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# Task-Space Control of Bilateral Human-Swarm Interaction with Constant Time Delay

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*Abstract*— **This paper presents system framework and control algorithm that enable a human operator to simultaneously interact with a group of swarm robots in a remote environment. In this control system, several characteristics of the configuration of the swarm robots are encoded as task functions, for which a human operator can specify desired values that are conveyed to the end-effector of the master robot. Stability and tracking performance of the proposed control system are investigated in the presence of communication delays so that the swarm robots can be manipulated remotely. Moreover, the swarm robots, which perform like a redundant robotic system, can also regulate their position to achieve secondary tasks autonomously. The proposed control algorithms are validated via numerical simulations on a 3-DOF robot manipulator with a group of mobile robots.**

## I. INTRODUCTION

Much recent research has been devoted to developing control architectures and algorithms for human operators to regulate a group of swarm robots [1], [2], [3], [4], [5]. However, most of the control systems proposed in the literature are difficult to fulfill complex missions in cluttered environments. In addition, few studies have addressed the issue of unreliable communication channels and the flexibility of the swarm robot. Therefore, this paper proposes a new control framework to accomplish bilateral human-swarm interaction by adopting the task-space teleoperation control system addressed in [6], [7].

Studies in recent decades have examined how to control multiple robots in maintaining formation, avoiding obstacles, and monitoring the environment [8], [9], [10], [11], [12], [13]. Based on artificial potential functions and sliding-mode control, [9] demonstrated that a group of swarm robots could achieve formation control and social foraging. It has been proposed recently to control a swarm of robots by positioning robots inside a desired region while maintaining a minimum inter-robot distance [10]. The redundant manipulator technique has also been proposed to control the motion of a swarm group [14] by designing different task functions with the use of null-space projection. In order to handle task functions having different priority and to avoid possible singularities in the system, [15] applied singularity-robust task-priority inverse kinematics [16] to redundant techniques for platoons of robots [14]. However, most of the

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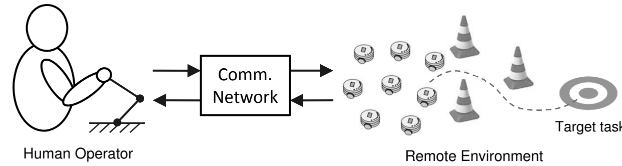


Fig. 1. Overall system for bilateral human-swarm interaction.

aforementioned work does not consider human input, and can be applied only for a predefined trajectory and formation.

Several studies have explored teleoperating a group of mobile robots remotely using a single human operator. The work [3] used passivity techniques to address the stability problem of controlling groups of mobile robots by employing one master robot. The teleoperation system with single-master and single-slave robots [17] was extended by [2] to support multiple mobile slave robots under constant, bounded communication delays. Assigning an identity to each slave robot enables coordinating their positions to maintain a rigid formation. Although the aforementioned control algorithms can provide a human operator to remotely command multiple robots, assuming either no delay in the communication channel or slave robots following a predefined static formation could limit the applications in practice.

This paper develops a control system for human-swarm interaction, see Fig. 1, by adopting the study of semi-autonomous teleoperation between kinematically dissimilar master and slave robots [7]. We formulate two types of tasks, teleoperation task and autonomous task, to enable the swarm robots to be teleoperated by the human operator and to autonomously achieve additional tasks. The teleoperation task is accomplished by utilizing the abstraction of group robots addressed in [14], [15]. Thus, the human operator can control the movement and formation of the swarm robots by commanding on the corresponding task functions. Considering that the augmented slave swarm exhibits high level of redundancy, the null-space of the group of swarm robots can be utilized to achieve several sub-tasks.

The paper is organized as follows. Section II presents the model of the master manipulator and the slave swarm robots. The proposed controller and stability analysis are addressed in Section III, which is followed by the discussion of teleoperation and autonomous task functions in Section IV. Section V illustrates numerical examples of the proposed bilateral human-swarm interaction system. Finally, Section VI summarizes the results and provides possible future research directions.

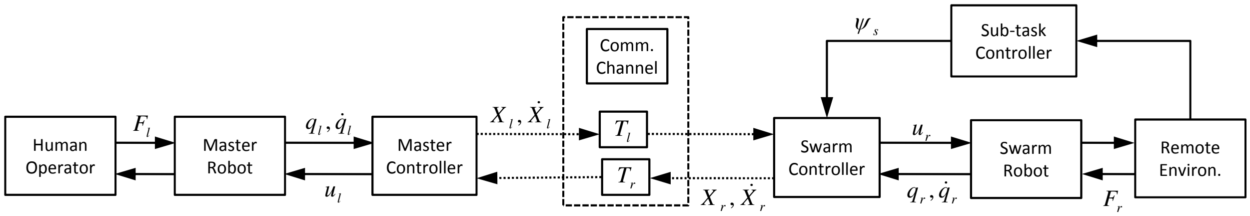


Fig. 2. Control framework of the proposed human-swarm system.

## II. HUMAN-SWARM SYSTEM

The proposed human-swarm system is composed of a robotic manipulator, which is commanded by a human operator, and a group of swarm robots, which are teleoperated to accomplish various tasks in a remote environment (Fig. 1). The control framework, illustrated in Fig. 2, is formulated subsequently in this section.

### *A. Master Side-Human Operator*

Without loss of generality, the robotic manipulator influenced by the human operator is assumed to be a non-redundant robot and described by [18]

$$M_l(q_l)\ddot{q}_l + C_l(q_l,\dot{q}_l)\dot{q}_l + g_l(q_l) = u_l + J_l^T F_l, \tag{1}$$

where $q_l \in R^m$ is the vector of generalized configuration coordinates, $M_l(q_l) \in R^{m\times m}$ is the inertia matrix which is symmetric and positive-definite, $C_l(q_l,\dot{q}_l) \in R^{m\times m}$ is the matrix of Coriolis/Centrifugal terms, which satisfies the property that the matrix $\dot{M}_l(q_l) - 2C_l(q_l,\dot{q}_l)$ is skew symmetric under an appropriate definition of the matrix $C_l(q_l,\dot{q}_l)$ [18], $g_l(q_l) = \partial G_l(q_l)/\partial q_l \in R^m$ is the gradient of the potential function $G_l(q_l)$, $u_l \in R^m$ is the vector of applied control torque, $F_l \in R^m$ is the force exerted by the human operator, and $J_l$ is the Jacobian matrix that will be defined subsequently. The subscript $l$ in (1) denotes local site of the human-swarm system.

Let $X_l \in R^m$ represents the end-effector of the robotic manipulator that is related to the robot joint-space as

$$X_l = f_l(q_l), \quad \dot{X}_l = J_l(q_l)\dot{q}_l, \tag{2}$$

where $f_l(q_l) \in R^{m\times m}$ denotes the differentiable forward kinematics of the manipulator, and $J_l(q_l) = \partial f_l(q_l)/\partial q_l \in R^{m\times m}$ denotes the differentiable Jacobian matrix that is assumed to be known in this paper. Additionally, the Jacobian matrix is a square matrix as the robot is considered a non-redundant manipulator for simplicity.

### *B. Slave Side-Swarm Robot*

The dynamics of the $i^{\text{th}}$ fully actuated mobile robot in the swarm can be described by [9], [10], [19]

$$M_{ri}(q_{ri})\ddot{q}_{ri} + C_{ri}(q_{ri},\dot{q}_{ri})\dot{q}_{ri} + g_{ri}(q_{ri}) = u_{ri} + u_{ei}, \tag{3}$$

where $q_{ri} \in R^r$ is the vector of generalized coordinates, $M_{ri}(q_{ri}) \in R^{r\times r}$ is the symmetric and positive-definite inertia matrix, $C_{ri}(q_{ri},\dot{q}_{ri}) \in R^{r\times r}$ is the matrix of Coriolis/Centrifugal terms where the matrix $\dot{M}_{ri}(q_{ri}) - 2C_{ri}(q_{ri},\dot{q}_{ri})$ is skew symmetric under an appropriate definition of the matrix $C_{ri}(q_{ri},\dot{q}_{ri})$ [2], [10], $g_{ri}(q_{ri}) = \partial G_{ri}(q_{ri})/\partial q_{ri} \in R^r$ is the gradient of the potential function $G_{ri}(q_{ri})$, $u_{ri} \in R^r$ is the vector of applied control torque, and $u_{ei} \in R^r$ is the external force applied to the $i^{\text{th}}$ robot. The subscript $r$ in (3) denotes the remote site of the proposed system.

In this paper, we consider that the group of swarm is constituted by $n$ mobile robots whose dynamics are described by (3). By denoting $q_r = [q_{r1}^T,\dots,q_{rn}^T]^T$, $M_r(q_r) = \text{diag}\{M_{r1},\dots,M_{rn}\}$, $C_r(q_r,\dot{q}_r) = \text{diag}\{C_{r1},\dots,C_{rn}\}$, $g_r = [g_{r1}^T,\dots,g_{rn}^T]^T$, and $u_r = [u_{r1}^T,\dots,u_{rn}^T]^T$, $u_e = [u_{e1}^T,\dots,u_{en}^T]^T$, the dynamics of the swarm robots are given by

$$M_r(q_r)\ddot{q}_r + C_r(q_r,\dot{q}_r)\dot{q}_r + g_r(q_r) = u_r + u_e, \tag{4}$$

where $M_r(q_r) \in R^{rn\times rn}$, $C_r(q_r,\dot{q}_r) \in R^{rn\times rn}$, $q_r \in R^{rn}$, $g_r \in R^{rn}$, $u_r \in R^{rn}$, and $u_e \in R^{rn}$. Following the dynamics (3), the matrix $\dot{M}_r - 2C_r$ is skew symmetric.

Therefore, the task-space function of the swarm robots (4), similar to a robotic manipulator, can be described as

$$X_r = f_r(q_r), \quad \dot{X}_r = J_r(q_r)\dot{q}_r, \tag{5}$$

where $X_r \in R^m$ is the vector of task functions that will be defined subsequently to represent the characteristic of the swarm robots, and $J_r(q_r) \in R^{m\times rn}$ is the Jacobian matrix corresponding to $X_r$. The relationship between the generalized coordinates of the swarm robots and the task-space function is given by $f_r(q_r) = [f_{r1}(q_r) \ \dots \ f_{rm}(q_r)]^T \in R^m$, where $f_{ri}(q_r)$ are differentiable scalar task functions that suitably depend on the state of swarm to describe the abstractions of the group of robots. Accordingly, the Jacobian matrix of the swarm robot can be represented by

$$J_r(q_r) = \frac{\partial f_r(q_r)}{\partial q_r} = \Big[\frac{\partial f_{r1}}{\partial q_r}^T,\dots,\frac{\partial f_{rm}}{\partial q_r}^T\Big]^T, \tag{6}$$

where $\partial f_{ri}/\partial q_r$ is the $i^{\text{th}}$ row of the Jacobian matrix $J_r$. Based on the definition of $J_r$, the external force applied on the swarm group can be represented by $u_e = J_r^T F_r$, where $F_r \in R^m$ denotes the external force corresponding to the task-space function of the swarm robots. Therefore, by assigning appropriate task-space function for the abstraction of the swarm robot, the group behavior of the swarm robots can be controlled by manipulating the corresponding task functions [8], [14]. The design of artificial task functions $f_{ri}(q_r)$ will be addressed in Section IV.

In this paper, we assume $rn > m$ such that the augmented dynamic model of swarm robots (4) performs like a redundant robotic system. Therefore, the Jacobian matrix $J_r$ is a non-square matrix so that the null-space of the swarm robots can be controlled to provide the swarm robot the ability to achieve several sub-tasks [20], [21].

## III. CONTROLLER DESIGN

The control algorithm is proposed and studied in this section for the system framework illustrated in Fig. 2, and system model formulated in the previous section. Following the dynamic model of the human-swarm system, the tracking errors in task space between the robotic manipulator and the group of swarm are defined by $e_l(t) = 1/\alpha X_r(t-T_r) - X_l(t)$ and $e_r(t) = \alpha X_l(t-T_l) - X_r(t)$, where $\alpha$ is the ratio between the end-effector of the manipulator and the task-space function of the swarm robots. As illustrated in Fig. 2, $T_i$ for $i = \{l, r\}$ in the tracking errors denote the communication delays for signals transmitted between the human operator and the swarm robots. The constant $\alpha$ is utilized to scale the workspace of the manipulator for the workspace of the swarm robot. Thus, if $\lim_{t\to\infty} e_l(t) = \lim_{t\to\infty} e_r(t) = 0$, the task functions that represent the characteristics of the swarm robots can track the scaled position of the robotic manipulator that is commanded by the human operator.

For the robotic manipulator (1), we define $s_l \in R^m$ as

$$s_l = -J_l^{-1} k_t e_l + \dot{q}_l, \tag{7}$$

where $k_t$ is a scalar control gain, and $J_l^{-1} \in R^{m\times m}$ denotes the inverse of $J_l$. Let $v_l = \dot{q}_l - s_l$, we have $v_l = J_l^{-1} k_t e_l$. By denoting $a_l$ the differential of $v_l$, it is obtained that $a_l = \ddot{q}_l - \dot{s}_l = \dot{J}_l^{-1} k_t e_l + J_l^{-1} k_t \dot{q}_l$.

Similarly for the dynamics of the swarm robot (4), we define $s_r \in R^{rn}$ as

$$s_r = -J_r^{+} k_t e_r + \dot{q}_r - (I_{rn} - J_r^{+} J_r)\Psi_s, \tag{8}$$

where $I_{rn} \in R^{rn\times rn}$ is an identical matrix, $J_r^{+} \in R^{rn\times m}$ denotes the pseudoinverse of $J_r$, which is defined by $J_r^{+} = J_r^T (J_r J_r^T)^{-1}$ and satisfy $J_r J_r^{+} = I_m$ [21], and $\Psi_s \in R^{rn}$ is the auxiliary function employed to control the additional degree-of-freedom (null space) of the swarm robot via the projection $(I_{rn} - J_r^{+} J_r)$. (The design of function $\Psi_s$ will be discussed in detail in Section IV.) Subsequently, we define $v_r = \dot{q}_r - s_r$ such that $v_r = J_r^{+} k_t e_r$. Then, $a_r$, the differential of $v_r$, can be given by $a_r = \ddot{q}_r - \dot{s}_r = \dot{J}_r^{+} k_t e_r + J_r^{+} k_t \dot{e}_r + \frac{d}{dt}[(I_{rn} - J_r^{+} J_r)\Psi_s]$.

From the definition of $v_i$ and $a_i$ for $i = \{l, r\}$, we get $\dot{q}_i = s_i + v_i$ and $\ddot{q}_i = \dot{s}_i + a_i$. By substituting $\dot{q}_i$ and $\ddot{q}_i$ into the dynamics model (1) and (4), we have

$$M_i(q_i)(\dot{s}_i + a_i) + C_i(q_i, \dot{q}_i)(s_i + v_i) + g_i(q_i) = u_i + J_i^T F_i. \tag{9}$$

Since the robot dynamics are linearly parameterizable [10], [18] such that $M(q)\dot{\xi} + C(q,\dot{q})\xi + g(q) = Y(q,\dot{q},\xi,\dot{\xi})\Theta$, where $\xi \in R^{\beta}$ is any differentiable vector, $\Theta \in R^{w}$ is a constant vector of dynamic parameters, and $Y(q,\dot{q},\xi,\dot{\xi}) \in R^{\beta\times w}$ denotes the matrix of known functions, the equation (9) can be written as

$$M_i(q_i)\dot{s}_i + C_i(q_i,\dot{q}_i)s_i = u_i + J_i^T F_i - Y_i(q_i,\dot{q}_i,v_i,a_i)\Theta_i. \tag{10}$$

By defining $p_i = J_i s_i = -k_t e_i + \dot{X}_i$, where the property that $J_r(I_{rn} - J_r^{+} J_r) = 0$ [21] is utilized for the swarm robots, the proposed control input for the human-swarm system is given as

$$u_i = Y_i\hat{\Theta}_i - k_{si}s_i - k_p J_i^T p_i + k_d J_i^T \dot{e}_i, \tag{11}$$

where $k_{si}, k_p, k_d$ are positive control gains, and $\hat{\Theta}_i$ is the estimate of the uncertain parameter vector $\Theta_i$. By substituting $u_i$ into the robot dynamics (10), the closed-loop dynamics of the proposed system becomes

$$\begin{aligned} M_i(q_i)\dot{s}_i + C_i(q_i,\dot{q}_i)s_i + k_{si}s_i \\ = Y_i\tilde{\Theta}_i - k_p J_i^T p_i + k_d J_i^T \dot{e}_i + J_i^T F_i, \end{aligned} \tag{12}$$

where $\tilde{\Theta}_i = \hat{\Theta}_i - \Theta_i$ for $i = \{l, r\}$ is the estimated error for the unknown vector $\Theta_i$, and the estimated dynamic parameter vector $\hat{\Theta}_i$ is updated by the adaptive law

$$\dot{\hat{\Theta}}_i = -\Gamma_i Y_i^T s_i. \tag{13}$$

Thus, the stability analysis of the presented closed-loop control system follows

*Theorem 3.1:* Consider the closed-loop human-swarm system described by (12) with the update law (13). Assume that the Jacobian matrix of the robotic system is full rank. If there exists no external force ($F_l = F_r = 0$), then all signals in the closed-loop system are bounded. Additionally, the scaled tracking errors $e_l$, $e_r$, velocities $\dot{X}_l$, $\dot{X}_r$, and $s_r$ asymptotically approach the origin independent of constant delays in the communication network.

*Proof:* Consider a positive-definite storage functional for the human-swarm system as

$$\begin{aligned} V = &\frac{\alpha}{2}\Big(s_l^T M_l s_l + \tilde{\Theta}_l^T \Gamma_l^{-1}\tilde{\Theta}_l + k_t k_d e_l^T e_l\Big) + \frac{1}{2\alpha}\Big(s_r^T M_r s_r \\ &+ \tilde{\Theta}_r^T \Gamma_r^{-1}\tilde{\Theta}_r + k_t k_d e_r^T e_r\Big) + \frac{\alpha k_d}{2}\int_{t-T_l}^{t} \dot{X}_l^T(\tau)\dot{X}_l(\tau)d\tau \\ &+ \frac{k_d}{2\alpha}\int_{t-T_r}^{t} \dot{X}_r^T(\tau)\dot{X}_r(\tau)d\tau. \end{aligned} \tag{14}$$

Taking time derivative of the functional $V$, we have

$$\begin{aligned} \dot{V} = &\frac{\alpha}{2}\Big(s_l^T \dot{M}_l s_l + 2s_l^T M_l \dot{s}_l + 2\tilde{\Theta}_l^T \Gamma_l^{-1}\dot{\hat{\Theta}}_l + 2k_t k_d e_l^T \dot{e}_l\Big) \\ &+ \frac{1}{2\alpha}\Big(s_r^T \dot{M}_r s_r + 2s_r^T M_r \dot{s}_r + 2\tilde{\Theta}_r \Gamma_r^{-1}\dot{\hat{\Theta}}_r + 2k_t k_d e_r^T \dot{e}_r\Big) \\ &+ \frac{\alpha k_d}{2}(\dot{X}_l^T \dot{X}_l - \dot{X}_l^T(t-T_l)\dot{X}_l(t-T_l)) \\ &+ \frac{k_d}{2\alpha}(\dot{X}_r^T \dot{X}_r - \dot{X}_r^T(t-T_r)\dot{X}_r(t-T_r)). \end{aligned} \tag{15}$$

Substituting the closed-loop system (12) and the adaptive law (13) into the above equation, we get

$$\begin{aligned} \dot{V} = &-\alpha k_{sl} s_l^T s_l - \frac{k_{sr}}{\alpha} s_r^T s_r - \alpha k_p p_l^T p_l - \frac{k_p}{\alpha} p_r^T p_r + \alpha k_d p_l^T \dot{e}_l \\ &+ \frac{k_d}{\alpha} p_r^T \dot{e}_r + \frac{\alpha k_d}{2}(\dot{X}_l^T \dot{X}_l - \dot{X}_l^T(t-T_l)\dot{X}_l(t-T_l)) \\ &+ \frac{k_d}{2\alpha}(\dot{X}_r^T \dot{X}_r - \dot{X}_r^T(t-T_r)\dot{X}_r(t-T_r)) + \alpha k_t k_d e_l^T \dot{e}_l \\ &+ \frac{k_t k_d}{\alpha} e_r^T \dot{e}_r. \end{aligned} \tag{16}$$

By substituting $p_i = -k_t e_i + \dot{X}_i$ to the terms of $p_i^T \dot{e}_i$ with $\dot{e}_l = \frac{1}{\alpha}\dot{X}_r(t-T_r) - \dot{X}_l$ and $\dot{e}_r = \alpha \dot{X}_l(t-T_l) - \dot{X}_r$, the above equation can be written as

$$\dot{V} = -\alpha(k_{sl}s_l^T s_l + k_p p_l^T p_l + \frac{k_d}{2}\dot{e}_l^T \dot{e}_l) - \frac{1}{\alpha}(k_{sr}s_r^T s_r + k_p p_r^T p_r + \frac{k_d}{2}\dot{e}_r^T \dot{e}_r) \le 0. \quad (17)$$

Since $V$ is positive-definite and $\dot{V}$ is negative semi-definite, $\lim_{t\to\infty} V$ exists and is finite. Hence, $s_i, p_i, \dot{e}_i \in \mathscr{L}_2$ and $s_i, e_i, \tilde{\Theta}_i \in \mathscr{L}_\infty$. From the closed-loop of the system (12), we get $\dot{s}_i \in \mathscr{L}_\infty$. By invoking Barbalat's Lemma [22], we obtain that $\lim_{t\to\infty} s_i(t) = 0$. Since $p_i = -k_t e_i + \dot{X}_i = J_i s_i$, we have $\dot{X}_i \in \mathscr{L}_\infty$. As $s_i, e_i \in \mathscr{L}_\infty$, $\dot{q}_i \in \mathscr{L}_\infty$. From the derivative of $p_i = J_i s_i$ that $\dot{p}_i = \dot{J}_i s_i + J_i \dot{s}_i$, we further obtain $\dot{p}_i \in \mathscr{L}_\infty$. Since $p_i \in \mathscr{L}_2$ and $\dot{p}_i \in \mathscr{L}_\infty$, by invoking Barbalat's Lemma again, we get $\lim_{t\to\infty} p_i(t) = 0$. Following the proof of Theorem 2.1 in [7], we conclude that $\lim_{t\to\infty} \dot{X}_r(t) = 0$. Similarly, we obtain $\lim_{t\to\infty} \dot{X}_l(t) = 0$. Consequently, from $p_i = -k_t e_i + \dot{X}_i$, we have $\lim_{t\to\infty} e_i(t) = 0$, which implies $\lim_{t\to\infty} \big(X_r(t) - \alpha X_l(t)\big) = 0$. ■

By considering the case when the external force $F_i$ is passive with respect to $p_i$ as the output, there exists a constant $c_i \in R^+$ such that [2], [17]

$$-\int_0^t F_i^T(\tau) p_i(\tau) d\tau \ge -c_i^2, \quad \text{for } i = \{l, r\}. \quad (18)$$

Hence, we have the next corollary.

*Corollary 3.2:* Consider the closed-loop system of the human-swarm framework described by (12) and the update law (13). Assume that the Jacobian matrix of the robotic system is full rank. If the external forces from the human operator and the remote environment satisfy the passivity condition (18), then the closed-loop system is stable, and the scaled tracking error $e_l$, $e_r$, velocity $\dot{X}_l$, $\dot{X}_r$, and $s_r$ asymptotically approach the origin independent of constant communication delays.

*Proof:* By considering the storage functional $V$

$$V = \frac{\alpha}{2}\Big(s_l^T M_l s_l + \tilde{\Theta}_l^T \Gamma_l^{-1} \tilde{\Theta}_l + k_t k_d e_l^T e_l\Big) + \frac{1}{2\alpha}\Big(s_r^T M_r s_r + \tilde{\Theta}_r^T \Gamma_r^{-1}\tilde{\Theta}_r + k_t k_d e_r^T e_r\Big) + \frac{\alpha k_d}{2}\int_{t-T_l}^t \dot{X}_l^T(\tau)\dot{X}_l(\tau) d\tau + \frac{k_d}{2\alpha}\int_{t-T_r}^t \dot{X}_r^T(\tau)\dot{X}_r(\tau) d\tau + \alpha c_l^2 - \alpha \int_0^t F_l^T(\tau) p_l(\tau) d\tau + \frac{c_r^2}{\alpha} - \frac{1}{\alpha}\int_0^t F_r^T(\tau) p_r(\tau) d\tau, \quad (19)$$

the proof can be completed by following the proof of Theorem 3.1. ■

## IV. TASK FUNCTIONS

The scaled position tracking of the proposed human-swarm system is demonstrated in the preceding section that $\lim_{t\to\infty} X_r(t) = \lim_{t\to\infty} \alpha X_l(t)$, which implies that the end-effector of the master robot and the task space of the swarm robot converge to each other under the scale $\alpha$. Therefore, the movement of the swarm robot can be teleoperated by the human operator through the design of an appropriate task function $f_r(q_r)$ for various applications. This type of task function, which is named as teleoperation tasks in this paper, provides a mechanism for the human operator to directly influence the group of swarm robots in a remote environment. In addition to the teleoperation tasks, the null space of the redundant swarm robots can be controlled to achieve additional tasks by properly choosing the auxiliary function $\Psi_s$, and this part of task function is called autonomous task. The designs of these two types of task are introduced in this section.

### A. Teleoperation Task

*1) Centroid of swarm robots:* The first teleoperation task function introduced in this section is a function representing the average position of the swarm robots. By denoting $q_{rij} \in R$ the $j^{\text{th}}$ coordinate of the $i^{\text{th}}$ robot in the swarm, the task function that represents the centroid of the swarm robots in the $j^{th}$ coordinate $f_{\sigma_j}(q_r)$ is defined by

$$f_{\sigma_j}(q_r) = \frac{1}{n}\sum_{i=1}^n q_{rij}, \quad (20)$$

where $n$ is the number of robots in the swarm. Based on the definition of the task function, the partial derivative of $f_{\sigma_j}(q_r)$ can be given by

$$\frac{\partial f_{\sigma_j}(q_r)}{\partial q_r} = [\ \dots\ \ 0_{j-1}\ \ \frac{1}{n}\ \ 0_{r-j}\ \ 0_{j-1}\ \ \frac{1}{n}\ \ 0_{r-j}\ \ \dots\ ], \quad (21)$$

where $0_j$ denotes a $j$-dimensional zero row vector, and $\partial f_{\sigma_j}(q_r)/\partial q_r \in R^{rn}$. For the use of this task function $f_{\sigma_i}(q_r)$ being one of the entry in task space $f_{ri}(q_r)$, the human operator can individually control the average position of the $j^{\text{th}}$ coordinate of the swarm robots through teleoperation.

*2) Variance of swarm robots:* Even though the task function for average position can provide the human operator to control the centroid of the swarm robots, only commanding the average position might lead to divergent swarm position. Therefore, the next teleoperation task considered is the variance of the swarm robots. The task function of the total swarm variance can be given by

$$f_\mu(q_r) = \frac{1}{rn}\sum_{j=1}^r \sum_{i=1}^n (q_{rij} - f_{\sigma_j}(q_r))^2, \quad (22)$$

where the value $r$ in the denominator is the number of degree-of-freedom of the individual swarm robot. Thus, the partial derivative of $f_\mu(q_r)$ is

$$\frac{\partial f_\mu(q_r)}{\partial q_r} = \Big[\ \dots\ \frac{\partial f_{\mu_{j-1}}(q_r)}{\partial q_{rij-1}}\ \frac{\partial f_{\mu_j}(q_r)}{\partial q_{rij}}\ \frac{\partial f_{\mu_{j+1}}(q_r)}{\partial q_{rij+1}}\ \dots\ \Big],$$

where $\partial f_{\mu_j}(q_r)/\partial q_{rij} = \frac{2}{rn^2}\big((n-1)q_{rij} - \sum_{l=1,l\ne i}^n q_{rlj}\big)$

With the use of swarm variance for teleoperation task, all entries of $\partial f_\mu(q_r)/\partial q_r$ are zero if only if $q_{rij} = q_{rlj}$, $\forall i \ne l$, which means that all swarm robots are located in the same position. This situation is unacceptable in practice and can be avoided by designing suitable autonomous task functions, i.e. inter-robot collision avoidance.

## B. Autonomous Task

The natural of redundancy for the swarm robots can be utilized to achieve additional tasks other than tracking the teleoperation task function received from the human operator. Since Theorem 3.1 and Corollary 3.2 have demonstrated that $\lim_{t\to\infty} s_r(t) = 0$, based on the definition in [6], [20], [21], the convergence of the sub-task tracking error for the redundant robotic system to the origin can be guaranteed. Hence, the null space of the swarm robots can be controlled to achieve additional tasks autonomously. The design of autonomous tasks for the human-swarm system is addressed in this section.

The sub-task control, which is considered as the autonomous task in the framework of human-swarm interaction system, can be achieved by designing appropriate auxiliary function $\Psi_s$. The vector $\Psi_s$, defined by a negative gradient of an auxiliary function $f_a(q_r)$ [23], is given as $\Psi_s^T = -\partial f_a(q_r)/\partial q_r$. According to [6], [20], [21], the auxiliary function $f_a(q_r)$ can be designed appropriately such that the lower value corresponds to more desirable configurations. Two examples of the autonomous task functions are introduced in this section for the control of swarm robots.

*1) Inter-robot distance:* For the swarm robotic system with $rn < m$, controlling the average position and variance of swarm robots has infinite solution for the swarm formation. Robots might gather together in several small groups while achieving the teleoperation tasks. However, in some applications such as coverage control of mobile robot network, increasing the covering area is necessary to improve the overall performance. Therefore, the first autonomous task considered in this paper is to maximize inter-robot distance in order to enlarge the area of coverage. By denoting $d(i,j)$ the distance between the $i^{th}$ robot and the $j^{th}$ robot, the task function are given by

$$f_d(q_r) = -\frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d(i,j), \tag{23}$$

which is the average of the inter-robot distance. Thus, the gradient of the function $f_d(q_r)$ can be shown as

$$\frac{\partial f_d(q_r)}{\partial q_r} = -\frac{2}{n(n-1)} \Big[ \cdots \ \frac{\partial f_d(q_r)}{\partial q_{rij-1}} \ \frac{\partial f_d(q_r)}{\partial q_{rij}} \ \frac{\partial f_d(q_r)}{\partial q_{rij+1}} \ \cdots \Big].$$

If $d(i,j)$ is given by Euclidean distance such that $d(i,j) = \left(\sum_{l=1}^{r}(q_{ril} - q_{rjl})^2\right)^{1/2}$, then the function $\partial f_d(q_r)/\partial q_{rij}$ can be obtained from (23) with $\partial d(i,j)/\partial q_{ril} = (q_{ril} - q_{rjl})/d(i,j)$. Thus, the swarm robot can regulate their formation by utilizing the auxiliary function $\Psi_s^T = -\partial f_d(q_r)/\partial q_r$ in the controller to maximize the inter-robot distance in order to increase the coverage area autonomously. It is worth pointing out that this autonomous task should be performed cooperatively with the use of variance for teleoperation task. Since the teleoperation task has higher priority than the autonomous task, the group of swarm robots will not diverge because of the control on swarm variance by the teleoperation task.

*2) Avoidance of obstacles:* Due to the limited information from the remote environment to the human operator and time delays in the communication channels, the ability of swarm robots to autonomously avoid obstacles within the environment in real-time is necessary to improve the performance of human-swarm interaction system. Based on the proposed control framework, the collision avoidance algorithm presented by [24] for multi-agent system can be adopted to the autonomous tasks for swarm robots. If there exist collision-free configurations and paths, the group of swarm robots can change their formation to avoid obstacles in the environment.

By denoting $d(i,o_j)$ the distance between the $i^{\text{th}}$ robot to the $j^{\text{th}}$ obstacle in the remote environment, the task function is given by

$$f_{ao_j}(q_r) = \sum_{i=1}^{n} \left( \min\left\{0, \frac{d(i,o_j)^2 - R_o^2}{d(i,o_j)^2 - r_o^2}\right\}\right)^2, \tag{24}$$

where $R_o \in R^+$ is the avoidance distance, and $r_o \in R^+$ denotes the smallest safe distance of $d(i,o_j)$ such that $R_o > r_o$. The avoidance function $f_{ao_j}(q_r)$ is zero for $d(i,o_j) \geq R_o$, and the aim of this function is to regulate the configuration of swarm robots to guarantee that $d(i,o_j) > r_o$. By using this function, we assume that the initial configurations of the swarm robot satisfy $d(i,o_j) \geq R_o, \ \forall i,j$. Hence, the auxiliary function $\Psi_s$ for the avoidance function can be written by $\Psi_s^T = -\partial f_{ao}(q_r)/\partial q_r$, where

$$\partial f_{aoj}(q_r)/\partial q_r = \sum_{i=1}^{n} 4 \Big[ \frac{\left(R_o^2 - r_o^2\right)^2 \left(d(i,o_j)^2 - R_o^2\right)}{\left(d(i,o_j)^2 - r_o^2\right)^3} \Big] (q_{ri} - q_{oj}),$$

if $r_o < d(i,O_j) < R_o$, and $\partial f_{ao}(q_r)/\partial q_r = 0$ if $d(i,o_j) \geq R_o$ and $d(i,o_j) < r_o$. Moreover, the partial derivative function is not defined if $d(i,o_j) = r_o$ [7], [24].

The autonomous function (24) can be utilized for inter-robot collision avoidance by simply replacing $d(i,o_j)$ by $d(i,j)$, the distance between robots. The efficiency of this autonomous task function will be demonstrated in Section V.

# V. NUMERICAL EXAMPLES

Numerical simulations are presented in this section to validate the efficacy of the proposed bilateral human-swarm interaction with time delays. The model of 3-DOF Phantom Omni robotic manipulator [25] is considered as the master robot, and the swarm robot is described by a group of 8 fully actuated mobile robots moving on the X-Y plane. According to the proposed system framework and controller, the end-effector of the master robot is able to operate three different teleoperation task functions. In the simulation, we consider the average X-position $f_{\sigma_1}(q_r)$, the average Y-position $f_{\sigma_2}(q_r)$, and the swarm variance $f_\mu(q_r)$ as the three teleoperation tasks for the swarm robot. In addition to the teleoperation tasks, the autonomous tasks for increasing inter-robot distance and collision avoidance (see Section IV-B) are utilized in the following simulation.

By following the notations in [25], the physical parameters for the master robot are chosen as $m_1 = 0.5$, $m_2 = 0.3$, $m_3 =$

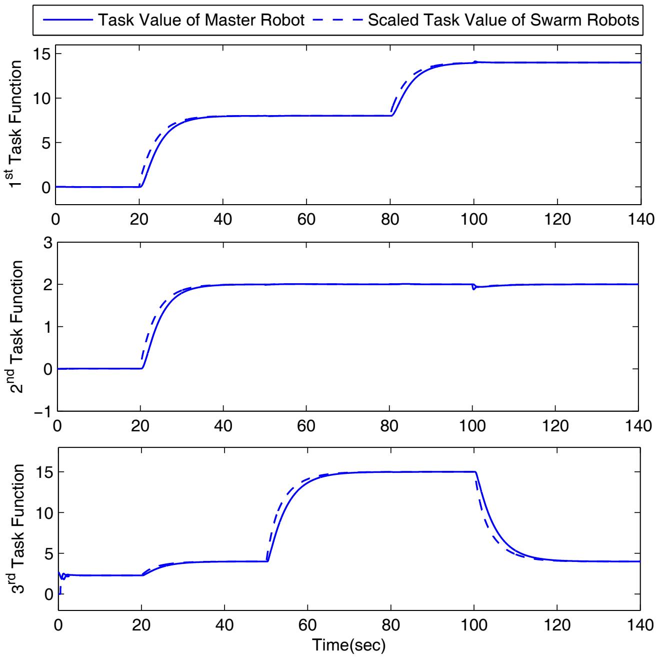


Fig. 3. Teleoperation task functions of the master robotic manipulator and the remote swarm robots.

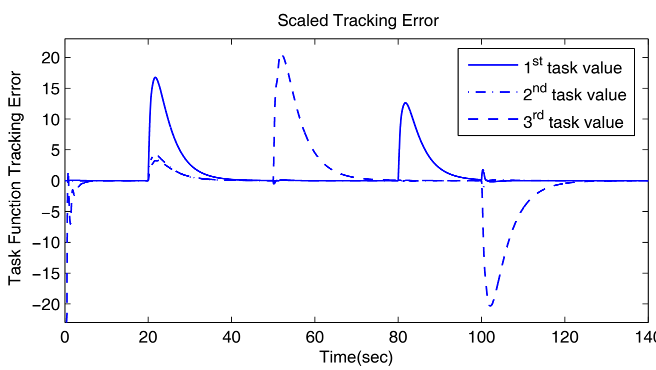


Fig. 4. Scaled tracking errors, $\alpha X_l(t) - X_r(t)$, for the human-swarm system.

0.6, $l_1 = 25$, $l_2 = 25$, $I_{1x} = 0.2$, $I_{1y} = 0.2$, $I_{1z} = 0.1$, $I_{2x} = 0.4$, $I_{2y} = 0.1$, $I_{2z} = 0.3$, $I_{3x} = 0.1$, $I_{3y} = 0.2$, $I_{3z} = 0.3$, and $g = 0$. Moreover, the augmented swarm model is given as time-invariant system with $M_r = 2I_{16}$ and $C_r = I_{16}$. Therefore, by choosing the control gains $k_t = 1$, $k_{sl} = 1$, $k_{sr} = 1$, $k_d = 1$, $k_p = 1$, $\Gamma_r = 0.01I_{32}$, and $\Gamma_l = 0.01I_8$, and selecting the communication delays $T_l = 0.3$sec, $T_r = 0.5$sec and the scaled constant $\alpha = 10$, the simulation results are illustrated in Fig. 3 to Fig. 6.

In this numerical example, the entire system is assumed to be in free-motion for $t < 20$sec. After $t = 20$sec, the human operator exerts a spring-damper force to manipulate the end-effector of the master robot towards $X_l = [8,\ 2,\ 4]^T$ for $t = 20 \sim 50$sec, $X_l = [8,\ 2,\ 15]^T$ for $t = 50 \sim 80$sec, $X_l = [14,\ 2,\ 15]^T$ for $t = 80 \sim 100$sec, and $X_l = [14,\ 2,\ 4]^T$ for $t = 100 \sim 140$sec. The swarm robots are teleoperated to attain the specified values of task functions, which are represent

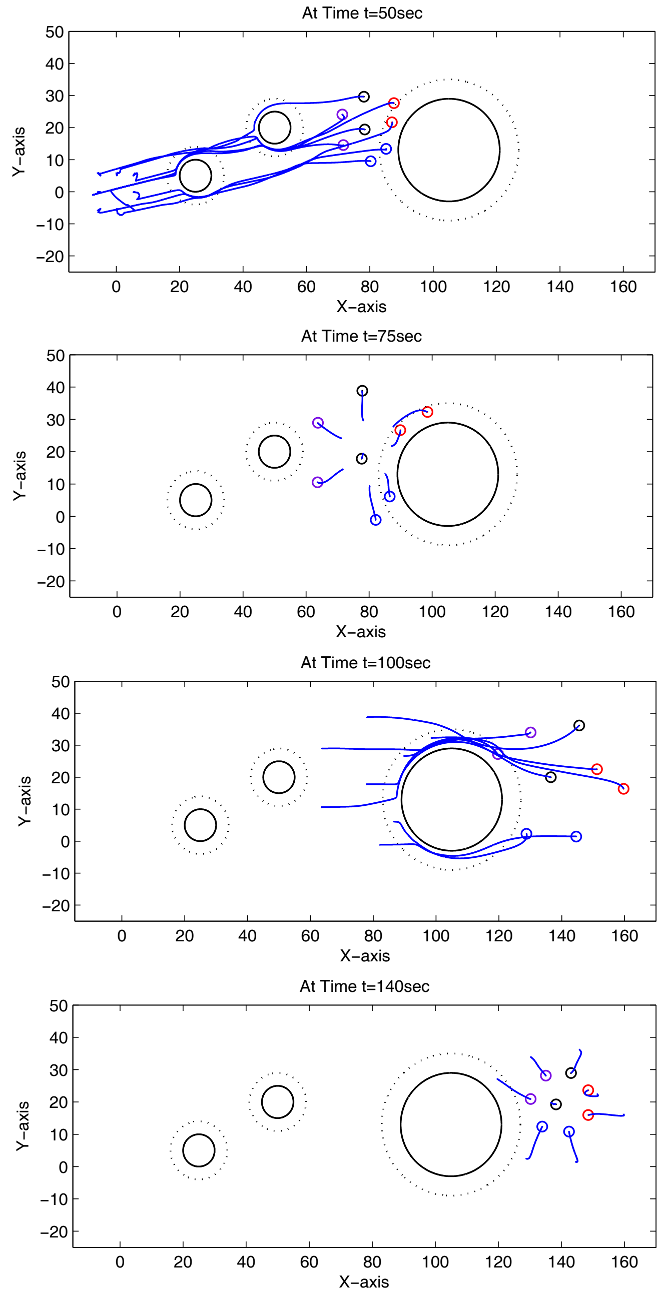


Fig. 5. Snapshots of the swarm robot in the remote environment with collision avoidance autonomous task control. The average X-position, average Y-position, and variance of the eight robots are corresponding to Fig. 3.

the X-average position ($1^{st}$ task function), Y-average position ($2^{nd}$ task function), and variance of the swarm robots ($3^{rd}$ task function). From Fig. 3, we can observe that the closed-loop system is stable in the presence of time delays, and the group of swarm robots can change its configuration to ensure that the task-space functions converge to the end-effector position of the master robot. The scaled tracking errors are shown in Fig. 4.

The snapshots of the remote swarm robots, corresponding to Fig. 3, are illustrated in Fig. 5. The circles with solid line denote the smallest safe distance $r_o$ that the robots should not enter, and the dashed circles denote the avoidance distance

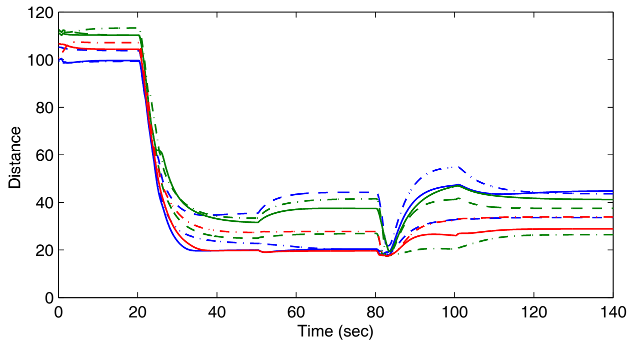


Fig. 6. Distances between the swarm robots to the center of the largest obstacle in the remote environment.

$R_o$ such that the autonomous tasks start to influence the movement of the swarm robots to avoid the obstacles. Based on the proposed control framework, the human operator can simultaneously teleoperate a group of swarm robots to regulate their position and variance. In addition, the swarm robots are able to change their configuration for avoiding obstacles and enlarging the inter-robot distance by utilizing the proposed autonomous tasks. Fig. 6 illustrates the distance between the eight swarm robots with the center of the largest obstacle. It can be observed that the robots start to change their configurations if the distance is less than $R_o = 22$ and keep away from the safe distance $r_o = 16$.

## VI. CONCLUSIONS AND FUTURE WORK

This paper proposed a system framework to enable a human operator to control a group of swarm robots in a remote environment. Task functions allow the operator to specify characteristics of the desired swarm-robot configuration and thereby teleoperate the swarm robots accordingly. A control algorithm was presented to guarantee position tracking between the task function of the master robot, controlled by a human operator, and the swarm robot under kinematic dissimilarity. Moreover, the control system has been proven to be stable independent of time delays in the communication channels. Since the group of swarm robots exhibits high level of redundancy, the additional degree-of-freedom was used to achieve several local tasks autonomously. Numerical examples demonstrated the efficacy of the proposed human-swarm interaction system. Future work will involve the study of non-passive external forces and time-varying delays in the communication network.

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## REFERENCES


[1] S. Bashyal and G. K. Venayagamoorthy, "Human swarm interaction for radiation source search and localization," in *IEEE Swarm Intelligence Symposium*, Sep. 2008.

[2] E. J. Rodriguez-Seda, J. J. Tory, C. A. Erignac, P. Murray, D. M. Stipanović, and M. W. Spong, "Bilateral teleoperation of multiple mobile agents: Coordinated motion and collision avoidance," *IEEE Transactions on Control Systems Technology*, vol. 18, no. 4, pp. 984–992, Jul. 2010.

[3] A. Franchi, C. Secchi, H. I. Son, H. H. Bülthoff, and P. R. Giordano, "Bilateral teleoperation of groups of mobile robots with time-varying topology," *IEEE Transactions on Robotics*, vol. 28, no. 5, pp. 1019–1033, Oct. 2012.

[4] D. Lee, "Semi-autonomous teleoperation of multiple wheeled robots over the internet," in *ASME Dynamic Systems and Control Conference*, Oct. 2008, pp. 147–154.

[5] M. A. Goodrich, B. Pendleton, P. B. Sujut, and J. Pinto, "Toward human interaction with bio-inspired robot teams," in *IEEE International Conference on Systems, Man, and Cybernetics*, Oct. 2011, pp. 2859–2864.

[6] Y.-C. Liu and N. Chopra, "Control of semi-autonomous teleoperation system with time delays," *Automatica*, vol. 49, no. 6, pp. 1553–1565, Jun. 2013.

[7] ——, "Semi-autonomous teleoperation in task space with redundant slave robot under communication delays," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Sep. 2011, pp. 679–684.

[8] G. Antonelli, F. Arrichiello, and S. Chiaverini, "Experiments of formation control with multirobot systems using the null-space-based behavioral control," *IEEE Transactions on Control Systems Technology*, vol. 17, no. 5, pp. 1173–1182, Sep. 2009.

[9] V. Gazi, "Swarm aggregations using artificial potentials and sliding-mode control," *IEEE Transactions on Robotics*, vol. 21, no. 6, pp. 1208–1214, Dec. 2005.

[10] C. C. Cheah, S. P. Hou, and J. J. E. Slotine, "Region-based shape control for a swarm of robots," *Automatica*, vol. 45, no. 10, pp. 2406–2411, Oct. 2009.

[11] H. G. Tanner, G. J. Pappas, and V. Kumar, "Leader-to-formation stability," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 3, pp. 443–455, Jun. 2004.

[12] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 243–255, Apr. 2004.

[13] M. J. Daigle, X. D. Koutsoukos, and G. Biswas, "Distributed disgnosis in formations of mobile robots," *IEEE Transactions on Robotics*, vol. 23, no. 2, pp. 353–369, Apr. 2007.

[14] B. E. Bishop, "On the use of redundant manipulator techniques for control of platoons of cooperating robotic vehicles," *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, vol. 33, no. 5, pp. 608–615, Sep. 2003.

[15] G. Anotnelli and S. Chiaverini, "Kinematic control of platoons of autonomous vehicles," *IEEE Transactions on Robotics*, vol. 22, no. 6, pp. 1285–1292, Dec. 2006.

[16] S. Chiaverini, "Singularity-robust task-priority redundancy resolution for real-time kinematic control of robot manipulators," *IEEE Transactions on Robotics and Automation*, vol. 13, no. 3, pp. 398–410, Jun. 1997.

[17] D. Lee and M. W. Spong, "Passive bilateral teleoperation with constant time delay," *IEEE Transactions on Robotics*, vol. 22, no. 2, pp. 269–281, Apr. 2006.

[18] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. New York: John Wiley & Sons, Inc., 2006.

[19] B. Ranjbar-Sahraei, F. Shabaninia, A. Nemati, and S.-D. Stan, "A novel robust decentralized adaptive fuzzy control for swarm formation of multiagent systems," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 8, pp. 3124–3134, Aug. 2012.

[20] E. Zergeroglu, D. M. Dawson, I. Walker, and P. Setlur, "Nonlinear tracking control of kinematically redundant robot manipulators," *IEEE/ASME Transactions on Mechatronics*, vol. 9, no. 1, pp. 129–132, Mar. 2004.

[21] Y.-C. Liu and N. Chopra, "Controlled synchronization of heterogeneous robotic manipulators in the task space," *IEEE Transactions on Robotics*, vol. 28, no. 1, pp. 268–275, Feb. 2012.

[22] H. K. Khalil, *Nonlinear Systems*. New Jersey: Prentice Hall, 2002.

[23] Y. Nakamura, *Advanced Robotics: Redundancy and Optimization*. Addison-Wesley, 1991.

[24] D. M. Stipanović, P. F. Hokayem, M. W. Spong, and D. D. Šiljak, "Cooperative avoidance control for multiagent systems," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 129, no. 5, pp. 699–707, Sep. 2007.

[25] Y.-C. Liu and N. Chopra, "Synchronization of networked mechanical systems with communication delays and human input," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 135, no. 4, Jul. 2013.