2013 IEEE/RSJ International Conference on
Intelligent Robots and Systems (IROS)
November 3-7, 2013. Tokyo, Japan

Assessing the Value of Coordination in Mobile Robot Exploration using a Discrete-Time Markov Process

Torsten Andre[1] and Christian Bettstetter[2]

Abstract— **With the introduction of multi-robot systems for exploration the question then arises, whether coordination among robots in such systems is required. We propose a model based on Markov processes to evaluate the need for coordination in multi-robot systems during the exploration of unknown environments and determine possible gains achievable through coordination. The model is illustrated by exploration of an indoor office environment. We qualitatively identify characteristics of environments which make coordination necessary and allow to quantitatively include them in the model. The expected gain through coordination highly depends on the environment. We further investigate the impact of team sizes. In favorable environments explicit coordination is not needed at the cost of increased team sizes. This helps to raise understanding of factors having an impact on coordination functions and making it possible to approximate a possible gain through coordination.**

Index Terms— **Coordination, Robot Exploration, Mobile Robot Teams, Indoor Exploration, Multi-robot systems**

I. Introduction and Objectives

The exploration of unknown environments is among the most basic tasks for robots in indoor applications. Generally, one may assume that an environment is unknown to a robot when first deployed. Robots have to learn their environment autonomously by creating a map. Depending on the application, time may be an important factor for map creation. Robots deployed at homes to assist in domestic work, for example, need to map the environment only once while working in the same environment possibly for months. The mapping may be done during robot setup before starting the intended task. In such cases, the time to set up the robot is not an issue. In other scenarios, however, such as search and rescue, time may become mission critical. Robots may be brought to an environment on demand, for example, in case of fire or natural disasters. The mapping of the environment is an inherent part of the mission.

Multiple robots may be utilized to speed up the task at hand by parallelizing the exploration of an unknown environment. Using multi-robot systems, the question for the necessity of coordination comes into play. For single robot systems, exploration efficiency depends on the *exploration strategy*, which in turn determines the next area that a robot shall move to. A *coordination method* may be required, which shall prevent multiple robots to move to the same area determined by the exploration strategy.

[1]T. Andre is with the Institute of Networked and Embedded Systems, University of Klagenfurt, Austria. `torsten.andre at aau.at`

[2]C. Bettstetter is with the Institute of Networked and Embedded Systems, University of Klagenfurt, Austria, and Lakeside Labs GmbH, Klagenfurt, Austria.

Various forms of coordination are discussed in the literature. Hollinger and Singh, for example, consider coordination of robots with periodic connectivity [1]. Butzke and Likhachev incorporate multiple utility functions allowing the adjustment of exploration priorities [2]. Heo and Varshney coordinate sensors in a wireless sensor network by distributing them spatially evenly [3].

Amigoni *et al.* quantify the expected gain of coordination of multi-robot teams [4]. They conclude that coordination is not as important as one might think in some scenarios and suggest to concentrate on optimizing robots' individual exploration processes instead of optimizing coordination among robots. The problem with evaluating the necessity of coordination in real-world scenarios is that the perception of the environment has strong impact on the assignment of yet unexplored areas (as will be illustrated below). Due to varying perceptions of the same environment caused by sensor noise, slightly different starting positions, or changes in the environment itself, a robot is unlikely to follow the very same path at all times when an environment is being explored. To get results of general validity, many runs in the same environment are needed. To circumvent such problems, we propose to use a Markov process to consider all possibilities how multi-robot systems may distribute during the exploration of an unknown environment. This way we aim to quantify the gain to coordinate robots and evaluate the impact of varying environments and team sizes.

The contribution of this paper is the quantitative assessment of coordination in multi-robot teams considering team sizes and environments' characteristics having an impact of its perception and thus on coordination.

II. Background on Robot Exploration

In exploration missions, robots typically start out in an environment unknown to them. The first task is to move along the unexplored area and create a map. The allocation of unexplored areas to robots is a continuous-time and continuous-space process at the low-level planner, which is responsible for continuous path planning. A high-level planner abstracts from the environment by segmentation [5], yielding discrete areas of the environment. In indoor environments, segments may map to rooms or parts thereof. As commonly referred to, we denote unexplored areas, here segments, next to already explored segments as frontiers [6].

We assume a set of segments $\mathfrak{S}$ with cardinality S for a given environment and a given segmentation algorithm. The exploration strategy determines the next frontier from the set of known frontiers $\mathfrak{F}(t) \subseteq \mathfrak{S}$ at time t based on

978-1-4673-6357-0/13/$31.00 ©2013 IEEE

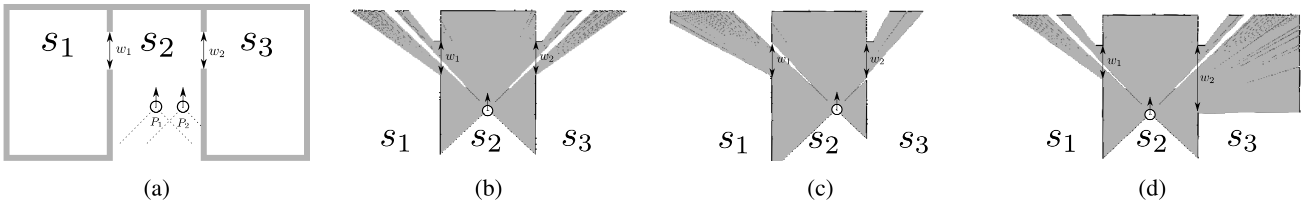


Fig. 1: (a) Layout of indoor environment with segments s_1, s_2, and s_3. Segments s_1 and s_2 are connected by a passage of width w_1, s_2 and s_3 with one of width w_2. P_1 and P_2 mark two possible positions of the robot. (b) and (c) show the robot's perception of the environment from position P_1 and P_2, respectively, for $w_1 = w_2$. (d) shows the robot's perception from position P_1 with $w_2 \gg w_1$.

the current map once a robot has finished exploration of a previous segment.

Numerous approaches have been developed to solve the problem of assigning available frontiers to robots. Such exploration strategies are driven by a utility function u assigning a utility value to frontiers, which enables us to compare and select the frontier expected to maximize system performance. Hence, the exploration strategy problem is to select a frontier $f \in \mathfrak{F}(t)$ maximizing the utility value returned by u: $\arg\max_{f \in \mathfrak{F}(t)} u(f)$.

Utility functions may include a cost function $c(f)$ and a gain function $g(f)$. Costs may include any combination of travel distance, used energy, time, and other constraints to reach a frontier. Costs are minimized either for individual robots, such as in [4], [7], or system wide such as in [5]. Gains may include information gain (see [8] and references therein), connectivity [9] between robots, or other gains.

Another class of utility functions combines the problems of simultaneous localization and mapping (SLAM) [10] and frontier allocation to allocate frontiers to robots on the next-best-view principle to reduce inaccuracies during map building. We do not consider this approach here but assume that map creation and area exploration are disjoint. An elaborate summary and comparison of exploration strategies can be found in [11].

During exploration robots perceive their environment by means of sensors, such as ultra-sound sensors, laser range scanners, or cameras using image processing. Based on these perceptions, robots iteratively create a map of their unknown environment by mapping frontiers assigned by the utility function [5]. A complete map of the environment is obtained upon exploration having traversed all S segments of the environment. The order in which segments are added to the map is not fixed and may be influenced by the perception of the environment and the characteristics thereof.

III. Perception Influences Map Creation

A given environment may be perceived differently by a robot at different points in time. The perception is mainly influenced by two factors: the robot's current position and noisy sensor readings. Furthermore, the interpretation of such perceptions may suffer from map inaccuracies. Let us explain with an example that environment perception has impact on map creation.

Fig. 1a illustrates the layout of a segment s_2 at a corridor's end connecting two segments s_1 and s_3. The openings to the segments s_1 and s_3 have widths w_1 and w_2, respectively. For illustration, the robot heading towards the corridor's end is positioned at either location P_1 or P_2, where P_1 is at the horizontal center of the corridor while P_2 is shifted to the right. To assign a utility value to frontiers, we assume without loss of generality that the robot is capable of deducing the information gain of a segment by its expected unexplored area, as suggested in [12]. If the robot is located at position P_1, the perceived sensor readings look like in Fig. 1b. In this case, the inferred areas are likely to be identical, because s_1 and s_3 are assumed to be of identical size and the same area of both segments is already explored. If the robot is located at P_2, under the same assumption of identical segment sizes, the information gain of segment s_3 will be larger, because a larger area of segment s_1 has already been explored (see Fig. 1c). Therefore, the assignment of the next frontier depends on the robot's position. The same holds for any utility function that considers costs (such as travel distance), gains, or combinations thereof.

Also the characteristics of the environment influence in which order segments are traversed. Fig. 1d shows the same environment, but this time with different door widths ($w_2 \gg w_1$). It can be seen that, independent of the robot's position P_1 or P_2, the explored area of segment s_3 is larger than that of s_1. Therefore, s_1 becomes more likely to be assigned as next frontier.

IV. Markov Model and Performance Metrics

A. Definition of States and State Transition Matrix

Let us use a Markov process to determine the likelihood of a robot's progress in its exploration task in an unknown environment. We consider the high level planner operating on spatially discrete segments. Robots change from segment to segment at discrete time steps. This is a common assumption used in many publications, such as [4], [7], [13]. We assume that exploration takes approximately the same time for each segment. This enables us to model the exploration process by a discrete-time, first-order Markov process.

Given is an environment consisting of S segments obtained through segmentation. At any discrete time instant t, a subset $\mathfrak{F}(t) \subseteq \mathfrak{S}$ with cardinality $F(t)$ frontiers is known. Exploration ends when $\mathfrak{F}(t) = \varnothing$. The environment's segments are the state space of the Markov process having cardinality $S = |\mathfrak{S}|$. The set of robots is denoted by $\mathfrak{A}$; the number of robots is $|\mathfrak{A}| = A \in \mathbb{N}$.

The state transition matrix $\vec{P}$ of dimension $S \times S$ models the utility function and the impact of the environment. It allows for a non-deterministic order of traversed segments as explained in Section III. If there is no path between two segments s_i and s_j, $i \neq j$, the corresponding transition probability in $\vec{P}$ is zero. Self-loops are not allowed in the model, $P_{i,i} = 0$, because robots may not stall in an area after having finished exploration. Each segment takes a given time period, here set to one time unit.

The position of a robot $a \in \mathfrak{A}$ at discrete time t is described by the row vector $\vec{v}_a(t)$ by setting the sth entry corresponding to the sth segment, $s \in \mathfrak{S}$, in which the robot is located, to one. The initial position of robot a is called $\vec{v}_a(0)$. With the Markov assumption, the probability of a robot's location at time $t = 1, 2, \ldots$ time unit(s) can then be computed by

$$\vec{v}_a(t) = \vec{v}_a(0)\, \vec{P}_a(t)\,. \tag{1}$$

The probability of a robot to be at a segment $s \in \mathfrak{S}$ at time t corresponds to the sth entry in $\vec{v}_a(t)$. Since we assume that no coordination between robots is present, robots move independently from each other making them stochastically independent and, therefore, it is possible to compute each robot's location distribution individually.

B. Definition of Performance Metrics

In order for a segment to be explored, it needs to be visited by at least one robot. Let $X(t)$ denote the binary random variable that segment s is being explored at time instant t by at least one robot. The probability $\mathrm{P}(X(t) = s)$ that at least one robot is at segment s at time t is

$$\mathrm{P}(X(t) = s) = 1 - \prod_{a=1}^{A}\left[1 - v_a^{(s)}(t)\right] \tag{2}$$

where $v_a^{(s)}(t)$ is the sth entry of robot a's state vector $\vec{v}$ at time t.

This model enables us to compute the expected level of parallelization by considering the expected number of segments explored in parallel. Generally robots shall be assigned uniquely to frontiers. Such motivation may be twofold. First, we would like to prevent possible self-interference among robots during mapping. Second, multiple robots in the same area may decrease efficiency during exploration due to redundant exploration of the same segment causing an overall delay. Therefore, we assume that the system performs most efficiently if a single robot only operates at any assigned frontier. The expected number of frontiers explored simultaneously at time t is

$$\mathrm{E}[X(t)] = \sum_{s=1}^{S} \mathrm{P}\left(X(t) = s\right)\,. \tag{3}$$

We normalize the total explored area to compute the efficiency of the exploration by

$$\eta(t) := \frac{\mathrm{E}[X(t)]}{O(t)} \in [0,1]\,, \tag{4}$$

where $O(t)$ is the number of segments which could have been explored at time t. The cumulative efficiency is

$$\theta(t) := \frac{\sum_{\tau=0}^{t} \mathrm{E}[X(\tau)]}{\sum_{\tau=0}^{t} O(\tau)}\,. \tag{5}$$

To compare uncoordinated and coordinated robots we use the coordination gain

$$g(t) := \frac{\min\{F(t), A\}}{\mathrm{E}[X(t)]}\,, \tag{6}$$

which is the ratio of frontiers explored assuming perfect coordination and the expected number of explored frontiers without coordination.

C. Transition Probabilities

Setting the probabilities in the transition matrix $\vec{P}$ allows to model the impact of the environment on the frontier assignments to robots. We consider robots with memory. Markov processes do not allow easily to keep track of already visited states, here segments, due to their Markov property. To model a robot's memory, the transition graph must not include any loops preventing robots to reach any state more than once. In principle (1) can be used to model heterogeneous robots running different utility functions leading to different transition probabilities. Here, we only consider homogeneous robots using the same utility function.

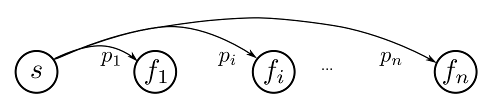
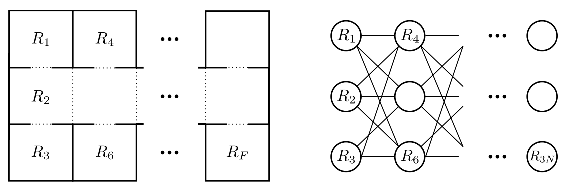


Fig. 2: Frontiers are weighted with transition probabilities.

To model the impact of the perception of the environment, the transitions from a given segment to all potential succeeding segments are weighted with the corresponding transition probabilities (see Fig. 2). Frontiers f_i $(i = 1 \ldots n)$ are possible succeeding frontiers to s with probabilities p_i $(i = 1 \ldots n,\ \sum_{i=1}^{n} p_i = 1)$.

We consider the non-deterministic order of traversed segments and distinguish two cases. We assume a robot's position in a segment to be uniformly distributed, making adjacent frontiers equally likely to be selected as next frontier (case I). The probabilities for each frontier to be assigned as next frontier are equally distributed: $p_i = p_j$, $i, j = 1 \ldots n$. In case II, it is assumed that, being at a segment s_i, one frontier is more likely to be selected than the others. We denote such frontier as *dominant frontier* and assume a single dominant frontier to ease analysis. A dominant frontier may be selected with probability p_{dom}, the remaining frontiers are assumed to be selected with a uniform distribution for the sake of simplicity. The following results may be adopted to realistic non-uniform distributions. In the example, let f_1 be the dominant frontier, thus $p_1 = p_{\mathrm{dom}}$, then $p_i = \frac{1-p_{\mathrm{dom}}}{n-1}$, $i = 2 \ldots n$.



(a) (b)

Fig. 3: (a) Map of a corridor with a total of S segments. (b) The state representation in the Markov process with a total of S segments.

D. Example Scenario: Office Corridor

We apply the model in an environment such as a corridor in an office building. Fig. 3 illustrates the corridor's map and its exemplified representation in the model. We assume the corridor to be equally segmented (illustrated by the dotted lines). Each segment is assumed to take approximately the same time to be explored/searched. Differences in travel costs for various robots are considered to be comparably small and are neglected. Segments are mapped to rooms, thus each room R_i corresponds to a segment s_i in the state space of the Markov process. The robots start out in the same segment positioned at the very left corridor's end and sweep to the right.

V. The Value of Coordination

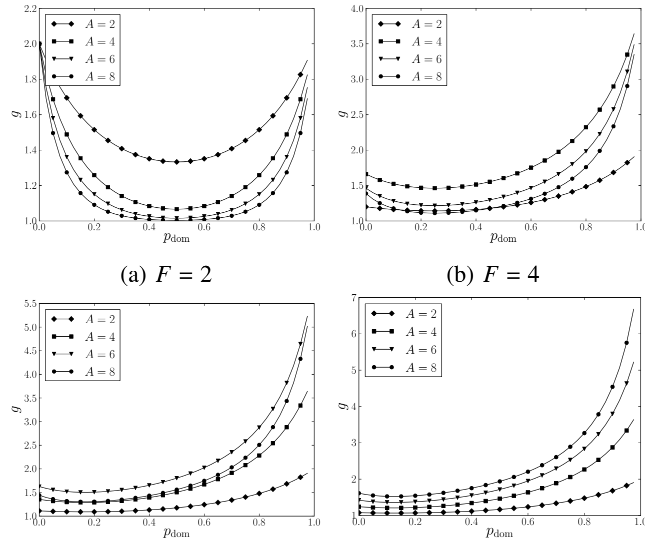
We first investigate the impact of the environment and the perception thereof and the number of robots for various number of frontiers F. For this we omit the index t. Afterwards we apply the model to the office corridor scenario.

A. Impact of Dominant Frontier on Coordination Gain

As mentioned before, some frontiers have in general higher likelihoods to be explored before others (see Fig. 1d). We determine the influence of such dominant frontiers.

Fig. 4 shows the expected coordination gain according to (6) for different F and A. The coordination gain is minimal if robots are uniformly distributed among available frontiers, which is the case for $p_{\text{dom}} = \frac{1}{F}$. The maximum gain is reached for $p_{\text{dom}} = 1$, i.e. the uncoordinated multi-robot system behaves like a single robot. In such cases coordination is most useful. In conclusion, robot systems tend to profit from coordination if the environment leads to multiple frontiers of which one or a few are more likely to be selected.

The authors of [4] come to similar qualitative conclusions comparing an indoor environment to an open environment. Robots in less structured environments tend to select the same frontier making one or a few frontiers more prominent than others, which prevents the robots to spatially spread. In comparison, structured environments are less likely to contain a dominant frontier allowing for implicit coordination by design of the utility function.



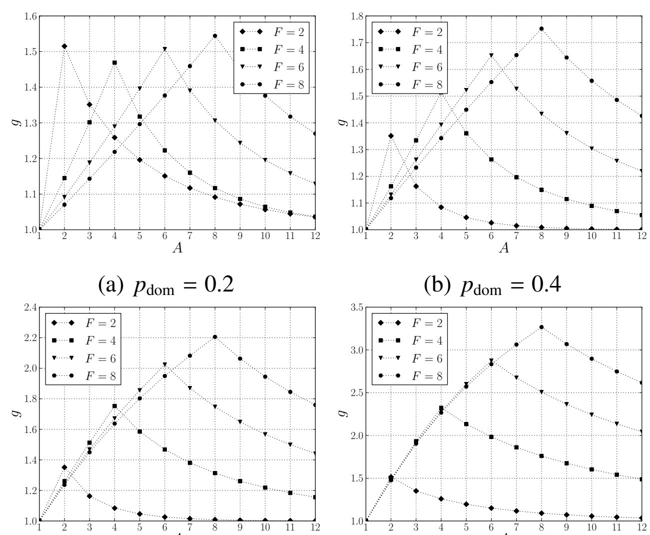
(a) $F = 2$ (b) $F = 4$

(c) $F = 6$ (d) $F = 8$

Fig. 4: Impact of dominant frontier on expected coordination gain for a team of A robots in an environment with with F frontiers.

B. Impact of Team Size on Coordination Gain

We evaluate the impact of the team size on possible gains through optimal coordination. We compare multi-robot systems with up to $A = 12$ robots. Fig. 5 shows the results for various p_{dom}.



(a) $p_{\text{dom}} = 0.2$ (b) $p_{\text{dom}} = 0.4$

(c) $p_{\text{dom}} = 0.6$ (d) $p_{\text{dom}} = 0.8$

Fig. 5: Coordination gain for various p_{dom} depending on the number of robots A. Markers are connected to improve readability.

In all cases, the gain is first strictly increasing to a maximum, after which it monotonically decreases. This can be explained by the robot density $d = \frac{A}{F}$. We need to distinguish three cases. For small robot densities ($d < 1$), the likelihood for robots to select the same frontier is comparably low. With increasingly many robots, this probability increases leading to higher coordination gains. Once A reaches F (i.e. $d = 1$), the coordination gain is maximized. In this case, each frontier is assigned to one robot. Further increasing number of robots ($d > 1$), the likelihood of frontiers left unexplored decreases, thus decreasing the expected gain.

It can be seen that, for $d < 1$, the increase in gain decreases with every additionally added robot. In case of $p_{\mathrm{dom}} = 0.8$ (see Fig. 5d), for example, adding a second robot increases the gain from 1.0 to 1.5, while adding a seventh (for $F = 8$) increases the gain by only 0.2. We will discuss this in more detail.

We determine the impact of adding additional robots to increase the efficiency. Fig. 6 shows the expected number of segments explored in parallel for various team sizes and F with $p_{\mathrm{dom}} = \frac{1}{F}$. With each additional robot added,

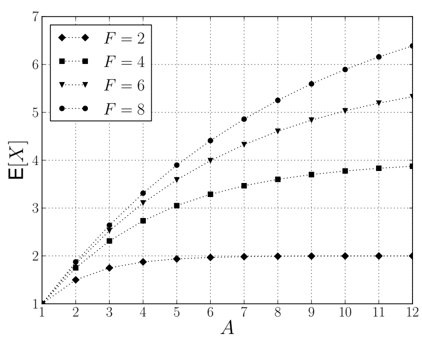


Fig. 6: Comparison of expected number of segments explored parallel for various number of uncoordinated type 2 robots.

the increase in efficiency for the next robot decreases. To address the issue how many robots to add, we can compute the number of uncoordinated robots required to achieve comparable results to optimally coordinated ones. We assume no dominant frontiers and homogeneous robots, thus all frontiers are explored with equal probability independent of the robot. It follows $\vec{v}_{a,f}$ is independent of a and f: $v_{a,f} = v = \frac{1}{F}$. We can rewrite (3) along with (2) to

$$\mathrm{E}\left[X\right] = F\,\mathrm{P}(X) = F\,\left(1 - \left(1 - \tfrac{1}{F}\right)^{A}\right), \tag{7}$$

which can be rearranged to

$$A = \frac{\log(1-\gamma)}{\log(1-\frac{1}{F})},\ \text{where}\ \gamma = \frac{\mathrm{E}[X]}{F} \leq 1 \tag{8}$$

is the ratio of the expected number of explored frontiers to available frontiers. For $\gamma \rightarrow 1$, A goes to infinity because an unlimited number of uncoordinated robots is required for $\mathrm{P}(X) \rightarrow 1$. For example, consider $\gamma = 0.75$ and $F = 4$, a total of $A = \lceil 4.82 \rceil = 5$ robots is required compared to 3 coordinated ones. This is a lower bound. If one considers dominant frontiers, the number of robots will increase further.

With increasing ratio γ, the number of robots required to achieve an expected level of exploration certainty (indicated by $\mathrm{E}[X]$) increases exponentially making exploration necessary without the need for a vast number of robots.

C. Impact of Dominant Frontiers and Team Size on Frontiers Explored in Parallel

Let the random variable (RV) Y_s model the number of robots in segment $s \in \mathfrak{S}$. We denote the tuple $(Y_1 \ldots Y_s)$ as a configuration and determine its probability with the multinomial distribution

$$P(Y_1 = y_1 \wedge \cdots \wedge Y_S = y_S) = \frac{A!}{y_1! \ldots y_S!} p_1^{y_1} \ldots p_S^{y_S}\ . \tag{9}$$

The RV Z models the number of segments explored in parallel in the absence of coordination, i.e. the number of configurations in which $1 \leq z \leq S$ segments contain at least one robot. Fig. 7 shows the resulting probabilities $P(Z = z)$ for $F = 4$. For each p_{dom} we compare the probabilities for robot densities $d = 1, \frac{3}{2}$, and 2 robots per segment. The

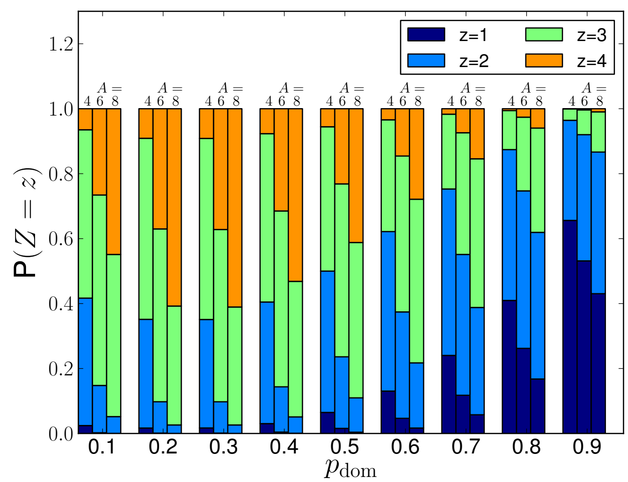


Fig. 7: Probability $\mathrm{P}(z)$ for densities $1, \frac{3}{2}, 2$ and $F = 4$. We omit figures for other sets of parameters. Qualitatively they are comparable with increasing probabilities for increasing p_{dom} for small s.

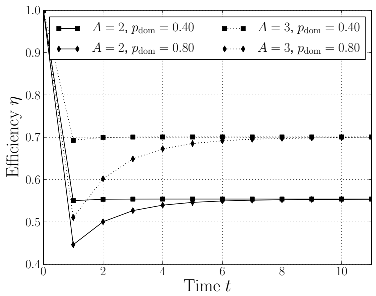
probabilities relate to the robot density and p_{dom} confirming previous results. With increasing p_{dom}, the probability of achieving a high γ decreases significantly. Especially for very high p_{dom}, the probability of exploring four segments $\mathrm{P}(z) = 4$ is negligibly small. Adding additional robots increases the probability, but with a decreasing effect for each additional robot, especially for small p_{dom}.

As discussed previously, $\mathrm{E}[X]$ is maximized if $p_{\mathrm{dom}} = \frac{1}{F}$. For $F = 4$ and $d = 1$, even in this most beneficial case $\mathrm{P}(Z = 4) = 0.14$. With a density $d = 2$, $\mathrm{P}(Z = 4)$ can be increased to 0.67. For increasing p_{dom}, $\mathrm{P}(Z = 4)$ decreases dramatically. Therefore, coordination will become unavoidable if all available frontiers shall be explored with high probability at a given time.

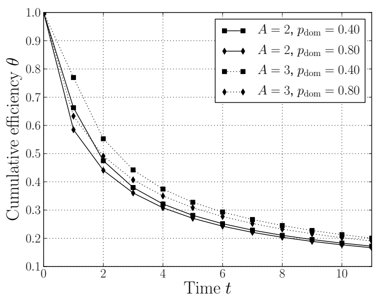
Note that apart the underlying assumptions for this model, all frontiers will be explored in a multi-robot exploration. Frontiers that were not explored at time t_1 will remain in the set $\mathfrak{F}(t)$ for $t_2 > t_1$. They will be explored at a later time, possibly increasing the costs. If these costs are not mission critical, it may be sufficient to have a high enough probability $\mathrm{P}(Z)$ to explore frontiers.

D. Multi-robot Systems in Office Corridors

We apply our model to the exploration of an office corridor depicted in Fig. 3 considering $A = 2$ and 3 robots allowing a complete exploration for coordinated robots in one case. Fig. 8 shows the efficiencies for the corridor environment depicted in Fig. 3. We consider the same probabilities for



(a)



(b)

Fig. 8: Efficiencies exploration office corridor with $A = 2, 3$ robots (see Fig. 3 for illustration).

p_{dom} with two and three robots having densities $d = \frac{2}{3}$ and 1. The cumulative efficiency θ decreases significantly for $t \to \infty$ requiring coordination if additional sweeps are not feasible. The efficiency η for three uncoordinated robots lies at $\eta = 0.71 > 0.67$, which is the achievable efficiency of a optimally coordinated two-robot system. Thus, expectedly three uncoordinated robots outperform two optimally coordinated robots for environments comparable to the office corridor. In the case of optimally coordinated robots the cumulative efficiency θ is 1.0 and 0.67 in case of three and two robots, respectively. In such cases less efforts may be spend on the coordination function while still obtaining comparable results at slightly increased deployment costs.

It is unlikely that a whole environment consists of dominant frontiers only, but at least single junctions with dominant frontiers may occur, thus reducing the efficiency of multi-robot systems. Note, however, that these analyses consider expected values. In the worst case, uncoordinated multi-robot systems may behave as single-robots, which is highly unlikely but not impossible. In comparison, explicit coordination eliminates the possibility of the worst case to occur increasing system efficiency in any case. Here we assume optimal coordination to be achievable at all times. This may not hold for scenarios where the optimal coordination cannot be determined due to missing knowledge of yet unexplored segments.

VI. Conclusions

There is a need for coordination in multi-robot systems, e.g. due to different interpretations of the perception of the environment leading to a non-deterministic order of traversed segments. Dominant frontiers make some orders of traversed segments more likely than others. To be able to qualitatively and quantitatively evaluate the impact of an environment's characteristics on the exploration process, we proposed a Markov process to model explorations using the transition probabilities to consider these characteristics. The gain of coordination mainly depends on two factors: the density of the robots and the environment or the perception thereof. We gave quantitative measures for these values in an office environment. Coordination is most efficient if the number of robots equals the average number of frontiers available. In contrast, if the robot density is very small or large, the small coordination gain may not justify the use of explicit coordination in the absence of dominant frontiers.

Acknowledgments

This work was partly supported by the ERDF, KWF, and BABEG under grant KWF-20214/24272/36084 (SINUS). It was performed in the research cluster Lakeside Labs.

References

[1] G. A. Hollinger and S. Singh, "Multirobot coordination with periodic connectivity: Theory and experiments," *IEEE Trans. Robot.*, vol. 28, no. 4, Aug. 2012.

[2] J. Butzke and M. Likhachev, "Planning for multi-robot exploration with multiple objective utility functions," in *Proc. IEEE/RSJ Int. Intelligent Robots and Systems (IROS) Conf.*, Sept. 2011.

[3] N. Heo and P. K. Varshney, "A distributed self spreading algorithm for mobile wireless sensor networks," in *Proc. IEEE Wireless Communications and Networking (WCNC)*, Mar. 2003.

[4] F. Amigoni, N. Basilico, and A. Quattrinie Li, "How much worth is coordination of mobile robots for exploration in search and rescue?" in *Proc. RoboCup International Symposium*, June 2012.

[5] K. M. Wurm, C. Stachniss, and W. Burgard, "Coordinated multi-robot exploration using a segmentation of the environment," in *Proc. IEEE/RSJ Int. Conf. Intelligent Robots and Systems (IROS)*, Sept. 2008.

[6] B. Yamauchi, "A frontier-based approach for autonomous exploration," in *Proc. Symposion IEEE Int. Computational Intelligence in Robotics and Automation (CIRA)*, July 1997.

[7] R. M. Zlot, A. T. Stentz, M. Dias, and S. Thayer, "Multi-robot exploration controlled by a market economy," in *Proc. IEEE International Conference on Robotics and Automation*, May 2002.

[8] A. Visser and B. A. Slamet, "Including communication success in the estimation of information gain for multi-robot exploration," in *Proc. Int. Symp. Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks and Workshops (WiOPT)*, Apr. 2008.

[9] A. Ghaffarkhah and Y. Mostofi, "A communication-aware framework for robotic field estimation," in *Proc. IEEE Conf. Decision and Control and European Control Conf. (CDC-ECC)*, Dec. 2011.

[10] S. Thrun, "Simultaneous localization and mapping," *Robotics and cognitive approaches to spatial mapping*, pp. 13–41, 2008.

[11] M. Juliá, A. Gil, and O. Reinoso, "A comparison of path planning strategies for autonomous exploration and mapping of unknown environments," *Autonomous Robots*, vol. 33, Nov. 2012.

[12] H. H. González-Baños and J.-C. Latombe, "Navigation strategies for exploring indoor environments," *International Journal of Robotic Research*, vol. 21, no. 10-11, Oct. 2002.

[13] W. Sheng, Q. Yang, J. Tan, and N. Xi, "Distributed multi-robot coordination in area exploration," *Robotics and Autonomous Systems*, vol. 54, Dec. 2006.