The 2010 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 18-22, 2010, Taipei, Taiwan

Comparative Analysis of the Repeatability Performance of a serial and parallel robot

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Abstract— **The paper proposes a new procedure to compare the repeatability of serial and parallel robots based on the stochastic ellipsoid theory. The ISO9283 position repeatability index is estimated but also other performance criteria built upon the stochastic ellipsoid geometrical characteristics. The choice of the best comparison criterion is investigated and different solutions are proposed, associated with the task specificity. For each criterion, maps are built to determine the set of workspace points where the serial robot is better than the parallel robot. The ratio of the workspace surface where one robot is better than the other is computed and the results are analysed. Contrary to the common opinion that parallel robots are more accurate than serial robots, we prove here that the repeatability performance depends mainly on the chosen performance criterion. Another result found is that the considered RRRRR parallel robot keeps the same repeatability in all its workspace.**
Keywords- **accuracy, repeatability, stochastic ellipsoids, industrial Robots.**

I. INTRODUCTION

Robots are more and more used in minute positioning industrial applications [1]. That encourages us to improve our knowledge on their precision and behavior on a microscopic scale.

When an industrial robot is to be used for a specific task, the question of the choice of the correct robot among a wide panel of robots is difficult to tackle because of the lack of information on the robot performance and the lack of modeling for the task description.

Most of the robot manufacturers will display the reach, the number of DOF, the workspace volume, the acceleration, the time cycle and the repeatability. But is it enough to choose the correct robot? Certainly not if the ISO9283 and ANSI R15 standards are to be considered [2, 3]. Many more performance indices are proposed but with their own pros and cons. A recent paper deals with the question of repeatability indices adequacy between the two norms and proposes a solution with the notion of “intrinsic repeatability” [4].

In the design stage of a robot, well described in [5], one of the first questions to answer to, is how to choose the robot topology. Is it better to have a serial, hybrid or parallel structure? Parallel robots suffer from some disadvantages especially their limited workspace, calibration difficulties, passive joint nonlinearities. But there is a common opinion that does not rely on scientific papers, which seems to consider parallel robots as more accurate than serial robots. Repeatability and accuracy depend not only on the quality of the robot control, but also and the topological structure and the sensors disposal. In [6], Corbel described the concept of actuation/measurement dissociation applied to robots. In [7], Briot and Bonev ask the question: "are parallel robots more accurate than serial robots?" In their introduction, they quote numerous papers or books giving many clues for better accuracy in favor of parallel robots. Indeed this opinion is shared and it is generally admitted that:

- Parallel robots offer more advantages than conventional machine tools, such as higher accuracy [8].
- Parallel manipulators are preferred to serial manipulators for their high positioning accuracy [9].
- A small measurement error of a serial robot in angular internal sensors quickly leads to a large error in the position of the end-effector while errors of the internal sensors of a parallel robot only slightly affect errors on the platform position [10].
- Compared to the traditional serial-chain mechanism, the parallel mechanism exhibits the following advantages: better accuracy due to non-cumulative joint error [11].
- The errors of parallel manipulators are averaged out in the parallel chains and the errors of serial manipulator are accumulated [12].

But repeatability modeling or experimental repeatability statistical work was not really performed to have a definitive position on this subject. This is the reason that motivated our research work. We want in this paper to test this hypothesis in the light of the stochastic ellipsoid method which is well-suited for repeatability modeling.

The robot precision errors depend on numerous factors, coming from the robot mechanism or the robot control. The traditional approach developed in the ISO or ANSI standard distinguishes accuracy and repeatability, and gives detailed procedures to estimate them.

To sum up, the robot has to perform a cycle 30 times coming from one position to a target where the global position and orientation coordinates of the end-effector are measured. Statistical estimators are then built: repeatability estimates the variability of the cloud points and accuracy is the distance between the mean of the points and the target.

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As precision performance indices vary in the workspace, the manufacturer must evaluate them in different locations of the robot workspace.

It is admitted that servocontroller errors are the main factor for repeatability error. On the contrary, errors in geometrical characteristics of the robot, leading to a incorrect forward kinematics function are the main factors for accuracy error [13]. Compliance, lost motion, gear backlash, load, workspace location are other influence factors for precision error, certainly all affecting both repeatability and accuracy to a certain point.

In previous works, we proved that for the Kuka IR384, the workspace location influence on repeatability was more important than the load [14]. In a recent paper, the influence factors for orientation repeatability of a Samsung robot are studied [15].

In section II, the methodology used in the comparison is clearly explicated. The choice of the two robots is detailed and the stochastic ellipsoid applied to parallel and serial robots. Five different comparison criteria are proposed.

In section III, for the five criteria, the performances of the two robots are computed and results are displayed.

The section IV is devoted to analysis and interpretation of the results.

II. Methodology for Repeatability Comparison

A. Choice of the robots to be compared

In order to compare the two types of architecture, serial or parallel, we considered the same methodology proposed by S. Briot and Ilian A. Bonev.

1. Choice of two serial and parallel robots with the same rotational actuators. Thus the angular errors can be considered to be distributed statistically in an identical way.
2. For the link dimensions, choice of robots with the same workspace.
3. To simplify the analysis, the choice is set upon two planar structures with 2 degrees of freedom, a parallel robot RRRRR displayed in Fig. 1 and a two-link planar robot derived from the traditional serial SCARA displayed in Fig. 2. The 3rd and 4th axes of the SCARA are not taken into account because the repeatability will be studied in the plane. The links have the same length in both robots. The main advantage of this choice is that both the workspace surface and the reach are identical. Here our choice is slightly different from Briot and Bonev who considered a parallel robot with a non-nil distance between actuated axes.

The choice of these robots is non just a theoretical choice but corresponds to some industrial robots. The Spider SCARA proposed by Epson can reach the workspace center. In the Samsung robot Faraman, there is an hybrid structure which includes a RRRRR but the parallelogram dimensions are unequal.

B. Stochastic Ellipsoid theory

The stochastic ellipsoid theory is now introduced to model the repeatability and takes into account the spatial distribution of the phenomenon. This theory was applied with success to many industrial robots (Kuka, Faraman, Epson….) in our laboratory to study their pose and orientation repeatability. For a serial robot, the forward kinematic function links angular coordinates θ with Cartesian coordinates X.

$$X = F(\theta)$$

By differentiating F, we obtain Jacobian matrix linking the Cartesian and angular errors:

$$dX = Jd\theta$$

We proved that the angular deviations dθ were following a Gaussian law characterized by a covariance matrix D and as the angular random variables are independent, the Cartesian random variables dX are then Gaussian vectors with the following covariance matrix C.

$$C = JDJ^T$$

The density of probability of the Cartesian positions is given by a Gaussian law:

$$g(dX) = k \exp(\frac{1}{2} dX^T C^{-1} dX)$$

Isodensity surfaces are ellipsoids described by equation:

$$dX^T C^{-1} dX = Cte \quad [16]$$

The reference stochastic ellipsoid is:

$$dX^T C^{-1} dX = 1$$

The lengths of semi-axes of the reference ellipsoid are the square roots of the eigenvalues of the covariance matrix C and directions of main axis of the stochastic ellipsoids are the eigenvectors of the covariance matrix C.

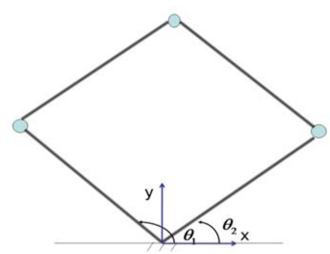

Fig. 1 Parallel robot RRRRR

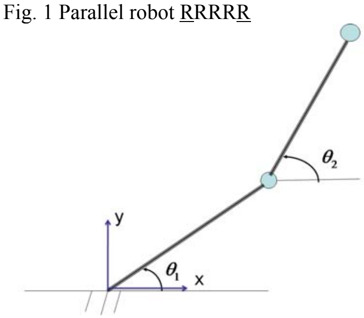

Fig. 2 Serial robot SCARA

All ellipsoids can be obtained from the reference ellipsoid by a central homothety whose ratio is linked to the risk. The risk is the probability that the robot endpoint fall outside the ellipsoid.

C. *Choice of the performance criteria*

To compare the repeatability, we will first use the repeatability index of the ISO9283 standard. It is possible to compute it directly from the stochastic ellipsoid modeling.

But then we introduce the spatial modeling of repeatability which gives the spatial confidence set for the robot final position taking into account the input errors modeled in the covariance matrix.

This leads us to introduce other criteria linked directly with the ellipse geometry as the ellipse surface and eccentricity. We want to take into account the isotropic or anisotropic nature of the task.

An isotropic task is illustrated in Fig. 3 and corresponds to the insertion of a peg in a hole. The robot must position the center of the peg in the clearance disk. The probability of success can be quite different depending if the ellipse is or not enclosed in the clearance disk.

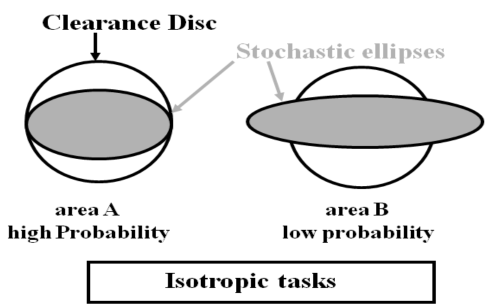

Fig. 3 Stochastic ellipses and clearance disc for an isotropic task

An anisotropic task is illustrated in Fig. 4 and corresponds to the insertion of a pin-cotter into a hole. The clearance area is not isotropic but one direction is far more important than the other one. The ellipse orientation is to be taken into account for the reliability of the assembly task.

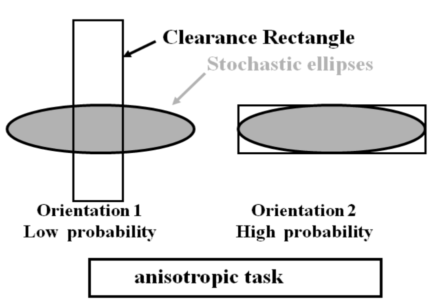

Fig. 4 stochastic ellipses and rectangle clearance for an anisotropic task

Ultimately among many parameters which determine the success or the failure of an assembly task, the dimensions of the semi-axes of the stochastic ellipsoids, named hereafter maximax and minimax, have a major importance. They are displayed on Fig. 5 and defined by:

1. The minimax error corresponds to the length of the minor semi-axis of ellipse; in this direction, the maximum error is the smallest.
2. The maximax error corresponds to the length of the major semi-axis of ellipse; in this direction, the maximum error is the largest.

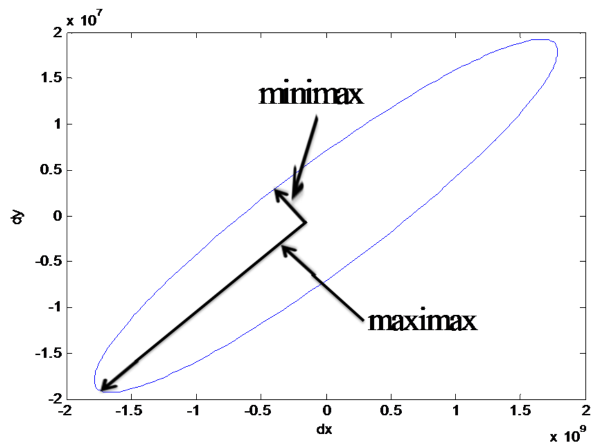

Fig.5 Maximax and minimax error

Because of the symmetry of the workspace obtained by a revolution around 1st axis, it is not necessary to draw the ellipses on the whole workspace but it is sufficient to draw them on a line corresponding to a radius.

To compare the two robots considering a performance criterion, we evaluate the workspace ratio α where the SCARA has a better performance than the RRRRR and consequently β corresponds to the workspace ratio where the RRRRR performance is better.

By convention, when the two robots have the same performance on one area, the area surface is not taken into account in the α or β evaluation,so that sometimes:

$$\alpha + \beta < 1$$

Most of the time, according to the ith criterion, the workspace will be separated in two sets where the robots performances are different. In this case, the radius of the frontier will be defined by R_i.

III. RESULTS OF COMPARISON

A. *Stochastic Ellipses evolution*

The Fig. 6 displays some stochastic ellipses for both robots and it is then possible to study their variation through the workspace. For a clear understanding, let us explain that the ellipses are computed on the same points of a workspace radius but are displayed one above another on the figure.

The level of risk is of course the same for the two robots, for instance 1% in our case and the angular standard deviation was set at 0.01 rad.

It is clear that the stochastic ellipses of both robots are changing in size, orientation and eccentricity from the center of the workspace to the periphery. But the evolution is different:

The stochastic ellipses of the SCARA are globally increasing if the surface is considered, and the orientation is changing completely. There is no isotropic point.

The stochastic ellipses of the RRRRR have an important eccentricity at center and at the periphery of the workspace and the surface does not change so much. It seems that there is an isotropic point which we will characterize more precisely later.

It is clear that the SCARA will be more repeatable in the vicinity of its workspace center.

It is not obvious at first glance to say if one robot is better than the other and it will certainly depend on the performance criterion.

The advantage of the stochastic ellipsoid theory is to give a clear spatial representation of the confidence interval. Then it is possible to choose the best location in the workspace to perform the task.

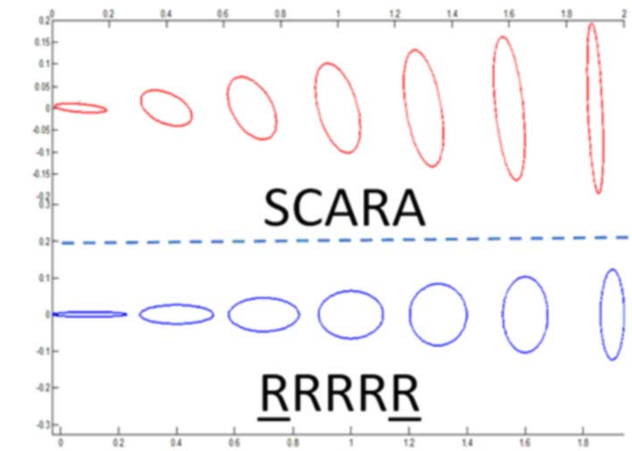

Fig. 6 Stochastic ellipses of SCARA and RRRRR robot

B. Comparison based on the repeatability index.

The repeatability index is defined in ISO 9283 standard by

$$REP_{iso} = \overline{D} + 3S_D$$

With

$$\overline{D} = \sqrt{\left(x_i - \overline{X}\right)^2 + \left(y_i - \overline{Y}\right)^2 + \left(z_i - \overline{z}\right)^2}$$

$\overline{D}$ is the random variable of the distance between the point (x_i, y_i, z_i) and barycenter $(\overline{X}, \overline{Y}, \overline{Z})$

S_D : The standard deviation of the random variable.

This index can be computed directly from the stochastic ellipsoid theory using the covariance matrix and the Jacobian matrix. The details of the computation can be found in [16].

Fig. 7 gives the curves of the computed repeatability index. The results are interesting and show that the SCARA is better than the RRRRR in the workspace center under the radius value R_{lim}=1.004m. Beyond this radius, it is the contrary.

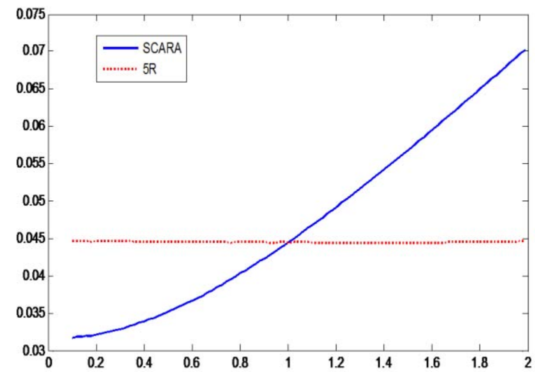

Fig. 7 ISO Repeatability index according to the workspace radius.

Another very interesting result is that the repeatability index of the RRRRR is nearly constant over the workspace.

It implies that the SCARA should be used for minute assembly task in the center of the workspace. But the difficulty is then to control the robot near a singularity.

C. Comparison based on the surface criterion.

We are now interested in the surface of the stochastic ellipsoids displayed on Fig. 8. Amazingly, the surfaces of the stochastic ellipses for the SCARA and the RRRRR are the same in the whole workspace! So considering this criterion, the robots have the same performance.

But this criterion suffers from the same drawback as the repeatability index. It is not possible to know if one task could be performed just considering the value of the surface, because the spatial distribution of the error is quite different if the ellipse is a circle or if the ellipse has an eccentricity near 1.

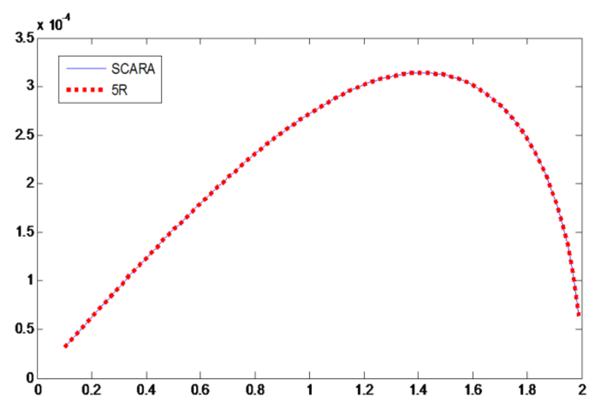

Fig. 8 Stochastic Ellipse Surfaces according to the workspace radius.

D. Comparison based on the minimax criterion.

From the analysis of the curves of Fig. 9, the SCARA has a better performance index than the RRRRR beyond Rlim=1.004 m and the RRRRR is better than the SCARA near the workspace center.

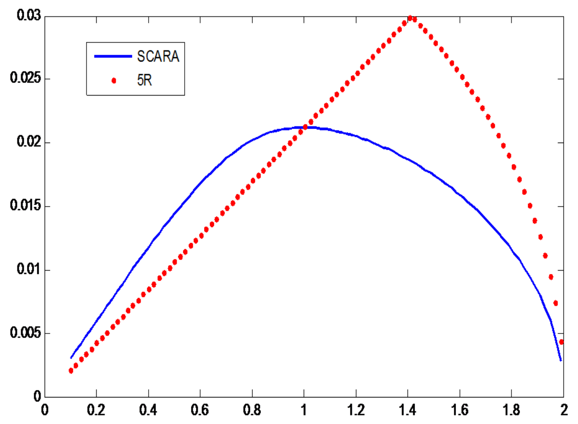

Fig. 9 Stochastic ellipses Minimax according to the workspace radius.

E. *Comparison based on the maximax criterion.*

The last criterion to be studied is the Maximax error and the results are displayed in Fig. 10. Here the SCARA is better than the RRRRR under the Rlim value. The results are then similar to the repeatability index performance.

This criterion is one of the criteria used in the paper of Briot and Bonev and the results are quite similar, because they computed the maximum position error in the section 3 of [7].

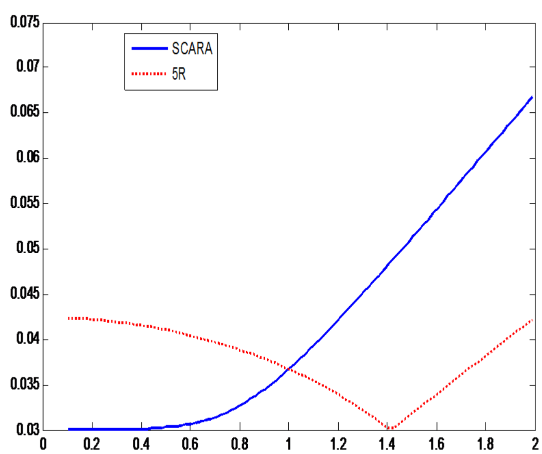

Fig. 10 Stochastic ellipses Maximax according to the workspace radius.

We display on Fig. 11 the minimax and maximax errors for the RRRRR. The point where the two curves intersect is an isotropic point corresponding to radius R_{iso}=1.43m. This proves the existence of a circular stochastic ellipse for the RRRRR and there is here a link with the dexterity index. Briot and Bonev found similar result in [7]

F. *Comparison of the eccentricity.*

The eccentricity e of the stochastic ellipses is defined by $e=\sqrt{1-\frac{a}{b}}$, where a and b are respectively the lengths of the minor and major semi-axes of the ellipse.

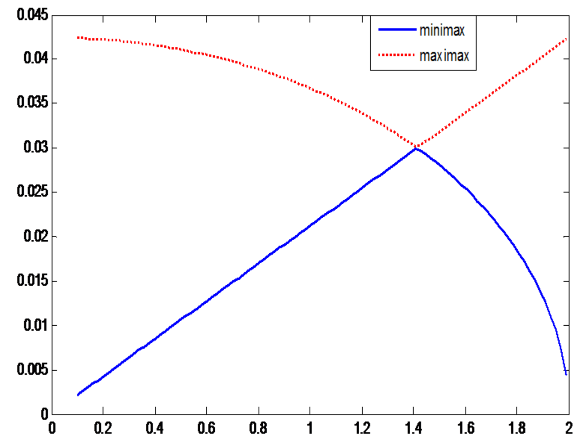

Fig. 11 Maximax and minimax of the RRRRR robot according to the workspace radius.

The variations of the stochastic ellipse eccentricity is displayed on Fig. 12 and the results corroborate the last comments on the previous section concerning the existence of an isotropic point for the RRRRR.

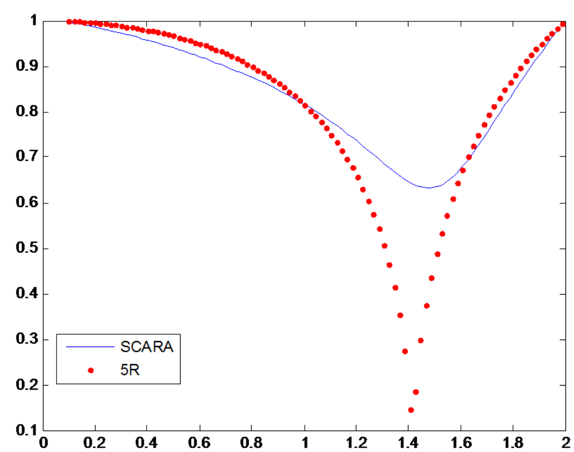

Fig. 12 Stochastic ellipses Eccentricities according to the workspace radius.

The eccentricity of the RRRRR ellipses is quite identical to the SCARA's in the center and the periphery of the workspace whereas in the vicinity of Riso, the SCARA value is higher. The SCARA does not present any isotropic point in its workspace.

IV. SYNTHESIS

Table I is a synthetic presentation of the results for the previous studied performance criteria.

First of all, the result of the comparison depends of the performance criterion. There are one case where the serial robot is better (minimax) and two cases where the parallel robot is better (maximax and repeatability index).

An interesting result is this case where the robots have the same performance when the stochastic ellipse surface is considered.

Now can we answer this question: are parallel robots more accurate than serial robots?

Yes, we have a lot of relevant information and analyzing the table I, it is clear that in most of the cases, parallel robots show better performances on a wider area of the workspace. There is one exception when the minimax criterion is considered. But in reality, this criterion is very specific and attached to the anisotropic tasks. Most of the time, robot design gives priority to a family of task [5] and concerning manipulation, the subdivision in isotropic and anisotropic tasks may seem artificial.

If we consider the maximax error performance, the SCARA is better on 25% of the workspace but this area is situated near the workspace center. Unfortunately, in most of the case, the mechanical design of industrial SCARAs prevents the robot end-effector to go in the workspace center because it is considered to be too close to the singularity. This has for consequence that the α ratio will be in reality smaller than 25%.

TABLE I
SYNTHESIS OF THE COMPARISONS

Performance criterion	SCARA	RRRRR
Repeatability index	α= 25%	β=75%
maximax	α= 25%	β= 75%
minimax	α = 75%	β = 25%
surface	equal performances	

V. CONCLUSIONS

The theoretical work developed in this paper considered two robots with different topology but with the same actuators, the same workspace surface and the same reach. It is the ideal case for a comparison between serial and parallel robots.

Different and motivated criteria have been introduced to evaluate the robot performance in accordance to the spatial characteristics of the stochastic ellipses.

The general conclusion is that the considered parallel robot is here more repeatable than the serial robot. But this result must be moderated: first the real workspace can be different from the theoretical workspace impacting the ratio α and β; secondly, the nature of the task is to be considered and it is not impossible to find some applications where the minimax criterion could have an interest.

More general conclusions are resulting from this paper. It is important to be able to choose the best location for the task in the workspace because the robot performance index depends strongly on the workspace location and task.

Some amazing specific results are obtained concerning the RRRRR constant repeatability across the workspace or the fact that Rlim is the same for three criteria. We are going to investigate thoroughly these points in our future work.

All these results open the way to innovative ideas to build new robot architecture and control in order to optimize the task orientation and workspace location.

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