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Dynamic State Feedback Control of Robotic Formation System

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Abstract—**This paper proposes a constructive control approach, dynamic state feedback formation control, for achieving the realization of the Multi-Robot Formation System (MRFS) with respect to the problem of dilation of a formation shape and stabilization issue in nonholonomic system simultaneously. Combining with theoretical analysis, the proposed control approach has been successfully to deal with the following design issues: one is to elude the switch control of the nonholonomic MRFS for preventing the divergence of the MRFS; another one is to stabilize the MRFS which is allowed to change the interconnection structure dynamically.**

Index Terms—**Formation Control, Dynamic Feedback Control, Multi-Robotic System, Stabilization.**

I. Introduction

A widely studied design concern about the MRFS is its interconnection stability, which is defined as the capability to create or maintain the desired formation shape while performing formation control. Another concern, following the interconnection stability issue of the MRFS, is how to design a formation control in terms of the nonholonomic subsystem, which seems to have become more highly regarded than before. Therefore we have to investigate the following design problems:

1. Find the stability criteria among subsystem stability, interconnection stability and formation system stability.
2. Stabilize the MRFS with respect to nonholonomic subsystems in terms of limited sensor/communication capability.

For the second statement of the problem, it particularly represents that the global nonholonomic path planning approach cannot be applied.

Numerous researches have concerned the nonholonomic subsystem and interconnection structure of the MRFS. A feedback linearization control approach for the nonholonomic MRFS with Leader-Follower is proposed in [1] and later, in [2], the author considers several formation strategies with different interconnection structure applied in indoor environment. With this control structure, Das *et al*.[3] practically design a vision-based system. The switching between decentralized controllers that allows for changes in formation with obstacle avoidance of the MRFS. Lawton *et al*.[4] conduct behavior-based formation control with respect to dynamics rather than kinematics of a WMR. In addition, for dealing with the measurement uncertainty, an observer based nonlinear Lyapunov based output feedback control is proposed in [5]. Recently, Ji *et al*.[6] focus on the connectedness issue using graph Laplacian in the multirobot coordination problem. Also, a constrained forced based control approach is proposed by Zou *et al*.[7] with respect the dynamics of a WMR. Ren and Soresen in [8] study the interconnection issues of the MRFS with respect to the physical limitation of the communication bandwidth and the stability of the interconnection structure as well as consensus problem in communication system. Do[9] uses the backstepping control scheme to control the reduced order dynamics of a WMR.

This research, based on the existing researches, intends to go further to investigate the stability and control issues of the MRFS. With this aspect, several design challenges shall be overcome: interconnection stability, subsystem stability and nonholonomic constraints. The former can be recast as the rigidity condition of a properly defined graph [6, 10]. Combining with the subsystem stability condition, one can yield it as a consensus problem[11] with respect to the communication issue.

This paper is organized as follows: initially, in Section II, the theoretical analysis of the MRFS is proposed. Then, the stability analysis of the formation system is defined and analyzed in Section III. The dynamic state feedback control of the MRFS is obtained in Section IV. In Section V, an experiment of the MRFS is examined. Finally, the conclusions are made in Section VI.

II. Theoretical Analysis

In this section, the general model of a MRFS is initially established. In addition to simplifying the problem, we temporarily assume that the rigidity condition of the interconnection structure of the MRFS is satisfied. Later the algebraic differential topology of the nonholonomic MRFS is obtained and the result is significantly helps us to understand the differential structure of the nonholonomic MRFS.

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A. General Formula of MRFS

Consider an MRFS with the formation state: $z=\left\{z_{ij}\in\mathbb{R}^2 \middle| i\neq j;1\leq i,j\leq n\right\}$ where n denotes the number of WMRs in the formation team, along with i and j being the sub-indices which represent the j^{th} connection of the i^{th} WMR. Following the definition of the formation variables in [7], the component form of the formation states defined in the Euclidean space of the global coordinate can be generally described as:

$$z_{ij}=\begin{bmatrix} l_{ij} \\ \varphi_{ij} \end{bmatrix} \triangleq \begin{bmatrix} \left\| q_{pj}-q_{pi} \right\| \\ q_{\theta j}-q_{\theta i} \end{bmatrix} \in Q . \quad (1)$$

Here, $q_i=\begin{bmatrix} q_{pi} & q_{\theta i} \end{bmatrix}^T \in \mathbb{R}^k$ and $q_j=\begin{bmatrix} q_{pj} & q_{\theta j} \end{bmatrix}^T \in \mathbb{R}^k$ where $q_{pi}\in N_i$ and $q_{pj}\in N_j$ denote the position state of the WMR with N_i and N_j being the subspace of the base space B with $N_i,N_j\subset B$, $q_{\theta i}$ and $q_{\theta j}$ denote the orientation state of the WMR, and $Q\in\mathbb{R}^m$ denotes the abstracted space with $k\geq m$. Additionally, the following property is satisfied: $N_i\bigcup N_j\subset B$ for all i,j. Following the definition, z_i is denoted as the sum of the j^{th} connection to the i^{th} WMR $z_i=\sum_j z_{ij}$ for all i. Following the definition, if there exists a smooth and differentiable map $h:N\rightarrow Q$, $N=\bigcup_i N_i$ for all i, with respect to the interconnected structure Ω of the MRFS, we say h is a diffeomorphism when $k=m$ and similarly, we say h is a surjection when $k>m$ such that $z=\begin{bmatrix} z_1 & \cdots & z_n \end{bmatrix}^T \triangleq h(q,\Omega)$ with $q=\begin{bmatrix} q_1 & \cdots & q_n \end{bmatrix}^T \in\mathbb{R}^{kn\times rn}$; $\Omega\in\mathbb{R}^{kn\times kn}$ with r being the dimension of the WMR control.

Suppose that a virtual center of the MRFS moves along a desired trajectory $c(t)\in\mathbb{R}^2$ in two-dimensional space so that the MRFS is driven from an initial state $q_{c0}=c(t_0)$ to a final state $q_{cf}=c(t_f)$ where t_0 and t_f denote the initial and final time, respectively. The control goal, thus, is to move the MRFS to follow $c(t)$ in addition to maintaining the given interconnection structure (resp. formation shape) simultaneously.

Therefore, initial configurations of the MRFS require the desired interconnection structure Ω_d, the desired formation state z_d, and a virtual formation center $q_c\in\mathbb{R}^2$, which is located inside the closed region of the formation shape. The kinematics of the i^{th} WMR for all i can be generally regarded as a driftless affine control system $\dot{q}_i=\sum_j g_{ij}(q_i)u_i$ with $j=1,\cdots,n$ or we can generally write $\dot{q}_i=g_i(q_i)u_i$ with $g_i\triangleq\begin{bmatrix} g_{i1} & \cdots & g_{ir} \end{bmatrix}^T\in\mathbb{R}^{m\times r}$ and $u_i\in\mathbb{R}^r$.

Generally, consider a MRFS:

$$\dot{q}(t)=f\left(q(t),u(t)\right). \quad (2a)$$

$$z=h(q,\Omega). \quad (2b)$$

where $f:\mathbb{R}^{kn\times rn}\times\mathbb{R}^{rn}\rightarrow\mathbb{R}^{kn}$ is a smooth function. We formally pose the problem of stabilizing the formation state z by means of state feedback. The desired states of the subsystems are to derive naturally via forwarding geometrical constraints such that the set of the desired state $\left\{q_{d_1},\cdots,q_{d_n}\right\}$ of the subsystems is well defined if the initial configurations of the MRFS are given, *i.e.*, $q_{di}=q_{c0}+l_{dic}\times\begin{bmatrix} \cos\varphi_{dic} & \sin\varphi_{dic} \end{bmatrix}^T$ for all i where l_{dic} and φ_{dic} denotes the formation states from i^{th} WMR to the virtual center of the MRFS.

Now we define the general control system in terms of the nonholonomic MRFS in Eq. (2).

Definition 2.1 *Control System*: A control system $S=(B,F)$ consists of the following.

- A fiber bundle $\pi:M\rightarrow B$ called the control bundle.
- A smooth map $F:M\rightarrow TB$ which is fiber preserving.

A map is fiber preserving if $\pi_b\circ F=\pi$, where $\pi_b:TB\rightarrow B$ is the tangent bundle projection. Considering the integral flow of the control system, an extensive definition is made in the sense of the fiber bundle structure.

Definition 2.2: A smooth curve $c_m:I\rightarrow M$ with $I\subseteq\mathbb{R}$ is called the trajectory of the control system (B,F) if there exists a curve $c:I\rightarrow B$ such that $\pi(c_m)=c$ and $c'(t)=F\left(c_m(t)\right)$.

In local coordinates, this definition is translated into the well-known concept of trajectories in a control system by $\dot{q}=f(q,u,t)$. Regarding the kinematics of the MRFS, the formation state z is reduced from $\mathbb{R}^{2n}$ to $\mathbb{R}^n$ if we consider the relative length only so that the constraint for the relative angle is embedded via a differential topology of the nonholonomic system. We rearrange Eq. (2a) and Eq. (2b):

$$\begin{aligned} \dot{q}_p &= f_p\left(q_p,u_1\right) \\ z &= h\left(q_p,\Omega\right). \end{aligned} \quad (3)$$

According to Eq. (3b), the differential equation in the oriented angle is able to be combined.

$$\begin{aligned} \dot{z} &= \mathcal{L}_{f_p}h \\ \dot{q}_\theta &= u_2 \end{aligned} \quad (4)$$

where $\mathcal{L}$ denotes the Lie derivative with $\mathcal{L}_{f_p} h \triangleq \langle f, \partial h/\partial q_p \rangle$. Notice that the MRFS is still nonholonomic and cannot be stabilized via a smooth static feedback control. Also, we may apply a smooth dynamic state feedback control for stabilizing the system in Eq. (4). If one gives the desired formation state z_d, Eq. (4) can be further obtained as:

$$\begin{aligned} \dot{\tilde{z}} &= \mathcal{L}_{f_p} h - \dot{z}_d \\ \dot{q}_\theta &= u_2. \end{aligned} \tag{5}$$

The control purpose of the MRFS in Eq. (5) is thus expressed to design the control u_1 and u_2 for zeroing the output of $\tilde{z}$ without setting the control output as zero only.

Remark 2.3: Consider two control systems, $S_B = (B_B, F_B)$ and $S_Q = (B_Q, F_Q)$, with respect to some submersion $\Phi : B \to Q$. Then, the vector fields in S_B maps from vector fields in S_Q and are found by the following process:

$$\begin{gathered} (q_p, q_\theta, u) \in M \\ \Downarrow F_B \\ (X_p, [X_p, u]) \in TB \\ \Downarrow T\Phi \\ (Z, [Z, u]) \in TQ. \end{gathered}$$

Parallel to the design a control of the MRFS, if we remove the assumption of the rigidity condition of the MRFS in this section, one yield a challenge in selection of the interconnection structure for achieving the rigidity condition of the formation shape. Consequently, we try to investigate the relationship among subsystem stability, interconnection stability, and formation system stability.

III. Stability of MRFS

Now, we release the assumption of the rigidity condition in Section II and begin by translating the stability problem into purely algebraic terms in a local sense. In the neighborhood of $q_i \in V_i$ and $z_i \in U_i$ with $V_i \subset B$; $U_i \subset Q$, suppose there exists $\delta_{q_i} > 0$ around q_i and similarly, there exists $\delta_{z_i} > 0$ around z_i so that small enough balls $B(q_i, \varepsilon_{q_i})$ and $B(z_i, \varepsilon_{z_i})$ with $\varepsilon_{q_i} > 0$; $\varepsilon_{z_i} > 0$ exist on V_i and U_i, respectively. Thus, the upper bound of the open balls $\sup B(q_i, \varepsilon_{q_i}) = r_{q_i}$ and $\sup B(z_i, \varepsilon_{z_i}) = r_{z_i}$, with r_{z_i} and r_{q_i} being the maximum radius of the balls can be found. Now, we set $\varepsilon_{q_i} = \min(\delta_{q_i}, r_{q_i})$ and $\varepsilon_{z_i} = \min(\delta_{z_i}, r_{z_i})$, and the following definitions can be given.

Definition 3.1 (Interconnection stable): Let z_{ij} be piecewise continuous in t, and suppose that z_{dij} is given such that the formation state error is defined as $\tilde{z}_{ij}(t) \triangleq z_{ij}(t) - z_{dij}(t)$. The system is *interconnection stable* if for every $\varepsilon_{zi} > 0$ there exists a $\delta_{zi} > 0$ such that if $\sum_j \|\tilde{z}_{ij}(t_0)\| \le \delta_{z_i}$ then $\lim_{t\to\infty} \sum_j \|\tilde{z}_{ij}(t)\| \le \varepsilon_{z_i}$ for all i.

In addition, the state error of the i^{th} WMR of the MRFS, $\tilde{q}_i(t) \triangleq q_i(t) - q_{di}(t)$, is defined for finding the conditions of the subsystem stability.

Definition 3.2 (Subsystem stable): Let z_{ij} be piecewise continuous in t and z_{dij} be a given. The equilibrium point $\tilde{z}_{ij}(0) = 0$ and $\tilde{q}_i(0) = 0$ for all i, j in the formation error state and subsystem error state respectively is

- *subsystem system stable*: if there exists $\sum_j \|\tilde{z}_{ij}(t_0)\| \le \delta_{z_i}$ and $\|\tilde{q}_i(t_0)\| \le \delta_{q_i}$ then $\lim_{t\to\infty} \|\tilde{q}_i(t)\| \le \varepsilon_{q_i}$, for all i;
- *asymptotically subsystem system stable*: if there exists $\sum_j \|\tilde{z}_{ij}(t_0)\| \le \delta_{z_i}$ and $\|\tilde{q}_i(t_0)\| \le \delta_{q_i}$ then $\lim_{t\to\infty} \|\tilde{q}_i(t)\| \to 0$, for all i;
- *subsystem system unstable*: if it is not subsystem stable.

The following lemma identifies the relationship between the interconnection stability and the subsystem stability of the MRFS.

Lemma 3.3: The following statements are true:

- The MRFS is interconnection stable if the subsystem stability is satisfied;
- The MRFS is subsystem unstable if and only if the interconnection stability is not held.

Proof:

For the first statement, we shall prove that, if $\lim_{t\to\infty} (\|\tilde{q}_i(t)\| + \|\tilde{q}_j(t)\|) \le \varepsilon_{q_i} + \varepsilon_{q_j}$, then $\lim_{t\to\infty} \sum_j \|\tilde{z}_{ij}(t)\| \le \varepsilon_{z_i}$ for all i. From Definition 2.2, we have $\|\tilde{q}_i(t_0)\| \le \delta_{q_i}$ and $\|\tilde{q}_j(t_0)\| \le \delta_{q_j}$. Hence, according to Figure 1, the inequality can be written as $\delta_{q_i} + \|z_{dij}\| + \delta_{q_j} \ge \|z_{ij}\|$ with $\delta_{q_i} + \delta_{q_j} > 0$ such that we have $\|\tilde{z}_{ij}(t_0)\| \le \delta_{q_i} + \delta_{q_j}$ which can be represented as $\lim_{t\to\infty} \sum_j \|\tilde{z}_{ij}(t_0)\| \le \lim_{t\to\infty} (\|\tilde{q}_i(t_0)\| + \|\tilde{q}_j(t_0)\|)$ by Definition 3.1 and Definition 3.2. With this result, the boundary $\varepsilon_{z_i} \le \varepsilon_{q_i} + \varepsilon_{q_j}$ can be found such that $\lim_{t\to\infty} \sum_j \|\tilde{z}_{ij}(t)\| \le \varepsilon_{z_i}$ for all i.

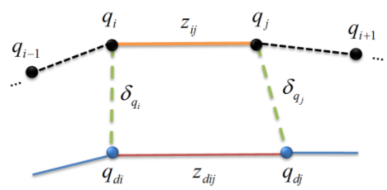

Fig. 1. The diagram from formation state space to the individual state space.

For the second statement, we first claim that the subsystem is unstable and the interconnection stability is held. From the preceding proof, the unstable subsystem implies that it is able to be written as $\lim_{t\to\infty}\left(\left\|\tilde{q}_i(t)\right\|+\left\|\tilde{q}_j(t)\right\|\right)\le\varepsilon_{q_i}+\varepsilon_{q_j}$, which implies that $\varepsilon_{q_i}+\varepsilon_{q_j}$ is unbounded. Hence, the bounded ε_{z_i} does not exist, so the claim is not true. On the contrary, we claim that the MRFS is interconnection unstable but the subsystems are locally stable. Mathematically, the interconnection being unstable demonstrates ε_{z_i} is unbounded such that $\varepsilon_{q_i}+\varepsilon_{q_j}$ is unbounded, which indicates that the subsystem is unstable, so the claim cannot be true. ❑

Lemma 3.3 clearly indicates that, if the subsystem is stable, the interconnection stability is guaranteed and there also exists a unique map $\Phi\left(q_{p_i}\right)$ from B to Q for all i. Following this result, a more strict definition with respect to the formation stability is needed.

Definition 3.4 (Formation system stable): Let z_{ij} be piecewise continuous in t and z_{dij} be a given. The equilibrium point $\tilde{z}_{ij}(0)=0$ and $\tilde{q}_i(0)=0$ in the formation error state and subsystem error state for all i,j is

- *Formation system stable*: if there exist $\sum_j\left\|\tilde{z}_{ij}(t_0)\right\|\le\delta_{z_i}$ and $\left\|\tilde{q}_i(t_0)\right\|\le\delta_{q_i}$ then $\lim_{t\to\infty}\sum_j\left\|\tilde{z}_{ij}(t)\right\|\le\varepsilon_{z_i}$ and $\lim_{t\to\infty}\left\|\tilde{q}_i(t)\right\|\le\varepsilon_{q_i}$ for all i;
- *asymptotically formation system stable*: if there exist $\sum_j\left\|\tilde{z}_{ij}(t_0)\right\|\le\delta_{z_i}$ and $\left\|\tilde{q}_i(t_0)\right\|\le\delta_{q_i}$ then $\lim_{t\to\infty}\sum_j\left\|\tilde{z}_{ij}(t)\right\|\to 0$ and $\lim_{t\to\infty}\left\|\tilde{q}_i(t)\right\|\to 0$, for all i;
- *formation system unstable*: if the aforementioned conditions do not exist.

IV. Dynamic State Feedback Control

In this section, a dynamic state feedback formation control with respect to the interconnection structure is proposed for the nonholonomic MRFS based on Lyapunov theory.

For connecting the rigidity matrix and the formation dynamics, we may introduce an adjacency matrix to describe the interconnected structure of the MRFS. The adjacency matrix[12] (or so-called interconnection matrix), A_G, is imposed and represented as a binary matrix which implies q_j acts on q_i if the element in i^{th} row and j^{th} column of the matrix equals "1" denoted as A_{Gij}. It is the fact that all of the connections of the i^{th} WMR to all neighbor interconnections are able to form a set: $a_{ij}=\left\{A_G(i,j)\middle|1\le j\le n\right\}$ where i and j denotes the i^{th} raw and j^{th} column in the adjacency matrix. Therefore, regarding with the interconnection structure, the kinematics of the MRFS could be rewritten as:

$$\dot{z}_i=\sum_j\left(\begin{bmatrix}q_{pij}^T\Omega_{ij}^I\dot{q}_{pij}\\ q_{pij}^T\Omega_{ij}^J\dot{q}_{pij}\end{bmatrix}-\begin{bmatrix}0\\ \dot{q}_{\theta i}\end{bmatrix}\right)\tag{6}$$

with $\Omega_{ij}^I=\dfrac{a_{ij}I_2}{l_{ij}}$; $\Omega_{ij}^J=\dfrac{a_{ij}J_2}{l_{ij}}$; $I_2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$; $J_2=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$; $q_{pij}\triangleq q_{pj}-q_{pi}\in\mathbb{R}^2$ where l_{ij} denotes the relative length and φ_{ij} denotes the relative oriented angle.

We select the Lyapunov function $L_i=\frac{1}{2}a_{ii}\tilde{q}_i^T\tilde{q}_i$ in each of the subsystems. Additionally, we limit the output of the angular velocity to prevent the behaviors of sliding and slipping. Nevertheless, this may lead the saturation in the control, so we have to carefully choose the operation point in the control design by means of manipulating the control gain.

To aid in the judgement of the stability criteria, we also impose an interconnection Lyapunov function: $L_{ij}=\sum_{j:j\ne i}\frac{1}{2}w_{ij}a_{ij}\tilde{z}_{ij}^T\tilde{z}_{ij}$ with w_{ij} being an element of the weight matrix $W\in\mathbb{R}^{3n\times 3n}$. By the additive property of the energy, the formation Lyapunov function, L_i^F can be simply split into two parts: the individual Lyapunov function of the i^{th} WMR and the normalized interconnection Lyapunov functions of the j^{th} WMR which acts on the i^{th} WMR

$$L_i^F=L_i+\frac{1}{n-1}\sum_j L_{ij}\tag{7}$$

In Eq. (7), L_i is generated from the i^{th} subsystem and $\sum_j L_{ij}$ is produced by the interconnection of the MRFS for the i^{th} subsystem. In the component form, it can be written as:

$$\begin{aligned}L_i^F=&\frac{1}{2}\begin{bmatrix}\tilde{q}_{pi}&\tilde{q}_{\theta i}\end{bmatrix}P_i\begin{bmatrix}\tilde{q}_{pi}\\ \tilde{q}_{\theta i}\end{bmatrix}\\&+\frac{1}{2}\bar{L}_{Gi}\begin{bmatrix}\tilde{z}_1^T\tilde{z}_1&\cdots&\tilde{z}_n^T\tilde{z}_n\end{bmatrix}^T\end{aligned}\tag{8}$$

where $P_i\in\mathbb{R}^{3\times 3}$ is positive definite; $\bar{L}_{Gi}$ denotes the i^{th} raw of the normalized weighted laplacian matrix with $\bar{L}_G\triangleq W\cdot\left(I-\dfrac{1}{n-1}A_G\right)$ with W being a weighted positive definite matrix. Hence, the necessary condition for the asymptotically formation stability is established via the following theorem:

Theorem 4.1: Considering the MRFS described in Eq. (11-12), the system is said to be asymptotically interconnection stable if

$$F\left(W\cdot\Omega^I\right)+\left(W\cdot\Omega^I\right)F=-G$$

with $F=\left[F_1 \quad \cdots \quad F_n\right]^T$ where $F_i=\partial f_{pi}/\partial q_i$ denotes a Jacobian matrix which is proceeded with the linearization scheme from the nonlinear function f_{pi} in Eq. (3) ; G being positive matrix.

Proof. The time derivative of Equation (8) can be reduced as the following formulation:

$$\begin{aligned}\sum_j \dot{L}_{ij} &= \frac{1}{2}\left(\sum_j \dot{\tilde{q}}_{pij}^T\left(W_{ij}\cdot\Omega_{ij}^I\right)\tilde{q}_{pij}+\tilde{q}_{pij}^T\left(W_{ij}\cdot\Omega_{ij}^I\right)\dot{\tilde{q}}_{pij}\right)\\ &= \frac{1}{2}\left(\sum_j \tilde{q}_{pij}^T\left(F_i\left(W_{ij}\cdot\Omega_{ij}^I\right)+\left(W_{ij}\cdot\Omega_{ij}^I\right)F_i\right)\tilde{q}_{pij}\right)\end{aligned} \tag{9}$$

Thus, we reformulate the result in Eq. (9) in associated with a matrix formula:

$$F\left(W\cdot\Omega^I\right)+\left(W\cdot\Omega^I\right)F=-G \tag{10}$$

where G is a positive matrix. According to the Lyapunov stability theorem, if Ω^I and G are positive definite, then the MRFS is asymptotically stable. ❑

Theorem 4.1 also provides the necessary condition of the formation system stability in association with the interconnection matrix. The condition is that the interconnection matrix Ω^I shall have full dimension. As a consequence, Ω^I has to be a non-singular matrix from the linear system theory. Practically, let us now consider a MRFS in terms of it interconnection structure in Eq. (6). The Lyapunov function in Eq. (9) can be further taken as the partial derivative:

$$\begin{aligned}\dot{L}_i^F &= \frac{\partial L_i}{\partial q_i}+\sum_j \frac{\partial L_{ij}}{\partial q_j}\\ &= \sum_j f_i^1\left(a_{ij},z_{ij},q_{pi},q_{pj},q_{\theta j}\right)\\ &\quad +\left(q_{pi}S_i+\sum_j f_i^2\left(a_{ij},z_{ij},q_{pi},q_{pj},q_{\theta i}\right)\right)v_i+q_{\theta i}w_i\end{aligned} \tag{11}$$

with $f_i^1=\left(\rho_{ij}\tilde{z}_{ij}\right)\left(\dot{z}_{dj}-\cos\gamma_{ij}\right)$; $f_i^2=\rho_{ij}\tilde{z}_{ij}\cos\varphi_{ij}$ with $\gamma_{ij}=\varphi_{ij}+q_{\theta i}-q_{\theta j}$; $S_i=\begin{bmatrix}\cos q_{\theta i} & \sin q_{\theta i} & 0\\ & 0 \;\; 0 & 1\end{bmatrix}^T$. Therefore, the formation control can be chosen by the following theorem:

Theorem 4.2: Considering that the MRFS follows Eq. (11-12), if the velocity and angular velocity is chosen by

$$\begin{aligned}v_i &= \begin{cases} -\sum_j f_i^1-K_{pi}L_i^F \Big/ \left(q_{pi}S_i+\sum_j f_i^2\right); \\ 0 \quad \text{if } \left(q_{pi}S_i+\sum_j f_i^2\right)=0; \end{cases}\\ w_i &= -K_{\theta i}q_{\theta i}.\end{aligned} \tag{12}$$

then the MRFS is asymptotically stable where $K_{pi}\geq K_{\theta i}\geq 0$ denotes the positive control gain.

Proof: After putting the controller in Eq. (12) into Eq. (11), the Lyapunov equation is obtained:

$$\dot{L}_i^F=-K_{pi}L_i^F-\left(K_{pi}-K_{\theta i}\right)q_{\theta i}^2\leq -K_{pi}L_i^F \tag{13}$$

Observing Eq. (13), the MRFS with the control of the i^{th} WMR for all i in Eq. (12) is exponentially stable. ❑

The result from Theorem 4.2 certainly reveals that, if the energy is exponentially decaying, then the reachability of the subsystems of the MRFS is guaranteed. Now, the overall control issues can be linked together: internal dynamics and the interconnected structure.

Remark 4.3 According to Theorem 4.2, the control of the MRFS satisfies the formulation of the smooth dynamic feedback control design

$$\begin{aligned}u &= \alpha\left(q,z\right)\\ \dot{z} &= \bar{h}\left(q,z\right)\end{aligned} \tag{14}$$

so that Brockett's necessary condition for the piecewise control design by means of the smooth static feedback of the nonholonomic system can be overcome by the proposed dynamic state feedback control design in the MRFS in Theorem 4.2. In the next section, an experiment is conducted for evaluating the theoretical results.

V. Experimental Result

An experiment is set up to evaluate the proposed approach. The MRFS with three WMRs that move on a free 2D space is the major application scenario.

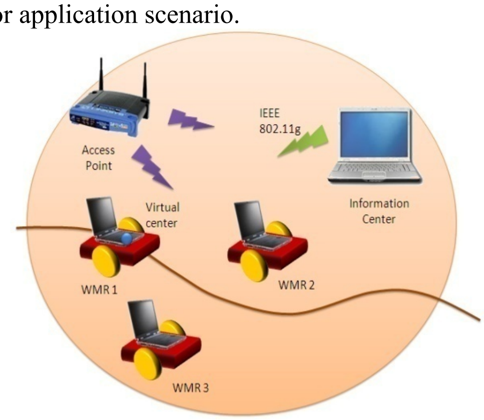

Fig. 2. Physical layout of the setup of the experiment.

Each of the mobile robots is equipped with encoders and is connected virtually with a wireless communication device shown in Fig. 2. Thus, three mobile robots (P3DX) made by Active Media Inc. play a role as an experimental platform. The average speed is $0.25\, m/\sec$ and the average acceleration is $1\ m/\sec^2$. In the initial setting, we put the three WMRs in their specified places and respectively regard the positions as their starting points. Also, the relative distance with each other is initially set to 2.5 (m). After 20 sec from the starting time, the desired relative distance is set to be 3.5 (m). 60 sec later, the relative distance is set back to 2.5 (m).

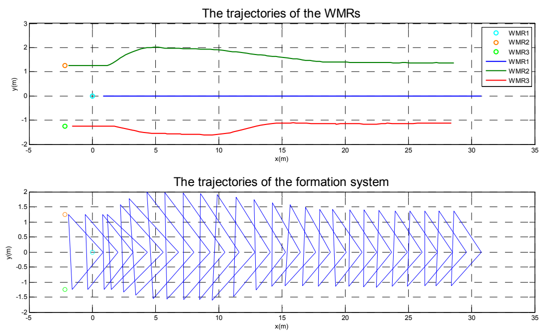

Fig. 3. The trajectories of the MRFCS.

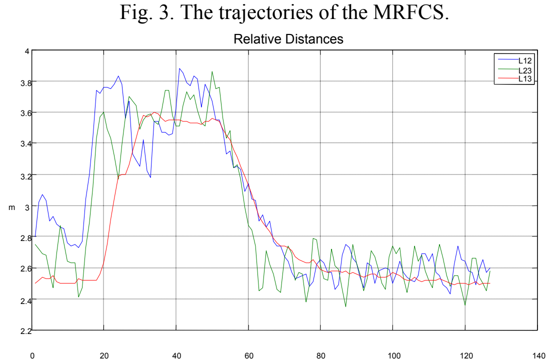

Fig. 4. The relative distances ($L12$, $L13$, $L23$) of the MRFS.

The experimental results are drawn in Fig. 3 and Fig. 4, respectively. The first drawing describes both the trajectory of the individual WMR and the relative distance from WMR *i* to WMR *j* which is denoted as Lij with $i, j = \{1,2,3\}$. In Fig. 4, it is significant that the relative distance between WMR 2 and WMR 3 is relatively more stable than the one from WMR 1 to WMR 2 and WMR 1 to WMR 3 due to the property of the nonholonomic system. The formation control parameter in K_{pi} and $K_{\theta i}$ are both selected as 0.01 and 0.001. This follows the analysis result in Theorem 4.2 precisely.

VI. Conclusion

In this research, the problem of dilation formation shape is successfully solved by means of smooth dynamic state stable feedback control. By the way of introducing the higher dimension variable, the proposed control approach of the MRFS is essentially used solve the following design problem: the stable control for the nonholonomic MRFS with respect to the interconnection structure for preventing the divergence of the MRFS; another one is that the global path planning is impossible to perform in large scale subsystems of a MRFS. Furthermore, the theoretical analysis is set up so that the necessary condition for the formation system stability is distinctly obtained with respect to the nonholonomic subsystems. Finally, the experiment is performed to evaluate the proposed control approach.

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