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Searching for a One-Dimensional Random Walker: Randomized Strategy with Energy Budget

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Abstract— **In this paper we study the problem of designing search strategies to find a target whose motion is described by a random walk along a one-dimensional bounded environment. The sensing model and the characteristic of the environment require the searcher and the target to be on the same site at the same time to guarantee capture. The objective is to optimize the searcher's motion, given by a sequence of actions (move right, left or remain stationary), so that the probability of capturing the target is maximized. Each action is associated with an energy cost. The searcher strategy is constrained by a total energy budget. We propose a class of randomized strategies for which we provide an analytical expression for the capture probability as a function of a single parameter. We then use this expression to find the best strategy within this class. In addition to theoretical results, the algorithms are analyzed in simulation and compared with other intuitive solutions.**

I. Introduction

Searching for a target is a fundamental task in mobile robotics with applications in monitoring, surveillance and rescue missions. In a very common scenario the target has not only an unknown position, but also an unknown motion model. Often, for non adversarial targets, such motion can be modeled as a simple random walk. This is the case, for instance, for many animal systems. Our motivating application is monitoring radio-tagged invasive fish. In recent years, we have been working on building a system of Autonomous Surface Vehicles (ASVs) to locate radio-tagged carp in inland lakes [14]. The purpose of the system is to collect data which can be used for studying carp behavior. Recent experiments have shown that in the Twin-Cities Metro-Area lakes, the fish tend to be near the shore most of the time. This observation allows us to restrict our attention to the boundary of the lake and reduce the problem to a one-dimensional (1D) setting. This setting is of interest in other practical applications such as monitoring national borders, corridors, rivers or, as in our case, shores (see e.g. [7] and [1]).

In developing a search strategy, a crucial aspect of the system (especially for battery powered robots) is energy. Even though the capabilities of field robots have increased significantly in recent years, most of them are subject to severe energy limitations. In particular, robot developers face trade-offs between payload capacity and battery life since current batteries are heavy and last a short amount of time under full actuation. In order to deal with this issue, it is desirable to extend the lifetime of a robotic system by energy-aware algorithms which use existing resources efficiently.

In this work, we focus on an energy-efficient search problem in which a robot with limited battery life is charged with finding a random-walker in a 1D, discrete, circular environment. The searcher is subject to an energy budget which is used for either moving or station keeping. These actions incur different costs. Typically station keeping is less costly but a random-walker can be found much more quickly by moving. Therefore, the crucial trade-off for an optimal strategy is between spending energy to explore the environment and remaining stationary to exploit the diffusive properties of the random walker. Moreover, in our model we assume the searcher is able to capture the target only if they are at the same site at the same time. This means that the searcher could cross the target in the segment between two adjacent nodes without capturing it. As a result, even with enough energy to explore the whole environment, the capture is not guaranteed and an optimal strategy has to take the crossing phenomenon into account.

In our previous work, we studied this search problem when crossing is not allowed. In this case, the optimal strategy has a simple structure: sweep of the environment up to a certain location followed by station-keeping at that location [9]. Crossing makes the problem much more challenging. In fact, when crossing is allowed, for any deterministic search strategy, one can find a target strategy which avoids detection even if the searcher can sweep the entire environment. This aspect also makes the design and analysis of a search strategy for a random walking target challenging.

In this paper, we study a class of natural randomized strategies in which the pursuer flips a biased coin at each step to decide whether to move or not. Such strategies are appealing for field applications because they do not require any state or history information and therefore can be implemented very easily. We study fundamental properties of one-dimensional random walks in bounded environments (such as expected capture time and survival probability) and show how an optimal solution for this class of strategies can be obtained.

In a parallel submission [10], we study a special case of this problem where the searcher is subject to a time budget (which is also equivalent to the case where station-keeping and moving has the same cost). For this special case, the problem can be formulated and solved using a partially observable Markov decision process which is globally optimal for a given resolution. In contrast, the present work studies

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a more general version and presents analytical solutions for a restricted class of strategies.

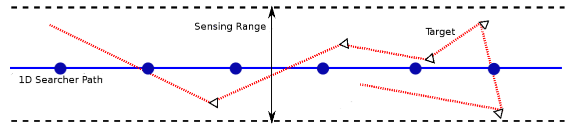
The rest of the paper is organized as follows: an overview of related work is presented in the next section. Section III formalizes the problem. In Section IV, some preliminary results which are in later sections are presented. In Section V, we propose a class of randomized strategies and show how to find the optimal one which maximizes the capture probability. Finally, in Section VI, simulation results are provided and analyzed to evaluate the proposed strategy.

II. Related Work

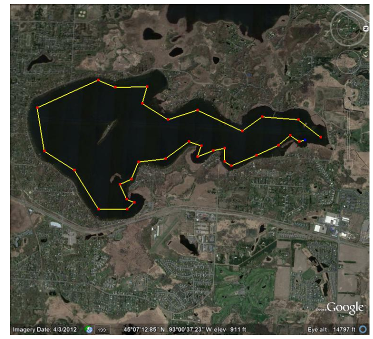
Random motions, both as discrete random walks and continuous diffusive motions, have been extensively studied as models of unknown animals motions or complex physical processes [3]. In particular they are widely used in the literature to simplify pursuit-evasion games and absorption (or search) processes. A large number of interesting properties closely related to searching missions including: first passage probability (the probability for the random walker of visiting for the first time a given point at a given time), survival probability (the probability that the random walk has not been found at a given time) and mean capture time (the expected time to be found) are collected in [12]. Various characteristics of random walks in general graphs have been studied in [8]. Examples are hitting time, which is the expected number of steps before a node is visited, and cover time, which is the expected number of steps to visit every node at least once.

Although one-dimensional random walks might seem to be simple processes, they present several interesting behaviors and properties and are still source of open problems. The survival probability of a particle that performs a random walk on a chain when traps are uniformly distributed with known concentration is studied in [2] and an asymptotically exact solution is provided. In [13] the authors study the survival probability of a prey chased by N diffusive predators on a line. In this case the capture dynamics is exactly solvable by probabilistic techniques when the number of predators is very small. For three or more predators the exact solution is still not known. The same problem but in a semi-infinite line where the boundary represents a haven for the prey is presented in [5]. Krapivsky and Redner studied the behavior of a random walk in a one dimensional bounded environment when absorption occurs whenever the random walk hits a boundary of the system [6]. In particular, the authors considered the case of an expanding *cage* and a receding *cliff*, i.e. with boundaries moving with a known motion law. Contrary to our paper, none of these works include any kind of constraint neither on the energy of the system, limiting the predator's autonomy, nor a maximum time for the chase.

In [11], the authors use random walks to tackle the coalescence problem, where the robots do not have any knowledge about the environment or positions of other robots. Each robot performs an independent random search and when two robots meet, they coalesce into a cluster which then moves as a single random walk.



(a)



(b)

Fig. 1. (a) The target moves randomly in a region of width less than the sensing range. This allows the searcher to restrict its searching domain to a 1D path, where the points are the sensing locations. In (b) it is shown a typical scenario in a lake where such path is next to the boundary.

The considered problem can also be included in the more general class of pursuit-evasion, where the pursuer's objective is to catch an evader whose motion might be also adversarial. An overview of recent results on pursuit evasion games can be found in [4].

Finally, in [10] we tackle a problem very similar to the one treated in this paper. In that version, the searcher is subject to a time (rather than energy) constraint. That work focuses on finding globally optimal strategies using Partially Observable Markov Decision Process (POMDP) models. Here, we study a more general problem and present an analytical solution for choosing the best randomized strategy among a class of memoryless strategies.

III. Motivation and Problem Formulation

Our general goal is to find radio-tagged invasive fish in lakes by using an Autonomous Surface Vehicle (ASV) equipped with a tracking system. Since they are almost always found very close to the boundary of the lake, we can assume that their motion is bounded in a corridor of width comparable to the sensing range. This allows us to limit the searching process along a 1D discrete path (see Fig. 1). The real fish position, which is moving in 2D, can be projected at every time on the closest node.

Thus, the problem considered in this paper can be formally described as follows. Let us consider a searcher and a target moving on a circular discrete environment with the same maximum speed, which for simplicity we assume unitary. The environment Ω is composed of N equally-spaced nodes $i = 0,\dots,N-1$. The target motion, which is assumed to be a simple random walk, can be expressed by the following Master equation:

$$P(i,t+1) = pP(i+1,t) + qP(i-1,t) \qquad (1)$$

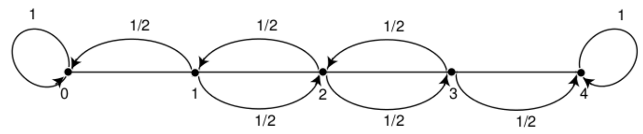


Fig. 2. Random walk moving on a chain with $N = 5$ nodes and absorbing boundary condition. This process is equivalent to a random walk moving on a circular environment composed of 4 nodes and a searcher located at $i = 0$, where the absorption represents the capture.

where $P(i,t)$ is the probability of finding the target at node i at time t, p and q are the probabilities of the target to move to the left and right node respectively, $P(N,t) = P(0,t)$ and $P(-1,t) = P(N-1,t)$. For simplicity, we consider only symmetric random walk, i.e. $q = p$. This assumption corresponds to neglecting the possibility of a preferential direction for the fish, which provides a starting point in the absence of additional information. A crucial point of our problem is that the searcher, which moves simultaneously and with the same speed, can sense the presence of the target only when it reaches a node, i.e. not while moving from one node to another. The practical motivation for this assumption comes from our fish searching problem: we observed that the reliability of detection decreases considerably when the boat is in motion. This is due to the sensitivity of the antenna to radio interference from the motors of the boat. The searcher motion can then be described by a sequence of action $\mathscr{S} = \{a_1, a_2, ..., a_k\}$ where $a_i \in$ {go left, go right, remain stationary}. Such set of actions is constrained by the searcher's limited initial energy budget E_0. Moving and stay actions have costs c_m and c_s respectively, which are assumed to be constant along the path. The search can continue until the energy of the system E remains greater than zero. Our intent is to optimize the probability P_c of eventually capturing the target, given the dimension of the circular environment, the initial energy budget and the action costs.

Formally, the optimization problem we would like to solve is:

$$\max_{\mathscr{S}} P_c(\mathscr{S} = \{a_1, a_2, ..., a_k\}) \quad s.t. \tag{2}$$

$$\sum_{i=1}^{k} cost(a_i) \leq E_0. \tag{3}$$

In this framework, the target's motion can be seen as a Markov chain on Ω, where the searcher position represents an absorption point (see Fig. 2). The distance between two adjacent nodes $d(i, i+1)$, which in our model corresponds to the distance between two successive measurements, plays a crucial role in the problem. In particular, we can identify two different cases.

1) The first case corresponds to the condition:

$$d(i, i+1) < R_s\,, \tag{4}$$

where R_s is the sensing range. In other words, from any node the searcher is able to sense also its two neighbors. This condition prevents the searcher from passing over the target while moving from a node to an adjacent node without sensing the target (we refer to this problem as the crossing problem).

2) In the second case, described by the condition

$$d(i, i+1) > R_s\,, \tag{5}$$

the searcher cannot sense its neighbors. As a result, it can find the target only if they simultaneously occupy the same node. This can occur in cases when the distance between two measurement spots cannot be controlled by the searcher or when the environment is too large compared to sensing range and imposing condition (4) is unfeasible.

In this paper, we focus on the second case, i.e. when crossing is allowed. In the succeeding sections, we show how this phenomenon forces the searcher to significantly change its strategy.

IV. Preliminary Results and Insights

Before describing the proposed strategy, we present some preliminary results which shed light onto the structure of the problem.

A. The No-crossing Case

We start by analyzing the problem considering the no-crossing case, i.e. when condition (4) holds. For details on this case, with searcher and target moving on a segment with reflecting boundaries, see [9]. The starting point to obtain an optimal strategy is the following proposition:

Proposition 1: Let $\mathscr{S}_1, \mathscr{S}_2$ be two different searcher strategies and $x_1(t), x_2(t)$ the location of the searcher at time t when executing $\mathscr{S}_1$ and $\mathscr{S}_2$ respectively. Then it holds:

$$x_1(t) \geq x_2(t)\ \forall t \quad \Rightarrow \quad P_c(\mathscr{S}_1) \geq P_c(\mathscr{S}_2)\,, \tag{6}$$

where $P_c(\mathscr{S}_i)$ is the capture probability executing the strategy $\mathscr{S}_i$.

This proposition is justified by observing that, imposing condition (4), any target captured by the strategy $\mathscr{S}_2$ is captured also by the strategy $\mathscr{S}_1$ with probability 1. On the other hand, the opposite is not always true. The consequence of this proposition was that the stay action is useful only to increase the searching time (assuming that $c_s < c_m$). Further, for a searcher starting from $i = 0$, an optimal strategy has to have the structure $R^j S^k$, i.e. the searcher moves for j consecutive steps to the right and then keeps the position for k steps. See [9] for more details.

The case treated in this paper is significantly different because Proposition 1 does not hold due to the crossing phenomenon and the different assumption on the environment. This implies that simply sweeping the entire environment does not assure the target detection. Indeed, let us consider the following scenario: the target is moving in a 1D circular environment composed by $2N$ nodes and its initial position n is on an even node, i.e. $n = 2s$ with $s \in [1, N]$. At the same time, the searcher starts the sweeping clockwise from $i = 1$ without any stop, i.e. its strategy is composed only of right actions. It is easy to see that at each time step, the searcher will be on a node with opposite parity with respect to the target. This means that, even though they will cross each other infinitely times, the searcher will never sense the target. In other words, considering the probability of the target's initial location uniformly distributed among the nodes, with probability $1/2$ the searcher will never catch the target.

As a result, a strategy of the form $R^j S^k$ is no longer suitable and a more complex structure is needed so as to take into account energy and to minimize crossing along the path. To achieve this, a certain number of S actions have to be distributed between the R actions. It is easy to see that increasing the number of S, the probability of crossing decreases. As a limit, if the searcher does not move at all from its initial position, the crossing phenomenon cannot occur. Further, thanks to the recurrence property of a random walk in 1D, it will eventually be captured with probability 1. For these reasons, a stationary strategy might be a trivial solution for the considered problem. However, as we are going to show, in this case the expected capture time is quadratic in the initial distance n between searcher and target. Due to energy constraints, this might not be a feasible strategy and a better approach is needed.

B. Stationary Searcher

In this section, we briefly present the analysis of the stationary strategy mentioned in the previous section. Even though this is a well-known result (e.g. see [12]), the technique will be useful for the analysis of the randomized strategy introduced in the next section.

The expected capture time t_n, where n is the random walker starting location, obeys to the following recursive equation [12]:

$$t_n = p(t_{n-1}+1) + q(t_{n+1}+1)\,, \quad n = 1,2,...,N-1\,, \tag{7}$$

where the time-step is 1 and the boundary conditions which correspond to absorption in $i=0$ and in $i=N$ are $t_0 = 0, t_N = 0$. For a symmetric random walk $p = q = 1/2$ and the previous equation becomes:

$$2t_n = t_{n-1} + t_{n+1} + 2\,, \tag{8}$$

with $n = 1,2,...,N-1$. The solution of the previous recursive relation is:

$$t_n = A + Bn - n^2\,, \tag{9}$$

where A and B are constants to fix imposing the boundary conditions. In our case the solution becomes:

$$t_n = n\,(N-n)\,. \tag{10}$$

This result proves that the expected capture time for a stationary searcher is quadratic in the initial distance from the target. The consequence is that, for very large environment the searcher could wait for the target only if taking measurements without moving has a zero cost, which is an unrealistic assumption. This strategy is then almost always unfeasible.

V. Randomized Strategy

To find a better solution, we restrict our attention to the following class of stochastic strategies: at each time step the searcher keeps the position with probability w and moves one step clockwise with probability $1-w$, with $w \in (0,1)$. Hereafter we refer to these strategies as w-strategies. Note that we do not include the possibility of left actions in the strategy. This is because a left action does not produce any benefit to prevent crossing and at the same time reduces the energy that can be employed in exploring the environment, reducing so the probability of capture.

The optimization problem (2) and the constraint (3) can now be expressed as follows:

$$\max_{w} P_c(w) \quad s.t. \tag{11}$$

$$c_m M + c_s S \le E_0\,, \tag{12}$$

where M is the total number of searcher movements and S the time steps it spends on the same position. In order to solve the problem (11), we firstly need to express the capture probability P_c as a function of w. Instead of computing P_c directly, we consider the survival probability P_s of a random walk moving in a bounded environment with an absorbing bound in $x = x^{(s)}(t)$, where $x^{(s)}(t)$ is the searcher position at time t. Then, P_c is simply $P_c = 1 - P_s$. Even if the exact analytical expression is very complicated (see [12] for the result of a symmetric simple random walk in a circular environment with absorption in $x=0$), it can be shown that the leading term of such probability decays exponentially in time, i.e. it assumes the form: $P_s(t) = e^{-t/\tau}$. Moreover, the characteristic time of decay τ can be identified with the mean capture time ([12], Chapter 2).

We start by computing this last quantity, the expected capture time, for a target starting from the node n. To do that we express the target's motion in the moving frame of reference in which the searcher location is $x^{(s)}(t) = 1, \forall t$ and the N nodes are $i = 1,...,N$. In such frame the target's Master equation becomes:

$$\begin{aligned} P(i,t+1) &= \frac{1-w}{2}P(i+2,t) + \frac{w}{2}P(i+1,t) \\ &+ \frac{1-w}{2}P(i,t) + \frac{w}{2}P(i-1,t)\,. \end{aligned} \tag{13}$$

The recursive equation for the expected capture time t_n is:

$$\begin{aligned} t_n &= \frac{1-w}{2}(t_{n-2}+1) + \frac{w}{2}(t_{n-1}+1) \\ &+ \frac{1-w}{2}(t_n+1) + \frac{w}{2}(t_{n+1}+1)\,. \end{aligned} \tag{14}$$

So we have

$$w t_{n+1} - (1+w)t_n + w t_{n-1} + (1-w)t_{n-2} = -2\,. \tag{15}$$

We firstly solve the homogeneous equation associated to (15). The characteristic polynomial is:

$$w\lambda^3 - (1+w)\lambda^2 + w\lambda + (1-w) = 0\,, \tag{16}$$

whose roots are:

$$\lambda_0 = 1,\ \lambda_{1,2} = \frac{1 \pm \sqrt{1+4w-4w^2}}{2w} \tag{17}$$

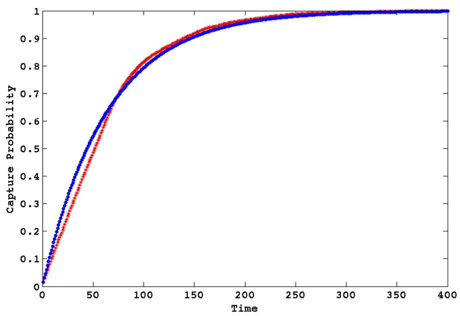


Fig. 3. Comparison between the approximations of the capture probabilities for a $w = 0.4$ strategy (in blue) with the behaviors obtained in simulation (in red). The environment is composed of $N = 50$ nodes.

and a particular solution of non-homogeneous equation (15) is $n/(1-w)$. Thus, the solution of (15) is:

$$t_n = A + B\lambda_1^n + C\lambda_2^n + \frac{1}{1-w}n\,, \tag{18}$$

where A, B and C are constants which can be fixed by imposing the boundary conditions. In our case the boundary conditions to impose are: $t_1 = 0, t_{N+1} = 0, t_0 = t_N$, where the first one represents the absorption on the searcher position and the other two express the periodic conditions. Then, since the initial target position is unknown, we compute the expected value $\langle t_n \rangle$ over n, assuming a uniform probability distribution:

$$\langle t_n \rangle = \frac{1}{N}\sum_{n=1}^{N} t_n\,. \tag{19}$$

The other quantity we need to compute in the survival probability expression for a given strategy is the duration T of the mission. Since our strategy is stochastic, we can compute the expected time $\langle T \rangle = \langle M \rangle + \langle S \rangle$, where $\langle M \rangle$ and $\langle S \rangle$ are given by:

$$\begin{cases} c_m\langle M \rangle + c_s\langle S \rangle = E_0 \\ \frac{\langle M \rangle}{\langle S \rangle} = \frac{1-w}{w} \end{cases} \tag{20}$$

whose solution gives

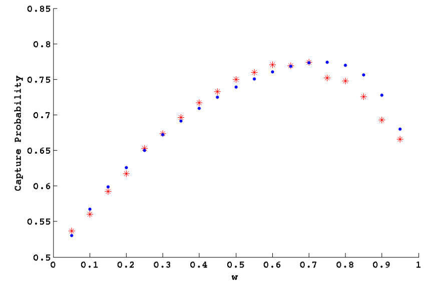
$$\langle T \rangle = \frac{E_0}{c_m(1-w)+c_s w}\,. \tag{21}$$

Note that the inequality in (12) has been substituted by an equality in (20) because there is no reason to use less energy than the maximum available.

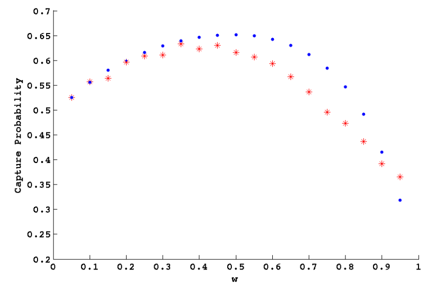
The final result for the capture probability by employing a w-strategy and with the energy constraint (12) is:

$$P_c(w; E_0, c_m, c_s) = 1 - e^{\frac{E_0}{c_m(1-w)+c_s w}\,\frac{1}{A+B\langle\lambda_1^n\rangle + C\langle\lambda_2^n\rangle + \frac{\langle n \rangle}{1-w}}}\,. \tag{22}$$

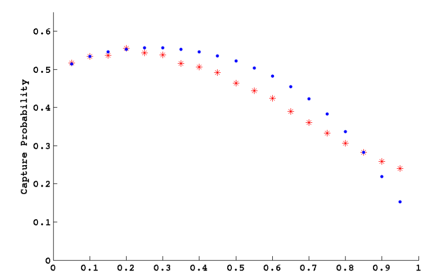
In Fig. 3 a comparison between the previous theoretical result and the capture probability obtained in simulation, as an average over 10^4 trials, is shown.



(a)



(b)



(c)

Fig. 4. Comparison between the approximations of the total capture probabilities (blue dots) with the behaviors obtained in simulation (red stars) in function of the staying probability w. The values considered are: $N = 50$, $E_0 = 50, c_m = 1$ and $c_s = 0.1, 0.5$, and 0.9 for (a), (b) and (c) respectively.

VI. Results

In the previous section we obtained, as a main result, an approximation for the probability of finding the target using a w-strategy (eq. (22)). Since such expression is a function of the only variable $w \in (0,1)$, it can be easily maximized finding the optimal w^*. In this section we provide the results for some selected scenarios. In particular, we show how, for a given scenario, a small variation of the cost c_s changes significantly the optimal strategy. Furthermore, since our result is an approximation, we also compare the probabilities provided by eq. (22) with the averaged values obtained by simulating the process.

The results shown in Fig. 4 correspond to an environment with $N = 50$ nodes. The searcher has an initial energy budget $E_0 = 50$ and the costs for moving and staying are respectively $c_m = 1$ and $c_s = 0.1, 0.5, 0.9$.

It is possible to see that, in terms of optimal strategy, as

Strategy	Capture probability
w-OPT	0.775
Stationary	0.504
Go-Right	0.484
Random Walk	0.228
Derandomized	0.789

(A)

Strategy	Capture probability
w-OPT	0.633
Stationary	0.254
Go-Right	0.484
Random Walk	0.223
Derandomized	0.695

(B)

TABLE I

COMPARISON BETWEEN DIFFERENT POSSIBLE STRATEGIES IN TERMS OF CAPTURE PROBABILITY. THE LENGTH OF THE ENVIRONMENT IS $N = 100$, THE ENERGY BUDGET $E_0 = 100$ AND THE ACTIONS COSTS ARE $c_m = 1$ AND $c_s = 0.1, cs = 0.4$ FOR (A) AND (B) RESPECTIVELY.

for the no-crossing model, increasing the ratio c_s/c_m implies a *faster* strategy, i.e. lower w^*. However, it is interesting to note that even when the cost of staying is very high and this ratio tends to 1, as in case (c), the optimal strategy still includes several stops and it does not simply keep moving in one direction (as in the no-crossing version). Indeed, in this case the optimal value of w is $w^* = 0.2$.

For a given scenario we compare our strategy with other possible strategies such as: stationary searcher (limit case $w = 1$), searcher always moving in the same direction (limit case $w = 0$) and random walk searcher. We consider also a deterministic strategy constructed by derandomizing the optimal w-strategy. To do that we uniformly distribute the stop actions such that their density among the path is equal to w. The results, shown in Table I, correspond to an environment of length $N = 100$, an initial energy budget $E_0 = 100$, a cost for moving $c_m = 1$ and a cost for staying of $c_s = 0.1, c_s = 0.4$ for (A) and (B) respectively. The optimal randomized strategies (w-OPT) for these two cases are defined by the values $w^* = 0.65$ (A) and $w^* = 0.35$ (B). The corresponding derandomized strategies are: *RSSRSS*... and *SSRSSR*... respectively. Also in this case the results are obtained in simulations averaging 10^4 trials and with a uniform distribution for the initial target position. A first interesting aspect is that for such similar scenarios, where only the cost to remain stationary is slightly different, the two resulting strategies are very different. It is also worth noting that the capture probability obtained by using the derandomized w-OPT strategy is the highest achieved. This result is not very surprising, since in every w-OPT strategy there is always a non-zero probability to find a sequence of actions where the density of stop actions is very far from the optimal w^*.

VII. CONCLUSIONS

In this paper, we studied a search problem where the target is a random walk confined on a one-dimensional circular chain. In our model, searcher and target move simultaneously, with the same speed, and the capture happens only if they occupy the same node at the same time. Furthermore, we considered a limited energy budget available for the searching process and a different cost for stay and moving actions. The problem is then to construct a strategy to maximize the probability of capturing the target. To do so we considered a class of randomized strategies and we proposed a solution for the constrained optimization problem from this class. The result is finally compared with other possible strategies to evaluate the performance.

As a next step, we will implement the proposed algorithm on our experimental platform, an Autonomous Surface Vehicle (ASV) carrying radio tracking equipment for detecting the target [14]. From a theoretical point of view we aim to better analyze the derandomization of the proposed strategy in order be able to construct a global optimal strategy and to better compare the obtained results with the MDP formulation presented in [10]. Finally, this paper should be seen as a first step toward developing an optimal search strategy in 2D environments.

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