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# Energy Efficient Trajectory Generation for a State-Space Based JPL Aerobot

Weizhong Zhang, Tamer Inanc and Alberto Elfes

*Abstract*— **The 40th anniversary of Apollo 11 project with man landing on the moon reminds the world again by what science and engineering can do if the man is determined to do. However, a huge step can only be achieved step by step which may be relatively small at the beginning. Robotic exploration can provide necessary information needed to do the further step safely, with less cost, more conveniently. Trajectory generation for a robotic vehicle is an essential part of the total mission planning. To save energy by exploiting possible resources such as wind will assist a robotic explorer extend its life span and perform tasks more reliably. In this paper, we propose to utilize Nonlinear Trajectory Generation (NTG) methodology to generate energy efficient trajectores for the JPL Aerobot by exploiting wind. The Aerobot model is decoupled into longitudinal and lateral dynamics with control inputs as elevator deflection $\delta_e$, thrust demand $\delta_T$, vectoring angle $\delta_v$ for the longitudinal motion, aileron deflection $\delta_a$, rudder deflection $\delta_r$ for the lateral motion. The outputs are the velocities and orientation of the Aerobot. The Aerobot state space model parameters are obtained from experimental identification on AURORA Airship since the actual JPL Aerobot is similar to the AURORA Airship. In this paper, the results show that with the state-space model, the proposed trajectory generation method can guide the Aerobot to take advantage of previously known wind profile to generate an energy-efficient trajectory.**

## I. INTRODUCTION

The outside far beyond our own planet is so enticing that can invoke so many imaginations, and make us think more about where we are truly come from. Maybe not all questions can be answered, but one thing is for certain, that is, humans can know more and more about surroundings and beyond. Robotic explorers such as ground mobile robots are convenient tools for assisting scientists and engineers to achieve such objectives. However, the main drawback of the current ground-based robotic planetary vehicles, such as Mars exploration rovers, is their limited range. The 2003 and 2006 Solar System Exploration Roadmap (SSE) [2] [3] indicate that aerial platforms will be required to explore planets and moons with atmosphere such as Mars, Venus and Titan.

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For aerial robotic planetary exploration, some aerial vehicles such as airplanes, gliders, helicopters, balloons [4] and airships [5][6][7][8] have been considered. Airplanes and helicopters require significant energy to just stay airborne, flight time of gliders depend mainly on wind, while balloons have limited navigation capabilities. Lighter-Than-Air (LTA) vehicles combine long term mission capability and low energy requirement of balloons with the maneuverability of airplanes. LTA systems, a.k.a. Aerobots or Robotic Airships, bring a new opportunity for robotic exploration of planets and their moons which have atmosphere. Aerobots can provide, due to their controllability, precise flight path execution for surveying, station-keeping for extended monitoring high-value science sites, long-range as well as near surface observations, and transportation of scientific equipments. They also are able to execute extensive surveys over solid as well as liquid-covered terrains, and reach essentially any point of the planet over multi-month time scales with minimal consumption of limited onboard energy sources. By taking avantage of suitable wind velocity [13], the aerobot can expand their range by less energy. As opportunistic trajectory generation methodology goes, the aerobot can go in a sense of energy efficient way.

Nonlinear Trajectory Generation (NTG) [14], developed at Caltech, is the state-of-the-art methodology to generate optimal trajectory in real-time for mechanical systems. The main advantage of NTG compared to other dynamic optimization methods is that it can quickly provide optimal or sub-optimal solutions, which makes it very useful for real-time applications. In addition, linear as well as nonlinear constraints and cost functions can be included in the problem formulation of NTG. The general NTG framework can handle both spatial and temporal constraints. NTG is based on a combination of nonlinear control theory, spline theory and sequential quadratic programming. NTG takes the optimal control problem formulation, characterization of trajectory space, and the set of collocation points, and transforms them into a Nonlinear Programming (NLP) problem. Transformed NLP problem is then solved using NPSOL [15], a popular NLP solver, which uses Sequential Quadratic Programming (SQP). It has been successfully applied for real-time trajectory generation of UAVs (Unmanned Air Vehicles) under the DARPA-MICA (Mixed Initiative Control of Automatic Teams) program [17] and for real-time trajectory generation of underwater gliders for the AOSN (Autonomous Oceanographic Sampling Network) project [18] [19].

In this paper, we propose to utilize this methodology, to generate energy efficient trajectory for the JPL Aerobot

Fig. 1. A JPL Aerobot [5].

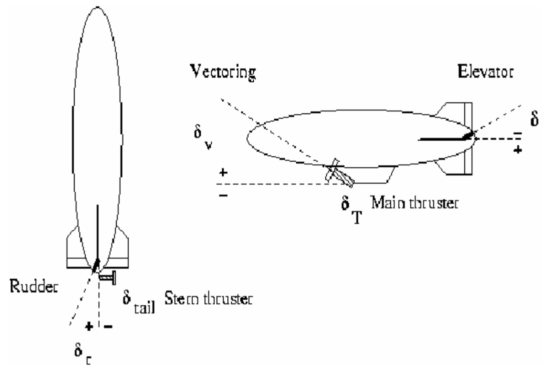


Fig. 2. The state-space based Aerobot controls [10]

with the model of AURORA (Autonomous Unmanned Remote Monitoring Robotic Airship) and previous known wind profile. The obtained minimizing-energy 3D trajectory in simulation is shown to save significant energy by taking advantage of previously known wind velocity in the field. The state space model is decoupled into longitudinal and lateral motions of equations for control purposes.

## II. JPL Aerobot Project

To develop autonomous robotic airships to explore planets and moons which have atmosphere, is one of NASA JPL Aerobot project objectives. Taking advantage of available wind velocity, the Aerobot can overcome many obstacles a ground vehicle can meet, and has the advantage of flying over difficult terrains to enter caverns and explore them with less energy consumption.

For a relative long period of time with limited energy on board, the Aerobot requires careful management of power because several other kinds of activities require energy consumption, such as scientific data gathering, surface sampling, and communications with Earth and/or with an orbiter. The Aerobot can use some possible external energy sources such as the solar power, however, for some planets like Titan, the Sun is blocked by its atmosphere. On the other hand, the atmosphere brings an opportunity of utilizing wind as energy source. Therefore, energy efficient trajectory generation algorithm is aimed to take advantage of available wind patterns to minimize energy consumption [13].

Wind patterns of some planets such as Mars are known to some degree through observations of previous space missions and atmospheric modeling. The NASA-JPL Aerobot has also an ultrasonic anemometer [23] which provides estimates of the 3D relative airspeed vector of the Aerobot. To generate opportunistic trajectory for the Aerobot, the model of Aerobot needs to be known in advance. In this paper, the decoupled dynamical model of an Aerobot is utilized. In this paper, the proposed NTG algorithm is shown to generate energy efficient trajectories for the Aerobot model. This trajectories are shown to take advantage of wind velocities even with the realistic complex state space Aerobot model.

## III. Problem Definition

As a type of optimal control problem, to generate energy efficient trajectory for the Aerobot with Nonlinear Trajectory Generation (NTG) [14] algorithm, the cost function and contraints are listed as in the following:

$$J = \Phi_0(q(t_0), f(t_0), t_0) + \int_{t_0}^{t_f} \Phi_t(q(t), f(t), t)dt + \Phi_f(q(t_f), f(t_f), t_f) \tag{1}$$

$$\begin{array}{lccccc} \text{Initial} & lb_0 & \leq & \Psi_0(q(t_0), \mathbf{f}(t_0), t_0) & \leq & ub_0 \\ \text{Trajectory} & lb_t & \leq & \Psi_t(q(t), \mathbf{f}(t), t) & \leq & ub_t \\ \text{Final} & lb_f & \leq & \Psi_f(q(t_f), \mathbf{f}(t_f), t_f) & \leq & ub_f \end{array} \tag{2}$$

where $q(t)$ is the state of the system and $f(t)$ is the control input. The cost function $J$ is composed of an initial condition cost $\Phi_0(.)$, an integral cost over the trajectory, $\Phi_t(.)$, and a final condition cost, $\Phi_f(.)$. $lb$ and $ub$ are lower and upper bounds for the constraint functions. $t_0$, and $t_f$ is the initial and final time, respectively.

## IV. State Space Model

As the Aerobot is modeled as a state-space model, it is adapted from AURORA (Autonomous Unmanned Remote Monitoring Robotic Airship) project [12] since the JPL Aerobot is similar to this Airship in the dynamics.

The state-space model is decoupled into longitudinal and lateral motions. The control inputs as elevator deflection $\delta_e$, thrust demand $\delta_T$, vectoring angle $\delta_v$ for the longitudinal motion, and aileron deflection $\delta_a$, rudder deflection $\delta_r$ for the lateral motion. The outputs are the velocities and orientation of the airship. The airship control inputs and their positive references are shown in Fig. 2.

The Aerobot is moving from the point $q_1 = 0, q_2 = 0, q_3 = 0$ to $q_1 = 200, q_2 = 200, q_3 = 200$, which are the coordinates in the Cartesian system. Still, the following assumptions are made:. The linearized state-space model is obtained from nonlinear dynamic equation of the airship given by [10], resulting into decoupled longitudinal and lateral motions. For the longitudinal motion, the output vector is

$$X_v(t) = [u, w, p, \theta] \tag{3}$$

where $u$ is the longitudinal component of the airship absolute speed which is relative to the air, $w$ its vertical component, $p$

is the pitch rate and $\theta$ is the pitch angle. The control vector for the longitudinal motion is

$$U_v(t) = [\delta_e, \delta_T, \delta_v] \quad (4)$$

where $\delta_e$ is the elevator deflection, $\delta_T$ is the thrust demand and $\delta_v$ is the vectoring angle. The equation of longitudinal motion is listed as

$$\dot{X}_v = A_v X_v(t) + B_v U_v(t) \quad (5)$$

where $A_v$ and $B_v$ are numerically linearized system matrices [10] as

$$A_v = \begin{bmatrix} -0.1569 & -0.0651 & 1.8059 & -0.4522 \\ -0.0965 & -0.6300 & 8.0737 & -1.3584 \\ 0.0178 & -0.1059 & -3.7053 & -0.8279 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_v = \begin{bmatrix} 1.83876 & 0.0531 \\ -1.5921 & -0.0003 \\ -1.1832 & 0.0074 \\ 0 & 0 \end{bmatrix}$$

For its lateral motion, the output vector is

$$X_h(t) = [v, s, r, \alpha] \quad (6)$$

where $v$ is the lateral component of the airship absolute velocity, $s$ and $r$ are the roll and yaw rates, $\alpha$ is the roll angle. The control vector is given by

$$U_h(t) = [\delta_a, \delta_r] \quad (7)$$

where $\delta_a$ is the aileron deflection, $\delta_r$ is the rudder deflection. Its lateral motion of equation is presented as

$$\dot{X}_h(t) = A_h X_h(t) + B_h U_h(t) \quad (8)$$

where $A_h$ and $B_h$ are numerically linearized matrices from [11] as

$$A_h = \begin{bmatrix} 0.0378 & 0.4037 & 1.8059 & -2.5864 \\ 1.5641 & -0.6429 & 8.0737 & -6.3747 \\ -0.4161 & -1.4674 & -6.2235 & -0.0225 \\ 0 & 1 & 0.0913 & 0 \end{bmatrix}$$

$$B_h = \begin{bmatrix} -7.1360 & 4.5273 \\ -13.4035 & 3.07573 \\ -0.2389 & -2.9211 \\ 0 & 0 \end{bmatrix}$$

For the state-space model, the wind profile is modeled as (9), the flying area is restricted in the cube from $(0,0,0)$ to $(300,300,300)$ with the start point as $(0,0,0)$ and the final point as $(200,200,200)$. Assuming the wind profile is layered horizontally, no upward or downward wind exists. The wind velocity vectors at each layer are considered as known.

$$(windu, windv) = \begin{cases} (-10,10), & \text{for } z \subseteq (0,50) \\ (10,-8), & \text{for } z \subseteq (50,100) \\ (5,8), & \text{for } z \subseteq (100,150) \\ (10,-10), & \text{for } z \subseteq (150,300) \end{cases} \quad (9)$$

where $z$ is the coordinate in the vertical direction. In Fig. 3, the $x, y, z$ are the coordinates of the system, respectively represent $q_1$, $q_2$, and $q_3$.

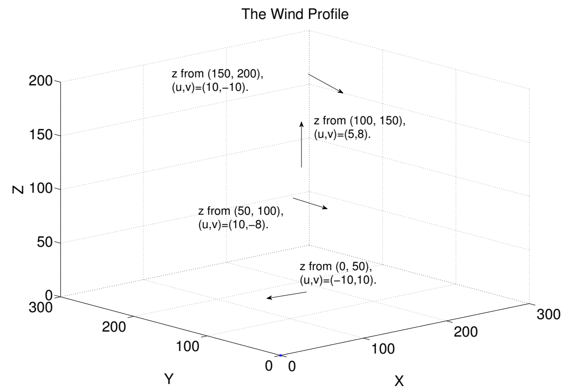


Fig. 3. The wind profile for the state-space based Aerobot.

### A. Problem Formulation

The cost function and constraints are listed in the following. The cost function $J$ is

$$J = W_t(t_f - t_0) + W_u \int_{t_0}^{t_f} ((\dot{x} - windu)^2 + (\dot{y} - windv)^2 + (\dot{z})^2) dt \quad (10)$$

where $W_t$, $W_u$ are the weights. For minimizing time trajectory, $W_t$ is equal to 1000, while $W_u$ are both 0. For minimizing energy only trajectory, $W_t$ is set to be 0, while $W_u$ is set to be 10. $t_f$ is the unknown final time for the trajectory. The constraints:

- (Linear) Initial Constraints:

$$0 - \varepsilon \le q_1(t_0) \le 0 + \varepsilon$$
$$0 - \varepsilon \le q_2(t_0) \le 0 + \varepsilon$$
$$0 - \varepsilon \le q_3(t_0) \le 0 + \varepsilon$$
$$0 \le t_f - t_0 \le 200\, s$$

- (Linear) Final Constraints:

$$200 - \varepsilon \le q_1(t_f) \le 200 + \varepsilon$$
$$200 - \varepsilon \le q_2(t_f) \le 200 + \varepsilon$$
$$200 - \varepsilon \le q_3(t_f) \le 200 + \varepsilon$$

- (Linear) Trajectory Constraints:

$$0 - \varepsilon \le q_1(t) \le 300 + \varepsilon$$
$$0 - \varepsilon \le q_2(t) \le 300 + \varepsilon$$
$$0 - \varepsilon \le q_3(t) \le 300 + \varepsilon$$

- (Linear) Control Inputs Constraints:

$$-1 \le \delta_e \le$$
$$-100 \le \delta_T \le 100$$
$$-0.5 \le \delta_v \le 0.5$$
$$-1 \le \delta_a \le 1$$
$$-1 \le \delta_r \le 1$$

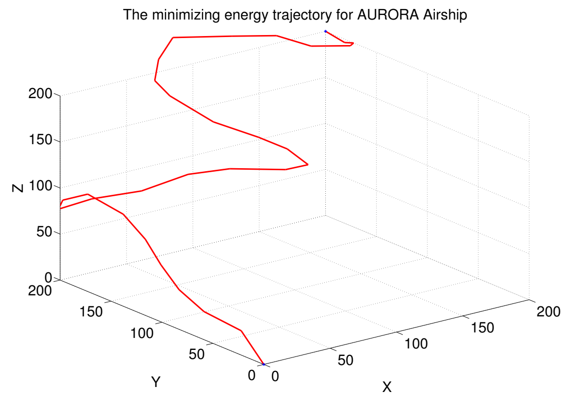


Fig. 4. The minimizing-energy trajectory for the state-space based Aerobot.

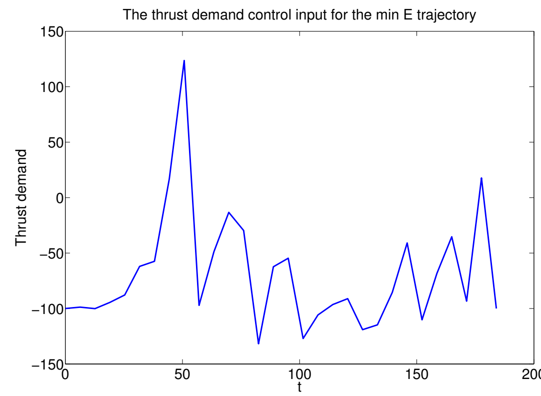


Fig. 6. The thrust demand $\delta_T$ for Fig. 4

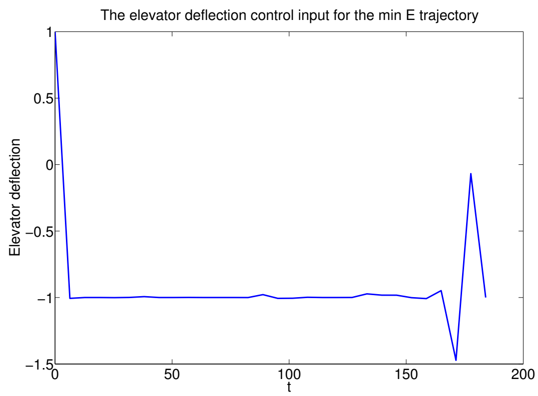


Fig. 5. The elevator deflection $\delta_e$ for Fig. 4

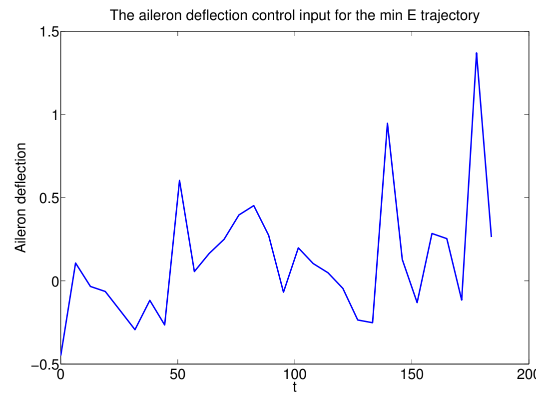


Fig. 7. The aileron deflection $\delta_a$ for Fig. 4

where $q_1(t_0)$, $q_2(t_0)$, $q_3(t_0)$, $q_1(t_f)$, $q_2(t_f)$, $q_3(t_f)$ are the initial and final location of the Aerobot. $q_1(t)$, $q_2(t)$ and $q_3(t)$ are the positions of the Aerobot in the trajectory. $\varepsilon$ is a small number.The other constraints are nonlinear constraints listed as (5) and (8).

### B. 3D Trajectories

When the Aerobot is modeled as (5) and (8), the wind profile is assumed to be known as in (9). NTG generated the minimizing-energy 3D trajectory in Fig. 4. The energy cost for this trajectory is $4.2297e3$, the final time is 183.97 seconds, the computation time is about 18 minutes. The longitudinal and lateral constraints make the computation time is as long as 18 minutes, which means that the trajectory has to be obtained by off-line with the available wind profile in advance. The control inputs elevator deflection $\delta_e$, thrust demand $\delta_T$, and aileron deflection $\delta_a$, rudder deflection $\delta_r$ are shown in the following Fig. 5 and Fig. 7. Fig. 5 shows that elevator deflection is quite small as the value from $-1.5$ to 1. While the thrust demand in Fig. 6 is changing from $-150$ to 150.

The aileron and rudder deflections shown in Fig. 7 and Fig. 8 are expectedly small in quantity in the trajectory.

The vectoring deflection $\delta_v$ is not shown here considering that it is not explicitly shown in the longitudinal and lateral dynamics constraints.

When the trajectory is trying to minimize the time, the trajectory is not going with the wind profile. It just go straight to the destination as it is shown in Fig. 9. For the minimizing-time trajectory, the final time is 100.30 seconds. The computation time of the NTG algorithm is 6 minutes. The energy cost is $4.6963e3$. The minimizing time trajectory control inputs elevator deflection $\delta_e$, thrust demand $\delta_T$, and aileron deflection $\delta_a$, rudder deflection $\delta_r$ are also shown in the following.

From Fig. 5 to Fig. 13, they show that deflections are comparably small and the thrust forces are changing around 0. Comparing Fig. 7 and Fig. 8 for minimizing energy trajectory with Fig. 12 and Fig. 13 for minimizing time trajectory, the minimizing time trajectory is a straight line, the aileron and rudder deflections are understandably almost unchanged while the ones in the minimizing energy trajctory are changing frequently in order to catch the wind velocity. The results show the method generated the minimizing energy trajectory as we expected.

The simulation platform is Ubuntu 7.10, Kernel Linux 2.6.22-14-386, Memory 2.0 GB, AMD Athlon(tm) $64 \times 2$ Dual Core Processor 3800+.

TABLE. I shows the trajectories generated by NTG for the modeled Aerobot are reasonable considering the minimizing-time trajectory is the straight line and the energy cost is larger

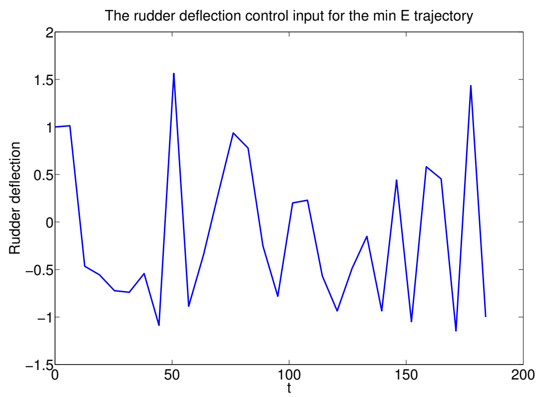


Fig. 8. The rudder deflection $\delta_r$ for Fig. 4

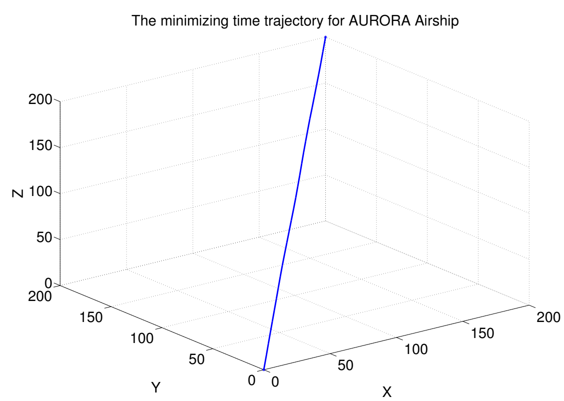


Fig. 9. The minimizing-time trajectory for the state-space based Aerobot

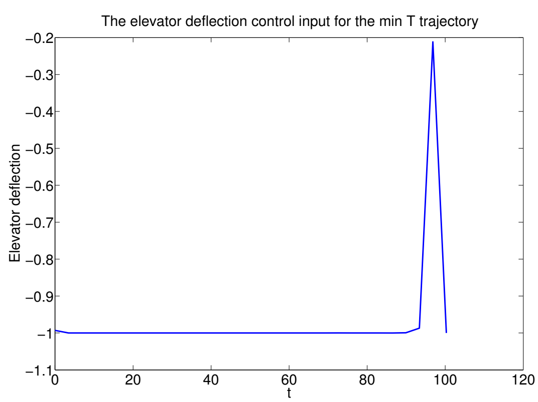


Fig. 10. The elevator deflection for Fig. 9

TABLE I

3D TRAJECTORIES GENERATED BY NTG FOR THE STATE-SPACE BASED AEROBOT

| *NTG* | $T_f(s)$ | *Time*$(m)$ | *Energy Cost*$(m^2/s)$ |
|---|---|---|---|
| *min E* | 183.97 | 18 | 4.2297$e$3 |
| *min T* | 100.29 | 5 | 4.6963$e$3 |

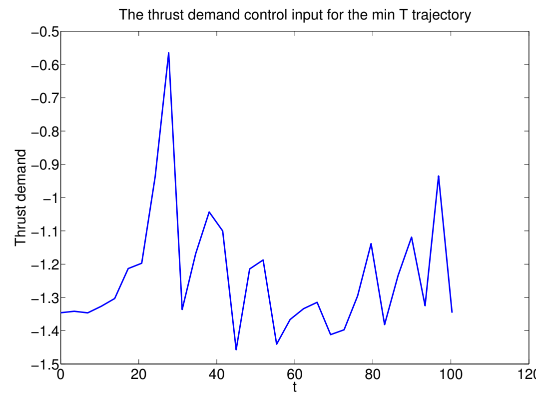


Fig. 11. The thrust demand $\delta_T$ for Fig. 9

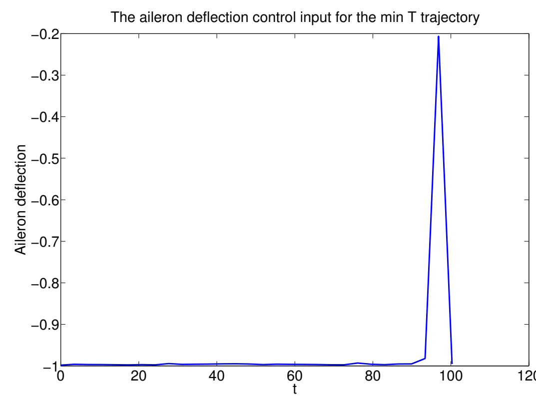


Fig. 12. The aileron deflection $\delta_a$ for Fig. 9

than the minimizing-energy trajectory.

## V. Conclusion

This paper proposes a framework to utilize NTG methodology to generate opportunistic 3D trajectory for the NASA-JPL Aerobot. The minimizing-energy trajectory use the less energy and more time than the minimizing-time trajectory. The energy efficient 3D trajectory generated for the Aerobot by NTG is promising for the future application with the available state space model.

## VI. ACKNOWLEDGMENTS

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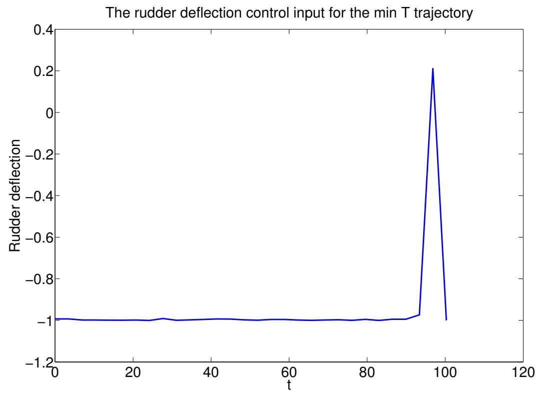


Fig. 13. The rudder deflection $\delta_r$ for Fig. 9


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