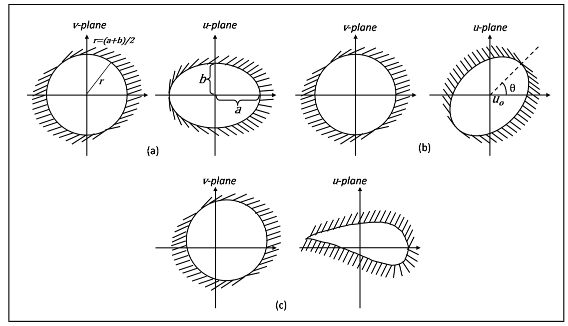


Fig. 6. Conformal mapping of exterior areas. (a) ellipse orentation fixed, (b) ellipse orientation, arbitrary and (c) aerofoil shape.

IX. Acknowledgments

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