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Cooperative Localization of Marine Vehicles using Nonlinear State Estimation

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Abstract— **This paper investigates the problem of cooperative navigation of autonomous marine vehicles using range-only acoustic measurements. We consider the use of a single maneuvering autonomous surface vehicle (ASV) to aid the navigation of one or more submerged autonomous underwater vehicles (AUVs), using acoustic range measurements combined with position measurements for the ASV when data packets are transmitted. The AUV combines the data from the surface vehicle with its proprioceptive sensor measurements to compute its trajectory. In previous work, we presented an experimental demonstration of this approach, using an extended Kalman filter (EKF) for state estimation. In the present paper, we analyze the observability properties of the cooperative ASV/AUV localization problem and present experimental results comparing several different state estimators. Using the weak observability theorem for nonlinear systems, we demonstrate that this cooperative localization problem is best attacked using nonlinear least squares (NLS) optimization. We present experimental results for this new approach and compare it to alternative state estimators, demonstrating superior performance.**

I. Introduction

Autonomous Underwater Vehicles (AUVs) are important in several marine applications, such as ocean mapping and exploration, ship inspection and environmental research. Accurate AUV localization is an important enabler for AUV navigation and autonomy. There are many different approaches to AUV navigation, including the use of inertial/Doppler velocity log systems [23], acoustic transponders [12], [13], and simultaneous localization and mapping (SLAM) [17], [8]. A key requirement for many AUV tasks is bounded error without the reliance on surfacing for GPS measurements.

This paper addresses cooperative localization of multiple AUVs which has been previously investigated by Vaganay *et al.* [20] and Bahr *et al.* [2]. In this concept, multiple AUVs equipped with acoustic modems help one another to navigate by sharing position information and acoustic range measurements. The original concept, named Moving Long Baseline (MLBL), envisioned two types of AUVs: a Search-Classify-Map (SCM) vehicles, whose task was oceanographic mapping [20] and a Comm-Nav-Aid (CNA) vehicles, whose task was to aid the SCM vehicle's navigation. The system design called for the CNA vehicles to be equipped with high-performance inertial/Doppler systems, and also for the CNA vehicles to surface frequently for GPS fixes, whereas the SCM vehicles would have low-cost proprioceptive sensors.

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(a)

(b)

Fig. 1. Our experiments used MIT's Scout kayaks (a). The proposed approach is targeted for future implementation on an OceanServer Iver2 AUV.

So as to simplify the problem at this stage, the CNA role has been taken by an autonomous surface vehicle (ASV) with continuous access to GPS. Using an acoustic modem for the ranging, the surface vehicle sends its GPS location estimate with each transmission. If the AUV receives such a message it combines the information with its own dead-reckoning (DR) within an estimation framework to improve its position estimate. The approach can either make use of round-trip acoustic ranging, or if the vehicles have synchronized clocks, using one-way acoustic travel time measurements.

Our earlier work relied on the use of two or more CNA vehicles, spaced sufficiently far apart to establish a wide baseline [20], hence the name MLBL. In more recent work, we have pursued the same goal using only a single surface vehicle [9] and exploiting its motion to achieve similar performance as when two ore more CNAs are used.

In the current paper, we analyze the observability of this configuration for marine cooperative localization and show that a single surface vehicle can localize the AUV, if its motion is chosen appropriately.

Our work is closely related to the work of Eustice *et al.*, who have investigated the localization of a submerged AUV using one-way acoustic travel time measurements between the AUV and a manned surface vessel [7], [22], [21]. In both approaches, GPS data and acoustic range measurements from a surface vehicle are fused onboard an AUV to compute the AUV's trajectory. Eustice and Webster's work has primarily targeted the use of higher cost AUVs for which a Doppler Velocity Log is available.

There are a number of estimation algorithms which can be used to solve this problem, including the Extended Kalman Filter (EKF), Particle Filtering (PF), and nonlinear Least Squares (NLS) optimization methods [7]. Also modern nonlinear theories such contraction theory [16] have motivated some researchers to investigate AUV navigation and control using nonlinear observers that take into account vehicle dynamics [15]. This approach has not yet been applied to this problem.

Cooperative AUV localization is a special category of a more general field of cooperative autonomy. The cooperative localization for ground vehicles has been extensively studied by Roumeliotis and colleagues [18], assuming reliable communications. In the AUV setting, we are encumbered by low bandwidth communications with high latency.

Moving forward from the use of two surface vehicles to aid one AUV [3], in this paper we analyze the observability of cooperative localization using a single ASV and demonstrate that accurate localization can be achieved if the ASV moves appropriately (Section II). Additionally, we expand on our previous work by implementing a nonlinear least squares (NLS) optimization algorithm which estimates the optimal AUV trajectory that minimizes the least square error relative to the measurements gathered (Section III). Finally in Section IV results are presented which illustrate that this approach outperforms other possible estimators.

II. AUV Cooperative Localization With a Single Surface Vehicle

A. Problem Definition

Within the field of cooperative marine robotics the cooperative localization problem has been developed as follows: acoustic modems are installed on one or more surface vehicles and one or more AUVs. The surface vehicle moves in formation with the AUV and continuously broadcasts messages that contain its GPS location and associated timestamp. If the AUV receives one of these messages, and assuming clock synchronization on every vehicle, the range between the two vehicles can be computed [6]. The AUV then combines this information with its own dead-reckoning filter, in a Bayesian framework, to estimate its position.

The AUV is equipped with sensors that can measure its forward and transverse velocities, $\hat{v}_m$ $\hat{w}_m$, and compass that measures its absolute heading, $\hat{\theta}_m$, with respect to the x and y axes as shown in Figure reffig:MLBL, at each time increment, m. A pressure sensor measures the vehicle depth precisely, which allows the 3-D problem to be simplified to 2-D

$$\begin{aligned} x_{1,[m+1]} &= x_{1,[m]} + \triangle_m(\hat{v}_m \cos\hat{\theta}_m - \hat{w}_m \sin\hat{\theta}_m) \\ y_{1,[m+1]} &= y_{1,[m]} + \triangle_m(\hat{v}_m \sin\hat{\theta}_m + \hat{w}_m \cos\hat{\theta}_m) \\ \theta_{1,[m+1]} &= \hat{\theta}_{1,[m+1]} \end{aligned} \quad (1)$$

The velocity measurements are considered to be observations of a combination of deterministic value and stochastic part with zero mean Gaussian noise ($n_{\hat{v}}$, $n_{\hat{w}}$ and n_θ). This propagation will operate with a frequency of 5Hz, thus $\triangle_m = 0.2$ sec.

As the vehicle dead-reckoning is propagated, the localization error and uncertainty grow. As mentioned above, a range measurement from the surface vehicle will occasionally be received (with frequency of at most 0.1Hz, thus $\triangle_i = 10$ sec) and will be used to bound this uncertainty. The surface vehicle trajectory at each such time i is denoted by $X_{2,[i]} = [x_{2,[i]},\ y_{2,[i]}]^T$, and the associated range measurement by $h_{3\mathrm{D},[i]}$. Utilizing the accurate depth information this range measurement is converted into a 2-D horizontal range, $h_{[i]}$. Again assuming zero mean Gaussian noise, the measurement function is given by:

$$h_{[i]} = \sqrt{(x_{1,[i]} - x_{2,[i]})^2 + (y_{1,[i]} - y_{2,[i]})^2} + n_r \quad (2)$$

Figure 2 illustrates this framework. Note that the heading is not considered in the observability analysis in the next section as it is directly observable. While further details of marine cooperative localization can be found in [3], [9], in this paper we will focus on the *observability* of the problem.

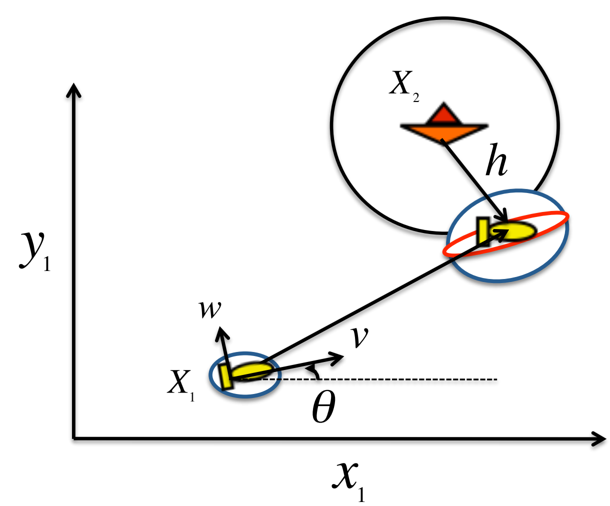

Fig. 2. An AUV navigates using measurements of its heading, θ, and velocities, v and w, however its uncertainty grows overtime (blue circle). Occasionally a surface vehicle provides the AUV with a range measurement (black circle), information that the AUV uses to reduce its uncertainty (red).

B. Observability Analysis

If the system described in Equations 1 and 2 is observable, the vehicle's position can be estimated by combining the dead-reckoning measurements with the range measurements. Observability depends on relative motion between the two vehicles. If the system is not observable, then there is no way the vehicle's position can be computed regardless of the estimation algorithm used.

Some previous work has considered observability of the cooperative localization framework. Gadre [11] performed an observability analysis for a similar system with a stationary ranging beacon. The approach taken linearized the system leading to important information being lost in the process. Gadre demonstrated that an AUV receiving range measurements from known locations can recover its states if the direction from which these measurements are taken varies over time. More recently, Antonelli *et al.* [1] examined locally weak observability of the marine cooperative localization framework, presenting results for an EKF used for cooperative localization, with simulations and using data from two surface vehicles.

Systems are linearized in both of the papers described above, and hence are treated as linear time-variant. The initial nonlinear system becomes a linear system that is observable — should the range measurements be received from a variety of directions during operation.

Consider Figure 3(upper), the AUV depicted receives one range measurement from the ASV. Subsequently, both the vehicles move to new positions, and the ASV provides the AUV with another measurement (Although for visual simplicity the figure illustrates AUV to be stationary). Linearization reduces the nonlinear range measurements to straight lines tangent to the linearization points. If the range measurements come from different directions, as is shown in Figure 3 (upper), then, the vehicle position can be recovered by solving for the intersection of two lines, and therefore the system is observable, in agreement with Gadre's results [11].

In Figure 3 (lower), the AUV depicted receives one range measurement from the ASV. Then, the ASV moves to a new colinear location and provides the AUV with another range and position measurement pair. According to Gadre's results [11], this system is unobservable. However, if the system is not linearized, the system should still be observable, as the position can be computed by solving for the intersection of two circles that intersect tangentially.

In this paper, we will use nonlinear observability theory to prove that the system has local weak observability. First we assume that the vehicle heading is directly observable. This leaves a second order system in x and y. Using Lie derivatives and the weak observability theorem for nonlinear systems [19], the observability matrix is given by

$$Obs = \begin{pmatrix} \frac{(x_1-x_2)}{h} & \frac{(y_1-y_2)}{h} \\ \beta_1 & \beta_2 \end{pmatrix} \quad (3)$$

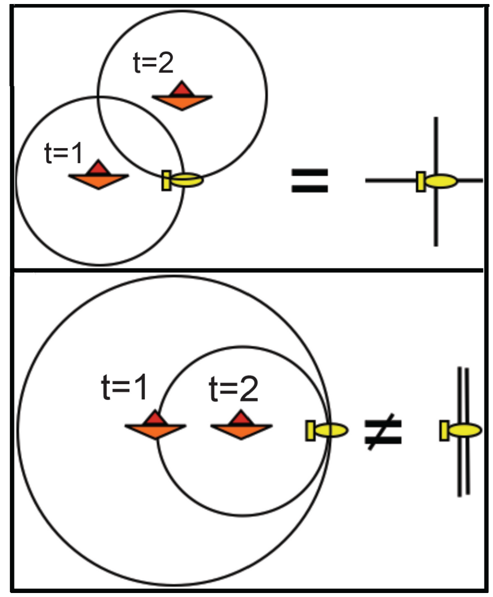

Fig. 3. The effects of linearization. In the upper figure, both the linearized and the nonlinear system are observable, however in the figure in the lower figure only the nonlinear system is observable.

where

$$\beta_1 = \frac{(y_1-y_2)^2 f_1 - (x_1-x_2)(y_1-y_2) f_2}{h^3}$$

$$\beta_2 = \frac{-(x_1-x_2)(y_1-y_2) f_1 + (x_1-x_2)^2 f_2}{h^3}$$

$$f_1 = \hat{v}_m \cos\theta_m - \hat{w}_m \sin\theta_m \quad (4)$$

$$f_2 = \hat{v}_m \sin\theta_m + \hat{w}_m \cos\theta_m \quad (5)$$

$$h = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2} \quad (6)$$

The system equations are presented in the continuous domain and the range measurement index i has therefore been dropped. Note that will the actual system is a discrete system, well-known observability theory for continuous systems can be used to make conclusions on the actual discrete system if the sample time is small enough.

This system is observable if the observability matrix is full rank. Thus, if

$$\det(Obs) = \frac{-f_1(y_1-y_2) + f_2(x_1-y_2)}{h^2} \neq 0 \quad (7)$$

Looking at the observability matrix, we can see that it is full rank (except some trivial cases) and therefore the system is observable, as long as the inter-vehicle range changes or the direction that the range measurements come from varies.

Thus nonlinear observability theory implies that it is possible to recover the states if a nonlinear observer such as a Particle Filter or nonlinear Least Squares (NLS) optimization is used. Conversely for some vehicle configurations a linearized observer (such the EKF) will fail. In addition to this, observability (as measured by the eigenvalues of Equation 3) can be improved with intelligent motion planning of the surface vehicle.

III. Least Squares Formulation

The classic solution to this problem can be obtained by using well-known techniques such as the EKF and Particle Filter. In the linear Gaussian case, the Kalman Filter gives the trajectory that minimizes the expected vehicle position error given the measurements obtained up to the current time. Particle Filtering can also give a solution close to the optimal one if the number of particles used is sufficiently large. In practice it is difficult to compute the number of particles that will result in convergence.

An alternative way to solve the problem is through an incremental batch optimization which computes the trajectory which minimizes the least square error of an entire state trajectory relative to the measurements. In the Gaussian (linear or nonlinear) case, this is equivalent to the Maximum Likelihood Estimator (MLE).

To consider the MLE estimator, all the vehicle position measurements should be included in the optimization. Given that velocity measurements are taken approximately 50 times more frequently than the range measurements; it is common to group all the velocity measurements between two sequential range measurements $(h_{[i-1]}, h_{[i]})$ into one cumulative dead-reckoning measurement $(z_{o,[i]} = [\Delta X_{[i]}, \Delta Y_{[i]}]^T)$. This allows us to reduce the number of states in the optimization to a reasonable number.

Figure 4 shows a series of poses of the AUV and the surface vehicle. At time $t = 0$, both the surface vehicle and the AUV have a certain initial location estimate with a known uncertainty. Some time later, $t = 1$, the AUV has moved to another location and receives a range measurement from the surface vehicle, $h_{[1]}$. Between these times the AUV computes an accumulative dead-reckoning vector using forward and starboard velocity measurements as well as compass heading estimates. This vector will be denoted $z_{o,[1]}$.

After n such periods, our aim is to compute the trajectory, $X_{1,[1:n]}$, that minimizes the least squares error of all cumulative dead-reckoning measurements, $z_{o,[1:n]}$, the range measurements, $h_{[1:n]}$ and surface vehicle GPS locations, $X_{2,[1:n]}$, up to the current time.

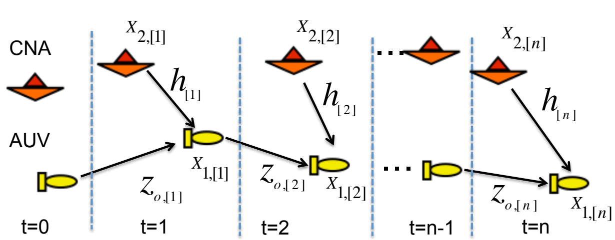

Fig. 4. Least Squares framework for a series of poses.

The trajectory corresponding to the least squares error can be found by minimizing

$$(X_{1,[1:n]})^* = \underset{X_{1,[1:n]}}{\operatorname{argmin}}(C_{(X_{1,[1:n]})}) \tag{8}$$

with the following cost function

$$C = \frac{1}{2}\sum_{i=1}^{n}(||(X_{1,[i]} - X_{2,[i]})|| - h_{[i]})^T \Sigma_{r,[i]}^{-1} (||(X_{1,[i]} - X_2[i])|| - h_{[i]}) + \frac{1}{2}\sum_{i=1}^{n}(X_{1,[i]} - X_{1,[i-1]} - z_{o,[i]})^T \Sigma_{o,[i]}^{-1} (X_{1,[i]} - X_{1,[i-1]} - z_{o,[i]})$$

where $\Sigma_{o,[i]}$ represents the covariance of the dead-reckoning measurements.

We perform this optimization iteratively, as follows:

1) Form an optimization problem that minimizes the least squares error for all the measurements taken up to the current time.
2) Use as the initial condition to the optimization, the previous optimal solution plus the extra pose from the current dead-reckoning measurement
3) Minimize the cost function using a conjugate gradient method.
4) When a new range measurement is received, compute the cumulative dead-reckoning vector, increase the number of poses, and repeat.

Admittedly the number of states included in NLS optimization increases continuously, and as a result the required computation increases accordingly. For this reason windowed versions of this approach have been considered [7] until this point. Future work will consider efficient incremental implementations of the NLS such as [14].

IV. Experiments

A set of experiments were carried out in Boston's Charles River to demonstrate this framework. In the experiments we used two MIT SCOUT autonomous surface vehicles [4]. We used one vehicle as a surrogate for an AUV performing a mapping mission, while the other acted as the CNA. Both kayaks were equipped with GPS, a WHOI acoustic modem [10], and a compass. More detail on the basic setup for the experiment can be found in Fallon *et al.* [9].

First we performed an experiment to characterize the modem range measurements. We determined the measurements to be approximately characterized by a Gaussian with $\sigma_r = 5m$. The vehicle heading uncertainty was taken to be $\sigma_\theta = 3^o$. As the vehicles did not have velocity sensors, differential GPS measurements were used to simulate actuation measurements. We envisage our solution to be used with a low cost AUV (such as the OceanServer Iver2 shown in Figure 1) with imprecise actuation sensors. For this reason the forward and starboard velocity measurement variances will be chosen to be $\sigma_{\hat{v}} = \sigma_{\hat{w}} = 0.5m/s$.

In this particular data set, we present both vehicles navigating autonomously. Figure 5 shows how the simulated AUV moved in a box pattern, while the surface vehicle followed a zig-zag pattern behind it (its path is not shown here). The supporting vehicles path was chosen so as to maximize the observability of the system.

In Figure 5 and Table I we can see a comparison between our proposed NLS algorithm and the commonly used EKF and particle filter. Two mean error values are presented for

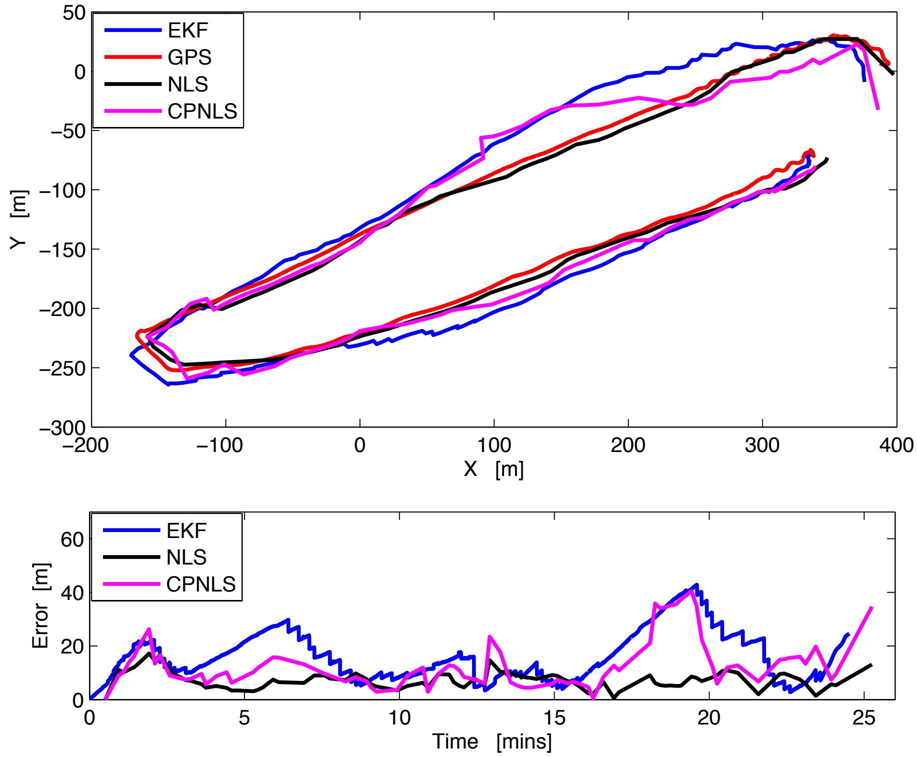

Fig. 5. Comparison of different localization methods. The top figure shows the estimated trajectory for the vehicle serving as the surrogate AUV, using the EKF, NLS, and CPNLS, compared against GPS which serves as ground truth. The CNA vehicle trajectory is not shown, but followed a zig zag pattern alternating back and forth across the other vehicle's path.

the NLS. One is the mean error for the entire optimization over the full trajectory (NLS) which would not have been available to the AUV for mission planning as it requires the full set of measurements — including future measurements to be calculated. The second is the mean error of each NLS estimate at the time it was first calculated, using data from the start of the mission up until that time step. We designate this the current point nonlinear least squares (CPNLS).

Our results show that the NLS and CPNLS estimators outperform the EKF. Post processing the data we gathered, we verified that the NLS localization algorithm runs in real-time for missions of approximately one hour duration. As mentioned above, future work will consider efficient incremental implementations of the NLS.

TABLE I

Comparison of Different Localization Methods

Method	Mean Error
extended Kalman filter	18.84m
particle filter	13.65m
nonlinear least squares	8.94m
current point nonlinear least squares	13.12m

In Figure 6, we present the localization error for this experiment (top) and the eigenvalues of the observability matrix (middle and lower).

V. Conclusions and Future Work

In this paper, we have studied the observability of the cooperative AUV localization problem. Using the weak observability theorem for nonlinear systems, we have shown that cooperative localization using a single surface vehicle aiding a submerged AUV is best performed using a nonlinear estimator. Because the observability cannot be guaranteed with a linearized observer, such as an EKF, a nonlinear estimator is the preferred approach for this situation.

Our algorithm recursively optimizes the AUV's path for each new dead-reckoning and range pair, using the NLS solution from the previous iteration, subject to a convergence criterion that depends on the relationship between the accuracy of the proprioceptive on-board sensors and the exteroceptive ranging sensor. We have demonstrated the performance of the approach with real data for an experiment with one AUV and one autonomous surface craft.

One potential alternative for future work is to reparameterize the cooperative localization framework in an alternative linear coordinate space, thus simplifying the required state estimation [5]. In the absence of such a reformulation, nonlinear methods present the state of the art for this problem. Since it is infeasible to operate a typical NLS optimization indefinitely as the number of states continually increases, future work will consider methods for on-line

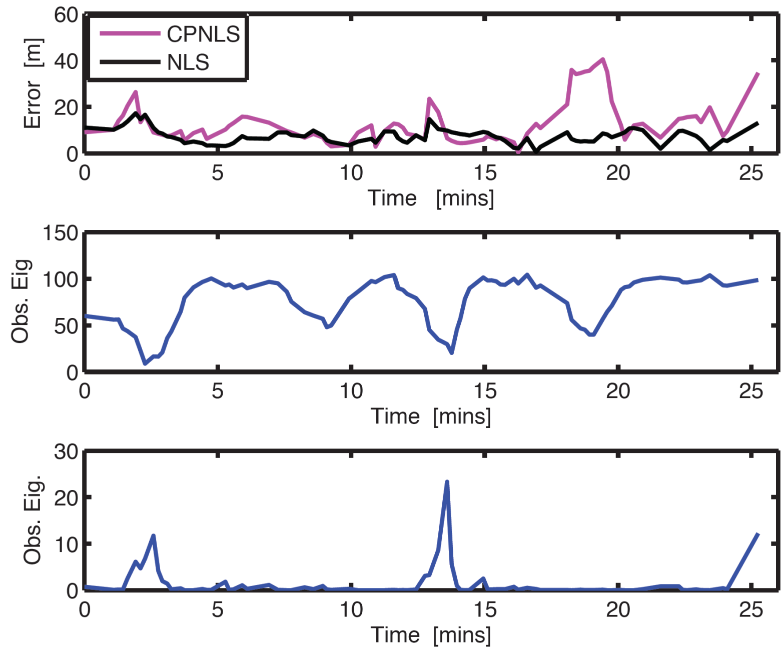

Fig. 6. Top: Error for Nonlinear Least Squares (NLS) and Continuous Point Nonlinear Least Squares (CPNLS) as a function of time; Middle, Lower: eigenvalues of the observation matrix as a function of time.

incremental optimization [14], followed by a full offline batch optimization after the mission.

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