Position Control Methods of Spherical Ultrasonic Motor

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Abstract—In this paper, we investigate the position control of spherical ultrasonic motor (SUSM). The generated torque of SUSM is influenced by the phase difference and the driving frequency of applied AC voltages. Therefore, the control strategy is classified into three types: (a) variable phase and fixed frequency, (b) fixed phase and variable frequency, and (c) variable phase and frequency. We formulate the position control rules for SUSM based on the above three types of control variables, and investigate the performances experimentally.

I. INTRODUCTION

Spherical motor is one of the multi-DOF actuators, and the promising applications are robot manipulator[1], actuator for a camera on robot[2], [3], mobile robot[4], [5], haptic device[6] and so on.

An ultrasonic motor[7] has several advantages such as a high output torque at low speed without a reduction gear, holding torque for braking, and high responsiveness. Also, it has a capability to construct the multi-DOF actuator with a compact and simple structure, as in [8], [9], [10].

A spherical ultrasonic motor (SUSM), which has the above advantages, has been developed and studied[8], [11], [12]. In order to control the position of SUSMs, several position control methods have been studied[2], [8], [11]. For example, Ref.[2] has proposed the hybrid control using the phase difference and frequency of input voltages. Ref.[8] has conducted the trajectory control. Ref.[11] has mentioned that the phase difference is changed for the position control. However, they have not formulated the control rules enough and clarified the differences of those control performances. In order to realize the better performance, the investigation of the position control methods of SUSM is required.

The generated torque of SUSM is influenced by the phase difference and the driving frequency. Therefore, the torque control strategy is classified into three types: (a) variable phase and fixed frequency, (b) fixed phase and variable frequency, and (c) variable phase and frequency.

The purpose of this study is to formulate the position control strategy for SUSM based on the above three types of control variables, and to investigate the performances. The rest of this paper is organized as follows. Section II depicts the principle of SUSM and the classification of the torque control strategy of SUSM. Section III describes three kinds of position control methods based on the torque control strategy. Section IV demonstrates the experimental system and the experimental results. Finally, Section V provides the conclusions and future works.

II. SPHERICAL ULTRASONIC MOTOR

A. Principle of Spherical Ultrasonic Motor

The SUSM used in this study consists of one spherical rotor and three ring-shaped stators. Figure 1 shows an overview of the SUSM. The geometric schemes are illustrated in Fig. 2.

The stator includes a metallic elastic body and piezoelectric elements. When an AC voltage is applied to the piezoelectric vibrator, a standing wave is generated on the elastic body. By applying two AC voltages with a phase difference to the positive and negative sections of the piezoelectric elements, a traveling wave is generated due to combination...
of the two standing waves[7]. The stators and the rotor are in pressure contact with each other, and the rotor is driven by the tangential force of the elliptical motion of the traveling wave. A single stator is shown in Fig. 3. Another piezoelectric element on the stator is used as a sensor detecting the resonance, and the signal is called the feedback signal. There are two inputs (AC voltage A and B), one output (Feedback) and FG (Frame Ground) terminals.

The stators, namely vibrators, are located as shown in Fig. 2. Geometric parameters (stators’ alignment) are \( \theta_1, \theta_2, \theta_3 \) and \( \phi \). Using the parameters, the moment vector of each stator, \( \mathbf{m}_i \), can be expressed as follows:

\[
\mathbf{m}_i = \begin{bmatrix} -\cos \theta_i \cos \phi \\ -\sin \theta_i \cos \phi \\ \sin \phi \end{bmatrix} \tau_i, \quad (i = 1, 2, 3) \tag{1}
\]

Here, \( \tau_i \) is the generated torque of each stator.

As a result, the output moment vector of the rotor, \( \mathbf{m}_{\text{rotor}} \), can be described as the summation of the vectors \( \mathbf{m}_i \) in Eq. (1).

\[
\mathbf{m}_{\text{rotor}} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \mathbf{m}_1 + \mathbf{m}_2 + \mathbf{m}_3 = \begin{bmatrix} -c\theta_1 \cos \phi \\ -c\theta_2 \cos \phi \\ -c\theta_3 \cos \phi \end{bmatrix} \tau_1 + \begin{bmatrix} -s\theta_1 \cos \phi \\ -s\theta_2 \cos \phi \\ -s\theta_3 \cos \phi \end{bmatrix} \tau_2 + \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = D \tau . \tag{2}
\]

Here, \( s\theta_i = \sin \theta_i, c\theta_i = \cos \theta_i, s\phi = \sin \phi, c\phi = \cos \phi \), and \( D \) is a constant matrix. From the Eq. (2), we can control the moment of rotor, \( \mathbf{m}_{\text{rotor}} \), with the torque of stators, \( \tau \).

B. Stator Torques Required for Rotor Moment

In this subsection, we derive the stator torques that require to realize the desired moment \( \mathbf{m}_d \). In the posture control strategy, we rotate the SUSM to the target position along the moment \( \mathbf{m}_d \). From the Eq. (2), the required stator torque \( \tau \) can be obtained as follows:

\[
\mathbf{m}_d = D^{-1} \mathbf{m}_d . \tag{3}
\]

Since the geometric parameter \( \phi \) is nearly zero in the used SUSM, we premeditate such the 2DOF motion so that \( m_z \) can be neglected, and we obtain the following relationship.

\[
\tau = \frac{1}{d c \phi} \begin{bmatrix} \sin \theta_2 - \sin \theta_3 \\ \sin \theta_3 - \sin \theta_1 \\ \sin \theta_1 - \sin \theta_2 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \end{bmatrix} \tag{4}
\]

where, \( d = \sin(\theta_1 - \theta_2) + \sin(\theta_2 - \theta_3) + \sin(\theta_3 - \theta_1) \).

As shown in Fig. 2(a), geometry of the each stator becomes \( \theta_1 = \pi/2, \theta_2 = -\pi/6 \) and \( \theta_3 = -5\pi/6, \) and the Eq. (4) can be expressed as follows:

\[
\tau = \frac{2}{3 \sqrt{3} c \phi} \begin{bmatrix} 0 & -\sqrt{3} & 0 \\ -3/2 & \sqrt{3}/2 & 0 \\ 3/2 & \sqrt{3}/2 & 0 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \end{bmatrix} \tag{5}
\]

\[
= \frac{2}{3 c \phi} \begin{bmatrix} \cos(\frac{\pi}{6}) & \sin(\frac{\pi}{6}) & 0 \\ \cos(\frac{-\pi}{6}) & \sin(\frac{-\pi}{6}) & 0 \\ \cos(\frac{-\pi}{3}) & \sin(\frac{-\pi}{3}) & 0 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \end{bmatrix} \tag{6}
\]

\[
= \frac{2 ||m_d||}{3 c \phi} \begin{bmatrix} \sin(\psi' - \theta_1) \\ \sin(\psi' - \theta_2) \\ \sin(\psi' - \theta_3) \end{bmatrix} , \tag{7}
\]

where, \( \psi = \tan2(m_y, m_x) \) is the direction of the target moment \( \mathbf{m}_d \) on the X-Y plane, and \( \psi' = \psi - \pi/2 \) is the direction of the motion of lever as shown in Fig. 4(a). \( ||m_d|| = \sqrt{m_x^2 + m_y^2} \) is a norm of the moment.

C. Torque Control Strategy of Stator

Generally, the single stator torque can be expressed as

\[
\tau_i = c_i(f_i) \sin \rho_i , \tag{8}
\]

where, \( f_i \) is the frequency and \( \rho_i \) is the phase difference of two AC voltages (A and B). Here, \( c_i(f_i) \) is a magnitude of torque, a function of the \( f_i \). When \( f_i \) to be tuned to resonant frequency \( f_{\text{res},i} \), \( c_i(f_i) \) becomes maximum value, \( c_{\text{max},i} \). Thus, we can obtain maximum value of the stator torque when the system satisfies the following condition.

\[
f_i = f_{\text{res},i}, \quad \text{and} \quad \rho_i = \pm \pi/2 . \tag{9}
\]

Eq. (7) expresses the relationship between desired moment and stator torque. On the other hand, Eq. (8) means that there are two ways to control the stator torque: the frequency and the phase difference. Thus, it is possible to classify the torque control strategy into three kinds of methods as follows.
1) Torque Control Based on Phase Difference (PH): Let the frequency \( f_i \) be the resonant frequency \( f_{\text{res},i} \), and \( c_{\text{max},i} = \frac{2m_{\text{max}}}{3c_\phi} \). Here, \( m_{\text{max}} \) is the maximum absolute value of the moment of rotor. Then, we can generate the target torque when the phase difference \( \rho_i \) satisfies a following condition.

\[
\begin{align*}
  c_i(f_i) = c_{\text{max},i} = \frac{2m_{\text{max}}}{3c_\phi} \\
  \sin \rho_i = \frac{\|m_d\|}{m_{\text{max}}} \sin (\psi' - \theta_i) 
\end{align*}
\]

(10)

We refer this type of torque control as PH method in this paper.

If Eq. (10) is substituted into Eq. (8), Eq. (7) can be obtained.

2) Torque Control Based on Frequency (FR): When we settle the phase difference \( \rho_i \) as \( \rho_i = \pi/2 \) or \( \rho_i = -\pi/2 \), —switches according to the direction of required torque—, we can generate the target torque with the frequency \( f_i \).

\[
\begin{align*}
  c_i(f_i) = 2\|m_d\|/3c_\phi \sin (\psi' - \theta_i) \\
  \sin \rho_i = \text{sgn} (\psi' - \theta_i)
\end{align*}
\]

(11)

This type of torque control is referred as FR method.

3) Torque Control Based on Phase Difference and Frequency (HB): We control both the phase difference and the frequency with Eq. (8). That is, we control \( f_i \) and \( \rho_i \) using the following condition.

\[
\begin{align*}
  c_i(f_i) = 2\|m_d\|/3c_\phi \sin (\psi' - \theta_i) \\
  \sin \rho_i = \text{sgn} (\psi' - \theta_i)
\end{align*}
\]

(12)

This may be called a hybrid control[2] (in this paper, refer to HB method).

III. POSITION CONTROL OF SUSM

A. Kinematics between Sensor Angle and Rotor Position

A picture of the SUSM used in this study was shown in Fig. 1. Potentiometers shown in the figure measure the angle of the lever \( (\theta_x, \theta_y) \) via guide rails (see Fig. 4(b) and Fig. 5). \( \theta_x \) and \( \theta_y \) are the angles between the axis Z and the lever projected on the X-Z plane and the Y-Z plane, respectively.

On the other hand, we express the position of lever as the posture of vector. The original position vector of the lever is set to \( k = [0, 0, 1]^T \) when it corresponds to Z-axis. The position vector of the lever \( p \) is expressed as follows:

\[
p = \begin{bmatrix} R_y(\theta_x)R_z(\theta_y)k \end{bmatrix}
\]

(13)

where, \( R_x(\theta) \) means the rotation matrix around the axis \( x \) through an angle of \( \theta \). By solving the above equations, when the angles \( \theta_x \) and \( \theta_y \) are obtained from the potentiometers, the position vector of the lever, \( p \), is expressed as follows:

\[
p = \begin{bmatrix} \sin \theta_x \cos \theta_y \\ \sin \theta_y \\ \cos \theta_x \cos \theta_y \end{bmatrix},
\]

(14)

where \( \theta_y = \tan^{-1}(\cos \theta_x \tan \theta_y) \).

B. Driving Frequency and Feedback Signal

Figure 6 shows an example of feedback signal obtained from a single stator while changing the driving frequency. As seen from this figure, the resonant frequency of the stator (vibrator) \( f_{\text{res},i} \) is about 92 [kHz].

The sudden change is observed at the lower frequencies than the resonant frequency. On the other hand, at the higher frequency, the feedback signal changes gently[8]. Therefore, it can be thought that the generated torque of the stator is changed smoothly along with shifting the driving frequency to the higher frequency. In the figure, the frequency should be changed between 92 and 94 [kHz].

C. Position Control Methods

1) Desired Moment Vector: The desired moment vector \( m_d \) that approaches the position vector of the lever \( p \) to the target position vector \( p_d \) can be written as the following equation:

\[
m_d = \begin{cases} 0 & \text{if } \|p \times p_d\| = 0 \\ k(\alpha) \frac{p \times p_d}{\|p \times p_d\|} & \text{otherwise} \end{cases}
\]

(15)

where \( \alpha \geq 0 \) is the angle between \( p \) and \( p_d \), \( k(\alpha) \) is the absolute value of the moment vector (torque) expressing as a function of the angle \( \alpha \). The absolute value of \( m_d \) is \( \|m_d\| = k(\alpha) \). Because the actual torque has an upper limitation, the maximum value becomes \( \max \{k(\alpha)\} = m_{\text{max}} \).

For example, in the case where the torque is proportional to the angle \( \alpha \), the value is simply expressed as below:

\[
k(\alpha) = K_P \alpha,
\]

(16)

where \( K_P \) is the proportional gain.

On the other hand, when the upper limitation (saturation) is considered, the value is expressed as follows:

\[
k(\alpha) = \begin{cases} K_P \alpha & \text{if } \alpha < \alpha_{\text{th}} \\ m_{\text{max}} & \text{otherwise} \end{cases}
\]

(17)
Here, $\alpha_{th} = m_{max}/K_P$ is a threshold angle of the torque saturation.

Furthermore, \( \beta \) is defined to normalize with \( m_{max} \)

\[
k(\alpha) = \beta m_{max} \quad (18)
\]

\[
\beta = \begin{cases} \frac{\alpha}{\alpha_{th}} & \text{if } \alpha < \alpha_{th} \\ 1 & \text{otherwise} \end{cases} \quad (19)
\]

This condition can be illustrated as Fig. 7. The slope $1/\alpha_{th}$ is relative to the proportional gain $K_P$.

In the above, only the proportional (P) control is considered for simplicity. It can, of course, include the integral (I) control and/or the derivative (D) control as well.

2) PH Method: According to II-C.1, the driving frequency $f_i$ is set to the predetermined resonant frequency $f_{res,i}$, and the phase difference $\rho_i$ is varied in order to control the rotor position. The second equation of Eq. (10), which is about the phase, can be rewritten as below based on Eq. (19).

\[
\sin \rho_i = \beta \sin (\psi' - \theta_i) \quad (20)
\]

3) FR Method: Based on II-C.2, the phase difference is fixed as $\rho = \pm \pi/2$, the driving frequency $f_i$ is varied. Although it is not easy to obtain the relation of $c_i(f_i)$ practically, it is assumed that the torque is proportional to the frequency[8]. Then, the control rule can be described as:

\[
f_i = f_{res,i} + f_{shift,i}(1 - \beta |\sin (\psi' - \theta_i)|) \quad (21)
\]

\[
\sin \rho_i = \text{sgn} \{\sin (\psi' - \theta_i)\} \quad (22)
\]

4) HB Method: According to II-C.3, both phase and frequency are changed. The phase can be obtained from the second equation of Eq. (12). On the other hand, the driving frequency is expressed similarly to the FR method.

\[
f_i = f_{res,i} + f_{shift,i}(1 - \beta) \quad (23)
\]

\[
\sin \rho_i = \sin (\psi' - \theta_i) \quad (24)
\]

For example, in case of Fig. 6, $f_{res,i} \approx 92$ [kHz] and $f_{shift,i} \approx 2$ [kHz]. The above equation is illustrated as Fig. 8.

IV. EXPERIMENTS

A. Experimental System

The system configuration in this study is shown in Fig. 9. The feedback signals of stators and the potentiometer voltages are measured with A/D converter. The position of lever on the rotor is calculated from the potentiometer voltages. From the difference between present and target positions, the desired moment is obtained. Finally, in order to realize the torque, the driving frequencies and phase differences of stators are determined by PH, FR or HB method, and they are set to the driver of SUSM.

An example of the control procedure based on PH method is presented as follows:

1) The control gain $K_P = 1/\alpha_{th}$ is predetermined.
2) The target position $p_d$ is set to a certain position.
3) The current position $p$ is calculated from Eq. (14) with the angles $\theta_x$ and $\theta_y$ measured by the potentiometers.
4) The angle $\alpha$ is obtained from $p_d$ and $p$. Namely, $\beta$ is calculated from Eq. (19).
5) The driving frequency, $f_i$, is fixed to $f_{res,i}$, and the phase difference $\rho_i$ is calculated from Eq. (20).
6) Two AC voltages with the frequency $f_i$ and the phase difference $\rho_i$ are input to each stator.
7) Continue (back to 3) procedure).

These procedure is conducted at a sampling time of 1 [ms] by PC with ART-Linux[13].

B. Step Responses

The step response experiments are examined using the control methods mentioned in the previous section. The desired angle of Y-axis is zero, and the desired angle of X-axis $\theta_{cd}$ is selected as $\pm 1, 3, 5$ [deg] since larger $\theta_{cd}$ tends to occur the saturation. The threshold $\alpha_{th}$ was changed to 1, 3 and 10 [deg], which is determined on trial-and-error. As a result, $\alpha_{th} = 3$ [deg] exhibits the best responses. Therefore, the results shown below are the ones in case of $\alpha_{th} = 3$ [deg]. The step responses with PH, FR and HB methods are shown in Figs. 10 to 12, respectively.

In the PH method (Fig. 10), at the steady state, the fluctuations and error are observed. In the FR method (Fig. 11), the fast responsiveness and a small error are demonstrated at the steady state. On the other hand, noisy response is also observed. In the HB method (Fig. 12), a good stability and small error are shown.

C. Circular Trajectory

By using the three methods, we make the SUSM trace a circular trajectory of radius of 3 [deg]. The periodic time is 4 [s]. The experimental results are shown in Figs. 13 to 15, respectively.
Fig. 10. Step responses (PH)

Fig. 11. Step responses (FR)

Fig. 12. Step responses (HB)

Fig. 13. Circular Trajectory (PH) of Radius 3 [deg]

Fig. 14. Circular Trajectory (FR) of Radius 3 [deg]

Fig. 15. Circular Trajectory (HB) of Radius 3 [deg]
The mean error of PH method is 0.108 degrees, with a standard deviation of 0.366 degrees. For FR method, the mean error is 0.028 degrees, and the standard deviation is 0.310 degrees. The HB method has a mean error of 0.033 degrees and a standard deviation of 0.206 degrees. The results show that HB method has the lowest error and deviation.

### Table I: Mean Error and Standard Deviation

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean Error [deg]</th>
<th>Standard Deviation [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PH</td>
<td>0.108</td>
<td>0.366</td>
</tr>
<tr>
<td>FR</td>
<td>0.028</td>
<td>0.310</td>
</tr>
<tr>
<td>HB</td>
<td>0.033</td>
<td>0.206</td>
</tr>
</tbody>
</table>

The result of PH method shows large error as seen from the step response. On the other hand, FR and HB methods trace the target circle with small error. Figs. 16 to 18 show the error from the target circle. And the mean error and the standard deviation of each methods are shown in Table I. As seen from the figures and table, HB method, in this experiment, has shown the lowest error and deviation.

### D. Discussion

The PH method is influenced by the characteristics of the phase difference. As seen from the characteristics of rotational speed and phase in [2], it exhibits nearly sinusoidal relationship. However, it may have lost motion and asymmetric diversity. The steady state error may be caused by the characteristics. Since the error can be reduced by adding the integral control, it is not a large problem.

The FR and HB methods have not only $\alpha_{th}$ but also $f_{\text{shift,i}}$ as the control parameters. In the above experiments, the same $\alpha_{th}$ and $f_{\text{shift,i}}$ are applied. However, they can be tuned for better responses.

### V. Conclusions and Future Works

#### A. Conclusions

In this paper, the classification and formulation of the position control strategy of SUSM are shown. The three methods, namely PH, FR and HB methods, are implemented. The position control experiments are conducted and the control performances are investigated.

PH method exhibits the steady state error and fluctuation. FR and HB methods show the faster response and lower error than PH method does although they include noisy response.

#### B. Future Works

This paper treats only the proportional control with constant parameters. The investigation of PID control performance and their parameters (gains, $\alpha_{th}$ and $f_{\text{shift,i}}$) tuning in detail must be a future work.

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### References


RT-Linux/