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Proposal of Augmented Linear Inverted Pendulum Model for Bipedal Gait Planning

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Abstract— **In this paper, we propose a new model called Augmented Linear Inverted Pendulum (ALIP) in which an augmented function F is added to the dynamic equation of the linear inverted pendulum. The purpose of adding the function F is to modify/adjust the inverted pendulum dynamics in such a way that disturbance caused by un-modeled dynamics (legs, arms, etc.) can be compensated or minimized. By changing the key parameters of the augmented function we can easily modify the inverted pendulum dynamics. The desired walking motion with maximized stability margin is achieved by optimizing the key parameters using genetic algorithm. The disturbance created by the un-modeled dynamics is minimized because full robot dynamics is considered in the optimization process. Simulations results show that the walking gait obtained using the proposed method is more stable than that obtained using the Linear Inverted Pendulum Mode (LIPM).**

I. INTRODUCTION

Many approaches have been used in walking pattern generation for biped robots[1-9]. One of the popular approaches is to use dynamic model analysis in designing walking patterns([1], [2], [9]). Due to its high-order, nonlinear and complex characteristics, it is difficult to analyze biped dynamics unless some simplification is made.

Simplification techniques have been used by many researchers. Golliday and Hemami used state feedback to decouple the high-order system dynamics of biped into independent low-order subsystems[2]. Kajita *et al.*[7] derived an ideal massless-leg biped model. In this model, the center of gravity (CG) of the body moves horizontally and the horizontal motion of the CG can be expressed by a simple linear differential equation. They called such a motion of the ideal model the *Linear Inverted Pendulum Mode (LIPM)* and used it to build the control framework for biped walking. The advantage of this method is that the dynamic model is simple hence it is easy to get analytical solution. Therefore, it is quite straightforward to design walking gait. However, since the model is too simple it may cause problem when controlling the real biped whose dynamics is not fully represented by the simple model. Kajita *et al.*[5] proposed using the ankle torque controller to compensate for the disturbance generated by swing leg dynamics. The ankle torque controller helps the biped to realize the desired walking motion generated using the LIPM model. However, the required ankle torque may need to be very large when the effect of swing leg dynamics is significant.

Park *et al.*[9] improved the LIPM model by introducing the term Gravity-Compensated Inverted Pendulum Mode (GCIPM). Their approach takes into account the gravity of the swing leg to generate biped locomotion pattern. The walking trajectory generated by this approach is more stable than that of the LIPM model. However, this method also has its own limitation since only gravity term of the swing leg is taken into account while the inertia term is not.

In this study, we further improve the LIPM model by proposing a new model called Augmented Linear Inverted Pendulum (ALIP). An augmented function F is added to the dynamic equation of the Linear Inverted Pendulum. The role of the augmented function is to adjust the inverted pendulum dynamics such that the disturbance caused by the un-modeled dynamics (legs and arms) is minimized. The inverted pendulum dynamics can be easily adjusted or modified by manipulating the key parameters of the augmented function. Our objective is to design a walking pattern that has the highest stability margin possible. In this study, the Zero-Moment-Point (ZMP)[12] is used as a stability criterion for dynamic walking. The desired walking motion with maximized stability margin is achieved by optimizing the key parameters of the augmented function using the genetic algorithm (GA). The advantage of this method over the above mentioned methods (LIPM and GCIPM) is that the disturbance caused by the un-modeled dynamics (swing legs, arms, etc.) is minimized during the optimization process. When the disturbance caused by un-modeled dynamics is minimized, better walking gait can be achieved. Simulation results show that the trajectory generated by our proposed method is more stable than that by the LIPM model and GCIPM model.

II. AUGMENTED LINEAR INVERTED PENDULUM

The Linear Inverted Pendulum Mode (LIPM) was proposed by Kajita *et al.*[7] in 1991. In this study, bipedal robot is modeled as an inverted pendulum in which total mass of the robot is concentrated into a point mass (see Fig. 1). The center of gravity (COG) is constrained to move on a horizontal plane. The dynamic equation of motion of the inverted pendulum is described as follows:

$$\ddot{x} = \frac{g}{z_o}x + \frac{\tau}{mz_o} \quad (1)$$

where x is the COG position, z_o is the constant height of the inverted pendulum, g is the gravity acceleration and τ is the ankle torque.

To design the nominal walking motion for bipedal robot, Kajita *et al.* ([6], [5], [7], [8]) assumed that the ankle torque is zero ($\tau = 0$). The dynamic equation becomes

$$\ddot{x} = \frac{g}{z_o}x \quad (2)$$

Fig. 2 shows an example of reference hip trajectory generated using the LIPM equation (2) with step length $S = 0.3$ m, step time $T = 1$ s. Ideally, if there is no disturbance (caused by the un-modeled dynamics such as legs or arms, foot impact and other external forces) this reference motion can be perfectly tracked and the zero-moment-point (ZMP) stays exactly at the ankle joint position. However, in most cases, at least one of these disturbances is present. These disturbances may cause the resulting motion to deviate from the reference motion.

In order to reduce or compensate for the disturbances caused by un-modeled dynamics (legs and arms), there are two possible solutions: i) Use the ankle torque to force the robot to follow the reference motion[5], [7]; and ii) Modify the reference motion of the center of mass (COM) so that the disturbing effect caused by un-modeled dynamics is minimized. The first solution works well when the disturbance caused by the un-modeled dynamics is small. However, when the effect of the un-modeled dynamics become too big, the reference motion may not be realized because the ankle torque required exceeds the acceptable limit. Whereas, if we can somehow modify the COM reference motion (second solution) in such a way that it is in harmony with legs and arms motions, a better walking behavior can be achieved. By "harmony" we mean that the motions of arms and legs have very little or no disturbing effect on the reference motion.

In this study, the determination of whether one trajectory is better than the other is based on the stability margin criterion. A trajectory is better than another one if it has larger stability margin. And by this way, we define that a trajectory having larger stability margin means that the disturbing effect on the reference motion caused by arms and legs dynamics is less.

One way to modify the reference trajectory is to modify the dynamic equation (2). In this study, we propose to modify the dynamic equation (2) by adding an augmented function F to the right hand side of the equation. The dynamic equation is as follows:

$$\ddot{x} = \frac{g}{z_o}x + F \quad (3)$$

where the augmented function F has the following characteristics:

• *F is continuous and is able to make gradual change to the dynamics of the inverted pendulum model in (2).*

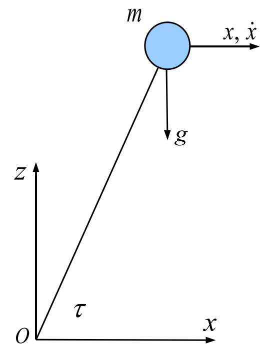

Fig. 1. The inverted pendulum model of humanoid robot.

• *F must satisfy the condition that (3) can be solved analytically.*

• *F should be as simple as possible.*

• *The value of F can be changed by changing some key parameters.*

The purpose of adding the function F to the inverted pendulum equation is to give us the ability to change or modify the dynamics equation. There can be many choices for the function F that has the above mentioned characteristics.

It can be seen that the dynamic equation (2) of the Linear Inverted Pendulum Mode is a special case of the second-order ordinary differential equation

$$a\ddot{x} + b\dot{x} + cx = 0 \quad (4)$$

where $a = 1$, $b = 0$, $c = -g/z_0$.

Equation (2) is the mathematical representation of the LIPM, a highly simplified model of bipedal walking robots. We suspect that Equation (4), a more general mathematical representation compared to (2), might be richer in representing the dynamics of bipedal walking. Therefore, we propose to choose the augmented function F to be

$$F = k_p x + k_v \dot{x} \quad (5)$$

where k_p and k_v are the constant parameters.

Substitute (5) into (3), we have

$$\ddot{x} = \frac{g}{z_o}x + k_p x + k_v \dot{x} \quad (6)$$

We call the dynamic model described by (6) the Augmented Linear Inverted Pendulum (ALIP).

Equation (6) can be re-written as follows:

$$\ddot{x} + b\dot{x} + cx = 0 \quad (7)$$

where $b = -k_v$, $c = -k_p - g/z_0$.

Solving the second-order linear differential equation (6), we have the following cases:

• If $b^2 - 4c > 0$:

$$x(t) = \frac{x(0)r_2 - \dot{x}(0)}{r_2 - r_1}e^{r_1 t} + \frac{\dot{x}(0) - x(0)r_1}{r_2 - r_1}e^{r_2 t} \quad (8)$$

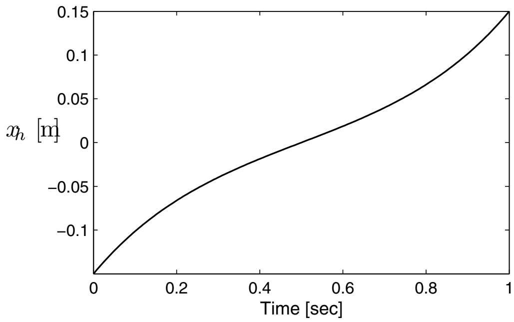

Fig. 2. A sample hip trajectory generated using the LIPM model (Eq. 2) where Step length S = 0.3 m, Step time T = 1 s.

where $x(0)$, $\dot{x}(0)$ are initial position and velocity conditions, respectively. r_1, r_2 are real roots of the auxiliary equation and are determined as below:

$$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4c}}{2} \tag{9}$$

$$\dot{x}(t) = \frac{x(0)r_2 - \dot{x}(0)}{r_2 - r_1} r_1 e^{r_1 t} + \frac{\dot{x}(0) - x(0)r_1}{r_2 - r_1} r_2 e^{r_2 t} \tag{10}$$

Since r_1, r_2 are functions of k_p and k_v, $x(t)$ is also function of k_p and k_v.

- If $b^2 - 4c = 0$:

$$x = [\dot{x}(0) - rx(0)]te^{rt} + x(0)e^{rt} \tag{11}$$

$$\dot{x} = [\dot{x}(0) - rx(0)](1 + tr)e^{rt} + x(0)re^{rt} \tag{12}$$

where $r = -b/2$.

- If $b^2 - 4c < 0$:

$$x = [x(0)\cos(\beta t) + \frac{\dot{x}(0) - x(0)\alpha}{\beta}\sin(\beta t)]e^{\alpha t} \tag{13}$$

$$\dot{x} = \dot{x}_0 \cos(\beta t)e^{\alpha t} + \frac{\alpha \dot{x}_0 - (\alpha^2 + \beta^2)x_0}{\beta}\sin(\beta t)e^{\alpha t} \tag{14}$$

where $\alpha = -b/2$, $\beta = \sqrt{4c - b^2}/2$.

In summary,

$$x(t) = \begin{cases} \frac{x_0 r_2 - \dot{x}_0}{r_2 - r_1} e^{r_1 t} + \frac{\dot{x}_0 - x_0 r_1}{r_2 - r_1} e^{r_2 t}, & \text{if } \Delta > 0 \\ [\dot{x}_0 - r x_0]te^{rt} + x_0 e^{rt}, & \text{if } \Delta = 0 \\ [x_0 \cos(\beta t) + \frac{\dot{x}_0 - x_0 \alpha}{\beta}\sin(\beta t)]e^{\alpha t}, & \text{if } \Delta < 0 \end{cases} \tag{15}$$

$$\dot{x}(t) = \begin{cases} \frac{x_0 r_2 - \dot{x}_0}{r_2 - r_1} r_1 e^{r_1 t} + \frac{\dot{x}_0 - x_0 r_1}{r_2 - r_1} r_2 e^{r_2 t}, & \text{if } \Delta > 0 \\ [\dot{x}_0 - r x_0](1 + tr)e^{rt} + x_0 r e^{rt}, & \text{if } \Delta = 0 \\ \dot{x}_0 \cos(\beta t)e^{\alpha t} + \frac{\alpha \dot{x}_0 - (\alpha^2 + \beta^2)x_0}{\beta}\sin(\beta t)e^{\alpha t}, & \text{if } \Delta < 0 \end{cases} \tag{16}$$

where $b = -k_v$, $c = -k_p - \frac{g}{z_o}$ and $\Delta = b^2 - 4c$.

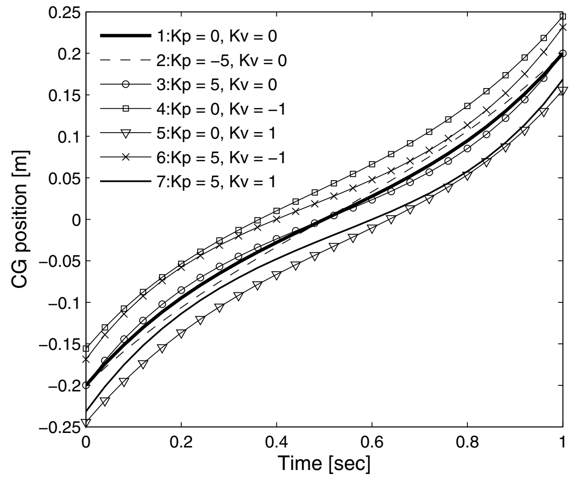

Fig. 3. Some sample trajectories generated using equations (15) and (16). The trajectories are numbered in sequence from 1 to 7 and each trajectory corresponds to a set value of k_p and k_v. When $k_p = 0$, $k_v = 0$ (trajectory 1 - the thick solid curve), the trajectory generated using our proposed approach will be the same as that generated using Kajita's method (LIPM). It can be seen that, the effect of k_p is to change the degree of curvature of the trajectory (see curve 2 and 3). Whereas, the effect of k_v is to offset the trajectory vertically (curve 4 and 5). .

Equations (15) and (16) will be used to plan reference trajectory for humanoid robot. Fig. 3 shows some sample trajectories generated using these equations when different values of k_p and k_v were used.

The proposed augmented function F satisfies all the required characteristics because: 1. F is able to make gradual change to the inverted pendulum dynamics; 2. Equation (3) can be solved analytically; 3. The function F is simple; 4. F can be changed by changing the key parameters k_p and k_v.

III. Trajectory Planning

To design walking gait for biped robots, we need to plan foot trajectory of the swing leg and hip trajectory. In this study, we use the same way of planning foot trajectory as in [9] (for comparison purpose later). Equations (15) and (16) will be used to plan the hip trajectory of the walking gait. $x(t)$ and $\dot{x}(t)$ are functions of time and the parameters k_p and k_v. Different choice of (k_p, k_v) results in different hip trajectory. Our objective is to find the optimal value of k_p and k_v such that the resulting walking gait has highest possible stability margin. Stability margin is defined as the shortest distance from the ZMP trajectory to the edges of the supporting foot polygon. This means that the maximum stability margin is achieved when the ZMP is located exactly at the middle of the foot polygon.

Let us assume that the robot is repeating single support phase of duration T and double support phase is instantaneous. Figure 4 illustrates one walking step in X-direction. At the beginning of the step ($t = 0$), the initial horizontal position of robot body is x_o. At the end of the step ($t = T$), the horizontal position of the robot body is x_f. S denotes

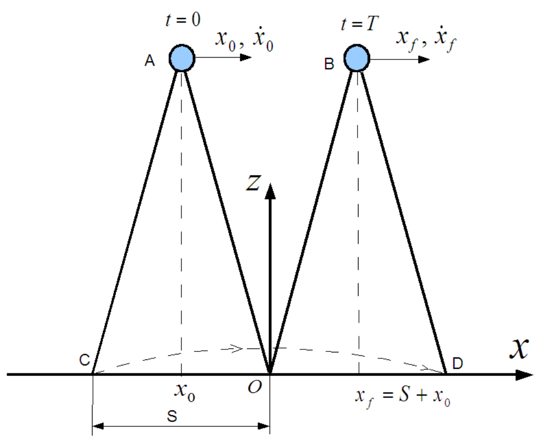

Fig. 4. *One walking step in the sagittal plane is illustrated. The body travels from A to B in the single support phase. While the body moves from A to B, the tip of the swing leg travels from C to D. x_o and x_f are the positions of the body at time $t = 0$ and $t = T$, respectively.*

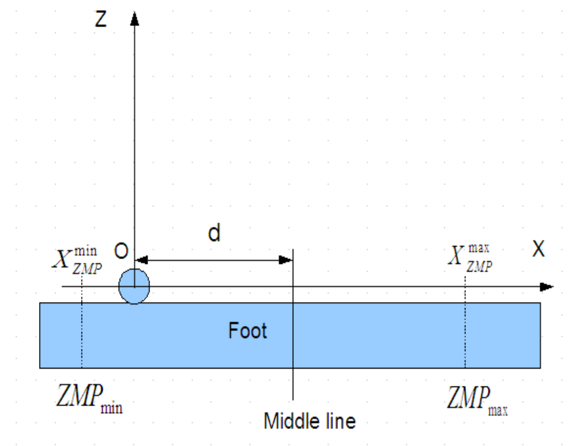

Fig. 5. *The supporting foot is shown. The origin O of the coordinate system is placed at the ankle joint.*

the walking step length. The origin O of the coordination system is placed at the ankle of the supporting foot.

The hip trajectory is computed using equations (15) and (16) together with the following conditions:

$$\begin{cases} x(0) = x_o \\ x(T) = x_f \\ \dot{x}(0) = \dot{x}(T) \quad \text{(velocity continuity condition)} \end{cases} \tag{17}$$

Genetic algorithm (GA)[11] is used to find the optimal value of k_p, k_v with the objective to maximize the stability margin. The cost function is described as follows:

$$CF = Max\{\left|x_{zmp}^{min} - d\right|, \left|x_{zmp}^{max} - d\right|\} \tag{18}$$

where CF is the cost function; d is the horizontal distance measured from the ankle joint to the middle point of the foot; x_{zmp}^{min}, x_{zmp}^{max} are the minimum and maximum ZMP position, respectively. *Note that in this study the ZMP is computed based on the full dynamics of the real biped. Therefore, we can make sure that swing leg and arms dynamics are included in the trajectory planning process which helps to minimize the disturbing effects of arms and legs.* The origin O of the coordinate system is placed at the ankle joint as shown in Fig. 5.

The fitness function is

$$FF = \frac{1}{CF} = \frac{1}{Max\{\left|x_{zmp}^{min} - d\right|, \left|x_{zmp}^{max} - d\right|\}} \tag{19}$$

GA will search for k_p, k_v that maximize the fitness function FF. When FF is maximized the stability margin will be maximized as well. Note that the optimal values of k_p and k_v may be different for different swing leg motions.

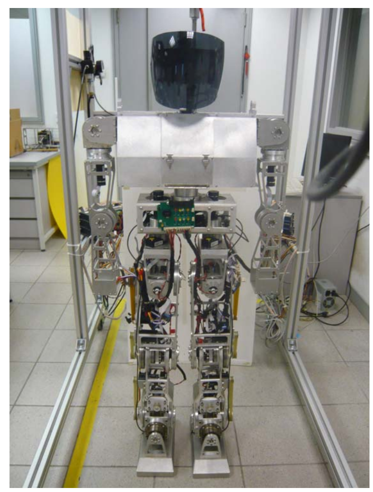

Fig. 6. *Picture of NUSBIP-III*

IV. Simulation Results

The specifications of the simulated biped was taken from a real biped, which was named NUSBIP-III and developed in our Legged Locomotion laboratory. Table I summarizes the specifications of the biped robot NUSBIP-III. A picture of NUSBIP-III is depicted in Fig. 6. In this study, we used Yobotics (http://yobotics.com/), a dynamic simulator, to simulate bipedal walking motion.

In this simulation, for no particular reason, the inputs were chosen as follows. The time for one walking step is $T = 0.8$ s, and the corresponding step length is $S = 0.3$ m. Genetic

TABLE I
Specifications of NUSBIP-II.

Length	Thigh		Shank		Foot Length		
(m)	0.32		0.32		0.21		
Weight	m_1	m_2	m_3	m_4	m_5	m_6	m_7
(kg)	1.52	1.68	2.68	10.45	2.68	1.68	1.52
Inertia	I_{1y}	I_{2y}	I_{3y}	I_{4y}	I_{5y}	I_{6y}	I_{7y}
(kgm^2)	0.1040	0.2739	0.3458	0.2426	0.3458	0.2739	0.1040

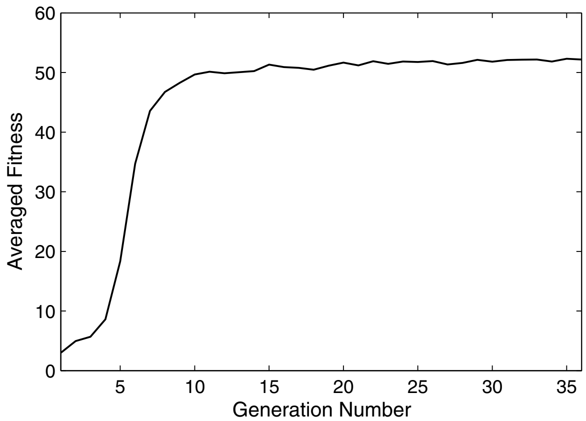

Fig. 7. *Averaged Fitness value of each generation is shown. It can be seen from the figure that GA converged after 50 generations.*

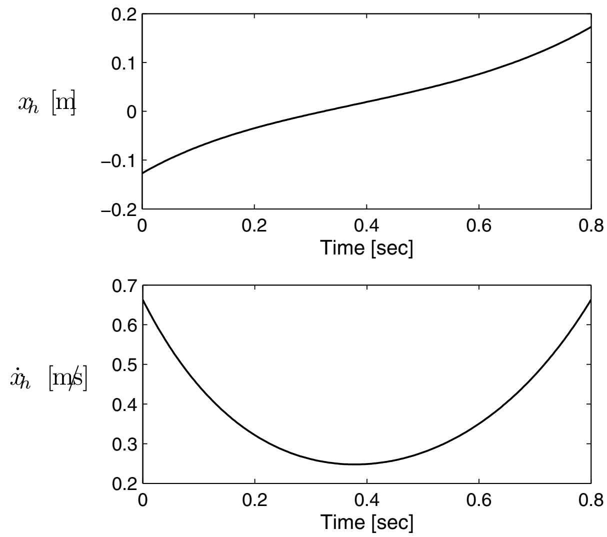

Fig. 8. *The resulting optimal hip trajectory is shown. The upper graph shows the position trajectory x_h of the hip while the lower graph shows the velocity trajectory.*

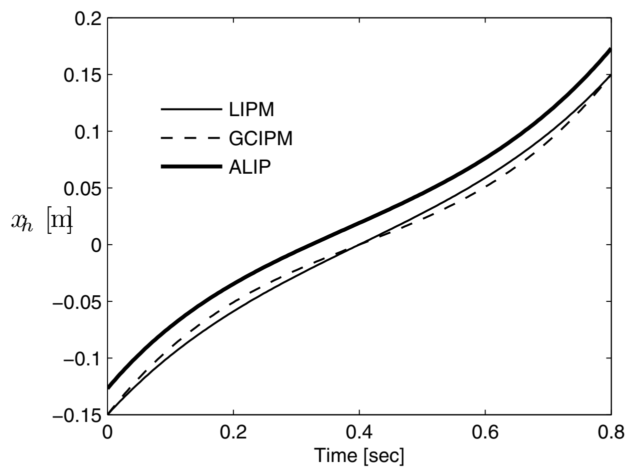

Fig. 9. *Hip trajectories obtained using different models*

algorithm (GA)[11] was used to determine the optimal value of k_p and k_v. The GA's parameters were chosen as follows: Number of Generations is 100, Population Number is 3000, crossover rate is 0.8 and mutation rate is 0.02.

In order for GA to start the optimization process, we need to define the ranges for the variables k_p and k_v. After some simple checks we found that when $k_p < -100$ or $k_p > 100$ the obtained trajectory is unstable (the ZMP stays outside the stable region) for any value of k_v. When $k_v < -30$ or $k_v > 30$ the obtained trajectory is always unstable for any value of k_p. Therefore, we select the ranges for k_p and k_v as follows: $-100 \leq k_p \leq 100$ and $-30 \leq k_v \leq 30$.

Figure 7 shows that GA converged after 30 generations. The converged value of k_p and k_v are: $k_p = 7.093$, $k_v = -0.94$. Since k_p and k_v were determined, the hip trajectory can be computed using (15), (16) and boundary conditions (17). Fig. 8 shows the resulting hip trajectory obtained by the proposed method (ALIP). The difference between the hip trajectory obtained using ALIP model and the other two models (LIPM and GCIPM) is illustrated in Fig. 9.

Fig. 10 shows the ZMP trajectories of the simulated biped in one walking step. The thick solid curve is the ZMP trajectory obtained using ALIP model while the dashed-curve is the ZMP trajectory obtained using GCIPM model and the thin solid curve is the ZMP trajectory obtained using LIPM model. The horizontal thick lines are the Foot Toe and Foot Heel which are the bounds of the stability region. $S1$, $S2$ and $S3$ are the stability margin of the walking trajectory generated using LIPM model, GCIPM model and ALIP model, respectively. Stability margin is defined as the shortest distance from a point on the ZMP trajectory to either the Heel or Toe of the supporting foot. From the figure it can be seen that $S3 \approx 12$ cm is much larger than $S1 \approx 1.35$ cm and $S2 \approx 6$ cm. Therefore, the walking gait generated using ALIP model is significantly more stable than that generated using the LIPM model and the GCIPM model.

Fig. 11 shows a comparison of the torque needed to apply at the ankle joint when different models are used. It can be seen from the figure that the ankle torque required for the ALIP model is about one half smaller compared to the ankle torque required for the LIPM model. Compared with the GCIPM model, the ankle torque required for the ALIP model is also smaller. Fig. 12 shows the stick diagram of the simulated walking motion.

V. CONCLUSION

In this study, we proposed the Augmented Inverted Pendulum (ALIP) model to generate walking patterns. In this method, the disturbance caused by the difference between mathematical model and real robots is minimized by incorporating an augmented function into the dynamic equation. Simulation results show that the walking pattern generated using ALIP model is more stable than that generated using LIPM and GCIPM models. Furthermore, the ankle torque required by the ALIP model is significantly smaller (about

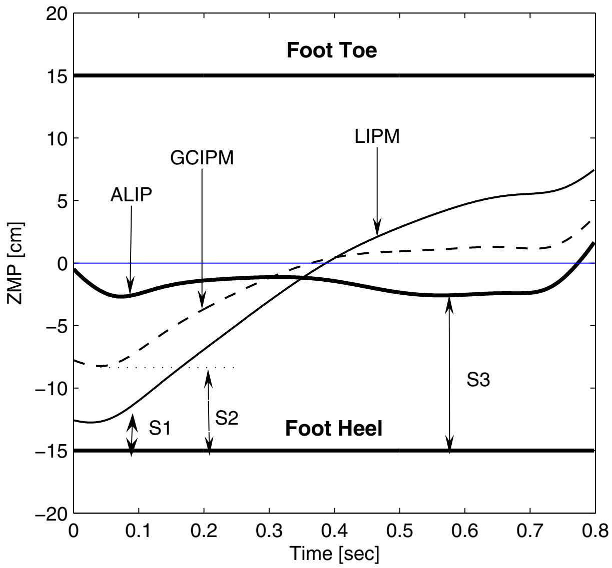

Fig. 10. *ZMP trajectories of one walking step are shown. The thick solid curve shows the ZMP trajectory in one walking step ($T = 0.8$ s) when ALIP model is used. While the thin solid curve shows the ZMP trajectory of the robot when LIPM model is used. And the dashed-curve shows the ZMP trajectory obtained using the GCIPM model. The thick horizontal lines are indications of Toe and Heel of the supporting foot. $S1$, $S2$ and $S3$ are the stability margins of the LIPM, GCIPM and ALIP models, respectively.*

one half) than that by the LIPM model. ALIP model is therefore a meaningful improvement of the LIPM and GCIPM model while still keeping the simplicity of the method. The significance of this proposed method is that although simple modeling was used, full dynamics of the robot was taken into account in the optimization process which resulted in better walking gait. The ALIP model can be used to design online walking gait. The result of online walking simulation will be reported in our future publications.

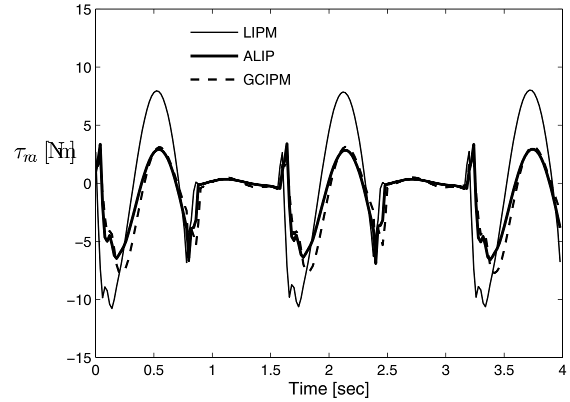

Fig. 11. *Comparison of the ankle torque generated by 3 different models. The thick solid curve represents the ankle torque generated by ALIP model, the dashed-curve represents the ankle torque generated by GCIPM model and the thin solid curve represents the ankle torque generated by LIPM model.*

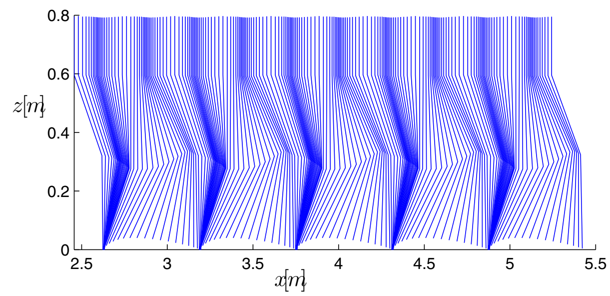

Fig. 12. *Stick diagram of the simulated biped in five walking steps is shown. Only the right leg is shown and the image is captured at 0.02 s apart.*

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