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Flight Formation of Multiple Mini Rotorcraft based on Nested Saturations

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***Abstract*— This paper addresses the coordination and trajectory tracking control design for a multiple mini rotorcraft system. The dynamic model of the mini rotorcraft is presented using the Newton-Euler formalism. Our approach is based on a leader/follower structure of multiple robot systems. A nonlinear controller based on nested saturations and a multi-agent consensus control are combined to obtain a flight formation control for a multiple mini rotorcraft system. The centroid of the coordinated control subsystem is used for trajectory tracking purposes. The analytic results are supported by simulation tests.**

I. INTRODUCTION

Unmanned Aerial Vehicles have become a vital platform in a wide variety of applications because they reduce cost and human life risk. Multiple spacecraft flying in formation has been intensively investigated during the last decades [1]-[4]. Different approaches for multiple spacecraft flying in formation have been proposed in the literature for coordination of multiple autonomous robot systems. There are mainly three approaches: Leader/Follower, Virtual Structure and Behavioral Control.

In the leader/follower architecture, one agent is designated as leader while the others are designated as followers which should track the leader. Leader/follower approaches are described in: [1], [2]. The virtual structure approach considers every agent as an element of a larger structure [3] and [4]. Finally, the behavioral control in [5] and [6] is based on the decomposition of the main control goal into tasks or behaviors. This approach also deals with collision avoidance, flock centering, obstacle avoidance and barycenter.

Generally, to analyze the communication between agents, directed or undirected graphs are used. Every node in a graph is considered as an agent which can have information exchange with all or several agents. In [4], [7], [8], and [9], the authors use algebraic graph theory in order to model the information exchange between vehicles. By using this technique, several control strategies have been developed, e.g. [9], [10], [11], [12] and [13]. In [9], the authors present several algorithms for consensus and obstacle avoidance for multiple-agent systems. [10] presents an algorithm for trajectory tracking of a time varying reference for a single integrator multi-agent system. [11] and [12] presents a passive decomposition approach for consensus and formation control. In [13], the authors present a bilateral teleoperation control approach for the multi-agent trajectory tracking problem.

We are interested in the problem of multiple mini rotorcraft flying in formation using a nonlinear control based on nested saturations and a coordination control strategy. A coordination algorithm assumes that there are n-agents which have some kind of information exchange between them. In this approach, every mini rotorcraft is considered as an agent in the multi-agent system with an information exchange topology. We propose a decoupled lateral dynamics coordination. Similarly, a decoupled longitudinal dynamics coordination is desired. Thus, the lateral and longitudinal dynamical systems of each mini rotorcraft are considered as agents to be coordinated and to follow a desired trajectory. To do this, combined with a nonlinear control, we use an algebraic graph theoretical approach to model information exchange between mini rotorcrafts. The work reported in the literature is by now quite vast and addresses different approaches for mini rotorcraft stabilization [14], [15], [16], [17], [18], [19], [20] and [21] among others. In [14] a nonlinear control based on nested saturations is presented. In this approach, the dynamics is decoupled into lateral and longitudinal dynamical subsystems. Thus, nested saturations control was used to stabilize each subsystem. In [20], the authors propose a robust linear PD controller considering parametric interval uncertainty. Here, the authors present a robust stability analysis and computes the robustness margin of the system with respect to the parameters uncertainty. In [21] a flight formation control based on a four integrators coordination control is presented. This approach is based on a forced consensus algorithm to achieve a multiple mini rotorcraft flight formation and tracking.

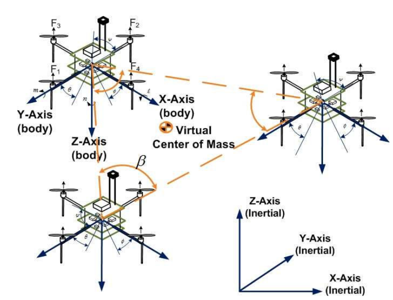

Fig. 1. Multiple mini rotorcraft flying in formation

This work addresses the nonlinear control for multiple mini rotorcraft flying in formation, shown in Figure 1, based

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on nested saturations and a single integrator coordination control strategy. In this approach we consider every mini rotorcraft as agents to be coordinated and follow a virtual reference. The proposed control scheme is based on the idea that lateral and longitudinal subsystems are decoupled which enable us to implement a decoupled coordination of the lateral and longitudinal subsystems. In this way, the multiple mini rotorcraft platoon can hover and thus keeping the desired formation by following a constant zero-reference. Another contribution of this work is that the centroid of a virtual center of mass can be used to follow a given smooth trajectory.

This paper is organized as follows: Section II presents some preliminary results on algebraic graph theory. Section III presents the dynamical model of the proposed architecture. In section IV the nonlinear control design is presented. Simulation results are presented in section V. Section VI presents the conclusions and future work.

II. Graph Theory

The interaction between mini rotorcraft can be modeled as a group of dynamical systems which has an information exchange topology represented by information graphs. A graph $\mathcal{G}$ is a pair $\mathcal{G}(\mathcal{N},\mathcal{E})$ consisting of a set of nodes (agents) $N = \{n_i \colon n_i \in N, \forall i = 1, ..., n\}$ together with their interconnections $\mathcal{E}$ on $\mathcal{N}$ [22]. Each pair (n_1, n_2) is called an edge $e \in \mathcal{E}$. An undirected graph is one where agents i and j can get information from each other. In a digraph, the i^{th} agent can get information from the j^{th} agent but not necessarily viceversa. One important characterization of graphs is their connectivity. A graph is said to be connected if for every pair (n_1, n_2) of distinct nodes or agents there is a path from n_1 to n_2. A connected graph allows the communication between all agents through the network. A directed graph is said to be strongly connected if any two vertices can be joined by a path. A graph is said to be balanced if its in-degree (number of communication links arriving at the node) is equal to its out-degree (number of communication links leaving the node).

III. Dynamic Model

To obtain the vehicle dynamical model, it will be assumed that it flies over a local area in the Earth. Then, the Flat-Earth model equations will be used [25]. The equations representing the kinematic and the moments are written as

$$\dot{\Phi} = H(\Phi)\,\omega^b_{b/e} \tag{1}$$

$$\dot{\omega}^b_{b/e} = \left(J^b\right)^{-1}\left[\mathbf{M}^b - \Omega^b_{b/e} J^b \omega^b_{b/e}\right] \tag{2}$$

The vehicle center of mass, CM, is coincident with the body frame origin, F_b. The angular velocity in terms of the body system is given by $\omega^b_{b/e} = \begin{bmatrix} P & Q & R \end{bmatrix}^T$ and its cross product matrix is denoted by $\Omega^b_{b/e}$. The angular velocity in the local inertial system has components $\dot{\Phi} = \begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T$. The matrix of rotation from the inertial frame F_e to F_b is denoted by $C_{b/n}$.

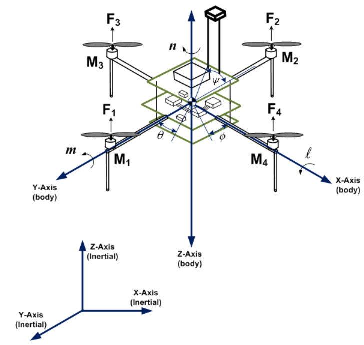

Fig. 2. Vehicle schematic for vertical flight mode

The set of attitude equations can be obtained using the equations (1) and (2). The transformation of the components of the angular velocity generated by a sequence of Euler rotations from the body to the local reference system is written as follows:

$$H(\Phi) = \begin{bmatrix} 1 & t\theta s\phi & t\theta c\phi \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix} \tag{3}$$

where s, c and t are used to denote the $\sin$, $\cos$ and the $\tan$ respectively.

The term J^b in (2) represents the inertia matrix. Since the X-4 prototype is symmetrical in the xz-plane and the xy-plane, the products of inertia J_{xy}, J_{yz} and J_{xz} vanish. Then J^b and its inverse can be written by

$$J^b = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \tag{4}$$

The aerodynamics and thrust moments can be denoted by $M^b = \begin{bmatrix} \ell & m & n \end{bmatrix}^T$, and are shown in Figure 2.

Then differentiating (1) we get

$$\ddot{\Phi} = \dot{H}(\Phi)\,\omega^b_{b/e} + H(\Phi)\,\dot{\omega}^b_{b/e} \tag{5}$$

Introducing the RHS of (2) into (5),

$$\ddot{\Phi} = \dot{H}(\Phi)\,\omega^b_{b/e} + H(\Phi)\left(J^b\right)^{-1}\left[\mathbf{M}^b - \Omega^b_{b/e} J^b \omega^b_{b/e}\right] \tag{6}$$

It is proposed that

$$\mathbf{M}^b \triangleq \Omega^b_{b/e} J^b \omega^b_{b/e} + J^b H(\Phi)^{-1}\left[\tilde{\tau} - \dot{H}(\Phi)\,\omega^b_{b/e}\right] \tag{7}$$

where $\tilde{\tau} = \begin{bmatrix} \tilde{\tau_\phi} & \tilde{\tau_\theta} & \tilde{\tau_\psi} \end{bmatrix}^T$. Then (5) can be rewritten as

$$\ddot{\phi} = \tilde{\tau}_\phi \tag{8}$$

$$\ddot{\theta} = \tilde{\tau}_\theta \tag{9}$$

$$\ddot{\psi} = \tilde{\tau}_\psi \tag{10}$$

Newton's second law is used to obtain the equations of translational motion in the inertial frame of reference as

$$\ddot{p}^n_{CM/T} = C_{n/b}\frac{\mathbf{F}^b}{m} + \mathbf{g}^n \tag{11}$$

where the position of CM in the North-East-Down (NED) coordinate system with respect to F_e, is given by $\mathbf{p}^n_{CM/T} = \begin{bmatrix} x & y & z \end{bmatrix}^T$. The aerodynamic and thrust force vector in the body system is represented by $F^b = [\ X \quad Y \quad Z\ ]^T$. The aerodynamic and thrust forces in the body frame of reference is given by

$$F^b = \sum_{i=1}^{4} F_i Z \tag{12}$$

Then (11) can be rewritten as

$$\begin{aligned} \ddot{x} &= -F^b sin\theta && (13)\\ \ddot{y} &= F^b cos\theta sin\phi && (14)\\ \ddot{z} &= F^b cos\theta cos\phi - 1 && (15) \end{aligned}$$

where the constant "-1" is the normalized product of the vehicle mass and the gravitational acceleration.

IV. Nonlinear Control Design

A. Vehicle Stabilization

In this section, a nonlinear controller with a coordination control strategy is developed. It will be proved that the proposed control scheme stabilizes the rotorcraft in hover flight. In order to stabilize the position for (13)-(15) and (8)-(10), the following control input is proposed

$$\begin{aligned} F^b &\triangleq \frac{-a_1\dot{z} - a_2(z - z^d) + 1}{\cos\phi\cos\theta} && (16)\\ \tilde{\tau}_\psi &\triangleq -a_3\dot{\psi} - a_4(\psi - \psi^d) && (17) \end{aligned}$$

where a_1, a_2, a_3 and a_4 are positive constants; z^d and ψ^d are the desired altitude and heading respectively.bad

Using (16) in (13)-(15) the lateral dynamic model is represented by the following set of equations:

$$\begin{aligned} \ddot{y} &= \tan\phi && (18)\\ \ddot{\phi} &= \tilde{\tau}_\phi && (19) \end{aligned}$$

Similarly, the longitudinal dynamic model is represented by

$$\begin{aligned} \ddot{x} &= \frac{-\tan\theta}{\cos\phi} && (20)\\ \ddot{\theta} &= \tilde{\tau}_\theta && (21) \end{aligned}$$

It is assumed that pitch angle and roll angle are operated in a neighborhood of the origin, i.e., $|\theta| < \pi/10$. Then, the lateral dynamical system (18)-(19) is reduced to

$$\begin{aligned} \ddot{y} &= \phi && (22)\\ \ddot{\phi} &= \tilde{\tau}_\phi && (23) \end{aligned}$$

and the longitudinal dynamic model is reduced to

$$\begin{aligned} \ddot{x} &= -\theta && (24)\\ \ddot{\theta} &= \tilde{\tau}_\theta && (25) \end{aligned}$$

Since the control inputs are bounded, we start our analysis introducing the following definition [26]

Definition 1: Given two positive constants L, M with $L \leq M$, a function $\sigma : R \rightarrow R$ is said to be a linear saturation for (L, M) if it is continuous, nondecreasing function satisfying

1) $s\sigma(s) > 0$ for all $s \neq 0$;
2) $\sigma(s) = s$ when $\mid s \mid \leq L$;
3) $\mid \sigma(s) \mid \leq M$ for all $s \in R$

In order to introduce the consensus algorithm we will review the longitudinal control design, which is a simplified dynamic model consisting of four integrators in cascade.

Now, we introduce the following variables

$$\begin{aligned} \xi_1(x) &= \xi_2 - x - 2\dot{x} + \theta && (26)\\ \xi_2(x) &= \xi_3 + \theta - \dot{x} && (27)\\ \xi_3(x) &= \xi_4 + \theta && (28)\\ \xi_4(x) &= \dot{\theta} && (29) \end{aligned}$$

To simplify the analysis, a recursive methodology is proposed. To do this, it is assumed that

$$\begin{aligned} \zeta_n &= \xi_n(x) + \sigma_{n-1}(\zeta_{n-1}(x)) && (30)\\ \zeta_1 &= \xi_1(x) && (31) \end{aligned}$$

and

$$u = -\sigma_n(\zeta_n) \tag{32}$$

Let us define the following positive definite function

$$V_n = (1/2)\xi_n^2(x) \tag{33}$$

Differentiating V with respect to time, we obtain

$$\dot{V_n} = \xi_n(x)\dot{\xi}_n(x) \tag{34}$$

from the fact that $\dot{\xi}_n(x) = -\sigma_n(\zeta_n)$, we have

$$\dot{V_n} = \xi_n(x)u = -\xi_n(x)\sigma_n(\zeta_n) \tag{35}$$

due to equation (30) we get

$$\dot{V_n} = -\xi_n(x)\sigma_n(\xi_n(x) + \sigma_{n-1}(\zeta_{n-1}(x))) \tag{36}$$

Using definition (1), we can see that $M_{n-1} < 0.5L_n$, it can be noted that if $|\xi_n| > 0.5L_n$ then $\dot{V_n} < 0$. This means that there exist a time T_n such that $|\xi_n| \leq 0.5L_n$ for $\forall t > T_n$ which implies that $|\xi_n + \sigma_{n-1}(\zeta_{n-1}(x))| \leq 0.5L_n + M_{n-1} \leq L_n$.

When $n = 1$ we have the base case of the recursion. This case is treated a little different, let us propose

$$V_1 = (1/2)\xi_1^2(x) \tag{37}$$

Differentiating V with respect to time, we obtain

$$\dot{V_1} = \xi_1(x)\dot{\xi}_1(x) \tag{38}$$

using (26)-(29) is possible to see that $\dot{\xi}_1(x) = -\sigma_1(\xi_1(x))$, then we have

$$\dot{V_1} = -\xi_1(x)\sigma_1(\zeta_1) \tag{39}$$

due to equation (31) we get

$$\dot{V_1} = -\xi_1(x)\sigma_1(\xi_1(x)) \tag{40}$$

As in the recursive case, it can be noted that if $|\xi_1(x)| > 0.5L_1$ then $\dot{V_1} < 0$. This means that there exist a time T_1

such that $|\xi_1(x)| \leq 0.5L_1$ for $\forall t > T_1$. It is important to note that $T_n < T_{n-1}$ for all $n > 2$.

Since $\dot{V_1} < 0$ then, from equations (31) and (40) implies that $\xi_1(x) = \zeta_1 \to 0$. It can be noted that starting from $i = 2$ until $i = n$ we have the following set of equations due to the recursion of the method

$$\dot{V_2} = -\xi_2(x)\sigma_2(\xi_2(x) + \sigma_1(\zeta_1(x))) \quad (41)$$
$$\dot{V_3} = -\xi_3(x)\sigma_3(\xi_3(x) + \sigma_2(\zeta_2(x))) \quad (42)$$
$$\dot{V_4} = -\xi_4(x)\sigma_4(\xi_4(x) + \sigma_3(\zeta_3(x))) \quad (43)$$

The recursion of equation (30) leads us to:

$$\zeta_2 = \xi_2(x) + \sigma_1(\zeta_1(x)) \quad (44)$$
$$\zeta_3 = \xi_3(x) + \sigma_2(\zeta_2(x)) \quad (45)$$
$$\zeta_4 = \xi_4(x) + \sigma_3(\zeta_3(x)) \quad (46)$$

After a time T_4, it can be seen that from (41), $\xi_2 \to 0$, (44) implies that $\zeta_2 \to 0$, in a recursive way (42), $\xi_3 \to 0$, from (45) $\zeta_3 \to 0$, from (43), $\xi_4 \to 0$, from (46), $\zeta_4 \to 0$. This means that, from (29) $\dot{\theta} \to 0$, from (28) $\theta \to 0$, from (27) $\dot{x} \to 0$, and finally from (26) $x \to 0$.

Due to the fact that the lateral dynamic model is also a four integrators in cascade, we use the same analysis to obtain a control design. Then, the lateral and longitudinal control laws are given by

$$\begin{aligned}\tilde{\tau}_\theta &= -\sigma_4(\dot{\theta} + \sigma_3(\dot{\theta} + \theta \\ &\quad +\sigma_2(\dot{\theta} + 2\theta - \dot{x} + \sigma_1(\dot{\theta} + 3\theta - 3\dot{x} - x)))) \end{aligned} \quad (47)$$
$$\begin{aligned}\tilde{\tau}_\phi &= -\sigma_4(\dot{\phi} + \sigma_3(\dot{\phi} + \phi \\ &\quad +\sigma_2(\dot{\phi} + 2\phi + \dot{y} + \sigma_1(\dot{\phi} + 3\phi + 3\dot{y} + y)))) \end{aligned} \quad (48)$$

B. Consensus Agreement

Now, we consider the case of having a multiple quadrotor vehicle system with either a cyclic topology or a chain topology of information exchange as shown in Figure 3.

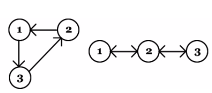

Fig. 3. Cyclic and chain topology of information exchange

The longitudinal kinematic model for the multi-quadrotor system is given by

$$\begin{aligned} \dot{x} &= -\mathcal{L}x + bu \\ y &= c^T x \end{aligned} \quad (49)$$

where $\mathcal{L}$ is the Laplacian matrix of the information exchange graph having the following properties:

1) $\mathcal{L}$ has a single eigenvalue at 0, $\lambda_1(\mathcal{L}) = 0$ with right eigenvector $w_1^T = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}, i.e.\ \mathcal{L}w_1 = 0$.
2) The remaining eigenvalues are all positive, i.e. $\lambda_i(\mathcal{L}) > 0$ and $\mathcal{L}w_i = \lambda_i w_i$ for $i = 2, ..n$, and $w_i \in R^n$.

We assume that the information exchange graph is balanced. Let us assume also that in the coordinating controller the gains multiplying the signals in between agents are all equal to 1. For the $i-th$ row of $\mathcal{L}$, the entries $l_{ij} = -1$ for $i \neq j$ correspond to the gains multiplying the signals from other agents coming to agent i. For the $i-th$ column of $\mathcal{L}$, the entries $l_{ji} = -1$ for $i \neq j$ correspond to the gains multiplying the signals going out of agent i towards the other agents. We then have the following property.

3) w_1 defined above is also the left eigenvalue of $\mathcal{L}$ corresponding to the eigenvalue 0, i.e. $w_1^T \mathcal{L} = 0$.

It is worth to mention that dynamics (49) can also be written as

$$\dot{x}_i = \bar{u}_i \quad (50)$$

with multiple agent consensus achieved using the following forced consensus algorithm

$$\bar{u}_i = -\sum_{j \in \mathcal{N}_i} (x_i - x_j) + u_i \quad (51)$$

where $\mathcal{N}_i$ is the set of agents transmitting their information to the agent i.

Then, we next propose a simple strategy for the position consensus of the multiple quadrotor vehicle system. We will consider the use of a nonlinear control using a nested saturations strategy which drives all the states to the origin. Since we want to have consensus to the origin (on x and y-axis) of a set of quadrotor vehicles, we propose a change of variable on (47) and (48)

$$x \triangleq \sum_{j \in \mathcal{N}_i} (\tilde{x}_j - \tilde{x}_i) \quad (52)$$
$$y \triangleq \sum_{j \in \mathcal{N}_i} (\tilde{y}_j - \tilde{y}_i) \quad (53)$$

Remark 1: On one hand a multiple mini rotorcraft consensus can be achieved by means of a single integrator consensus algorithm, then (52) provides a simple way to solve the coordination problem. On the other hand, we may think of the neighbors position of a mini rotorcraft as the position reference and thus the stability of every mini rotorcraft is guarantied using the nonlinear control based on nested saturations.

As it was presented in the previous section, after a time T_4 we have that $x \to 0$. Then, (52) becomes

$$0 = \sum_{j \in \mathcal{N}_i} (\tilde{x}_j - \tilde{x}_i) \quad (54)$$

which implies that all $\tilde{x}_j \to \tilde{x}_i$, similarly, $\tilde{y}_j \to \tilde{y}_i$. Therefore, the control laws $\tilde{\tau_\theta}$ and $\tilde{\tau_\phi}$ for the longitudinal and lateral subsystems of the i^{th}-minirotorcraft becomes

$$\begin{aligned}\tilde{\tau}_{\theta,i} = &-\sigma_4(\dot{\theta}_i + \sigma_3(\dot{\theta}_i + \theta_i + \sigma_2(\dot{\theta}_i + 2\theta_i - \dot{x}_i \\ &+ \sigma_1(\dot{\theta}_i + 3\theta_i - 3\dot{x}_i - \sum_{j \in \mathcal{N}_i} (\tilde{x}_j - \tilde{x}_i))))) \end{aligned} \quad (55)$$
$$\begin{aligned}\tilde{\tau}_{\phi,i} = &-\sigma_4(\dot{\phi}_i + \sigma_3(\dot{\phi}_i + \phi_i + \sigma_2(\dot{\phi}_i + 2\phi_i + \dot{y}_i \\ &+ \sigma_1(\dot{\phi}_i + 3\phi_i + 3\dot{y}_i - \sum_{j \in \mathcal{N}_i} (\tilde{y}_j - \tilde{y}_i))))) \end{aligned} \quad (56)$$

C. Formation Control

In the practice, a coordination to the origin implies that every mini rotorcraft will converge to the same position in the 3D space. Then, we propose a leader-relative position consensus (UAV formation) for the multi quadrotor system, i.e. the quadrotor vehicles will converge to a position with respect to the leader of the group. In this case, the following geometric formations are proposed.

1) Triangular Formation: A triangular formation around a circle of radius r for the team of three quadrotor vehicles is proposed. Assuming a cyclic information exchange topology, the relative position is given by

$$\tilde{x}_1 - \tilde{x}_2 = r\cos(\pi/6) \quad (57)$$
$$\tilde{x}_3 - \tilde{x}_1 = -r\cos(\pi/6) \quad (58)$$
$$\tilde{x}_2 - \tilde{x}_3 = r\cos(\pi/2) \quad (59)$$
$$\tilde{y}_1 - \tilde{y}_2 = r\sin(\pi/6) \quad (60)$$
$$\tilde{y}_3 - \tilde{y}_1 = -r\sin(\pi/6) \quad (61)$$
$$\tilde{y}_2 - \tilde{y}_3 = 2r\sin(\pi/6) \quad (62)$$

Assuming a chain information exchange topology, the relative position is given by

$$\tilde{x}_1 - \tilde{x}_2 = \cos(\pi/6) \quad (63)$$
$$\tilde{x}_2 - \tilde{x}_3 = \cos(\pi/2) \quad (64)$$
$$\tilde{y}_1 - \tilde{y}_2 = \sin(\pi/6) \quad (65)$$
$$\tilde{y}_2 - \tilde{y}_3 = 2\sin(\pi/6) \quad (66)$$

Therefore, we can use either (57)-(62) or (63)-(66) as a relative position reference with respect to each other.

2) Line Formation: For the team of three quadrotor vehicles assuming chain information exchange topology, the relative position for a line formation over the y-axis is given by

$$\tilde{x}_i - \tilde{x}_j = 0 \quad (67)$$
$$\tilde{y}_i - \tilde{y}_j = d_{ij} \quad (68)$$

where d_{ij} is a fixed distance between any two mini rotorcraft. Similarly, the relative position for a line formation over the x-axis is given by

$$\tilde{x}_i - \tilde{x}_j = d_{ij} \quad (69)$$
$$\tilde{y}_i - \tilde{y}_j = 0 \quad (70)$$

Therefore, we can use either (67)-(68) or (69)-(70) as a relative position reference with respect to each other.

Using a relative position reference for the flight formation of multiple mini rotorcraft, equations (55) and (56) are rewritten as

$$\tilde{\tau}_{\theta,i} = -\sigma_4(\dot{\theta}_i + \sigma_3(\dot{\theta}_i + \theta_i + \sigma_2(\dot{\theta}_i + 2\theta_i - \dot{x}_i + \sigma_1(\dot{\theta}_i + 3\theta_i - 3\dot{x}_i - (\sum_{j\in\mathcal{N}_i}(\tilde{x}_j - \tilde{x}_i) - \tilde{x}_i^d))))) \quad (71)$$

$$\tilde{\tau}_{\phi,i} = -\sigma_4(\dot{\phi}_i + \sigma_3(\dot{\phi}_i + \phi_i + \sigma_2(\dot{\phi}_i + 2\phi_i + \dot{y}_i + \sigma_1(\dot{\phi}_i + 3\phi_i + 3\dot{y}_i - (\sum_{j\in\mathcal{N}_i}(\tilde{y}_j - \tilde{y}_i) - \tilde{y}_i^d))))) \quad (72)$$

where $\tilde{x}_i^d$ and $\tilde{y}_i^d$ represent the position reference of agent i as described in previous section. (71) and (72) are such that the geometric flight formation of a multiple mini rotorcraft system is guarantied.

D. X4 Trajectory Tracking Control

Now, we will consider the case of trajectory tracking of a multiple vehicle system. It is assumed that the leader of the group is always vehicle 1. Then, (49) is rewritten as

$$\dot{\tilde{x}} = -\mathcal{L}\tilde{x} + bu_1 \quad (73)$$

where $b^T = [\, 1 \quad 0 \quad \ldots \quad 0 \,]$ and u is the input given to the leader. Define $\tilde{x}_{CM} = \frac{1}{N}\sum_{i=1}^{N}\tilde{x}_i$ where N is the number of agents in the formation. Let $\tilde{x}_{CM}^d$be the desired value for $\tilde{x}_{CM}$. Assume for simplicity that agent 1 is the leader, i.e. $c^T = b^T = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$ and that the control law is

$$u_1(x) = Nk\sigma(\tilde{x}_{CM}^d - \tilde{x}_{CM}) \quad (74)$$

where $\sigma(\cdot)$ represents the saturation function and k is a positive gain. Note that $\tilde{x}_{CM}$ may not be directly measurable for the leader (agent 1). It should be noticed that for cyclic and chain topologies of information exchange the system is observable from the input and output of the leader, see [27]. The state can therefore be observed from the input and output of agent 1. Introducing (74) into (73) we get

$$\begin{array}{rcl} \dot{\tilde{x}}_{CM} & = & k\sigma\left(\tilde{x}_{CM}^d - \tilde{x}_{CM}\right) \\ v_i^T\tilde{x} & = & -\lambda_i(v_i^T\tilde{x}) + v_i^T bu_1 \ ; \ i = 2,..,N \end{array}$$

The modes in the last equation above are all stable. When $u_1 = 0$, these modes converge to zero which means that $(\tilde{x}_i - \tilde{x}_j) \to 0$ for $i \neq j$. This property is obtained by using the coordinating control algorithm that leads the position dynamics to (73). These modes are uncontrollable when $v_i^T b = 0$. There is a trade-off in the choice of gain k in (74). For smaller values of k, the speed of convergence of x_{CM} is slower, but the transient in the errors $(\tilde{x}_i - \tilde{x}_j)$ for $i \neq j$, will be smaller.

Again, we can use the same control strategy for the lateral dynamics by replacing the x position by the y position.

Then the trajectory tracking control for the leader of the group is given by

$$\tilde{\tau}_{\theta,1} = -\sigma_4(\dot{\theta} + \sigma_3(\dot{\theta} + \theta + \sigma_2(\dot{\theta} + 2\theta - \dot{x} + \sigma_1(\dot{\theta} + 3\theta - 3\dot{x} - \sum_{j\in\mathcal{N}_i}(\tilde{y}_j - \tilde{y}_i) - x_i^d - u_1(x))))) \quad (75)$$

$$\tilde{\tau}_{\phi,1} = -\sigma_4(\dot{\phi} + \sigma_3(\dot{\phi} + \phi + \sigma_2(\dot{\phi} + 2\phi + \dot{y} + \sigma_1(\dot{\phi} + 3\phi + 3\dot{y} - \sum_{j\in\mathcal{N}_i}(\tilde{y}_j - \tilde{y}_i) - y_i^d - u_1(y))))) \quad (76)$$

V. Results

A. Simulation

To illustrate the proposed methodology, this section presents the simulation results concerning the multiple mini quadrotor formation control. We consider three mini quadrotors evolving in the 3D space. Extensive simulations were run on a platoon of three rotorcrafts considering the 6-DOF nonlinear dynamical model. Cyclic and chain topologies of information exchange were considered. The initial conditions for inertial position and velocity are [2,-1,0](m) and [-0.1,-0.1,0.2](m/s) for the first vehicle; [-1,2,0](m) and [-0.1,-0.2,0.3](m/s) for the second vehicle and [-1,-1,0](m) and [0.2,0.3,-0.5](m/s) for the third vehicle The results of simulation show that the proposed nonlinear control strategy can be used to achieve a geometric formation as well as formation flying of multiple mini rotorcrafts. Thus, using control inputs (71), (72), (16) and (17) on the mini rotorcraft acting as followers and (75), (76) (16) and (17) on the mini rotorcraft acting as leader, on the 6-DOF nonlinear dynamical model (13)-(15) in simulation, we get the results shown in Figure 4.

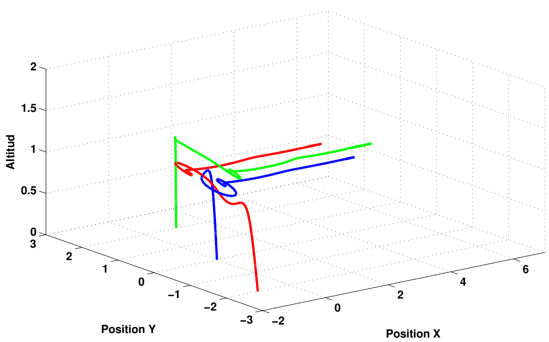

Fig. 4. 3D position

VI. Conclusions and Future Work

A nonlinear dynamical model of the mini rotorcraft has been presented using the Newton-Euler formulation. Nonlinear control based on nested saturations and a single integrator consensus control for flight formation of mini rotorcraft was developed. The x-position and the y-position of each mini rotorcraft were considered as dynamical agents with full information access. Trajectory tracking for the group of mini rotorcraft was achieved by using the virtual center of mass of the agents formation. Extensive simulations were run in order to show the performance of the developed control scheme. Future work in this area includes experimental tests on mini rotorcraft using real-time embedded control systems.

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