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Climbing and Descending Control of a Snake Robot on Step Environments based on Kinematics

Motoyasu Tanaka and Kazuo Tanaka

***Abstract*— This paper proposes a snake robot control method for climbing and descending a step. On a multi-plane step environment, it is necessary for locomotion to transfer from one plane to another. A snake robot can move with touching several planes as its body is long and thin. In this paper, we propose a control method for accomplishing trajectory tracking of a snake robot on a step environment. The control method consists of the tracking controller, the shifting method of the robot's part connecting between the planes, and the active lifting for controlling the shape of the robot. Simulations and experiments demonstrated the effectiveness of the proposed controller and the shifting method of the connecting part of the robot's body.**

I. INTRODUCTION

A snake can locomote in various environments, e.g., on uneven terrain, on walls, underwater, and on trees, despite the simple limbless body. Snake robots are expected to be able to locomote in the same types of environment, but it is difficult to control a snake robot because the number of actuators is very large and it moves by utilizing the friction force of its body. This paper deals with the snake robot which move by using the friction of its body and bending motion without generating propulsion by active wheels or active crawlers.

Recently, there has been an increase in number of studies involving intended three-dimensional rather than two-dimensional motion. Backborn curve [1] which represents a curve of the body of the snake robot has been usually used to generate three-dimensional motions. Backborn curve and its extension were proposed for multiple three-dimensional gaits, e.g. sidewinding, pole climbing, and helical rolling [2]–[5]. Date and Takita proposed a continuum model and the control method by discretizing the optimal bending moment [6]. However, these methods have not presented mathematical proofs of tracking of the desired trajectory.

Tracking controllers on a two-dimensional plane were proposed in [7]–[13]. However, there exists a trade-off between the singularity avoidance and the convergence of the tracking error in [7]–[9]. Matsuno found that introduction of links without wheels made the robot a kinematically redundant system and proposed the controller which accomplishes both the trajectory tracking of the robot's head and sub-tasks like a singularity avoidance using redundancy [10]–[12]. In the case of the three-dimensional motion on a plane, the main problem is which parts of the body of the snake robot should be grounded which should be raised. We proposed the

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M. Tanaka and K. Tanaka are with Department of Mechanical Engineering and Intelligent Systems, The University of Electro-Communications, Tokyo 182-8585, Japan mtanaka at uec.ac.jp, ktanaka at mce.uec.ac.jp

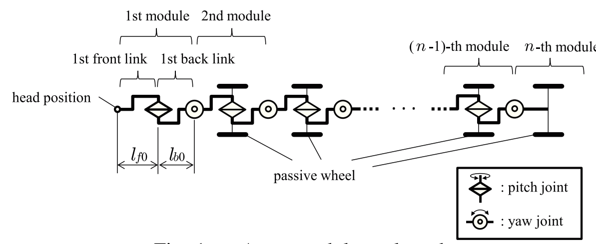

Fig. 1. An n-module snake robot

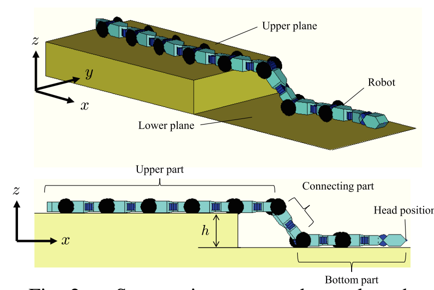

Fig. 2. Step environment and a snake robot

model and controller considering the switching of grounding parts and obtained the tracking motion with moving obstacle avoidance [13]. With respect to control in a smooth three-dimensional environment, trajectory tracking control on a cylindrical surface is proposed [14]. On the other hand, as control of snake robots in three-dimensional discrete environments, there have been no reports of tracking of the desired trajectory.

This paper proposes a control method to accomplish trajectory tracking of the robot's head and moving up and down in a step environment consisting of two parallel planes. We model the snake as a two-dimensional snake robot in which the grounding condition of the wheels switches dynamically and the length of the projections of links changes on a plane, by devising the motion of the "connecting part," which is the part of the robot's body connecting the two planes. We propose a controller to accomplish trajectory tracking considering the condition of touching the two planes, and the shifting method of the robot's part connecting the planes to accomplish moving from the upper plane to lower plane and vice versa. The proposed shifting condition can prevent the collision between the robot and the step in the descending case. Simulations and experiments demonstrated the effectiveness of the proposed control method.

II. MODELING

The controlled object and the setup environment are described and the kinematic model is derived by introducing

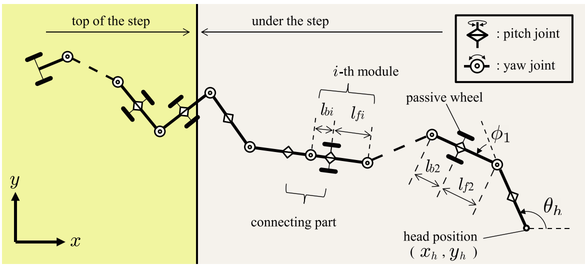

Fig. 3. xy projection model of the snake robot

some assumptions.

A. Snake robot

We consider an n-module snake robot as shown in Fig. 1. The wheeled module of this robot has a pitch rotational joint and is connected in series via a yaw rotational joint. The n-th module does not have a pitch joint. The module has the pitch joint and the passive wheel coaxially and the velocity constraint meaning no side slip occurs if the wheel touches the ground. The snake robot can perform the same locomotion as a living snake by bending its joints appropriately considering the velocity constraint. The passive wheel of the head link is removed to control the position and attitude of the snake head at the same time [10]. l_{f0} is the length from the anterior end of the link to the wheel axis, l_{b0} is the length from the posterior end of the link to the wheel axis, and l_{fi} and l_{bi} are the projections of l_{f0} and l_{b0} in xy plane, respectively. Let ψ_i be the pitch joint angle of the i-th module, ϕ_i be the yaw joint angle between i-th and $i+1$-st module, and let us define $\boldsymbol{\psi} = [\psi_1, \cdots, \psi_{n-1}]^T$, $\boldsymbol{\phi} = [\phi_1, \cdots, \phi_{n-1}]^T$.

B. Step environment

Figure 2 shows the step environment and the snake robot discussed in this paper. The environment consists of a step of height h against xz plane in the absolute coordinate system Σ_{xyz} in the step environment and the environment is spatially uniform to the y-axis. The robot moves down the step if $h > 0$, and moves up the step if $h < 0$. In the case where the snake robot moves on the step environment consisting of a number of planes, it is necessary for the robot to generate propulsion force by touching the wheels on each plane. The snake robot can locomote in this environment by moving both parts touching the planes and parts connecting the planes, but it is difficult to control as the model is a hybrid system involving the switching of touch conditions between wheels and planes dynamically.

For the robot, we call the part whose wheel touches the upper plane of the step environment the "upper part," that whose wheel touches the lower plane is called the "lower part," and that between the upper part and the lower part is called the "connecting part." It is preferred that the connecting part is shifted back and forth depending on the motion. For example, we consider locomotion in the direction of the x-axis from the state as shown in Fig. 2 with fixing of

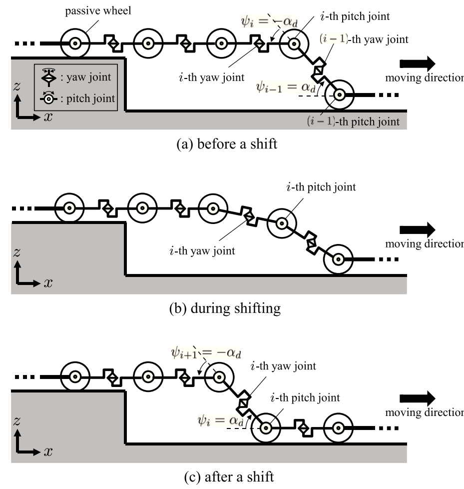

Fig. 4. Shift of the connecting part

the connecting part. The load of pitch joints increases with increasing number of ungrounded wheels along the increase in distance of locomotion. As many wheels touch the ground and the load on joints becomes small, it is necessary to shift the connecting part in the posterior direction.

Complicated motion is needed to plan the joint angle of the connecting part and to shift it with the wheels touching each plane. Thus, the following assumptions are introduced.
[Assumption 1]: The height of the step satisfies $|h| < l_{f0} + l_{b0}$.
[Assumption 2]: The initial configuration is that in which some links touch the lower plane and others touch the upper plane.
[Assumption 3]: The yaw angle of the connecting part is zero.
Assumptions 1 and 2 are introduced to simplify the control problem. By assumption 3, the direction of the wheel axis of the anterior link to the connecting part becomes the same as that of the posterior link, and the connecting part can connect the upper and lower parts such that each wheel touches each plane. In this case, planning of the connecting part only requires appropriately controlling the angles of the pitch joints.

By introducing these assumptions, the shift of the connecting part can be planned based only on the pitch joint angles. Therefore, it becomes possible to treat the three-dimensional motion divided into motion parallel to the xy-plane and that parallel to the z-axis.

C. Kinematic model

The robot can be treated as a two-dimensional snake robot by projecting onto the xy plane as shown in Fig. 3. In Fig. 3 ungrounded wheels are not shown (e.g., the wheel of the $i+1$-st module). Note that the length of the module changes with the pitch joint angle because it is the projection on a xy plane and the state of whether each wheel touches the ground is switched dynamically by the position and body shape of the robot. On the xy plane, let $\boldsymbol{w} = [x_h, y_h, \theta_h]^T$

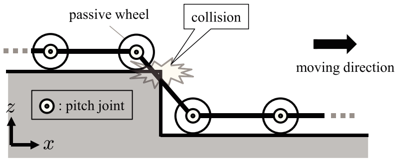

Fig. 5. A collision with the step

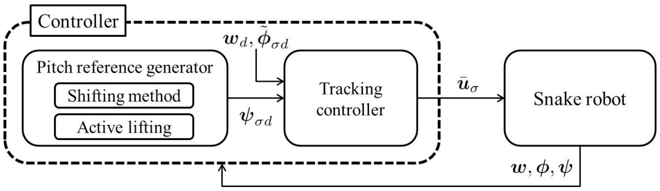

Fig. 6. Controller

be the position and attitude of the robot's head, l_{fi} be the length from the anterior end of the i-th link to the wheel axis, l_{bi} be the length from the posterior end of the i-th link to the wheel axis, and ψ_h be the absolute attitude of the robot's head, and let us define $\Psi_i = \psi_h + \Sigma_{j=1}^{i-1}\psi_j$. l_{fi} and l_{bi} are expressed as

$$\begin{aligned} l_{fi} &= l_{f0}\cos\Psi_i \\ l_{bi} &= l_{b0}\cos\Psi_{i+1}. \end{aligned} \tag{1}$$

The kinematic model of the snake robot on a xy plane changes with both the state of whether each wheel touches the ground and the state of the connecting part. In the case that wheels of m_a modules ($i_1, \cdots, i_{m_a}$-th) are ungrounded, we express the corresponding yaw angles as $\tilde{\boldsymbol{\phi}}_a = [\phi_{i_1-1}, \cdots, \phi_{i_{m_a}-1}]^T \in \mathbf{R}^{m_a}$ and the yaw angles of the connecting part as $\tilde{\boldsymbol{\phi}}_b = [\phi_{j_1}, \cdots, \phi_{j_{m_b}}]^T \in \mathbf{R}^{m_b}$. We call the unique integer depending on both the state of allocation of wheeled links and the state of allocations of the connecting part as "modes" and set the discrete mode number as σ. On the xy plane, the velocity constraints meaning no side slip, caused by wheels touching the ground, are expressed as

$$\begin{aligned} A_\sigma(\boldsymbol{\theta}, \boldsymbol{\psi})\dot{\bar{\boldsymbol{w}}}_\sigma &= B_\sigma(\boldsymbol{\theta}, \boldsymbol{\psi})\bar{\boldsymbol{u}}_\sigma \\ A_\sigma &\in \mathbf{R}^{(2n-m_b-2)\times(2+m+n)} \\ B_\sigma &\in \mathbf{R}^{(2n-m_b-2)\times(2n-m_b-2)} \end{aligned} \tag{2}$$

where $\boldsymbol{\theta} = [\theta_h, \boldsymbol{\phi}^T]^T$, $\bar{\boldsymbol{w}}_\sigma = [\boldsymbol{w}^T, \boldsymbol{\psi}^T, \tilde{\boldsymbol{\phi}}_\sigma^T]^T$, $\tilde{\boldsymbol{\phi}}_\sigma \in \mathbf{R}^m$ is the vector excepted $\tilde{\boldsymbol{\phi}}_b$ from $\tilde{\boldsymbol{\phi}}_a$, and $m = m_a - m_b$. $\tilde{\boldsymbol{\phi}}_\sigma \in \mathbf{R}^m$ called "shape controllable point (SCP)" are the joint angles which can be controlled directly, caused by introducing the ungrounded wheel [10] [11] [12]. $\bar{\boldsymbol{u}}_\sigma$ is $\bar{\boldsymbol{u}}_\sigma = [\dot{\bar{\boldsymbol{\phi}}}_\sigma^T, \dot{\boldsymbol{\psi}}^T]^T$ and $\bar{\boldsymbol{\phi}}_\sigma$ is the vector excepted $\boldsymbol{\phi}_b$ from $\boldsymbol{\phi}$. We set $\psi_h = \dot{\psi}_h = 0$ as the robot's head continues to move on the same plane. The control input $\bar{\boldsymbol{u}}_\sigma$ does not contain $\dot{\boldsymbol{\phi}}_b$ as the yaw angle of the connecting part is $\tilde{\boldsymbol{\phi}}_b = \mathbf{0}$ and is not allowed to rotate due to assumption 3. We assume that $\tilde{\boldsymbol{\phi}}_b = \mathbf{0}$ is fixed by the position control. Note that matrices and vectors in eq.(2) switch with mode σ.

This paper treats eq.(2) as the kinematic model of the snake robot on the xy plane.

III. CONTROLLER DESIGN

The control input related to the locomotion of the direction of the xy-axis is designed based on the kinematic model (2). Moreover, let the connecting part be shifted in the posterior direction by designing the desired value of the pitch joints in order that the ungrounded module touches the ground and vice versa sequentially.

Figure 6 shows the proposed controller. Pitch reference generator designs the desired value of the pitch joint angles based on the shifting method and the active lifting, and tracking controller calculates the input velocity by which the controlled variable converges to the desired value. In this section, we refer only to the descending case with regard to pitch reference generator.

A. Tracking controller

We set the desired vector of the controlled variable $\bar{\boldsymbol{w}}_\sigma$ as $\bar{\boldsymbol{w}}_{\sigma d}$. Let us define the control input $\bar{\boldsymbol{u}}_\sigma$ as

$$\bar{\boldsymbol{u}}_\sigma = B_\sigma^{-1}A_\sigma\left\{\dot{\bar{\boldsymbol{w}}}_{\sigma d} - K\left(\bar{\boldsymbol{w}}_\sigma - \bar{\boldsymbol{w}}_{\sigma d}\right)\right\} \tag{3}$$

where $K > 0$ is the feedback gain. The closed-loop system is expressed as

$$A_\sigma\left\{\left(\dot{\bar{\boldsymbol{w}}}_\sigma - \dot{\bar{\boldsymbol{w}}}_{\sigma d}\right) + K\left(\bar{\boldsymbol{w}}_\sigma - \bar{\boldsymbol{w}}_{\sigma d}\right)\right\} = \mathbf{0}. \tag{4}$$

If the matrix A_σ is full column rank, the uniqueness of the solution is guaranteed. The solution of (4) is given as

$$\left(\dot{\bar{\boldsymbol{w}}}_\sigma - \dot{\bar{\boldsymbol{w}}}_{\sigma d}\right) + K\left(\bar{\boldsymbol{w}}_\sigma - \bar{\boldsymbol{w}}_{\sigma d}\right) = \mathbf{0} \tag{5}$$

and $\bar{\boldsymbol{w}}_\sigma$ converges to the desired trajectory $\bar{\boldsymbol{w}}_{\sigma d}$ at $t \to \infty$. If the robot is the singular configuration in which all wheels are parallel or arc-like, the matrix A_σ is not full column rank. In this paper, θ_{hd} is set as a sinusoidal wave to avoid the singular configuration [11]. In addition, it is necessary that A_σ is not the horizontally long matrix to become full column rank, and the following condition should be satisfied.

$$\begin{aligned} 2n - m_b - 2 &\geq 2 + m + n \\ m_a &\leq n - 4. \end{aligned} \tag{6}$$

Condition (6) means that there is an upper limit to the number of ungrounded modules and it shows the necessity for the ungrounded module to touch the ground sequentially in the descending case.

Equation (5) has a feature that the controlled variable is switched depending on the mode. $\boldsymbol{w}$, which is involved in all modes, converges to the desired value because all the closed-loop systems corresponding to $\boldsymbol{w}$ are equal independent of the mode. However, in the case of arbitrary switching, the convergence of the SCP $\tilde{\boldsymbol{\phi}}_\sigma$ is not ensured because the elements of the SCP are varied by switching the mode. If the dwell time of each modes is sufficiently large, it is expected that $\tilde{\boldsymbol{\phi}}_\sigma$ converges sufficiently in each modes [15] [16]. There are times when the wheel of the non-grounded link is switched between contact and non-contact, but if the head of the robot moves forward a certain distance, the non-grounded link can continue to maintain a non-ground state because the non-grounded link cannot reach the upper plane geometrically. By maintaining the non-grounded state, the SCP corresponding to the non-grounded link is also kept as an element of the controlled variable, and finally converges to the desired value.

The desired values of the SCP $\tilde{\phi}_\sigma$ and the pitch angle ψ are proposed in the next section for utilizing the shift of the connecting part.

B. Shifting method of the connecting part

It is preferred that the connecting part is smoothly shifted from forward to backward depending on the motion. Let us define the connecting part is the part between $i-1$-st module and i-th module and let the shifting method be designed. In this paper, let the connecting part be shifted as shown in Fig. 4. Figure 4 shows (a) before a shift, (b) while shifting, and (c) after a shift of the connecting part and is described for the case in which all yaw angles are zero for readability.

1) Shifting condition: Let the border of the step be on the line $x = x_{step}$ and the direction of motion be the positive direction of x. Let us define the shifting condition of the connecting part as follows:

$$|\phi_i| < \epsilon \tag{7}$$

$$x_h - \{i(l_{f0} + l_{b0}) + l_{f0}\} > x_{step} \tag{8}$$

where $\epsilon > 0$ is a small value. ϕ_i is the yaw angle of the part that become the connecting part at next switching. Condition (7) means that ϕ_i becomes nearly zero and assumption 3 is approximately satisfied if this condition is satisfied. The left-hand side of (8) means the minimum position of x-axis in which the wheel of the $i+1$-st module can arrive. Therefore, condition (8) guarantees that the wheel of the $i+1$-st module does not touch the step. This is a conservative condition to prevent the robot from colliding with the step, as shown in Fig. 5. It would appear that shifting of the connecting part can be obtained by introducing (7) and (8) without collision with the step.

2) Preparation for shifting: From assumption 3 and condition (7), it is necessary to control ϕ_i to zero before shifting the connecting part. If condition (8) is satisfied, the wheel of the $i+1$-st module becomes ungrounded and ϕ_i becomes one of the SCPs. As the SCP can be controlled directly, we set the desired value of the SCP $\tilde{\phi}_{\sigma d}$ as

$$\tilde{\phi}_{\sigma d} = \mathbf{0}. \tag{9}$$

3) Desired value of the pitch joint: The connecting part can be shifted by moving the pitch angles $\psi_{i-1}, \psi_i, \psi_{i+1}$ as shown in Fig. 4. Let the appropriate angle of the pitch joint against the step be $\alpha_d = \sin^{-1} \frac{h}{l_{f0}+l_{b0}}$, the beginning time of the shift be t_{st}, the ending time of the shift be t_ψ, and let us define the desired value of the pitch joints $\psi_{(i-1)d}, \psi_{id}, \psi_{(i+1)d}$ as follows:

$$\psi_{id} = \alpha_d \left(2t' - 1\right) \tag{10}$$

$$\psi_{(i-1)d} = \alpha_d \left(1 - t'\right) \tag{11}$$

$$\psi_{(i+1)d} = -\alpha_d t' \tag{12}$$

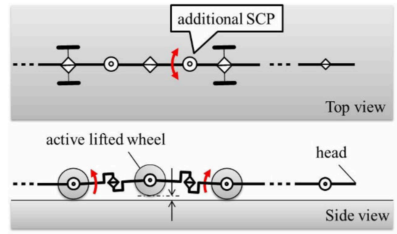

Fig. 7. Active lifting of the wheel

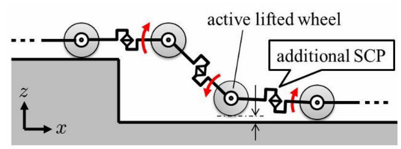

Fig. 8. Active lifting for descending a step

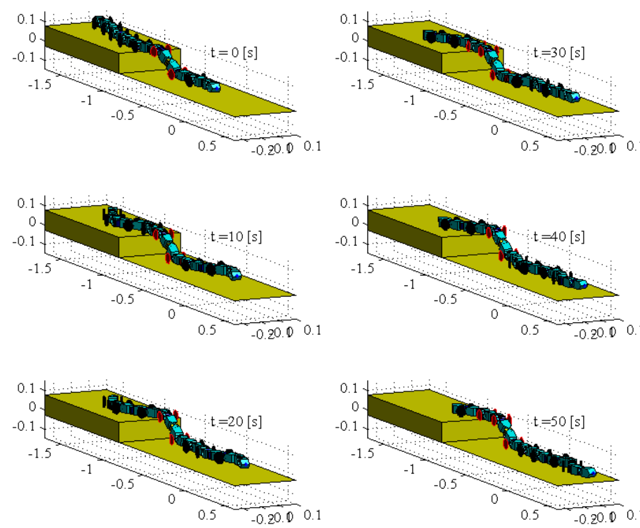

Fig. 9. Motion of the snake robot (simulation)

where t is the time, $t' = \frac{t-t_{st}}{t_\psi}$, and $t_{st} \leq t \leq t_{st} + t_\psi$. If the pitch joints satisfy (10), (11), and (12), the z length of the connecting part is h and the upper and lower parts of the robot can keep touching the ground.

C. Active lifting for the additional SCP

We consider to lift up arbitrary wheels actively as shown in Fig. 7. If one wheel is lifted up, the correspondent velocity constraint disappears. The number of SCPs increases by decreasing the number of velocity constraints [10]. These additional SCPs can affect the shape of the robot by controlled directly. If the lifting height is minute, the lifting angle of the pitch joint does not affect the motion of the yaw angles. In the descending case, in our motion design the robot lifts up the wheel of the connecting part as shown in Fig. 8 and controls the absolute angle of the additional SCP.

IV. Descending control

A. Simulations

Simulations were performed to demonstrate the effectiveness of the proposed control method. We consider an n-

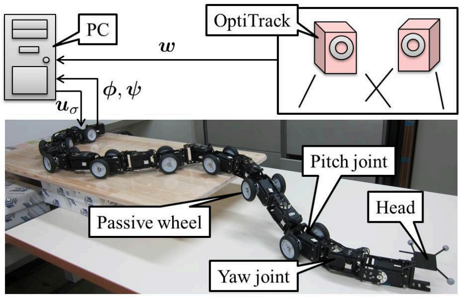

Fig. 10. The experimental system

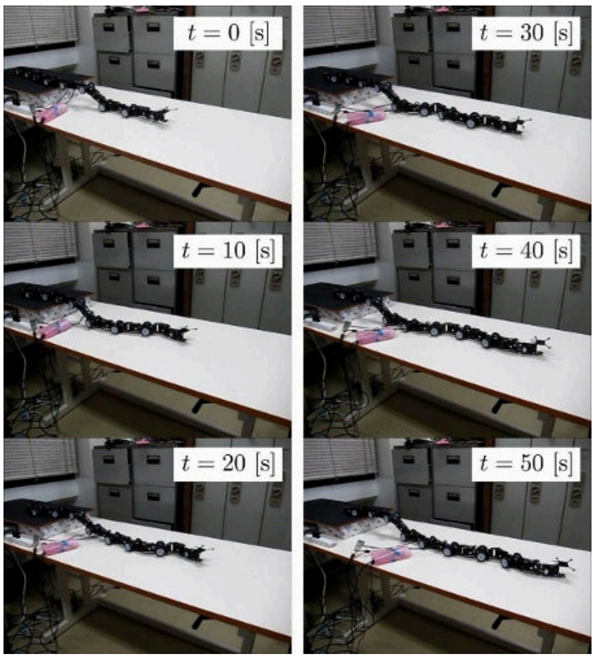

Fig. 11. Descending motion of the snake robot (experiment)

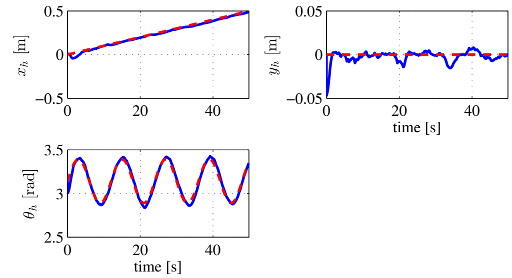

Fig. 12. Time responses of x_h, y_h, θ_h (experiment)

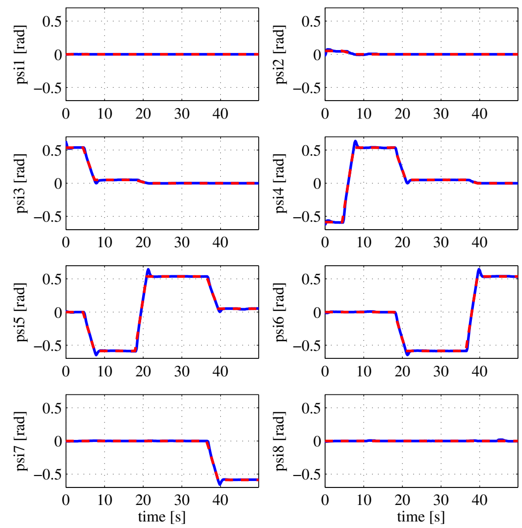

Fig. 13. Time responses of ψ_i (experiment)

module snake robot ($n = 9$) as shown in Fig. 2 and let us define $l_{f0} = l_{b0} = 0.088$[m], and $h = 0.105$[m]. Each parameter is equal to one of the experimental systems shown in the next section. Figure 9 shows the motion of the robot in the simulation. This means that the red wheel is the wheel that does not touch the ground, the red angle is the SCP, and the yellow angle is the angle of the connecting part in Fig. 9.

As shown in Fig. 9, the connecting part of the robot shifts backward sequentially without collision with the step.

B. Experiments

Experiments were performed to confirm the effectiveness of the proposed control method and the validity of simulation results. The experimental system is shown in Fig. 10. The position and attitude of the robot were measured by *OptiTrack* which is an optical motion capture systems and tracking software.

Figure 11 shows the motion of the robot in the experiment. Figures 12 and 13 show time responses of the controlled variables, and Figure 14 shows time responses of ϕ_i. Figures 11–13 show that the connecting part of the robot shifts backward sequentially without collision with the step and the controlled variable converges to the desired state along with the simulation. As shown in Fig. 14, there are periods when the values of $\phi_3, \phi_4, \phi_5, \phi_6$ converge to zero. ϕ_3 is zero from the start to $t = 7$[s] at which the connecting part shifts backward because ϕ_3 is the connecting part at the start time. ϕ_4, ϕ_5, ϕ_6 become the SCP when the wheel of the corresponding module leaves the ground, and converge to zero, which is the desired value. From Figs. 13 and 14, it can be seen that the connecting part shifts backward after the corresponding SCP converges to nearly zero. The attitude of the connecting part is constantly normal to the step in Fig. 11. This is because the wheel of the connecting part is lifted and the absolute angle of the connecting part is controlled by using the additional SCP.

As outlined above, the experimental results demonstrated the effectiveness of the control method and the validity of simulation results.

V. Climbing Control

The climbing motion can be obtained by modifying the shifting condition (8) and the position of the additional SCP, and by setting $h < 0$. Instead of (8), let us define the shifting condition for climbing as

$$n_{nw} < a_{nw} \tag{13}$$

where n_{nw} is the number of the non-contact wheel in the upper part and $a_{nw} > 0$ is an arbitrary integer. If the

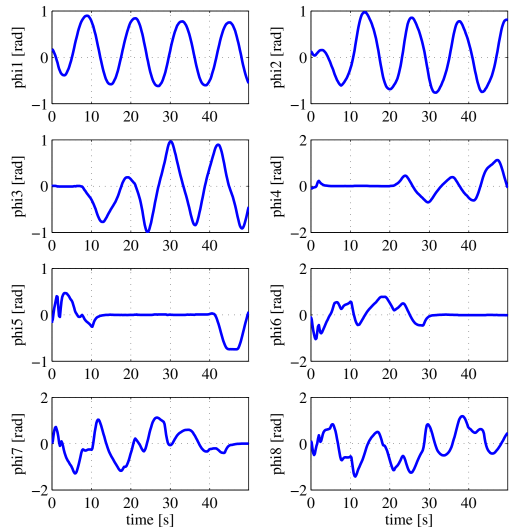

Fig. 14. Time responses of ϕ_i (experiment)

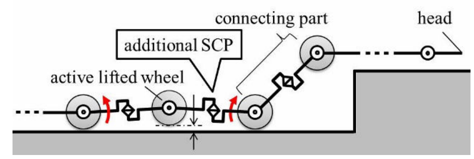

Fig. 15. Active lifting for climbing a step

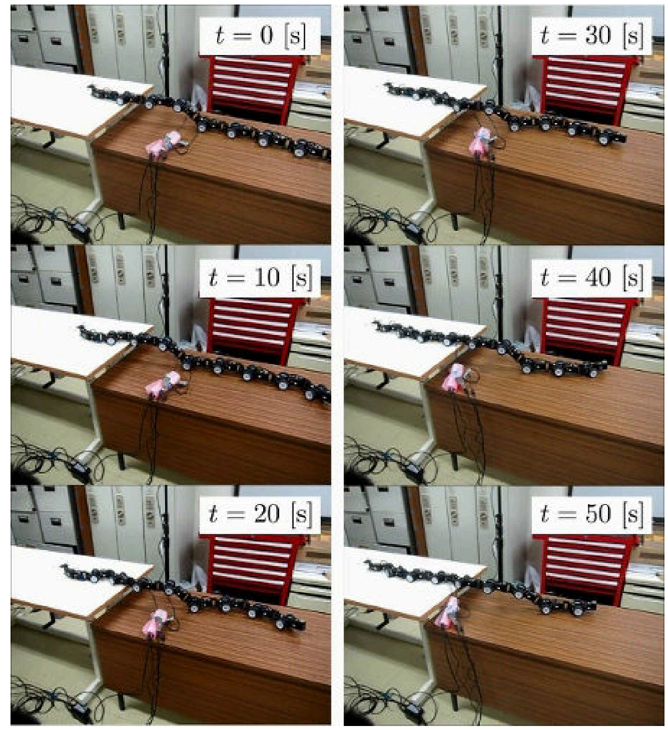

Fig. 16. Climbing motion of the snake robot (experiment)

connecting part is the part between $i-1$-st module and i-th module, it is necessary for the preparation for shifting that the robot makes ϕ_i into the SCP and controls it. We design the position of the active lifted wheel as shown Fig. 15 and makes ϕ_i into the additional SCP.

Figure 16 is the experimental result in $a_{nw} = 1$. It can be seen that the robot accomplishes to climb the step and to shift the connecting part.

VI. CONCLUSION

This paper proposed a control method to accomplish trajectory tracking of the robot's head and climbing and descending locomotion on a step environment consisting of two parallel planes. We derived the model of the system, and proposed a controller which is consist of tracking controller and pitch reference generator based on the shifting method of the connecting part and the active lifting for controlling the shape of the robot. Simulations and experiments demonstrated the effectiveness of the proposed control method.

In future studies, we will consider the control method of climbing and descending on stairs.

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