Topological Mapping of Unknown Environments using an Unlocalized Robotic Swarm

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Abstract—**Mapping and exploration are essential tasks for swarm robotic systems. These tasks become extremely challenging when localization information is not available. In this paper, we explore how stochastic motion models and weak encounter information can be exploited to learn topological information about an unknown environment. Our system behavior mimics a probabilistic motion model of cockroaches, as it is inspired by current biobotic (cyborg insect) systems. We employ tools from algebraic topology to extract spatial information of the environment based on neighbor to neighbor interactions among the biologically inspired agents with no need for localization data. This information is used to build a map of persistent topological features of the environment. We analyze the performance of our estimation and propose a switching control mechanism for the motion models to extract features of complex environments in an effective way.**

I. INTRODUCTION

Swarm robotic systems have been considered as the subject of intense research in recent years. Behaviors such as formation, coverage, flocking, and consensus are adopted from groups of moving animals in order to develop behavioral-based distributed motion coordination algorithms. Cockroaches, for example, while not exhibiting highly organized and complicated individual and group behaviors, still demonstrate weak cooperation and information sharing with each other. Furthermore, they are shown to manifest interesting individual and local behaviors such as diffusive random walks [1], wall following [2], climbing and tunneling [3]. An overview of studies on analysis of emergent self-organized motion coordination algorithms based on local interactions in biological groups is provided in [4] and [5]. Application of such algorithms on distributed and embedded systems makes the execution of distributed sensing missions including monitoring, surveillance, exploration, and search and rescue feasible.

An inevitable factor in designing such distributed swarm systems is to provide them with capability of stable, robust, and optimized performance in uncertain and adverse environments. Consider for example a disaster zone response scenario, in which an earthquake or a hurricane has taken place. Our aim is to explore and obtain a map of an unstructured unknown environment using a swarm robotic system to find and rescue survivors. However, under such rough conditions of the terrain, odometry information received from the robotic

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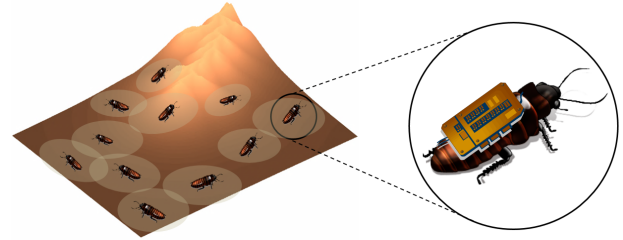
Fig. 1. A physical environment with a swarm of biobotic (cyborg insect) agents and their corresponding local sensing neighborhoods.

agents contains a high amount of uncertainty as a result of the abrupt structure of the environment. One may choose to employ a swarm of biologically inspired robots equipped with complex locomotion capabilities (such as climbing and crawling [6]), that enables them to explore and survive in such harsh environments. Alternatively, we can make use of existing biological agents via biobotic systems [7], where the natural locomotion capabilities of insects are fused with neuromuscular stimulation for their navigation (see figure 1). However, biobotic agents are provided with weak odometry capacities. Moreover, due to bandwidth constraints, vision sensing is also impractical. Additionally, by virtue of power constraints, long range RF communication and localization are unfeasible. Consequently, the system cannot be provided with strong and accurate localization information, and traditional localization and mapping algorithms such as SLAM [8] would fail to perform effectively. Alternatively, characterization of such scenarios can be accomplished by taking advantage of the randomness presented in the locomotion algorithm and employing topological representation techniques to estimate an abstract model of the environment rather than a complete metric mapping. Methods from algebraic topology can extract topological features from an environment without requiring localization information, which differentiates them from traditional approaches, and as a result, these methods are more suitable for scenarios in which weak or no localization is provided.

In this paper, we study exploration and mapping of unknown environments using a distributed swarm robotic system. In particular, we consider swarms of biobotic agents [7], which according to the above discussion, cannot make use of traditional localization techniques as a result of weak odometry information and limited sensing. Hence, we develop algorithms that do not depend on odometry or any other type of traditional localization schemes. Instead, we consider estimation of a topological model for the environment based

on limited information retrieved from the agents. Specifically, we assume that biobots are able to detect other agents in their proximity, and record their IDs and times of encounter and send these information to a base station. We consider biobotic agents whose motion model adopts to the locomotion model of cockroaches, including wall-following and random walk behaviors. Then, we employ algebraic topology to extract persistent features of the environment, such as connectivity and holes, based on agent encounter information. Finally, we explore the use of parameters from the motion models as external inputs for efficient mapping of the environment.

The remainder of the paper is organized as follows: Section II provides an overview of the related previous work. In Section III, we describe the motion model of agents based on the natural behavior of cockroaches. The mathematical tools for the analysis and estimation of the topological models are introduced in Section IV. In Section V, simulation studies are presented including the exploration of a swarm control mechanism based on switches on motion behavior. Finally, conclusion and future extensions of the presented work are discussed in Section VI.

II. RELATED WORK

In this section, we present the state of the art on: motion models inspired by insects, hole detection in sensor networks, and feature extraction based on topological representation of an environment.

Cockroach research: During the past decade, researchers in experimental biology and biomedical engineering have investigated models for the motion of individual cockroaches [1] as well as groups of them based on experimental data [9], [10]. Hardware platform developments include: a matched size miniature robot simulating the individual and group behavior of cockroaches based on hierarchical potential fields [11]; and a small autonomous robot equipped with an onboard camera and antenna-like contact sensors [12].

Hole detection in sensor networks: Finding the topology of a domain embedded in $\mathbb{R}^d$ is closely related to detecting its holes [13]. In sensor networks, a coverage hole is described as a region which is not covered by any agent [14]. Corke et al. [15] developed distributed algorithms for detecting location and borders of holes in a wireless sensor network. They implemented a local algorithm based on iterative convex hull method as well as a remote hole detection algorithm. However, their algorithm can only be applied to static and symmetric ad-hoc networks. In [16] the existence of holes are investigated by identifying the nodes where packets may get stuck.

Topological characterization of a domain: Traditional methods that require localization of agents for coverage characterization are not designed for large position uncertainty. Hence, methods that extract topological information were introduced. Ghrist et al. [17] developed an algebraic topological approach for stationary sensor networks based on persistent homology. The main feature of their approach was employing homology to characterize the coverage of a network by using only knowledge of which nodes are within
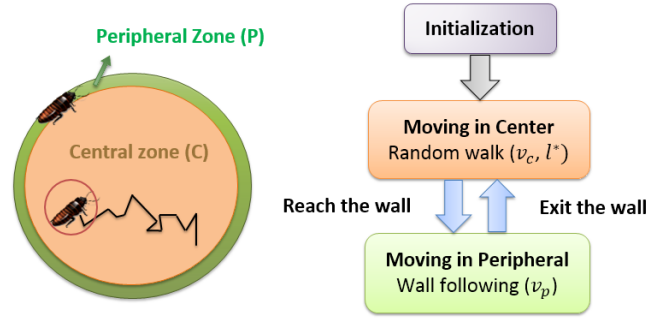

Fig. 2. Peripheral and central partitions (left) and the finite state machine representing the individual motion model of a cockroach (right)

a neighborhood [18]. In contrast to much of the previous work, in which a unit-disk model for the sensor coverage was assumed, E. Lobaton et al. [19] extended these topological models to incorporate long-range directional sensors in the presence of occluding objects, and demonstrated their approach in real networks of cameras. Muhammad et al. [20] developed distributed algorithms for the computation of these homologies through the use of gossip algorithms. The aforementioned studies mostly focused on static sensor networks.

Providing the nodes in a network with mobility increases the difficulty of the problem. One way to reduce this complexity is to look for patterns created by tracing the encounters of the nodes instead of investigating the mobility data itself. As such, Walker [21] employed persistent homology to compute topological invariants from encounter data of the mobile nodes in Mobile Ad-Hoc networks in order to infer global information regarding the topology of a physical environment. However, the nodes are assumed to follow a simple mobility model.

III. MOVEMENT MODEL OF COCKROACH-LIKE AGENTS

In this section, we describe the probabilistic model representing agents' motion in a bounded space based on the movement model of Blattella germanica cockroach, which is mainly adopted from [1] and [9].

It has been shown that individual cockroaches, when detecting an edge in the arena, perform wall following. Once they are far from the walls, their movements can be fitted on a diffusive random walk model. During agents' movement they might decide to have short or long stops. In a group level behavior, as interacting with other agents, cockroaches stop with higher probabilities when they encounter an aggregation and join the aggregated agents, and probabilistically leave the aggregation when they collide with a moving agent. In this paper, we limit the agents in our swarm model to follow only the individual motion characteristics of cockroaches for simplicity. A schematic of a finite state machine summarizing the states of the agents and their transitions based on occurring events is presented in figure 2 (left).

Consider a bounded environment $\mathcal{D} \in \mathbb{R}^d$, e.g. the one shown in figure 2. We split the environment into *peripheral* and *central* partitions. Peripheral partition, $\mathcal{P}$, is defined as

a subset of the points which are within a distance of r_p with respect to the boundary of $\mathcal{D}$, i.e. $\mathcal{P} = \{p \in \mathcal{D} \mid \|p - q\| < r_p, \text{for some } q \in \partial \mathcal{D}\}$. The central partition consists of the set of the rest of the points in the environment.

The movement model of an individual agent could be obtained by incorporating the motion models in the two regions.

1) Peripheral Partition (P): When agents are close enough to the boundary of the environment, they perform a wall following behavior with a constant average velocity of v_p. During their movement, they can either make a probabilistic stop for a period of time and then continue their movement, or leave the peripheral partition towards the central one. The agents' stop is modeled as a memory-less process characterized by an exponential distribution with a characteristic time $\tau_{\text{stop,p}}$ representing the average time elapsed before an agent stops. Once an agent stops, it could either remain active with a probability of $p_{\text{sh,p}}$, characterized by short stops, or inactive with the probability of $1 - p_{\text{sh,p}}$, described as long stops. Each of these two stopping events are characterized by exponential distributions with characteristic times $\tau_{s,p}$ and $\tau_{l,p}$, respectively. When the agents are following the boundary of the arena, they leave the peripheral zone after an average of τ_{exit} seconds, which denotes the characteristic time of the corresponding exponential distribution. They leave the peripheral partition with an angle of θ_{exit} with respect to the tangent vector to the boundary at their current position distributed as a uniform density $U(0, \pi)$.

2) Central Partition (C): Motion characteristics of the agents in the central partition is modeled as a diffusive random walk with piecewise fixed orientation movements, characterized by line segments, interrupted by direction changes. The average length of the line segments l^*, is considered to be the characteristic length of an exponential distribution for the path lengths. As for angular reorientation, we assume an isotropic diffusion, where $p(\theta_{\text{new}}|\theta_{\text{current}}) = p(\theta_{\text{new}}) \sim U(0, 2\pi)$, resulting in a uniform distribution of angle reorientation independent of the previous angle.

As in the peripheral partition, the agents stop their random walk on average after a characteristic time of $\tau_{\text{stop,c}}$. The short and long stops are characterized with probabilities of $p_{\text{sh,c}}$ and $1 - p_{\text{sh,c}}$ and characteristic times $\tau_{s,c}$ and $\tau_{l,c}$, respectively.

IV. TOPOLOGICAL CHARACTERIZATION OF ENCOUNTERS

Consider a network of N finite dimensional moving agents in a bounded environment $\mathcal{D} \in \mathbb{R}^d$ and denote the set of IDs
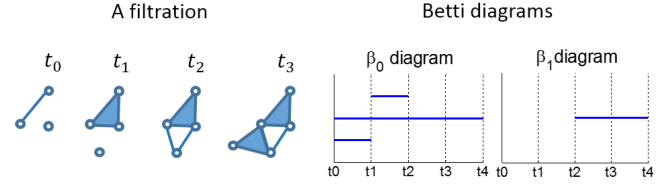

Fig. 3. An example of a filtration (left) and the corresponding Betti diagrams in dimensions 0 and 1 (right). The diagrams confirm the existence of a persistent connected component in the interval $[t_0, t_4]$ and a 1-dimensional hole which is born at t_2 and persists for the rest of the filtration time

assigned to all agents by $\mathcal{I}$. We assume that motion dynamics of the agents mimics the movements of cockroaches as described in Section III. The agents are provided with weak localization information, i.e. they can only identify the other agents within their radius of detection. However, they are provided with their own tags, and are able to capture the tags of the other agents coming into their detection radius, as well as the time at which this encounter event occurs. This information is sent to a base station.

Algebraic topology, in contrary to geometric techniques, which aim to estimate shapes and distances, deals with the way that objects are connected. Using tools from algebraic topology, we can construct a model of the environment using combinatorial objects called simplicial complexes. We investigate persistent features of the environment like connected components and holes via *persistent homology*, a mathematical tool from computational topology.

In the next subsection, we briefly go over the concepts we use from persistent homology throughout the rest of the paper. For more details the reader is encouraged to refer to [22].

A. Persistent Homology

A standard method to map a collection of data points to a combinatorial object in order to analyze their topological structure is to represent them as a *simplicial complex*. Given a set of vertices $\mathcal{V}$, a k-simplex is defined as a set $\{v_1, v_2, \ldots, v_{k+1}\}$ where $v_i \in \mathcal{V}$, $\forall i$ and $i \neq j, \forall i, j$. For example, a graphical interpretation corresponds to thinking of a 0-simplex as a vertex, a 1-simplex as an edge, and a 2-simplex as a triangle. A finite collection of simplices, is called a *simplicial complex*.

A *filtration* is a collection of complexes $X(t)$ such that if $t < s$ then $X(t) \subset X(s)$, meaning that all simplices in $X(t)$ are included in $X(s)$.

Topological invariants are mappings which aim to classify equivalent topological objects into the same classes. Betti numbers are ranks of a special type of topological invariants, called *homology groups*. The k-th Betti number, denoted by β_k, is a coarse measure of the number of k-dimensional holes in the topological space. In particular, β_0 is the number of connected components and β_1 is the number of *holes* in the complex.

Persistent homology is a topological technique that enables us to extract topological information of an environment via

TABLE I

RANDOM WALK AND WALL FOLLOWING MODEL PARAMETERS

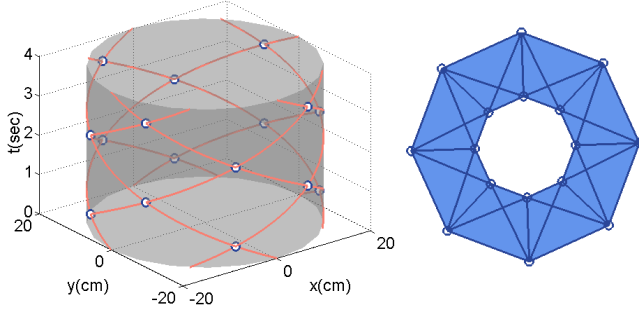| Zone	Mean Speed	l^*	$\tau_{\text{stop}}(n)$	$p_{\text{sh}}(n)$	$\tau_s(n)$	$\tau_l(n)$
C	15 mm/sec	30 mm	20	0.93	0.5	20
P	10 mm/sec	-	20	0.93	0.5	20

Fig. 4. Example of encounter events for 8 agents moving on a circle: the trajectories over time as orange curves and encounter points as white circles (left), and the corresponding encounter complex (right)

filtered simplicial complexes. It describes the topology of these complexes by means of a collection of *Betti intervals*, or *persistent intervals*. Each k-dimensional Betti interval $[t_{b_i}, t_{d_i}]$ can be associated to a k-dimensional hole with an appearance or *birth* time t_{b_i} and closing or *death* time of t_{d_i}. β_k diagrams are used to visualize k-dimensional Betti interval signals over filtration time.

As an example, consider the filtered simplicial complex shown in figure 3 as well as its Betti diagrams for dimensions 0 and 1. At time t_0, there exist three vertices and one edge resulting in two connected components. At t_1, one more vertex, two more edges, and one 2-complex are added which makes one of the components join the other one giving death to the corresponding connected component in β_0 diagram of figure 3; but another connected component appears which gives birth to a line in the same diagram. Adding more edges at t_2 results in one connected component and gives birth to a cycle which is shown in β_1 diagram of figure 3. At t_3 more simplices are added yet there is no change in the number of connected components and cycles. Clearly, $X(t_1) \subset X(t_2) \subset X(t_3) \subset X(t_4)$.

Persistent homology has been used to extract coverage hole information out of stationary sensor networks in [17]. However, this approach mainly concerns about the coverage holes in the sensing domain of the collection of the nodes as a sensor network, while we do not care about the holes in coverage but the topology of the physical environment itself. Hence, this technique by itself cannot be applied to our study case. On the other hand, for a network of moving agents, we can make use of a subset of data associated with encounters between agents instead of dealing directly with the collection of points $(p_i(t), t)$, where $p_i(t)$ is the position for the i-th agent at time t. This enables the construction of a combinatorial structure called the *encounter complex*.

B. Encounter Complex

For moving agents in the environment, we say that an encounter event E_i occurs at time t_i if the Euclidean distance between two agents with ID's $I_1, I_2 \in \mathcal{I}$ and position vectors $p_{I_1}(t_i)$ and $p_{I_2}(t_i)$, $\|p_{I_1}(t_i) - p_{I_2}(t_i)\|$, is less than a predefined encounter threshold, T_e. Accordingly, we construct the weighted encounter graph $\mathcal{G}_E$ with vertex set $\mathcal{V} = \{v_i\}$ with

v_i corresponding to the i-th encounter event E_i defined as

$$v_i = [t_i, I_1, I_2]. \qquad (1)$$

Two vertices v_i and v_j are connected with edge weights $w_{ij} = |t_i - t_j|$, if the events E_i and E_j involve a common agent. Having the set of all pairwise distances, we define a metric on the set of events as a matrix $\mathcal{G}_E$ whose elements are the length of shortest paths between two event nodes. Given a distance threshold ϵ, a $(k-1)$-dimensional simplex is formed if there is a subset of k points in the graph that are within ϵ distance from each other. The Rips complex (also referred to as the *Encounter complex* [21] in this application) for this ϵ value is the collection of all such simplices. A filtration is obtained by varying the value of ϵ from 0 to the diameter of the weighted graph.

Consider, for example, four agents moving with constant speed on the boundary of a circle with radius $r = 20cm$ in clockwise direction, and four other agents move under the same conditions but in counterclockwise direction. The trajectories of the agents over time as well as the encounter events are shown in figure 4 (left), and the encounter complex based on the metric defined above is depicted in figure 4 (right). It can be observed that although the aforementioned metric is independent of the positions of the encounter events, using filtered complexes, we can extract topological information from the environment.

Given a finite point cloud, or a finite metric space representing a point cloud, there are several ways to construct such a filtered simplicial complex. However, extracting topological information from the encounter complex can be computationally expensive due to the large number of events present. This is the motivation behind the development of the witness complex [23], in which a small subset of the points, called *landmark points*, is selected on which a smaller filtration, the witness complex, is constructed such that it possess the same topological properties as the original one. A well-known method for selecting landmark points, is the *maxmin* algorithm. The methodology of the maxmin algorithm and the construction of the witness complex can be found in detail in [23]. We make use of the implementation available in [24].

V. Results

In this section, we perform an exploratory study on how to robustly extract topological information of an unknown environment based on the weak encounter information of biobotic agents. We carry out our analysis though numerical simulations of a swarm of insect like agents with parameters described in section III.

We first explore topological characterization of two test environments, and investigate the effectiveness of the proposed topological mapping technique on the extraction of features from the environments. A motion model parameter is exploited as an exploration controller input to switch from interior mapping to boundary estimation of our test environments. Particularly, we vary the characteristic time before exiting the wall following behavior to its default value

Fig. 5. Block diagram of the topological estimation of an environment based on encounter events and witness complex
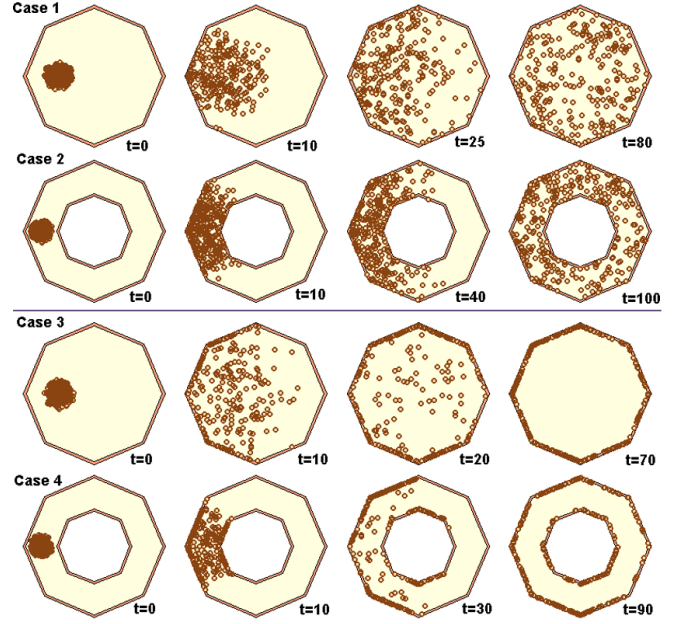

Fig. 6. Dispersion of the swarm in the environment over time under four different cases. Cases 1 and 2 correspond to the natural motion (NM) behavior, and cases 3 and 4 refer to the wall following (WF) behavior

of $\tau_{\text{exit}} = 7.69s$, corresponding to the agents' natural motion (NM), to the extreme values of ∞ and 0, corresponding to a pure wall following (WF) or a pure random walk (RW) behavior. We will employ switching to WF in order to map the boundary of the environments. Then we propose an alternatively switching swarm controller between RW and WF in order to perform interior and boundary mapping of more complex environments, which cannot be modeled with a fixed topological estimation set-up. To compute the persistent homology of the filtered encounter complexes, we use javaPlex [24] software package, aimed at implementing persistent homology and related techniques.

Figure 5 overviews the general process of estimating a topological model for an unknown environment as a block diagram. For visualization of the point cloud data corresponding to the distance metric $\mathcal{G}_E$, we make use of Multi-Dimensional Scaling (MDS) technique to obtain corresponding coordinates in the Euclidean space [25].

A. Topological analysis of encounter complex

We consider two octagonal environments, $\mathcal{D}_1$ (a simply connected region) and $\mathcal{D}_2$ (a region with a hole). Topological features of the interior of $\mathcal{D}_1$ and $\mathcal{D}_2$ can be described as both having only one connected component, and $\mathcal{D}_2$ having one 1-dimensional hole while $\mathcal{D}_1$ has no holes in its interior.

1) Cases 1 & 2: A total of 150 agents are used in both scenarios, which are initialized in close proximity from each other (see t = 0 for cases 1 and 2 in figure 6). In addition to parameters presented in tables I, we set our control parameter to the value corresponding to the natural motion, i.e. $\tau_{\text{exit}} = 7.69s$. For case 1, the configuration of the agents over three time samples, $t = 10s, 25s$ and $80s$ are presented in figure 6. It can be observed that at $t = 80s$ the agents cover almost equally the whole environment. Simulation results for environment $\mathcal{D}_2$ are also shown as case 2 in the same figure. Because of the topology of $\mathcal{D}_2$ it takes longer for the agents to disperse in the whole area. The encounter events are recorded to be used to produce the metric graph $\mathcal{G}_E$. However, we only select $n_E = 3000$ encounter points to extract topological information from, and out of these points only $n_l = 100$ landmark points are selected for construction of the witness complex using the maxmin algorithm. The trajectories of the agents as well as the corresponding encounter events over time for case 2 are shown in figure 7(a). The point cloud obtained using MDS corresponding to the events in case 2 are plotted in figure 7(b). It can be observed that although these points do not correspond to the real position of the events in the physical space, they represent the topological features of the

environment correctly. Corresponding landmark points are depicted in figure 7(c).

The resulting witness complexes as well as β_0 and β_1 diagrams are presented in figure 8. In case 1, the witness complex represents a single connected component with no holes. One can come to the same conclusion looking at the persistent diagrams of dimension 0 and 1. There is one persistent connected component in β_0 diagram and no persistent features in β_1 diagram. On the other hand, for case 2, a single connected component with a hole is observed in the witness complex, agreeing with a single persistent feature in the corresponding β_0 and β_1 diagrams.

2) Cases 3 & 4: Now we aim to perform a boundary mapping for $\mathcal{D}_1$ and $\mathcal{D}_2$. To this end, we exploit the control parameter to switch to WF behavior. Corresponding witness complexes and barcodes (see figure 8) confirm the existence of one connected component and one hole for the boundary of $\mathcal{D}_1$ and two connected components and two holes for the boundary of $\mathcal{D}_2$, corresponding to its inner and outer octagonal boundaries.

TABLE II
PERSISTENT FEATURES EXTRACTED FOR THE SWITCHING PHASES

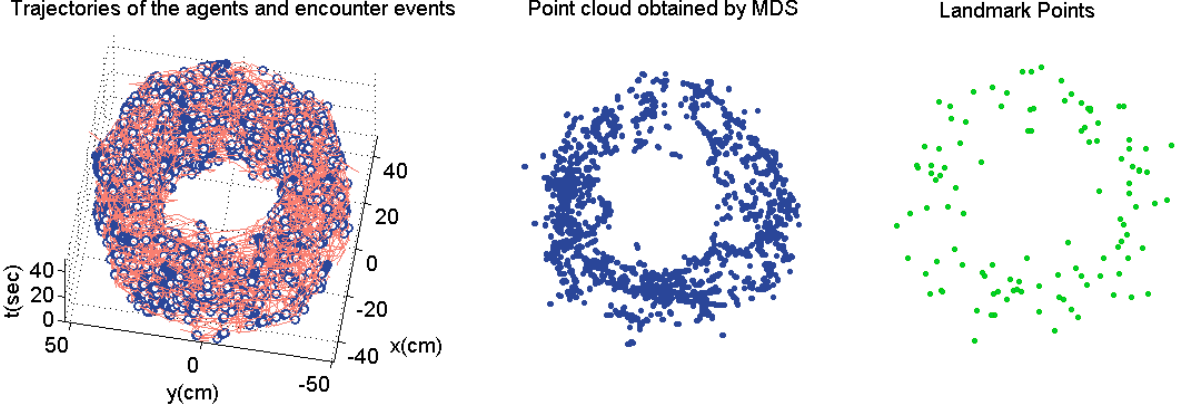|	Phase I	Phase II	Phase III	Phase IV
Time interval	[0, 40]	[40, 150]	[150, 300]	[300, 400]
Motion Model	NM	WF	RW	WF
β_0	1	1	1	2
β_1	0	1	1	2

Fig. 7. (a) Trajectories of the agents and the encounter events in Euclidean space for case 2, (b) corresponding point cloud of the metric space using MDS, and (c) the corresponding landmark points.

B. Switching swarm behavior for topological mapping

Now we consider a more complex environment, $\mathcal{D}_3$, consisting of two polygons, one simply connected and the other with one hole inside, connected via a narrow passage as shown in figure 9 (left). We start the simulation with the same initial configuration as in Case 1 with an NM behavior in the left polygon and $n = 250$ agents. The resulting configuration

after a transient time of 40 seconds is shown in figure 9 (a). It can be observed the agents are not able to pass the passage as they are bouncing back and forth between the edges of the passage due to the nature of their random walk motion. As a result, most of the agents cannot penetrate the right side of $\mathcal{D}_3$. The resulting persistent diagrams are shown in the same figure, exposing one simply connected component, which is incorrect.

In order to overcome this problem, we adopt a switching controlling scheme to extract as much information as possible out of the encounter data. We propose to switch controller parameter τ_{exit} between its two extreme cases. The type of motion models, the intervals of duration, and the persistent β_0 and β_1 features of the different phases considered are shown in Table II. The length of the phases are selected manually to ensure enough time for dispersion of all the agents. Figure 9 shows the corresponding physical configurations and persistent diagrams for each phase. As
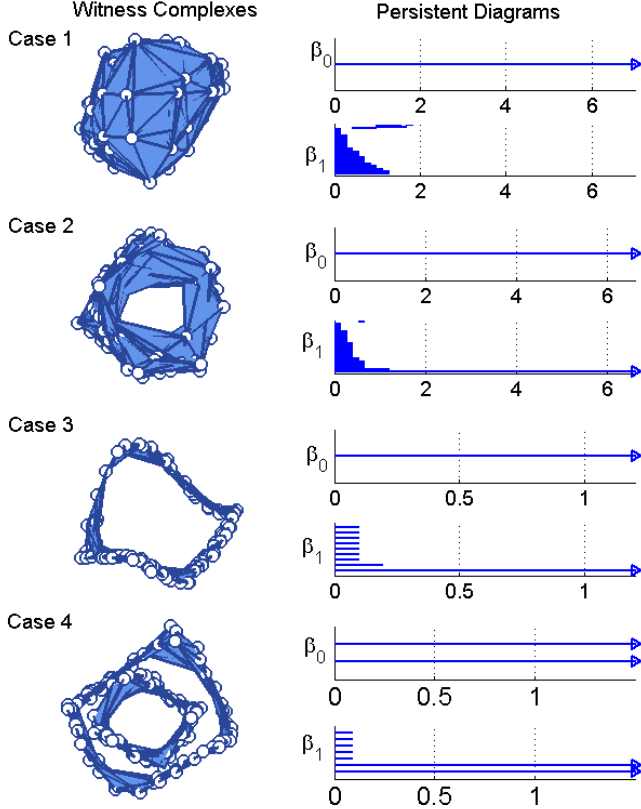

Fig. 8. Witness Complexes, and β_0 and β_1 persistent diagrams for the four scenarios in figure 6. The coordinates of the landmark points are obtained using the MDS algorithm.
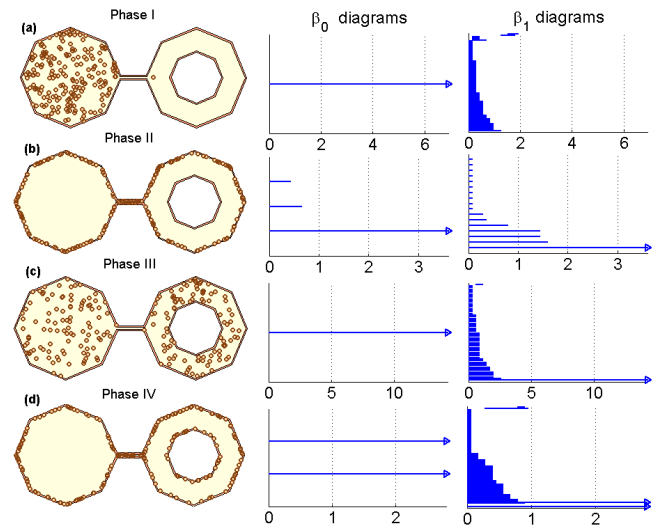

Fig. 9. Configuration of the swarm at the end of each phase (left) and the corresponding β_0 and β_1 diagrams (middle and right).

described before, at the end of phase I, we observe a single persistent connected component in the β_0 diagram, and no persistent β_1 holes. At the end of phase II, we observe that a single connected components and a hole are found due to the wall-following behavior of the system. At the end of phase III, a single connected component and a single hole are found, which leads us to believe that a new region has been discovered since this hole was not observed during phase I. This also indicates the existence of a narrow passage. Finally, at the end of phase IV, two connected components and two holes are observed, which supports our observations from phase III. As a summary, we can say that the environment consists of a simply connected region and a region with a hole connected by a narrow passage, which is a description for the environment.

VI. CONCLUSION AND DISCUSSION

In this study, we explored a topological estimation approach under weak localization assumptions. We used encounter information of agents with motion models inspired by cockroach behavior to extract topological information of the environment from persistent features. We then made use of a motion parameter to switch between different swarm behaviors in order to perform both interior and boundary mapping of the environment.

During our experiments, we observed that this estimation process is sensitive to the number of agents, number of time samples, number of encounter events, and landmark selection algorithm. If few agents are used, then the events will take place at sparse locations over the time-space domain, which does not permit for proper estimation of topological features. It was also observed that a higher number of encounters was needed when performing wall following, which we believe is due to the lack of change on direction of the agents. Moreover, the number of landmarks needed for proper estimation increases with the number of holes.

We plan to address the aforementioned sensitivity issues by investigating any lower bounds required for a provably accurate estimation of the topological features. This may also lead to better landmark selection methodologies for data reduction when computing the persistent diagrams.

The proposed switching strategy for exploration and mapping from Section V.B, needs to be further investigated. As future work, we will investigate a more careful control of the motion parameters. For example, the length and frequency of switches between the different phases can lead to faster exploration and more accurate mapping.

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