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Classical Grasp Quality Evaluation: New Algorithms and Theory

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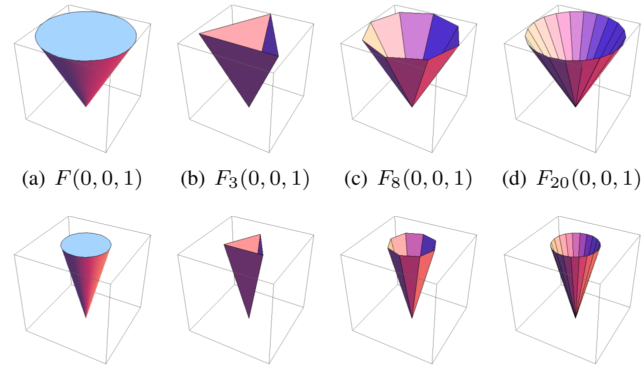
***Abstract*— This paper investigates theoretical properties of a well-known L^1 grasp quality measure Q whose approximation Q_l^- is commonly used for the evaluation of grasps and where the precision of Q_l^- depends on an approximation of a cone by a convex polyhedral cone with l edges. We prove the Lipschitz continuity of Q and provide an explicit Lipschitz bound that can be used to infer the stability of grasps lying in a neighbourhood of a known grasp. We think of Q_l^- as a lower bound estimate to Q and describe an algorithm for computing an upper bound Q^+. We provide worst-case error bounds relating Q and Q_l^-. Furthermore, we develop a novel grasp hypothesis rejection algorithm which can exclude unstable grasps much faster than current implementations. Our algorithm is based on a formulation of the grasp quality evaluation problem as an optimization problem, and we show how our algorithm can be used to improve the efficiency of sampling based grasp hypotheses generation methods.**

I. INTRODUCTION

One of the key problems in robotics is the question of how to generate *viable grasps*. Many aspects, such as the kinematics of a robot, collision avoidance, the object representation, resistance to noise and task constraints for a particular manipulation problem need to be investigated to form a useful grasp. This work focusses on grasping at the contact level which forms the foundation for many state of the art grasp synthesis algorithms [1], [2], [3]. Grasping at the contact level is concerned with the generation of point-contacts on an object of interest such that physical constraints, formulated in terms of the forces acting on an object, are satisfied. In their pioneering work two decades ago, Ferrari and Canny [4] introduced several grasp quality measures which have found a wide application in robotics [2] and which have sparked the creation of several closely related grasp quality measures [3]. We investigate an L^1 grasp quality measure Q introduced in [4] which is a function of the contact points and contact surface normals formed by the grasp and which, for a particular friction coefficient, takes positive values if the resulting grasp can withstand wrenches in an arbitrary direction. Importantly, the magnitude of Q furthermore yields a measurement which allows to rank grasps by their physical stability. In this paper, we establish some of the basic – as of yet unstudied – properties of the function Q which can be estimated by a function Q_l^- defined in terms of an approximation of the Coulomb friction cones by regular polyhedral cones with l edges (see Fig. 1). All major robot grasp simulation environments such as GraspIT [5], OpenGrasp [6] and Simox [7] implement such an approximation. Based on our theoretical insights, we devise an algorithm for the rejection of unstable grasp configurations which we show in experiments to be up to 600 times faster than an implementation based solely on Q_l^-. Our contributions can be summarized as follows:



(a) $F(0,0,1)$ (b) $F_3(0,0,1)$ (c) $F_8(0,0,1)$ (d) $F_{20}(0,0,1)$

(e) $F(0,0,1)$ (f) $F_3(0,0,1)$ (g) $F_8(0,0,1)$ (h) $F_{20}(0,0,1)$

Fig. 1. Friction cone $F(0,0,1)$ in normal direction $(0,0,1)$ in a) and uniform polyhedral cone approximations of that cone with 3, 8, and 20 edges in b)-d). The top row of figures assumes a friction coefficient of $\mu = 1$ while the bottom row displays the corresponding cones for $\mu = \frac{1}{2}$.

- We rephrase a characterization of Q as a minimization problem described by [8] and extend their work by providing a coordinate independent efficient description of the objective function.
- We suggest and evaluate a new optimization method for determining an upper bound Q^+ for Q.
- We prove results on the continuity properties of Q_l^-.
- We prove that Q is Lipschitz continuous and provide an explicit Lipschitz constant.
- We relate the classical approximation Q_l^- of grasp quality to Q and provide a worst-case error bound.
- We introduce a *very fast* algorithm for removing the majority of unstable grasp hypotheses from a set of grasps candidates which gives rise to a new efficient method for finding stable grasps.

Our paper is structured as follows: in Section II, we review related work and recall the basic definition of Q_l^- and Q. In Section III, we begin our investigation of the theoretical properties of L^1 grasp quality and devise a novel efficient algorithm for the rejection of unstable grasps and for the estimation of upper bounds. In Section IV, we present an evaluation of the theory and the algorithms developed in Section III. Finally, we conclude our work and describe future work in Section V.

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II. Background and Related Work

Consider a grasp configuration g with m contacts $c_1, \ldots, c_m \in \mathbb{R}^3$ located on some surface with corresponding inward pointing unit normal vectors $n_1, \ldots, n_m \in \mathbb{S}^2 = \{x \in \mathbb{R}^3 : \|x\| = 1\}$ and centre of mass $z \in \mathbb{R}^3$:

$$g = (c_1, \ldots, c_m, n_1, \ldots, n_m, z). \tag{1}$$

For any given inward pointing normal direction $n \in \mathbb{S}^2$, the Coulomb friction model with friction coefficient $\mu > 0$ states that any force $f \in \mathbb{R}^3$ that can be exerted without slippage at a point p on a surface and with inward pointing normal n has to lie in the friction cone $F(n) = \{f \in \mathbb{R}^3 : \|f^t\| \leqslant \mu f^\perp\}$, where $f = f^\perp n + f^t$, $f^\perp \in \mathbb{R}$ and $f^t \in \mathbb{R}^3$ is a vector tangent to the object (*i.e.* satisfying $\langle f^t, n\rangle = 0$).

In the robotics literature, this constraint is often made explicit by fixing an arbitrary orthonormal basis of the plane orthogonal to n, *i.e.* t_1, t_2, such that $f^t = \alpha t_1 + \beta t_2$ and $F(n) = \{f = \lambda n + \alpha t_1 + \beta t_2 : \sqrt{\alpha^2 + \beta^2} \leqslant \mu\lambda\}$. Here, we shall avoid this choice t_1, t_2 and work instead with a coordinate-independent formulation of the friction cones. We thus have $F(n) = \{f \in \mathbb{R}^3 : \|f - \langle f, n\rangle n\| \leqslant \mu\langle f, n\rangle\}$, where $\langle , \rangle$ denotes the standard inner product and $\|.\|$ the corresponding norm on $\mathbb{R}^3$. To every force f at contact c and for a centre of mass z, we can associate the wrench $w_{c,z}(f) = (f, (c - z) \times f) \in \mathbb{R}^6$. In [4], the space of all wrenches $(w_1, \ldots, w_m)$ which can be exerted at $(c_1, \ldots, c_m)$ using the grasp g and such that $\sum_{i=1}^m \left|f_i^\perp\right| \leqslant 1$ is considered. Any allowable such wrench lies in the convex hull $W = \mathrm{Conv}(\{0\} \cup S(g))$, where $S(g) = \{\sum_{i=1}^m \alpha_i w_{c_i,z}(f_i) : f_i \in D(n_i), \alpha_i \geqslant 0, \sum_{i=1}^m \alpha_i = 1\} = \mathrm{Conv}(\{w_{c_1,z}(D(n_1)) \cup \ldots \cup w_{c_m,z}(D(n_m))\})$, and where

$$D(n) = \{f \in \mathbb{R}^3 : \langle f, n\rangle = 1, \text{ and } \|f - \langle f, n\rangle n\| \leqslant \mu\}$$

denotes the disc obtained by intersecting the cone $F(n)$ with the plane normal to n and 'at height one'. Since this disc is the convex hull of a circle $D(n) = \mathrm{Conv}(C(n))$, where $C(n) = \{f \in \mathbb{R}^3 : \langle f, n\rangle = 1 \text{ and } \|f - \langle f, n\rangle n\| = \mu\}$, we finally have that

$$S(g) = \mathrm{Conv}(w_{c_1,z}(C(n_1)) \cup \ldots \cup w_{c_m,z}(C(n_m))).$$

Exact L^1 grasp quality: Q

Let S be a bounded convex set and denote by $d(0, S)$ the signed distance from the origin to S, so that $-d(0, S)$ is the radius of the largest ball around the origin which still fits inside S if such a ball exists and it is the negative of the radius of the largest ball entirely outside $\mathrm{Int}\, S(g)$ otherwise. The support function $h_S(z) = \sup_{s \in S}\langle s, z\rangle$ is a convex function and $-d(0, S)$ can be expressed as (*c.f.* [8] up a change in sign and notation):

$$-d(0, S) = \min_{\|z\|=1} h_S(z). \tag{2}$$

Up to a change in sign, the work [8] defines the quality of a grasp by considering $q(g) = -d(0, S(g))$. The function q can take negative values and $Q(g) = \max(0, q(g)) = -d(0, \mathrm{Conv}(\{0\} \cup S(g))$ is positive (indicating a stable grasp) precisely if arbitrary small wrenches of maximal norm $Q(g)$ can be resisted by the grasp. The main downside of the above approach is that there currently exists no analytic way of determining $Q(g)$. The earlier work of [4] studied Q using a different method by introducing an approximation by a function Q_l^- which we shall discuss next. An alternative approach of [8] showed that a numerical optimization of Eq. 2 with a standard out-of-the-box matlab optimization function can also be used to determine $q(g)$ approximately, while [9] approximate $q(g)$ by iteratively growing a polyhedral convex hull approximation to $S(g)$.

Approximate L^1 grasp quality: Q_l^-

Ferrari and Canny [4] defined an algorithm that provides an approximation to Q. All popular grasp simulation environments such as GraspIt [5], Simox [7] and OpenGrasp [6] implement a version of this algorithm. To approximate $D(n)$, one proceeds by using $l \geqslant 3$ edges $f_1, \ldots, f_l \in C(n)$ such that $D(n) \approx \mathrm{Conv}(\{f_1, \ldots, f_l\})$; see Fig. 1 for an illustration. If we space these edges uniformly, so that $f_j = n + \mu(\cos(\frac{2\pi j}{l})t_1 + \sin(\frac{2\pi j}{l})t_2)$, we obtain a regular approximation $D_l(n) = \mathrm{Conv}(\{f_j : j = 1, \ldots, l\})$ of the disc $D(n)$, so that $D(n) \approx D_l(n)$ and an approximation $F_l(n) = \mathrm{Conv}(\{0\} \cup D_l(n))$ of the truncated friction cone, so that $F_l(n) \approx \mathrm{Conv}(\{0\} \cup D(n))$ as displayed in Fig. 1. Note that this definition of f_j depends on a choice of orthonormal basis t_1, t_2 for the tangent space orthogonal to the line spanned by n. Using the above approximation, one then computes an approximation of $S(g)$ by $S_l^-(g) = \mathrm{Conv}\left(\{w_{ij} = w_{c_i,z}(f_{ij}) : i \in \{1, \ldots, m\}, j \in \{1, \ldots, l\}\}\right)$, where f_{ij} for $j = 1, \ldots, l$ are equally spaced edges on $C(n_i)$. A quality measure Q_l^- is then defined as the distance from the origin to the closest facet of $S_l^-(g)$ if the origin is contained in the interior. If this is not the case, we define the quality to be equal to zero. In order to compute $Q_l^-(g)$, all major state of the art implementations then use the Quickhull library [10] which converts the description of the convex hull $S_l^-(g)$ into its dual form as an intersection of affine half-spaces, where $\lambda_j \in \mathbb{R}, v_j \in \mathbb{R}^6$, $\|v_j\| = 1$:

$$S_l^-(g) = \bigcap_{j=1}^{N} \{x \in \mathbb{R}^6 : \langle x, v_j\rangle \leqslant \lambda_j\}, \tag{3}$$

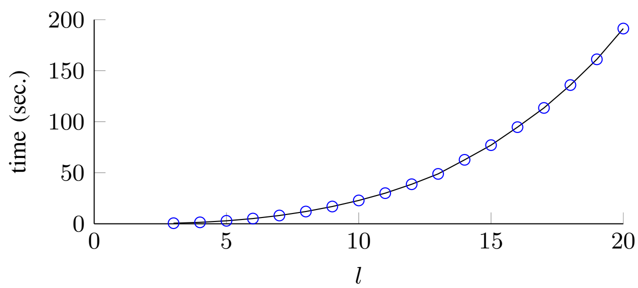
and which can then be used for computing grasp quality since $Q_l^-(g) = \max(0, \min_j \lambda_j)$. Recent work that uses Q_l^- includes [2], where a sampling based strategy towards the determination of a stable grasp is presented. The work of [11] uses the approximation Q_l^- together with a gradient based approach to find such grasps, while [8] investigates q instead. Recent examples of the usage and development of grasp quality measures includes the work of [12] and [13] where kinematic constraints are also incorporated. More classically, the work of [14] defines a quality measure that relies on a polyhedral cone approximation, but which decouples moments from forces.

III. Theoretical contributions

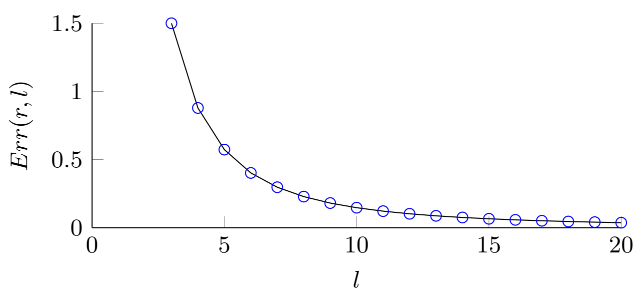
In this section, we investigate some of the basic but previously unstudied properties of the grasp quality measures Q_l^- and Q.

Computational complexity and error estimates

Let us begin our discussion of Q_l^- and Q by having a look at the time-complexity for computing Q_l^- in practice. As can be seen in Fig. 2(a), the time required to compute Q_l^- scales rather unfavourably as the number l of edges per friction-cone is increased. Since sampling based grasp hypothesis generation methods such as [2] typically require thousands of evaluations of Q_l^-, a low number of edges between 6 and 8 is often chosen in real-world applications. Our first task shall be to describe worst-case error bounds for the approximation of Q by Q_l^-.



(a) Mean computation time in seconds for Q_l^- for 10000 uniformly sampled grasps with three contact points c_1, c_2, c_3 such that $\|c_i\| \leqslant 2$, for uniformly sampled corresponding normals n_i in the sphere $\mathbb{S}^2$ and for friction coefficient $\mu = 1$. The centre of mass z was set to 0 for each grasp. The mean is obtained from 10 runs of the experiment.



(b) Maximal approximation error bound $Err(r, l)$ (vertical axis) for approximating each friction cone using various numbers of uniformly spaced edges (horizontal axis), for friction coefficient $\mu = 1$ and for $r = 2$.

Fig. 2.

Let us first state a rather simple fact:

Lemma 3.1. *For any grasp g, we have $0 \leqslant Q_l^-(g) \leqslant Q(g)$. Furthermore, $\|Q(g) - Q_l^-(g)\| \to 0$ as $l \to \infty$ when $Q_l^-(g)$ is computed using a uniform approximation of the friction cones with l edges.*

Proof. Note that $Q(g)$ ($Q_l^-(g)$ respectively) is equal to the radius of the largest ball centred at the origin and inside $S(g)$ ($S_l^-(g)$ respectively) if such a ball exists and zero otherwise. Furthermore, $S_l^-(g) \subset S(g)$ for all $l \in \mathbb{N}$ which implies the first statement. Finally, the convex hull $S_l^-(g)$ clearly converges to $S(g)$ as $l \to \infty$, since the friction cone approximations F_l converge to F as the number l of edges is increased. This yields the second statement. $\square$

Note that, since $Q(g) = \max(0, q(g))$, the above lemma tells us also that $q(g)$ can be approximated by $Q_l^-(g)$ for sufficiently large l if $q(g) \geqslant 0$. The function q does however contain additional information when it is negative. Note here that some implementations define $Q_l^-(g) = \min_j \lambda_j$, using the affine half-plane description of $S_l^-(g)$. In this case, negative values of Q_l^- can be attained. When the grasp quality is defined in this way and when it is negative, $Q_l^-(g)$ is the signed distance to the closest hyperplane $\{x \in \mathbb{R}^6 : \langle x, v_j \rangle \leqslant \lambda_j\}$ among the hyperplanes describing $S_l^-(g)$ which is not necessarily a good approximation to $q(g)$ for small l. We have the following result:

Lemma 3.2. *Suppose $Q_l^-(g)$ is computed using l uniformly spaced edges to approximate the friction cones, and suppose that the friction coefficient is given by $\mu > 0$. Define*

$$Err(r, l) \stackrel{\text{def}}{=} \max_{g : \|c_i - z\| \leqslant r, \, \forall i = 1, \ldots, m} \left| Q_l^-(g) - Q(g) \right|,$$

where $r > 0$ and g denotes a grasp with m contact points c_i, normals $n_i \in \mathbb{S}^2$ and with centre of mass z as in Eq. 1. Then $Err(r, l) \leqslant \mu(1 + r)(1 - \cos(\frac{\pi}{l}))$.

Proof. Note that $\mu(1 - \cos(\frac{\pi}{l}))$ is the maximal distance from any point on the disc $D(n)$ to the approximation $D_l(n)$ for arbitrary $n \in \mathbb{S}^2$. Given wrenches $w = (f, (c - z) \times f)$ and $w' = (f', (c - z) \times f')$ with $\|f - f'\| \leqslant \varepsilon$, we observe that

$$\begin{aligned} \|w - w'\| &\leqslant \|f - f'\| + \|c - z\| \|f - f'\| \\ &\leqslant \varepsilon(1 + \|c - z\|). \end{aligned}$$

To prove the result, we only need to consider g for which $q(g) > 0$ since otherwise both $Q_l^-(g)$ and $Q(g) = \max(0, q(g))$ are zero. Suppose there exists g such that $\|c_i - z\| \leqslant r$ and $\left|Q(g) - Q_l^-(g)\right| = q(g) - Q_l^-(g) = \lambda > \mu(1 + r)(1 - \cos(\frac{\pi}{l}))$. Consider $z^* \in \mathbb{R}^6$, $\|z^*\| = 1$ such that $-d(0, S_l^-(g)) = h_{S_l^-(g)}(z^*)$. Then, by definition, $Q_l^-(g) = \max(0, h_{S_l^-(g)}(z^*))$ and hence $S_l^-(g) \subseteq \{x \in \mathbb{R}^6 : \langle x, z^* \rangle \leqslant h_{S_l^-(g)}(z^*)\} \subseteq H \stackrel{\text{def}}{=} \{x \in \mathbb{R}^6 : \langle x, z^* \rangle \leqslant Q_l^-(g)\}$. Consider $p = q(g) z^*$. We have $\|p\| = q(g)$, so $p \in S(g)$, and furthermore, since $d(p, H) = \lambda$, we have $d(p, S_l^-(g)) \geqslant \lambda$. Now p can be expressed as $p = \sum_{j=1}^m \alpha_j (f_j, (c_j - z) \times f_j)$, for some $f_j \in D(n_j)$ and $\alpha_j > 0$, $\sum_j \alpha_j = 1$. Let $\hat{f}_j$ be the closest point to f_j in $D_l(n_j)$ and define $\hat{p} = \sum_{j=1}^m \alpha_j (\hat{f}_j, (c_j - z) \times \hat{f}_j)$. Clearly, $\hat{p} \in S_l^-(g)$, but

$$\begin{aligned} \|p - \hat{p}\| &\leqslant \sum_j \alpha_j \left(\|\hat{f}_j - f_j\| + \|\hat{f}_j - f_j\| \|c_j - z\| \right) \\ &\leqslant \mu(1 + r)(1 - \cos(\frac{\pi}{l})) < \lambda. \end{aligned}$$

Hence, we have arrived at a contradiction. $\square$

Figure 2(b), displays the convergence of these errors. For $l = 20$, we obtain a maximal error of 0.037 for $r = 2$. Note that, while the computational complexity blows up quickly as $l \to \infty$, the error bound converges rather slowly to zero.

Continuity

While one might think that the approximation $S_l^-(g)$ of $S(g)$ which determines Q_l^- varies continuously with respect to the parameter g, we will now show that this is in fact *not* the case. Recall that $S_l^-(g)$ is a convex hull determined by approximating the truncated friction cones $\mathrm{Conv}(\{0\}\cup C(n))$ using $C(n) \approx \mathrm{Conv}(\{f_1(n), \ldots, f_l(n)\})$. We now show that there cannot exist any continuous map $n \mapsto f_j(n)$ assigning normal directions $n \in \mathbb{S}^2$ to edges $f_j(n)$ of the truncated friction-cone $\mathrm{Conv}(\{0\} \cup C(n))$:

Theorem 3.3. *There does not exist any continuous edge-assignment $f : \mathbb{S}^2 \to \mathbb{R}^3$ such that $f(n) \in C(n)$ for all $n \in \mathbb{S}^2$, and where f is defined on the whole sphere of unit normal directions $\mathbb{S}^2$.*

Proof. Suppose there was such an assignment. The vectors $v(n) = f(n) - n$ are non-zero and lie in the tangent plane $T_n\mathbb{S}^2$ of the sphere $\mathbb{S}^2$ at n and hence form a non-vanishing continuous vector-field on $\mathbb{S}^2$. However, since the Euler-characteristic $\chi(\mathbb{S}^2) = 2$, any continuous vector field on $\mathbb{S}^2$ must have at least one zero. This result is also known as the 'hairy ball theorem'. This results in a contradiction. $\square$

The above result implies in particular that there is no continuous way of approximating $S(g)$ (that is continuous as a map $g \mapsto S(g)$) using a polyhedral approximation of the truncated cones $\mathrm{Conv}(\{0\}\cup C(n_i))$. While the maximal 'jumps' in the approximation clearly tend to zero as the number of edges of the polyhedral approximation of friction cones is increased, such an approximation scheme might cause problems when methods depending on continuity, such as gradient based methods, are to be used for grasp synthesis.

Let us now study the discontinuity of this map in the case of a particularly nice assignment of cones to normal-directions. We define the standard approximate disk with friction coefficient μ, with l defining edges, and with inward pointing normal direction $(0,0,1)$ by $D_l(0,0,1) = \mathrm{Conv}\left(\left\{\left(\mu\cos\left(\frac{2\pi j}{l}\right), \mu\sin\left(\frac{2\pi j}{l}\right), 1\right) : j \in \{1,\ldots,l\}\right\}\right)$, so that $F_l(0,0,1) = \mathrm{Conv}(\{0\} \cup D_l(0,0,1))$. See Fig. 1 for a visualization. In order to define a cone for a new normal direction n, we rotate $D_l(0,0,1)$ about the axis $(0,0,1)\times n$ by an angle of $\arccos(\langle(0,0,1),n\rangle)$. An explicit formula for this process can be given elegantly in screw coordinates by $D_l(n) = R(n)D_l(0,0,1)$, where

$$R(n) = \exp\left(\arccos(\langle(0,0,1),n\rangle)\xi_{\frac{(0,0,1)\times n}{\|(0,0,1)\times n\|}}\right),$$

the exponential denotes the matrix exponential, and

$$\xi_{(x,y,z)} = \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}.$$

Note that, for $\|(0,0,1)\times n\| \neq 0$, the above map is continuous and well-defined. So the only potential problem occurs at the poles $(0,0,1)$ and $(0,0,-1)$. As $n \to (0,0,1)$,

$$\frac{\arccos(\langle(0,0,1),n\rangle)}{\|(0,0,1)\times n\|} = \frac{\arccos(\langle(0,0,1),n\rangle)}{|\sin(\arccos(\langle(0,0,1),n\rangle))|} \to 1,$$

so that the argument of the exponential tends to zero and is continuous, resulting in the identity matrix after exponentiation. The argument of the exponential is however not continuous as $n \to (0,0,-1)$ which can be seen by considering a normal on a small circle around the south-pole: let $n = (\sin\varepsilon\cos\theta, \sin\varepsilon\sin\theta, -\cos\varepsilon)$. Then we can compute that, for small $\varepsilon > 0$, $R(n) = e^{(\pi-\varepsilon)\xi_{(-\sin\theta,\cos\theta,0)}}$. Since this formula still depends on θ as $\varepsilon \to 0$, we see that there is no unique limit as $\varepsilon \to 0$ which illuminates the discontinuity predicted by the theorem above. We shall use one of the possible limits,

$$e^{\pi\xi_{(-1,0,0)}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad (\text{using } \theta = \frac{\pi}{2}),$$

as the definition of $R(0,0,-1)$. Recall also that, by Rodriguez' formula, a rotation about a unit vector v by an angle θ is given by $e^{\theta\xi_v} = I + \xi_v\sin\theta + \xi_v^2(1-\cos\theta)$, for $v \in \mathbb{R}^3$ such that $\|v\| = 1$. This formula can then be used to efficiently evaluate $R(n)$ for all $n \in \mathbb{S}^2$ without the south-pole, where we define $R(n)$ as discussed above. To summarize, we have found that

Lemma 3.4. *The above assignment of uniform polyhedral cones with l edges given by $n \mapsto F_l(n)$ is a continuous map on the punctured sphere $\mathbb{S}^2_+ = \mathbb{S}^2 - \{(0,0,-1)\}$. When considered as a map on the whole sphere $\mathbb{S}^2$, the map has a discontinuity at $(0,0,-1)$.*

In Fig. 3, we exemplify how the above results leads to a discontinuity of the function $g \mapsto Q_3(g)$. Consider fixed contact points $c_1 = (0, -\sin(\frac{2\pi}{3} - 0.6), \cos(\frac{2\pi}{3} - 0.6))$, $c_2 = (0, -\sin(\frac{4\pi}{3} - 0.6), \cos(\frac{4\pi}{3} - 0.6))$ and a variable contact $c_3(\varepsilon,\theta) = (-\sin(\varepsilon)\cos(\theta), -\sin(\epsilon)\sin(\theta), \cos(\varepsilon))$. We chose corresponding normals given by $n_j = -c_j$ to form a grasp $g(\varepsilon,\theta)$. Fig. 3 displays the quality $Q_3(g(\varepsilon,\theta))$, for $\theta = -\frac{3\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ depicted in red, black (dashed) and blue respectively and for $\varepsilon \in [-0.01, 0.01]$ (horizontal axis). At $\varepsilon = 0$, $c_3(0,\theta) = (0,0,1)$ for all approach directions θ, but we can observe that $Q_3(g(\varepsilon,\theta))$ has a limit depending on the approach angle θ as $\varepsilon \to 0$.

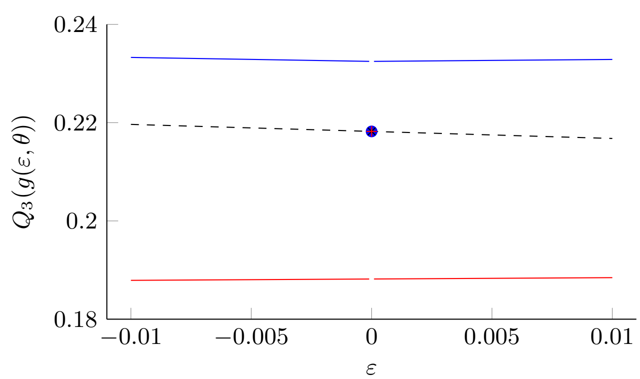


Fig. 3. Illustration of the discontinuity of $Q_3(g(\varepsilon,\theta))$ for three contacts on a sphere and where we continuously vary one contact $c_3(\varepsilon,\theta)$ near the north-pole for three different approach directions $\theta \in \{-\frac{3\pi}{4}, \frac{\pi}{2}\,\frac{3\pi}{4}\}$. At $\varepsilon = 0$, $c_3(0,\theta) = (0,0,1)$ for all $\theta \in [0, 2\pi)$.

Properties of q and Q

In [8], q is characterized (up to changes in notation) as:

$$\begin{aligned} q(g) &= \min_{u\in\mathbb{R}^6:\|u\|=1} h_{S(g)}(u) \\ &= \min_{u\in\mathbb{R}^6:\|u\|=1} \max_{i=1,\dots,m} h_{W_i(g)}(u), \end{aligned}$$

where $S(g) = \mathrm{Conv}(\cup_{i=1}^m W_i(g))$, $W_i(g) = w_{c_i,z}(C(n_i)) \subset \mathbb{R}^6$, and $h_{W_i(g)}(u) = \sup_{w\in W_i(g)}\langle u, w\rangle$ is the support function for the convex set $W_i(g)$. By picking an orthonormal basis t_1, t_2 of the tangent space at each contact, the authors then simplified the above by determining an explicit formula for $h_{W_i(g)}(u)$. We will now provide a basis independent formula for $h_{W_i(g)}(u)$.

Let us observe that $w_{c,z}$ is a linear map and that $\langle w_{c,z}(x), u\rangle = \langle x, w_{c,z}^t(u)\rangle$, where for $u = (a, b) \in \mathbb{R}^3 \times \mathbb{R}^3 = \mathbb{R}^6$, we have the adjoint $w_{c,z}^t(a, b) = a + b \times (c - z)$. Hence, $h_{W_i(g)}(u) = \sup_{x\in C(n_i)}\langle x, w_{c_i,z}^t(u)\rangle$. This problem can however be solved without a choice of coordinate frame:

Lemma 3.5. *For $w \in \mathbb{R}^3$, we have, for $n \in \mathbb{S}^2$ and for friction coefficient $\mu > 0$,*

$$\sup_{x\in C(n)}\langle x, w\rangle = \langle n, w\rangle + \mu\|n \times w\|.$$

Hence, for $u = (a, b) \in \mathbb{R}^3 \times \mathbb{R}^3 = \mathbb{R}^6$, we have

$$h_{W_i(g)}(a, b) = \langle n_i, a+b\times(c_i-z)\rangle + \mu\|n_i\times(a+b\times(c_i-z))\|.$$

Proof. We can verify directly that the formula for $h_{W_i(g)}(x) = -q_{W_i(g)}(-x)$ found on p.6, [8] yields the above result for an arbitrary choice of orthonormal basis $\{e_1, e_2, e_3\}$ such that $e_1 = n$. $\square$

Theorem 3.6. *We have*

$$q(g) = \min_{u\in\mathbb{R}^6,\|u\|=1} h_{S(g)}(u) = \min_{u\in\mathbb{R}^6,\|u\|=1} \max_{i=1,\dots,m} h_{W_i(g)}(u),$$

where $h_{S(g)}$ is convex on $\mathbb{R}^6$. Observe that q is invariant under fixed translation of the grasp centre and contact positions. Furthermore, let $\mathbb{B}(r) = \{x \in \mathbb{R}^3 : \|x\| \leqslant r\}$. Then q is Lipschitz continuous on grasps with m contact points lying in the set $X = \{(c_1, \dots, c_m, n_1, \dots, n_m, z) : (c_i - z) \in \mathbb{B}(r), n_i \in \mathbb{S}^2\}$ with a Lipschitz constant given by $L = (1 + \mu)(1 + r)$ and where we choose the distance measure

$$d(g, g') = \sum_i \|(c_i - z) - (c'_i - z')\| + \sum_i \|n_i - n'_i\|$$

for grasps $g = (c_1, \dots, c_m, n_1, \dots, n_m, z)$ and $g = (c'_1, \dots, c'_m, n'_1, \dots, n'_m, z') \in X$. We hence have

$$|q(g) - q(g')| \leqslant Ld(g, g') \quad \textit{for all } g, g' \in X.$$

Since $Q(g) = \max(0, q(g))$, Q is also Lipschitz continuous with the same constant L on X.

Proof. $h_{S(g)}$ is a support function of a convex set and hence convex. To prove the statement about Lipschitz continuity, consider the function $l_{i,a,b}(g) = h_{W_i(g)}(a, b)$. We have, for $\|(a, b)\| \leqslant 1$, that $|l_{i,a,b}(g) - l_{i,a,b}(g')|$ is bounded above by $|\langle n_i, a + b \times (c_i - z)\rangle - \langle n'_i, a + b \times (c'_i - z')\rangle| + \mu|\|n_i \times (a + b \times (c_i - z)\| - \|n'_i \times (a + b \times (c'_i - z')\||$. A simple calculation involving the facts that $\|a\| \leqslant 1$, $\|b\| \leqslant 1$, $\|c_i - z\| \leqslant r$, $|v \times w| \leqslant \|v\|\|w\|$ and $|\langle v, w\rangle| \leqslant \|v\|\|w\|$ yields that the first summand above is bounded by $\|n_i - n'_i\|(1 + \|c_i - z\|) + \|(c_i - z) - (c'_i - z')\| \leqslant (1 + r)d(g, g')$, while the second summand is bounded by $\mu(\|n_i - n'_i\|(1+r) + \|(c_i - z) - (c'_i - z')\|) \leqslant \mu(1+r)d(g, g')$. Hence, $|l_{i,a,b}(g) - l_{i,a,b}(g')| \leqslant (1 + \mu)(1 + r)d(g, g')$ and $l_{i,a,b}$ is Lipschitz continuous with constant L for fixed i, a, b. It follows now that $\max_i l_{i,a,b}(g)$ is Lipschitz continuous and

$$q(g) = \min_{(a,b)\in\mathbb{R}^6,\|u\|=1} \max_{i=1,\dots,m} l_{i,a,b}(g)$$

is also Lipschitz with the same constant. This is true because, in general, $\varphi(x) = \inf_{\alpha\in A} f_\alpha(x)$ and $\varphi(x) = \sup_{\alpha\in A} f_\alpha(x)$ are Lipschitz with constant L if all f_α are Lipschitz with constant L and φ is bounded. To see this, observe that $f_\alpha(x) \leqslant f_\alpha(y) + Ld(x, y)$ for all α. Then $\sup_{\alpha\in A} f_\alpha(x) \leqslant \sup_{\alpha\in A} f_\alpha(y) + Ld(x, y)$. Swapping the roles of x, y then yields the result in the $\sup$ case. The $\inf$ case is similar. The last claim regarding Q also follows from the above. $\square$

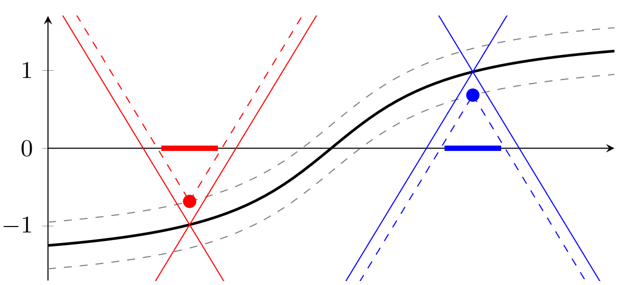


Fig. 4. Illustration of the usage of a Lipschitz constant L for a function $q : \mathbb{R} \to \mathbb{R}$ which we might not be able to evaluate, but for which we have computable bounds $q^-(x) \leqslant q(x) \leqslant q^+(x)$. The Lipschitz condition $|q(x) - q(x')| \leqslant L\,|x - x'|$ forces the graph of q to the left and right of the depicted blue and red cones. Regions for which $q < 0$ (red bar), or for which $q > 0$ (blue bar), can be deduced from points where $q^+(x) < 0$ (red point) and $q^-(x) > 0$ (blue point) respectively.

Note that $q(g)$ is now described by a minimization of a convex function $h_{S(g)}$ over the non-convex sphere $\mathbb{S}^5 = \{u \in \mathbb{R}^6 : \|u\| = 1\}$. Furthermore, our Lipschitz constant L allows us to infer a whole stable (unstable) *region* around any point for which $q > 0$ ($q < 0$). While we cannot easily visualize the concept of Lipschitz continuity in 6 dimensions, the reader unfamiliar with Lipschitz continuity is referred to Fig. 4 which provides an example of Lipschitz bounds in one dimension. Given our Lipschitz constant L, we can *globally* bound possible values of q using $|q(g) - q(g')| \leqslant Ld(g, g')$. This inequality forces the graph of q to lie inside certain regions around any known value $q(g)$ as depicted in Fig. 4. In particular, if we have a lower-bound estimate to $q(g)$ at a grasp g, the Lipschitz bound implies that $q(g')$ is globally lower-bounded by the inequality as indicated in the figure. From a single grasp with $q(g) > 0$ ($q(g) < 0$), we can hence infer the existence of a whole region around g where $q > 0$ ($q < 0$ respectively). As illustrated in Fig.4, Lipschitz bounds can hence be used to improve the efficiency of sampling based grasp synthesis approaches, *e.g.* by quickly determining stable and unstable parameter regions.

Detecting unstable grasps quickly

Note that, since q is given as a minimization problem, any $u \in \mathbb{R}^6$ such that $\|u\| = 1$ yields an upper bound $q_u^+(g) = \max_{i=1,\ldots,m} h_{W_i(g)}(u)$ which can be computed quickly for fixed u. Observe furthermore that we have the following:

Lemma 3.7. *$q(g) \geqslant 0$ if and only if $0 = \min_{u \in \mathbb{R}^6 : \|u\| \leqslant 1} h_{S(g)}(u)$.*

Proof. If $0 = \min_{u \in \mathbb{R}^6 : \|u\| \leqslant 1} h_{S(g)}(u)$, then $q(g) \geqslant 0$ by definition. Suppose $q(g) \geqslant 0$. Note that $h_{S(g)}(0) = 0$ and any $x \in \mathbb{R}^6$ such that $\|x\| \leqslant 1$ is of the form tw for some $t \in [0, 1]$, $w \in \mathbb{S}^5$. Now we have $h_{S(g)}(tw) = t h_{S(g)}(w) \geqslant 0$, and the result follows. □

The lemma above is very handy since the minimization of a convex function over the bounded convex set $\{x \in \mathbb{R}^6 : \|x\| \leqslant 1\}$ is a standard problem. In particular, we can search for minima of $h_{S(g)}$ using the subgradient method [15] and stop once we find $u \in \mathbb{R}^6$ such that $h_{S(g)}(u) < 0$ since then $q(g) \leqslant h_{S(g)}(\frac{u}{\|u\|}) < 0$, and we can conclude that the grasp g is unstable.

It is clear from the definition that $h_{S(g)}$ is not smooth everywhere. However, due to its explicit description as a convex enveloping function, we can attempt to apply the projected subgradient approach to find a minimizer.

Let us first recall that for any convex function $f : \mathbb{R}^n \to \mathbb{R}$, a subgradient at $x \in \mathbb{R}^n$ is a vector $s \in \mathbb{R}^n$ such that $f(y) \geqslant f(x) + \langle s, y - x\rangle$ for all $y \in \mathbb{R}^n$. The set of all subgradients at x is denoted by $\partial f(x)$. If f is differentiable at x, $\partial f(x) = \{\nabla f(x)\}$. Importantly, if $f(x) = \max(f_1(x), \ldots, f_m(x))$, $\partial f(x)$ is given by the convex hull of $\bigcup_{i : f_i(x) = f(x)} \partial f_i(x)$.

The subgradient method [15] for minimizing a function f, updates x_k to $x_{k+1} = x_k - \alpha_k s_k$, where s_k is *any* subgradient of f at x_k and $\alpha_k > 0$ is small. To solve $\min_{x \in C} f(x)$ for a convex set C, this step is replaced with $x_{k+1} = P(x_k - \alpha_k s_k)$, where $P(x)$ is the projection of x onto C. To apply this method, we now simply need to compute subgradients. Observe that, if $\|n_i \times (a + b \times (c_i - z))\| > 0$, $h_{W_i(g)}$ is smooth with $\partial h_{W_i(g)}(a, b) = \{\nabla h_{W_i(g)}(a, b)\}$, and where the partial derivatives are given by $\frac{\partial h_{W_i(g)}}{\partial a_j}(a, b) = (n_i)_j + \mu \frac{\langle n_i \times e_j, n_i \times (a + b \times (c_i - z))\rangle}{\|n_i \times (a + b \times (c_i - z))\rangle\|}$ and $\frac{\partial h_{W_i(g)}}{\partial b_j}(a, b) = ((c_i - z) \times n)_j + \mu \frac{\langle n_i \times (e_j \times (c_i - z), n_i \times (a + b \times (c_i - z))\rangle}{\|n_i \times (a + b \times (c_i - z))\rangle\|}$. If $\|n \times (a + b \times (c_i - z))\| = 0$, it can be easily seen that $h_{W_i(g)}$ has a subgradient given by $s = (n_i, (c_i - z) \times n_i)$.

We thus propose Algorithm 1 to determine quickly if a grasp is unstable. The method $Subgradient(h_{S(g)}, u)$ denotes any subgradient of $h_{S(g)}$ at $u \in \mathbb{B}(1) = \{x \in \mathbb{R}^6 : \|x\| \leqslant 1\}$. The algorithm starts at $u = 0$ and applies a projected subgradient algorithm for $h_{S(g)}$ on $\mathbb{B}(1)$ to find a minimum. It follows from the proof of Lemma 3.7 that $q(g) < 0$ if this minimum is negative, and this fact is used in the algorithm. $Subgradient(h_{S(g)}, u)$ returns a subgradient

Algorithm 1 Returns "unstable" if a grasp can be determined to satisfy $q(g) < 0$ and "undetermined" otherwise.

Require: $N \in \mathbb{N}$, grasp g
$i \leftarrow 0$, $u \leftarrow 0 \in \mathbb{R}^6$, $q \leftarrow \infty$
while $q \geqslant 0$ and $i < N$ **do**
  $q = \min(q, h_{S(g)}(u))$
  $s \leftarrow Subgradient(h_{S(g)}, u)$
  $u \leftarrow u - \frac{1}{1+i} \frac{s}{\|s\|}$
  if $\|u\| > 1$ **then**
    $u \leftarrow \frac{u}{\|u\|}$
  end if
  $i \leftarrow i + 1$
end while
return "unstable" **if** $q < 0$, **else return** "undetermined"

Algorithm 2 Determine an upper bound $q^+(g) \geqslant q(g)$.

Require: $M, N \in \mathbb{N}$, grasp g
$q^+ \leftarrow \infty$
for $r \in \{1, 2, \ldots, M\}$ **do**
  $u \leftarrow RandomSample(\mathbb{S}^5)$
  for $i \in \{1, 2, \ldots, N\}$ **do**
    $q^+ = \min(q^+, h_{S(g)}(u))$
    $s \leftarrow Subgradient(h_{S(g)}, u)$
    $u \leftarrow u - \frac{1}{4\sqrt{i}} \frac{s}{\|s\|}$
    $u \leftarrow \frac{u}{\|u\|}$
  end for
end for
return q^+

of the function $h_{W_i(g)}$ for the first $i \in \{1, \ldots, m\}$ such that $h_{S(g)}(u) = h_{W_i(g)}(u)$. At points where $h_{W_i(g)}$ is differentiable, the gradient is returned, otherwise, we return $(n_i, (c_i - z) \times n_i)$ as discussed above. The decay rate $\frac{1}{1+i}$ and normalization by $\|s\|$ was chosen empirically based on satisfactory performance in our experiments (see Section IV).

Determining the upper bounds q^+, Q^+

We propose Alg. 2 to determine an upper-bound for $q(g)$. Here, $RandomSample(\mathbb{S}^5)$ returns a uniform random sample on the sphere $\mathbb{S}^5 \subset \mathbb{R}^6$. We chose a decay factor given by an inverse square root term in order to slowly decrease the step-size of the subgradient descent method. We define $Q^+(g) = \max(0, q^+(g))$. If one is only interested in the computation of $Q^+(g)$ rather than $q^+(g)$, Alg. 2 could in future be optimized by terminating as soon as $q^+(g) \leqslant 0$. Note that we now have $Q_l^-(g) \leqslant Q(g) \leqslant Q^+(g)$.

IV. Experiments

Lower and upper bounds

Let us now experimentally study the convergence properties of Q_l^- and Q^+. We generated a set U of 100000 random grasps with three contacts by sampling uniformly from the set $\mathcal{D}(2) = \mathbb{B}(2)^3 \times (\mathbb{S}^2)^3 \times \{0\}$, so that an element $g = (c_1, c_2, c_3, n_1, n_2, n_3, 0) \in \mathcal{D}(2)$ determines a grasp with three contact points $c_i \in \mathbb{B}(2) = \{x \in \mathbb{R}^3 : \|x\| \leqslant 2\}$, corresponding normals $n_i \in \mathbb{S}^2$, and with centre of mass at the origin. We computed the value of Q_l^- on these grasp sets for $l \in \{3, 4, \ldots, 20\}$ edges. Given our error estimate, the maximal error for $l = 40$ edges is at most $Err(2, 40) = 0.0092$ and we used $Q_{40}^- \approx Q$ to calculate an 'approximate ground truth' for the data set. We used the errors $d^-(g) = Q_l^-(g) - Q_{40}^-(g)$ to benchmark the convergence towards $Q(g) = \max(0, q(g))$ as the number of edges is increased. To evaluate Algorithm 2, we used $M \in \{5, 10, 20, 30, 40, 50\}$ random samples and $N = 1000$ iterations and recorded timings and the differences $d^+(g) = Q^+(g) - Q_{40}^-(g)$. We then split U into 10 equally sized sets and studied the variation in mean errors in Fig. 5, while Fig. 6 displays the worst-case performance on the whole set U. We observe that both Q_l^- and Q^+ converge quickly towards Q. While our implementation of Algorithm 2 is currently not parallelized, it can be easily parallelized in future due to its simplicity. Quickhull's [10] algorithm which is used to compute $Q_l^-(g) = \max(0, \min_j \lambda_j)$ (see Eq. 3) is quite complicated in comparison and currently not thread-safe. For our algorithm, we determined a good decay rate of $\frac{1}{4\sqrt{i}}$ empirically, but alternative rates and an implementation using line-search could potentially boost performance further. In applications where a precise value of q needs to be approximated, our results show that our algorithm can quickly provide an upper-bound, so that $Q_l^-(g) \leqslant Q(g) \leqslant Q^+(g)$ enabling us to make precise statements about the unknown quantity $Q(g)$.

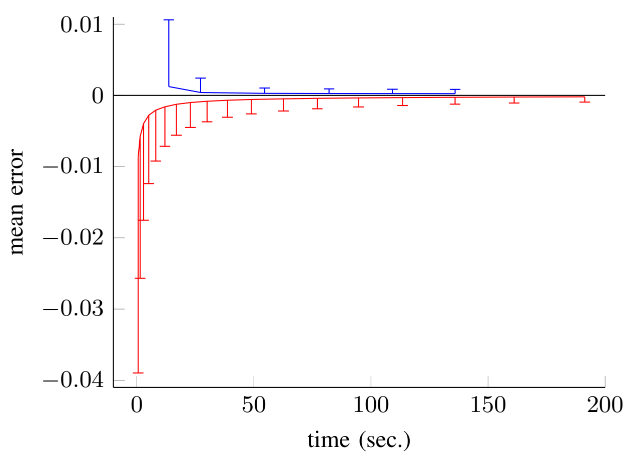


Fig. 5. Mean errors relating Q^+ and Q^- to Q_{40}^- and computation time per 10000 grasp quality evaluations are shown. The red curve displays the mean and standard deviation (indicated by bars below the curve) for the errors $d^-(g) = Q_l^-(g) - Q_{40}^-(g)$, for $l = 3, 4, \ldots, 20$ increasing from left to right, while the blue curve displays the mean and standard deviation (indicated by bars above the curve) for the errors $d^+(g) = Q^+(g) - Q_{40}^-(g)$ when using Alg. 2 with $M \in \{5, 10, 20, 30, 40, 50\}$ increasing from left to right and for $N = 1000$ iterations.

Fast unstable grasp rejection

We generated a set U' of approximately unstable grasps with three contacts by sampling uniformy in $\mathcal{D}(2) = \mathbb{B}(2)^3 \times (\mathbb{S}^2)^3 \times \{0\}$ until we obtained a set of 100000 sample grasps $g = (c_1, c_2, c_3, n_1, n_2, n_3, 0) \in \mathcal{D}(2)$ with contacts

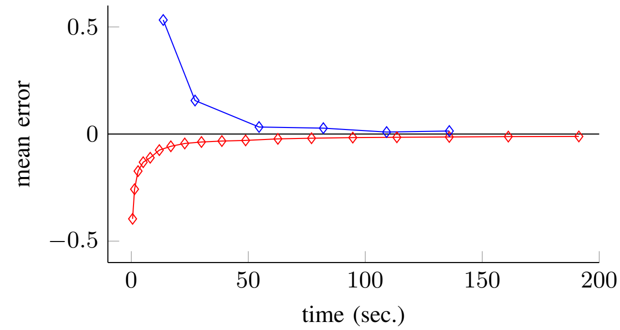


Fig. 6. Worst-case errors for our data-set of 100000 random grasps for Q^+ and Q^- are shown against mean computation time per 10000 grasp quality evaluations for 10 repetitions of our experiment with 10000 random grasps each. The red curve displays the errors $\min_{g \in U} Q_l^-(g) - Q_{40}^-(g)$, for $l = 3, 4, \ldots, 20$ increasing from left to right, while the blue curve displays the errors $\max_{g \in U}(Q^+(g) - Q_{40}^-(g))$ using Alg. 2, for $M \in \{5, 10, 20, 30, 40, 50\}$ increasing from left to right and for $N = 1000$ iterations.

c_i, normals n_i and centre of mass at the origin, and which satisfied $Q_{20}^- = 0$, i.e. they were unstable with respect to the quality measure Q_{20}^-. We now partition U' into ten equally sized subsets $U_1', \ldots, U_{10}'$ and tested the performance of our unstable grasp determination Algorithm 1 on these subsets. As can be seen in Fig. 7, our algorithm can quickly identify the majority of unstable grasps in U', with 97% of all grasps in U_i' correctly classified after 0.086 seconds on average.

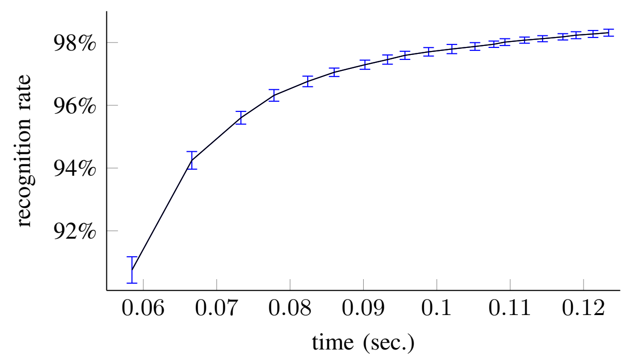


Fig. 7. The percentage of correctly recognized unstable grasps and the average computation time together with the standard deviation for 10 repetitions of the experiment are displayed. Data-points are ordered according to the number of maximal iterations $N = 50, 100, 150, \ldots, 1000$ increasing in increments of 50 from left to right. Note that the average computation time for evaluating Q_8 on 10000 unstable grasps is about $12s$, while our algorithm can correctly rule out 97% of those grasps as unstable in about *0.086 seconds*, resulting in an approximate 140-fold improvement in performance. If the same calculation was carried out using Q_{20}, our algorithm results in an approximate 2224-fold performance gain.

Finally, we tested our unstable grasp rejection algorithm as follows: we consider the same set of 100000 random grasps considered in Fig. 2(a). This time, we first, run Algorithm 1 with $N = 400$ maximal iterations. We then evaluate Q_l^- only on those grasps which our algorithm does not classify as unstable. This results in an algorithm that is between 49% and 503% faster than using Q_l^- by itself, as can be seen in Fig. 8 and 9. Our grasp quality evaluation algorithm is approximately 5 times faster than a simple evaluation of Q_l for $l \geqslant 8$ while yielding exactly the same quality values on stable grasps. Next, we considered the same set of grasps, but reduced the friction coefficient to $\mu = 0.2$ (*e.g.* polythene plastic against steel). While we obtained approximately 17.8% stable grasps under Q_{20}^- for $\mu = 1$ (*e.g.* copper against copper), we now only obtain 0.09% stable grasps with respect to Q_{20}^- and on our random grasp set U. As can be seen by the blue crosses, the computation of Q_l^-, takes even slightly longer for $\mu = 0.2$, while our algorithm with a first pass of Alg.1 now performs very well as indicated by the blue circles. Our algorithm computes Q_3 in $0.037s$, Q_8 in $0.056s$ and Q_{20} in $0.397s$ per batch of 10000 grasps. This amounts to an improvement of a factor of 15 (vs Q_3^-), 241 (vs. Q_8^-) and 600 (vs. Q_{20}^-) as indicated by Fig. 10. Our algorithm could hence be used to speed up grasp synthesis algorithms, and it could in particular enable such algorithms to quickly find grasps on objects with low friction coefficients for which sampling based methods were up to now unsuitable.

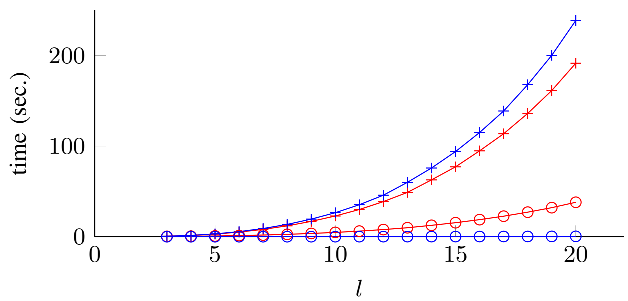


Fig. 8. Computation time when using a first pass of Alg. 1 with $N = 400$ (red circles) as compared a direct evaluation of Q_l^- (red crosses) on our randomly sampled grasps as in Fig. 2(a) and for a friction coefficient of $\mu = 1$. The blue circles and crosses depict the analogous plot for $\mu = 0.2$. We observe a substantial speedup when using our algorithm, especially for a lower friction coefficient of $\mu = 0.2$.

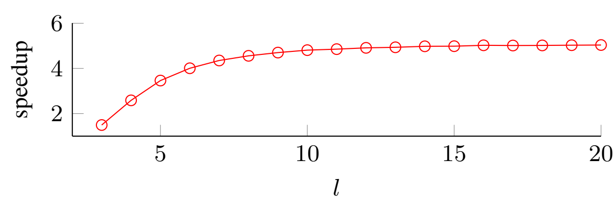


Fig. 9. Relative speedup improvement (vertical axis) multiple when using a first pass of Alg. 1 before evaluating Q_l^- rather than evaluating Q_l^- by itself for friction coefficient $\mu = 1$. The horizontal axis displays the l parameter for the computation of Q_l^-. The graph was obtained by evaluating grasp quality on a set of 100000 uniform grasp samples as in Fig. 2(a).

V. Conclusion

In this paper, we have extended the theoretical foundation for the study of a popular L^1 grasp quality measure which is based on an approximation of a friction cone by a convex polyhedral cone. Several related grasp quality measures that have been proposed in the literature [4], [14] are also dependent on such an approximation. While we have chosen to concentrate on the L^1 grasp quality measure introduced by [4], it is clear that our algorithm can be adapted to any grasp

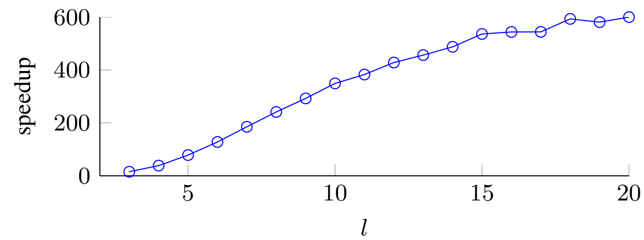


Fig. 10. Relative speedup improvement (vertical axis) multiple when using a first pass of Alg. 1 before evaluating Q_l^- rather than evaluating Q_l^- by itself for friction coefficient $\mu = 0.2$. The horizontal axis displays the l parameter for the computation of Q_l^-. The graph was obtained by evaluating grasp quality on a set of 100000 uniform grasp samples as in Fig. 2(a).

quality measure that needs to compute largest inscribing balls for a convex hull of wrenches such as the L^∞ metric of [4]. Furthermore, the algorithm can easily be adapted to scaled wrench metrics of the form $\|(f,p)\| = \sqrt{\|f\|^2 + \lambda\|p\|^2}$, for $\lambda > 0$ and wrenches $(f,p) \in \mathbb{R}^3 \times \mathbb{R}^3 = \mathbb{R}^6$. We have shown that our algorithms can dramatically improve the efficiency of grasp quality evaluation. This improvement is particularly evident in the case of low friction coefficients which poses a challenge to existing approaches. We are currently working on an optimized implementation of our approach and are in particular investigating a further optimization of the subgradient methods used in Algorithm 1 and 2.

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