2013 IEEE/RSJ International Conference on
Intelligent Robots and Systems (IROS)
November 3-7, 2013. Tokyo, Japan

Synthesis and Evaluation of Non-circular Gear that Realizes Optimal Gear Ratio for Jumping Robot

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Abstract— **For the effective use of motor power, an appropriate gear ratio has to be selected according to the robot task and motion. Because a jumping robot, in particular, requires both high torque and high velocity properties through the motion, a varying gear ratio is necessary. Moreover, it has to be optimally designed and realized. In this paper, we design a jumping robot with a non-circular gear that changes the gear ratio through the motion for the higher jumping. Based on statics, the gear ratio is optimized and change of the gear ratio is obtained by a simulation-based design considering the robot dynamics. So far, we have developed a design method of no-circular gear, and its effectiveness was evaluated by simulation. In this paper, a jumping robot is prototyped, and the effectiveness is evaluated by experiment containing the robustness for the perturbation of robot parameters.**

I. INTRODUCTION

For mechanical design of a robot system, a reduction gear is utilized to change the actuator property depending on the requirement of high torque output or high velocity realization. DC (Direct Current) motor is classified based on its power P[W] and the motor current and angular velocity have high correlation. The motor current yields output torque τ[Nm] and the relationship between τ and angular velocity ω[rad/sec] is represented by $\tau\omega = P$, which means that the motor torque τ will be small for the larger ω. By assuming the gear ratio as G, the output torque is multiplied by $1/G$ and the angular velocity is multiplied by G. Therefore, an appropriate gear ratio has to be selected considering the robot tasks.

We consider a jumping robot. It requires (i) a high torque output to kick the ground in the beginning of the motion, on the other hand, (ii) high velocity realization in the end of the motion as shown in Fig.1. The lower gear ratio in (i) and the higher gear ratio in (ii) are required. From these considerations, a varying gear ratio will be necessary and appropriate nonlinear property of varying ratio has to be optimally designed to jump higher with an effective use of a DC motor.

Many researches on jumping robots have been reported so far. Niiyama [1] realized a jump motion of a humanoid robot with a musculoskeletal mechanism. Ishikawa [2] proposed a motion stabilization method of hopping robots based on hybrid-system control theory. Ugurlu [3] proposed a ZMP-based jumping controller design method. These are researches on control. Shimoda [4] and Sakaguchi [5] developed a jumping principal and mechanism using inertia force of a mass inside the body. Kovac [6], Curran [7] and Tsuda [8] proposed jumping mechanism using potential energy of a spring. These researches are on development of a mechanism and its control. It is true that the usage of a spring yields large height of jumping, however, it may consume large energy of actuator because not all the accumulated energy of the spring will be available.

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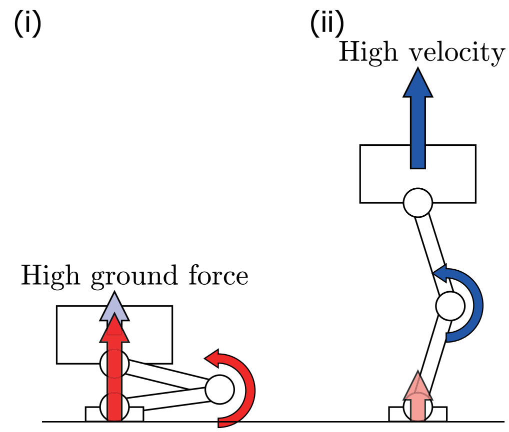

Fig. 1. Jumping robot property

On the other hand, we have proposed a design method of the optimal varying gear ratio for a jumping motion aiming at the optimal use of actuators power without charging energy, which enables the jumping robot to perform immediately. The obtained gear ratio is realized by a non-circular gear, and a design method of the pitch curve of the non-circular gear has been proposed. Moreover, the effectiveness of the non-circular gear has been evaluated by simulations. Because the pitch curve is designed based on simulation-based method, namely, forward dynamics analysis, it is effective only for a specified robot with a specified initial posture. In this paper, we prototype a jumping robot with a non-circular gear based on the proposed method and evaluate the effectiveness of the non-circular gear containing robustness for a perturbation of the robot parameters. The purposes of this paper are as follows;

1) Based on a simulation-based design method, we design an non-circular gear which has a optimal varying gear ratio maximizing ground force.
2) Considering a constrain of pressure angle, the pitch curve is modified and a non-circular gear is prototyped.
3) By the experiments using the prototyped robot, the effectiveness of the proposed method is evaluated. Moreover, the robustness for the perturbation of the

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initial posture of the robot is evaluated.

CVT (Continuously Variable Transmission) is used for a automobile transmission that realizes a varying gear ratio, however, it is for a multiple rotational actuator with high angular velocity. Hagiwara [9] and Takaki [10] proposed special mechanisms that realize load sensitive varying gear ratio. The proposed method in this paper focuses on a fixed but position depending gear ratio for a robot motion.

II. Optimal gear ratio and pitch curve design of non-circular gear

A. Jumping Robot

In this section, the optimal value of the gear ratio is introduced based on statics. Consider a jumping robot as shown in Fig.2. The input is a fixed voltage E. The motor

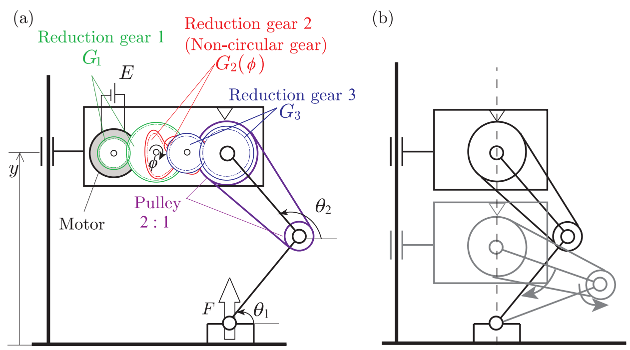

Fig. 2. Jumping robot with a non-circular gear

torque is transmitted to the ground force through three reduction gears. The gear ratio $G_2(\phi)$ changes depending on the input angle of the non-circular gear ϕ. The reduction gear 1 has a small constant gear ratio $G_1(\phi)$ to generate a large torque. Because the robot legs rotates 90° at maximum, the reduction gear 3 with a constant gear ratio is set in order to rotate the non-circular gear widely. The leg consists of two links whose length are ℓ and the under leg is chained to the robot body by a timing belt. The under leg rotates revolving around the hip joint as shown in Fig.2(b) because the gear ratio of timing pulley is set as 2 : 1. This gives a constrain so that the relationship between θ_1 and θ_2 is represented by:

$$\theta_1 + \theta_2 = \pi \tag{1}$$

Moreover, the body motion is constrained vertically by a prismatic joint. The motor current is limited to $i_{\ell im}$.

B. Optimal gear ratio

The longer time and larger acceleration will be required for the higher hump. However, because of kinematic constraint of robot mechanism (limitation of mobile area) and high correlation between the motor torque and angular velocity, the optimal use of the motor power is necessary. So far, we have developed an optimal design method of gear ratio that realizes efficient motor power and a realization method of the gear ratio by a non-circular gear [11]. They are summarized as follows. From the principle of virtual work, the relationship between the ground force F and the motor output torque τ is represented by:

$$F = \frac{\tau}{2G_1G_2G_3\ell\cos\theta_1} \tag{2}$$

The motor torque is represented by;

$$\tau = K_t i \tag{3}$$

where K_t is the torque constant and i is the motor current. The relationship between E and i is represented by;

$$E = Ri + K_e\dot{\phi}_m \tag{4}$$

where R is the motor resistance and K_e is the coefficient of back electro motive force ($= K_t$). The motor inductance is assumed to be negligible small. $\dot{\phi}_m$ represents the angular velocity of the motor. The relationship between $\dot{\phi}_m$ and the body velocity $\dot{y}$ is represented by;

$$\dot{\phi}_m = \frac{\dot{y}}{2G_1G_2G_3\ell\cos\theta_1} \tag{5}$$

By substituting equations (3), (4) and (5) into (2), the ground force is obtained by:

$$F = \frac{K_t}{2G_1G_2G_3R\ell\cos\theta_1}\left(E - \frac{K_t\dot{y}}{2G_1G_2G_3\ell\cos\theta_1}\right) \tag{6}$$

By assuming $\dot{y} > 0$, without loss of generality, equation (6) is a quadratic function of convex upward with respect to G_2^{-1}, and there exists an optimal G_2 that maximize F as:

$$G_2 = \frac{K_t\dot{y}}{EG_1G_3\ell\cos\theta_1} \tag{7}$$

C. Simulation-based design of pitch curve

Because the non-circular gear of the jumping robot in this paper does not rotate 360° and over, we can design the change of gear ratio considering only the shape of pitch curve.

Equation (7) represents that the optimal G_2 is a function of $\dot{y}$. However, because G_2 is realized by a non-circular gear, G_2 has to be a function of ϕ. Thus the relationship between ϕ and $\dot{y}$ has to be obtained as $\dot{y} = \dot{y}(\phi)$. In previous research, we proposed a simulation-based method to connects statics (optimal gear ratio) and dynamics (rerationship between ϕ and $\dot{y}$) as Fig.3. At first, an optimal gear ratio $G_2[0]$ is calculated from the initial attitude of the robot $\theta_1[0]$,$\phi[0]$ and initial velocity $\dot{y}[0]$. Setting a minimum gear ratio G_{2min}, because of a limitation of gear ratio. Angular acceleration of legs and ground force are obtained with the robot equations of motion using $G_2[0]$. With the angular acceleration, $\theta_1[1], \phi[1], \dot{y}[1]$ inthe next step are calculated. Based on these values the optimal gear ratio $G_2[1]$ is calculated. By iterating these steps until the robot takeoffs at $F < 0$, the a relationship between ϕ and G_2 is obtained as Fig.4. From the right hand side of Fig.4, the pitch curve of non-circular gear is calculated. By setting the distance between the rotational axes of the input and output non-circular gear as d, the pitch radius of the input gear $r_{in}(\phi)$ is obtained by:

$$r_{in}(\phi) = \frac{G_2(\phi)\,d}{1 + G_2(\phi)} \tag{8}$$

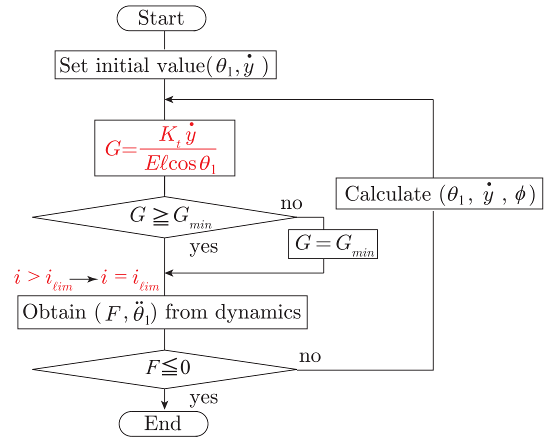

Fig. 3. Design of optimal change of gear ratio with simulation-based method

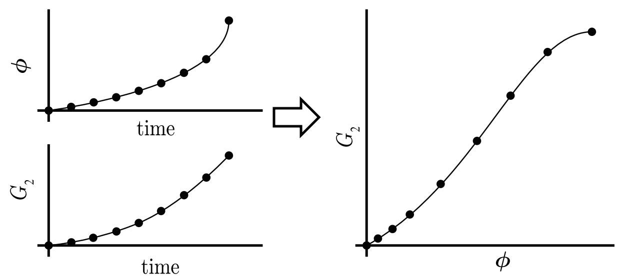

Fig. 4. Design of optimal gear ratio change by simulation

Because the rotational angle of the out put gear ϕ_{out} is represented by:

$$\phi_{out} = \int G_2(\phi)\, d\phi \tag{9}$$

The pitch radius of the output gear r_{out} is represented by:

$$r_{out} = d - r_{in}(\phi) \tag{10}$$

From equation (8) the pitch curve of the input gear is obtained. From equations (9) and (10) the pitch curve of the output gear is obtained as shown Fig.5.

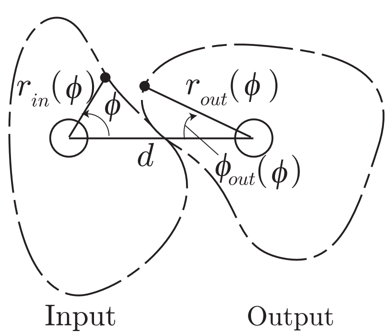

Fig. 5. Design method of pitch curve

From these considerations, the optimal gear ratio is realized by a non-circular gear. However, there is a maximum limitation for the pressure angle of a gear, which is a constraint for smooth meshing of input and output gear. The pressure angle is recommended to be less than 50° in [12]. In this paper, by adjusting minimum value of $G_2(G_{2min})$ to an appropriate value, the constraint is satisfied.

III. Prototype of Jumping Robot and Non-Circular Gear

A. Design of jumping robot

To design a non-circular gear with the simulation-based method, we design a jumping robot as shown in Fig.6(a). The three gears in Fig.2 are realized as shown in Fig.6(c). Reduction gear 1 is a HarmonicDrive gear and a timing belt. Reduction gear 3 is realized by a spur gear. Two linear bushes are utilized as a prismatic joint. Each parameter is shown in

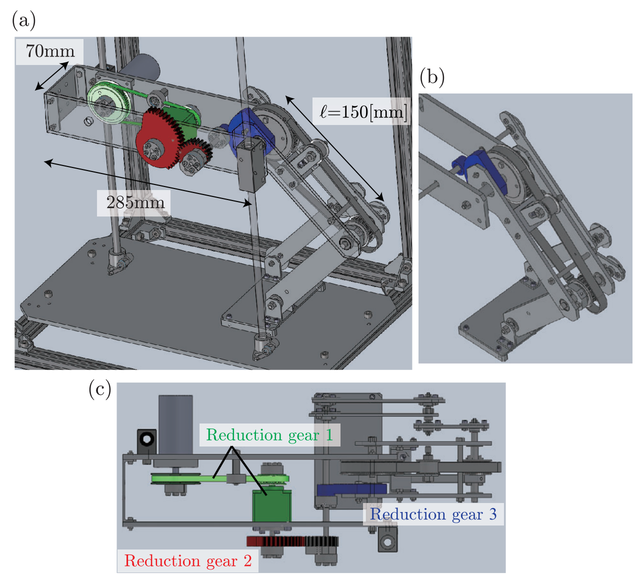

Fig. 6. Design of jumping robot

Table I. A 90W DC servo motor (MAXON Corp.) is used, whose torque constant and resistance are $K_t = 52.5$mNm/A and $R = 2.07\Omega$ respectively. The input voltage is supplied from power supply.

TABLE I

Designed Robot Parameters and Initial Value

G_1(Gear ratio of reduction gear 1)	1/28
G_3(Gear ratio of reduction gear 3)	1/5
ℓ(Leg length)	0.15m
M(Total mass)	2.27kg
$\theta_1[0]$(Initial value θ_1)	10°
$i_{\ell im}$(Current limitation)	12A
E(Input voltage)	48V

B. Design of non-circular gear

We design a non-circular gear based on the robot parameters in the previous subsection. By setting $G_{2min} = 0.1$, the change of the gear ratio is obtained as shown by the black line in Fig.7, and the pitch curve is obtained as shown Fig.8(a) where the dashed line is an arbitrary shape because it is unused for jumping. The pitch radius of this non-circular gear changes largely to realize large velocity with respect to rotation. It causes large pressure angle as shown by black line in Fig.9. By adjusting the minimum value of gear ratio, so as to satisfy the constraint, $G_{2min} = 1$ is selected. By a spline interpolation of every 10°s data, the curvature is obtained, which is shown by red line in Fig.7. Fig.8(b) shows the pitch curve of the non-circular gear. Because the pressure angle is less than 50° as shown by red line in Fig.9, this non-circular gear is realizable. Fig.10 shows the prototyped non-circular gear, whose module is 1.5, the distance between rotational axes of input and output gear is 56mm, the thickness is 10mm. They are made of die steel.

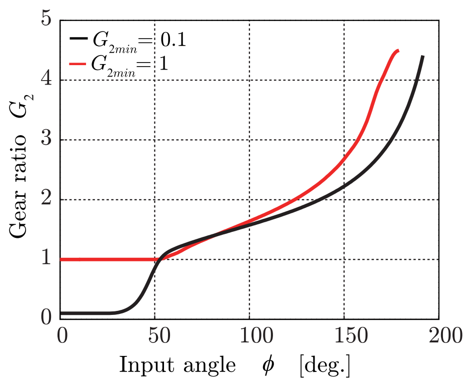

Fig. 7. Gear ratio of non-circular gears

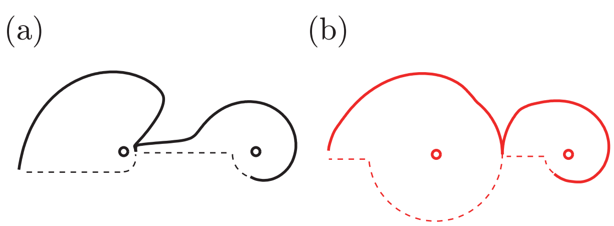

Fig. 8. Pitch curves of designed non-circular gears

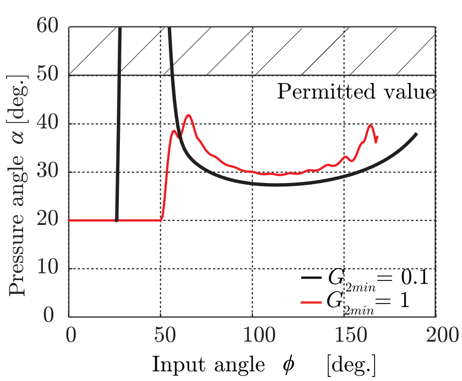

Fig. 9. Operating pressure angle

C. Evaluate of effectiveness by jumping simulation

To evaluate of the effectiveness of the prototyped non-circular gear, we compare jumping simulations using the non-circular gear and the optimal circular gear. The optimal circular that maximizes the jumping height gear is obtained as $G_2 = 3$ by simulations. The robot jumping velocities

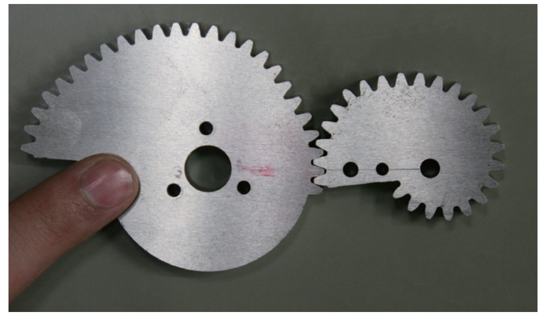

Fig. 10. Prototyped non-circular gear

with the non-circular gear $G_{2min} = 0.1$, the prototyped non-circular gear and constant gear ratio $G_2 = 3$ are shown in Fig.11. In the simulation with the prototyped non-circular gear, because of the limitation $G_{2min} = 1$ the robot accelerates as well as with the circular gear until $t = 0.1$. On the other hand, after $t = 0.1$, the robot accelerates so much because of the large gear ratio in associate with the rotation of the non-circular gear. The jumping heights with the non-circular gear $G_{2min} = 0.1$, the prototyped non-circular gear and constant gear ratio $G_2 = 3$ are 0.120m, 0.108m, 0.046m respectively. The robot using prototyped non-circular gear jumps 2.3 times higher than using the circular gear.

Because the non-circular gear is optimized for a specified initial posture, the robustness of the proposed method for the perturbation has to be evaluated. Fig.12 shows the jumping height with respect to $\Delta\theta[0]$ which is the perturbation of $\theta[0]$. The red line shows the jumping height of the robot using the prototyped non-circular gear, and blue line shows using a circular gear ($G_2 = 3$) for comparison. When $\Delta\theta[0] < 0$, the jumping height becomes large because the accelerate time changes longer. However, even though $\Delta\theta[0] \neq 0$, the jumping height of the robot with the non-circular gear is larger than using circular gear, which shows the robustness of the proposed method. Moreover, $\Delta\theta[0] > 5°$ is very large. The difference of the postures between $\Delta\theta[0] = 0$ and $\Delta\theta[0] = 5$ is visibly large.

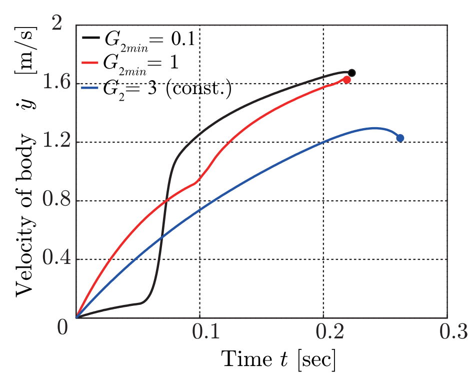

Fig. 11. Jumping simulation

D. Discussions

Fig.13 shows the motor current i. In the start of the motion, the motor current of the motor of the robot with

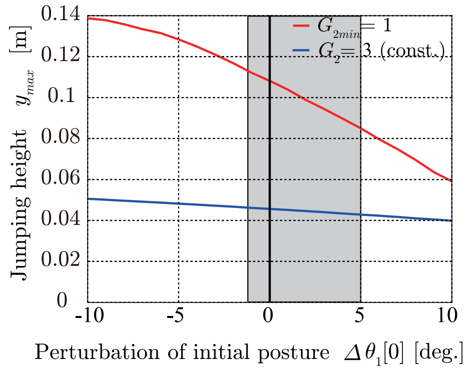

Fig. 12. Jumping height with respect to perturbation of initial posture

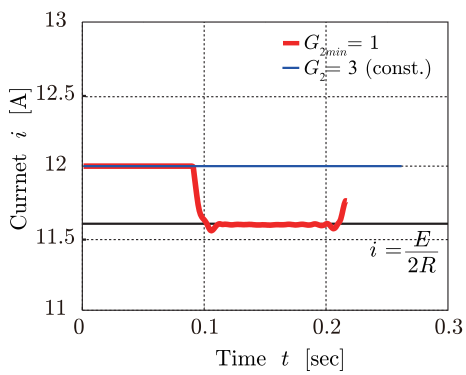

Fig. 13. Current value of the motor in jumping motion

the non-circular gear takes maximum value. However, after $t = 0.1$, the motor current changes to a constant value which is smaller than the maximum value. On the other hand, the optimal circular gear requires the maximum current through the motion. This result shows that the non-circular gear consumes smaller current than circular gear. This result is discussed as follows. By multiplying both sides of equation (4) by i, the motor power is obtained by:

$$Ei = Ri^2 + K_t\dot{\phi}i \tag{11}$$

By substituting equation (3) into equation (11), the output power is derived as;

$$\tau\dot{\phi} = Ei - Ri^2 \tag{12}$$

Because equation (12) is a quadratic function of convex upward with respect to i, the current maximizing output power is obtained as:

$$i = \frac{E}{2R} \tag{13}$$

The motor current in equation (13) maximize the motor power. This value is represented by the black line in Fig.13. By using the non-circular gear, the motor current takes the value of equation (13). Fig.14 shows the motor output power through the motion. The power is small in the beginning of the jump because of the current limitation. The limited current causes voltage drop and motor power becomes small.

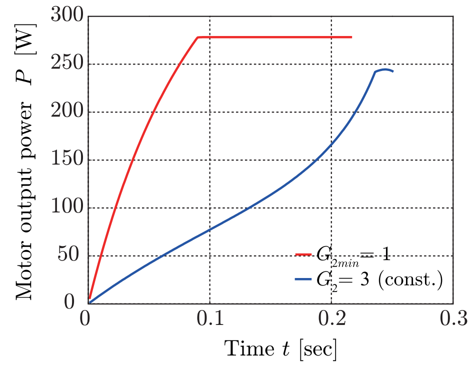

Fig. 14. Power of motor output

IV. Experimental evaluation

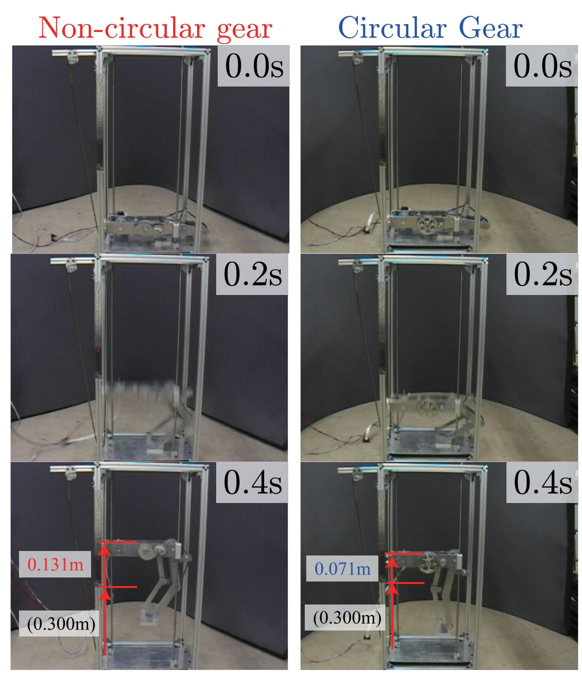

Fig. 15. Experiment of jumping robot with non-circular gear and with circular gear

We make experiments of the jumping robot using the prototyped non-circular gear and the optimal circular gear. Fig.15 and the attachment movie (jump.mp4) show the robot jump with the non-circular gear and the circular gear. The body is attached to a timing belt, and a one-way clutch is set in the pulley so that the robot dose not fall down, which realizes a measurement of jumping hight.

Jumping height with each gear is 0.131m and 0.071m. The non-circular gear makes the robot jump 1.8 times higher than the circular gear. Fig.16 shows the body velocity through the motion, which is measured by the encoder of the motor. Because the experimental data is not measured with a real-time operating system, sampling time is not ensured and the data has some noises. Because of uncertainties (friction term, link inertia, body weight and so on), the experimental result shows a little faster motion than the simulation. However, this result shows the effectiveness of the proposed method, because the robot accelerates greatly. Fig.17 shows the experimental results of the jumping height with respect to the perturbation of the initial position. The range of $\Delta\theta[0]$ corresponds to gray area in fig.12. Same as Fig.12, the robustness of the proposed method is evaluated.

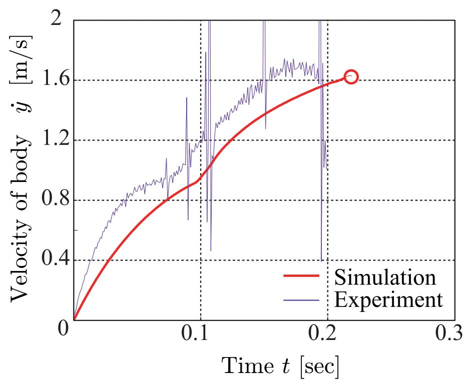

Fig. 16. Experimental result of of body velocity of the robot

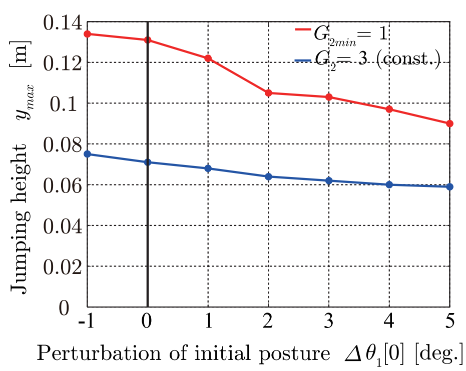

Fig. 17. Experimental result of jumping height

V. Conclusions

In this paper, we focus on a jumping robot and design a non-circular gear based on the conventional optimal design method. Considering the constraint of pressure angle, the non-circular gear and jumping robot is prototyped. The results are as follows;

1) Based on the constraint of the pressure angle, the optimal gear ratio is modified by adjusting the minimum value of the gear ratio.
2) The effectiveness of the proposed method is discussed from the effective use of the motor power point of view.
3) The non-circular gear and jumping robot is prototyped and the effectiveness of the proposed method is evaluated by experiments.
4) The robustness of the proposed method is evaluated by changing the initial posture of the robot.

Acknowledgment

This research is supported by the “Research on Macro / Micro Modeling of Human Behavior in the Swarm and Its Control” under the Core Research for Evolutional Science and Technology (CREST) Program (Research area : Advanced Integrated Sensing Technologies), Japan Science and Technology Agency (JST). Moreover, we are advised on designing non-circular gear by Dr. Hideo KATORI, who is president and CEO of Tecpha Japan.

References

[1] R. Niiyama and Y. Kuniyoshi : “Design of a Musculoskeletal Athlete Robot : A Biomechanical Approach”, Proc. of 12th International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines (CLAWAR 2009), pp.173–180, 2009.

[2] M. Ishikawa, A. Neki, J. Imura and S. Hara : “Energy preserving control of a hopping robot based on hybrid port-controlled Hamiltonian modeling”, IEEE Conference on Control Applications (CCA2003), Paper No. CF-002507, 2003.

[3] B. Ugurlu and A. Kawamura : “Real-time Running and Jumping Pattern Generation for Bipedal Robots based on ZMP and Euler’s Equations”, Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, pp.1100–1105, 2009.

[4] S. Shimoda, T. Kubota and I. Nakatani : “New Mobility System Based on Elastic Energy under Microgravity”, Proc. of the 2002 IEEE International Conference on Robotics and Automation, pp. 2296–2301, 2002.

[5] K. Sakaguchi, T. Sudo, N. Bushida, Y. Chiba, Y. Asai and K. Kikuchi : “Wheel-Based Stair-climbing Robot with Hopping Mechanism - Fast Stair-climbing and Soft-landing by Vibration of 2-DOF system -”, JSME Journal of Robotics and Mechatronics, Vol.19, No.3, pp.258–263, 2007.

[6] M. Kovac, M. Fuchs, A. Guignard, J.C. Zufferey and D. Floreano : “A miniature 7g jumping robot”, Proc. of the 2008 IEEE International Conference on Robotics and Automation, pp.373–378, 2008.

[7] S. Curran and D. E. Orin : “Evolution of a Jump in an Articulated Leg with Series-Elastic Actuation”, Proc. of the 2008 IEEE International Conference on Robotics and Automation, pp.352–358, 2008.

[8] T. Tsuda, H. Mochiyama and H. Fujimoto : “A Compact Kick-and-Bounce Mobile Robot powered by Unidirectional Impulse Force Generators”, Proc. of the 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp.3416–3421, 2009.

[9] T. Hagiwara and S. Hirose : “Development of Dual Mode X-Screw : A Novel Load-Sensitive Linear Actuator with a Wide Transmission Range”, Proc. of the 1999 IEEE International Conference on Robotics and Automation, pp.537–542, 1999.

[10] T. Takaki, K. Sugiyama, T. Takayama and T. Omata : “Development of two DOF finger using load-sensitive continuously variable transmissions and ultrasonic motors”, Advanced Robotics, Vol.20, No.8, pp. 897–911, 2006.

[11] M. Okada and Y. Takeda : “Optimal Design of Nonlinear Profile of Gear Ratio using Non-circular Gear for Jumping Robot”, Proc. of the IEEE International Conference on Robotics and Automation (ICRA’12), pp.1958–1963, 2012.

[12] H. Katori : “Design, fabrication and application of non-circular gear”,Nikkan Kogyo Shimbun (Business & Technology Daily News), 2001 (in Japanese)