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Bilateral Teleoperation of Flexible-Joint Manipulators with Dynamic Gravity Compensation and Variable Time-Delays

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Abstract— **It is widely known that the problem of controlling a rigid bilateral teleoperator with time-delays has been effectively addressed since the late 80's. However, the control of flexible joint manipulators in a bilateral teleoperation scenario, with dynamic gravity compensation, remains an open problem. This work aims at filling this gap by presenting a new controller for bilateral teleoperators composed of a rigid local manipulator and a flexible-joint remote manipulator with dynamic gravity compensation and asymmetric variable time-delays in the communication channel. In order to dynamically compensate the gravity term, in the flexible joint manipulator, a change of coordinates which accounts for the joint and link gravity position drift is used. The rest of the controller is a simple PD scheme. Assuming that the human operator and the environment define passive maps from velocity to force, it is proved that velocities and local and remote position errors are bounded. Additionally, if the human operator and remote environment forces are zero then velocities asymptotically converge to zero and position tracking is established. Some simulations are presented in order to show the performance of the proposed controllers.**

I. INTRODUCTION

Commonly, bilateral teleoperators are composed of the following five systems: i) the *human operator*; ii) the *local manipulator*; iii) the *communication channel*; iv) the *remote manipulator* and v) the remote *environment*. Generally speaking, the main objective of such a scheme is to extend the human manipulation capabilities to a remote environment. On one hand, the physical interaction between the human operator and the environment with the local and the remote manipulators, respectively, is mechanical, i.e., the human grasps the local manipulator and the remote manipulator contacts/grasps the environment. On the other hand, the local and the remote manipulators exchange control signals through the communication channel such that the bilateral teleoperator can be rendered as *transparent* as possible. Controlling these systems has become a highly active research field. For a recent historical survey on this research line the reader may refer to [1], for a teleoperators control tutorial, to [2] and, for an experimental evaluation comparison, to [3].

Passivity based control, via the scattering transformation originally proposed by Anderson and Spong in [4], has been one of most outstanding breakthroughs in the control of teleoperators with time-delays. Niemeyer and Slotine [5] introduced the wave variables, following the former scattering approach, and proved that by matching the impedances of the local and remote robot controllers with the impedance of the *virtual* transmission line, *wave* reflections are avoided. However, one of the main drawbacks of these schemes is position drift (an exception is [6]). In order to improve performance, in scattering based schemes, several approaches have been reported: transmitting *wave* integrals [7], [8], *wave* filtering [9], *wave* prediction [10], power scaling [11], amongst others. Recently, without the use of the scattering transformation, Chopra et al. [12] have proposed the use of adaptive schemes to overcome the effects of position drift. Along the same line Nuño et al. [13] report a different adaptive scheme that is capable of synchronizing the local and remote positions despite constant time-delays. Recently, similar to [14], [15], based on the small gain theorem and assuming that the physical parameters are known, [16] has proposed a controller for the asymptotical stabilization of a cooperative teleoperation system with variable time-delays. The use of simple PD controllers, in the local and the remote manipulators, has been proposed in [6], [17], [18] for the case of constant time-delays. Later, in [19], Nuño et al. show that these proportional plus damping controllers are capable of providing position tracking for bilateral teleoperators with variable time-delays.

All these previous developments deal with bilateral teleoperators composed by rigid joints manipulators. Nevertheless, it should be underscored that in diverse applications, including space and surgical telerobotics, the use of thin, lightweight and cable-driven manipulators is increasing. These manipulators are known to exhibit link and/or joint flexibility ([20] shows that the lumped (linear) dynamics of a flexible link is identical to the (linear) dynamics of a flexible joint). To the authors knowledge, most of the previous schemes deal with rigid manipulators and only few exceptions treat the problem of joint flexibility, but for linearized teleoperators and without time-delays, [20], [21], [22], [23], [24]. In this scenario, the control problem increases in difficulty, since joint flexibility (caused by transmission elements such as harmonic drives, belts, cables or long shafts) is a major source of oscillatory behaviors in robot manipulators. Joint flexibility is modeled using the motor rotor and the link positions and velocities, hence, the order of the model is twice the size of the corresponding rigid joint manipulators.

The present work, reports a new controller for nonlinear

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teleoperators with joint flexibility and variable time-delays. Unlike the authors' previous work along this line, i.e., [25], this paper does not assume the absence of gravity or its compensation in the flexible joint manipulator dynamics. Instead, the idea of the exact gravity compensation scheme is borrowed from the fundamental work of De Luca and Flacco [26], [27]. In this way, once the flexible joint dynamics are transformed, by using a proportional plus damping controller, and assuming that the human operator and the environment behave as passive systems, it is proved that all velocity and position error signals are bounded. Moreover, if the human operator and the environment do not inject forces on the local and on the remote manipulators, the velocities and the position errors are shown to be asymptotically convergent to zero. The proposed scheme is proved to be robust to variable time-delays provided that the local and remote damping is sufficiently large. The paper also presents some numerical simulations to show the effectiveness of the proposed dynamic gravity compensation controller.

The following notation is used throughout the paper. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $||\mathbf{A}||$ denotes the matrix-induced 2-norm of matrix $\mathbf{A}$. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty := \underset{t \geq 0}{sup} |\mathbf{f}(t)|$, and the square of the $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2^2 := \int_0^\infty |\mathbf{f}(t)|^2 dt$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively.

II. Modeling the Teleoperator with Remote Manipulator Flexibility

The local manipulator is modeled as a rigid *n*–degree of freedom (DOF) manipulator composed by revolute joints. Its nonlinear dynamic behavior is given by

$$\mathbf{M}_l(\mathbf{q}_l)\ddot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l + \mathbf{g}_l(\mathbf{q}_l) = \boldsymbol{\tau}_h - \boldsymbol{\tau}_l. \quad (1)$$

The remote manipulator is assumed to be a *n*–DOF manipulator with revolute flexible joints, whose dynamical behavior is governed by

$$\begin{aligned} \mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{g}_r(\mathbf{q}_r) + \mathbf{S}_r(\mathbf{q}_r - \boldsymbol{\theta}_r) &= -\boldsymbol{\tau}_e \\ \mathbf{I}_r\ddot{\boldsymbol{\theta}}_r + \mathbf{S}_r(\boldsymbol{\theta}_r - \mathbf{q}_r) &= \boldsymbol{\tau}_r \end{aligned} \quad (2)$$

where, using the subindex $i \in \{l, r\}$ for local and remote manipulators, respectively, $\mathbf{q}_i \in \mathbb{R}^n$ is the link position and $\boldsymbol{\theta}_r \in \mathbb{R}^n$ is the remote joint (motor) angular position. $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal effects matrix, defined via the Christoffel symbols of the first kind, $\mathbf{g}_i(\mathbf{q}_i) \in \mathbb{R}^n$ is the gravity vector force, $\mathbf{I}_r \in \mathbb{R}^{n\times n}$ is a symmetric and positive definite matrix corresponding to the remote actuator inertia, $\mathbf{S}_r \in \mathbb{R}^{n\times n}$ is a diagonal and positive definite matrix that contains the remote joint stiffness, $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the control signal and $\boldsymbol{\tau}_h \in \mathbb{R}^n$, $\boldsymbol{\tau}_e \in \mathbb{R}^n$ are the joint torques corresponding to the forces exerted by the human operator and the environment interaction, respectively. Note that, unlike the authors' previous work [25], this paper does not assume that gravity is absent from (2).

Throughout the paper, the following standard assumption is made: $\mathbf{M}_i(\mathbf{q}_i)$ is symmetric positive definite and bounded for all $\mathbf{q}_i$. Further, it is well-known that dynamics (1) and (2) enjoy the following properties [28], [29], [19]:

- P1. Matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric.
- P2. For all $\mathbf{q}_i, \dot{\mathbf{q}}_i \in \mathbb{R}^n$, there exists $k_{ci} \in \mathbb{R}_{>0}$ such that $|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i| \leq k_{ci}|\dot{\mathbf{q}}_i|^2$.
- P3. For all $\mathbf{q}_i \in \mathbb{R}^n$, there exist $k_{gi}, k'_{gi} \in \mathbb{R}_{>0}$ such that $|\mathbf{g}_i(\mathbf{q}_i)| \leq k_{gi}$ and $\left\| \frac{\partial \mathbf{g}_i(\mathbf{q}_i)}{\partial \mathbf{q}_i} \right\| \leq k'_{gi}$.
- P4. If $\dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ then $\frac{d}{dt}\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is a bounded operator.

Regarding the interconnection time-delays, the human operator and the environment, this paper employs the following standard assumptions:

- A1. The variable time-delay $T_i(t)$ has a known upper bound ${}^*T_i$, i.e., $0 \leq T_i(t) \leq {}^*T_i < \infty$, and its first and second time-derivatives are bounded.
- A2. The human operator and the environment define passive (velocity to force) maps, that is, there exists $\epsilon_i \in \mathbb{R}_{\geq 0}$ such that, for all $t \geq 0$,

$$-\int_0^t \dot{\mathbf{q}}_l^\top(\sigma)\boldsymbol{\tau}_h(\sigma)d\sigma + \epsilon_l \quad \geq \quad 0, \quad (3a)$$

$$\int_0^t \dot{\mathbf{q}}_r^\top(\sigma)\boldsymbol{\tau}_e(\sigma)d\sigma + \epsilon_r \quad \geq \quad 0. \quad (3b)$$

III. Proposed Controller with Dynamic Gravity Compensation

Before presenting the teleoperator controllers, inspired by the gravity compensation scheme of De Luca and Flacco [26], [27], let us define a new variable $\mathbf{x}_r \in \mathbb{R}^n$ as

$$\mathbf{x}_r := \boldsymbol{\theta}_r - \mathbf{S}_r^{-1}\mathbf{g}_r(\mathbf{q}_r). \quad (4)$$

Using this change of coordinates, the flexible–joint robot manipulator dynamics (2) can be written as

$$\begin{aligned} \mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{S}_r(\mathbf{q}_r - \mathbf{x}_r) &= -\boldsymbol{\tau}_e \\ \mathbf{I}_r\ddot{\mathbf{x}}_r + \mathbf{S}_r(\mathbf{x}_r - \mathbf{q}_r) &= \boldsymbol{\tau}_r - \mathbf{g}_r(\mathbf{q}_r) - \mathbf{I}_r\mathbf{S}_r^{-1}\ddot{\mathbf{g}}_r(\mathbf{q}_r). \end{aligned}$$

Hence, defining the remote controller as

$$\boldsymbol{\tau}_r = \bar{\boldsymbol{\tau}}_r + \mathbf{g}_r(\mathbf{q}_r) + \mathbf{I}_r\mathbf{S}_r^{-1}\ddot{\mathbf{g}}_r(\mathbf{q}_r) - d_r\dot{\mathbf{x}}_r, \quad (5)$$

where $d_r \in \mathbb{R}_{>0}$ is the damping injection gain and $\bar{\boldsymbol{\tau}}_r \in \mathbb{R}^n$ is a local-remote interconnection term that will be defined later, yields the remote closed-loop system

$$\begin{aligned} \mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{S}_r(\mathbf{q}_r - \mathbf{x}_r) &= -\boldsymbol{\tau}_e \\ \mathbf{I}_r\ddot{\mathbf{x}}_r + d_r\dot{\mathbf{x}}_r + \mathbf{S}_r(\mathbf{x}_r - \mathbf{q}_r) &= \bar{\boldsymbol{\tau}}_r. \end{aligned} \quad (6)$$

Note that, by using (4), the transformed dynamics (6) does not explicitly contain any gravity term.

Now, in order to achieve the desired local and remote position tracking objective, let us define $\bar{\boldsymbol{\tau}}_r$ as

$$\bar{\boldsymbol{\tau}}_r = -k_r(\mathbf{x}_r - \mathbf{q}_l(t - T_l(t))) \quad (7)$$

where $k_r > 0$ is the proportional local-remote interconnection gain. The local controller follows the simple P+d scheme proposed in [18], [19] and it is given by

$$\boldsymbol{\tau}_l = k_l(\mathbf{q}_l - \mathbf{x}_r(t - T_r(t))) + d_l\dot{\mathbf{q}}_l - \mathbf{g}_l(\mathbf{q}_l), \quad (8)$$

where $k_l, d_l \in \mathbb{R}_{>0}$ are the local controller gains.

The complete closed-loop system (1), (2), (8), (5) and (7), is given by

$$\begin{aligned}\ddot{\mathbf{q}}_l &= \mathbf{M}_l^{-1}\left[\boldsymbol{\tau}_h - (\mathbf{C}_l + d_l)\dot{\mathbf{q}}_l - k_l(\mathbf{q}_l - \mathbf{x}_r(t - T_r(t)))\right]\\ \ddot{\mathbf{q}}_r &= \mathbf{M}_r^{-1}\left[\mathbf{S}_r(\mathbf{x}_r - \mathbf{q}_r) - \mathbf{C}_r\dot{\mathbf{q}}_r - \boldsymbol{\tau}_e\right]\\ \ddot{\mathbf{x}}_r &= \mathbf{I}_r^{-1}\left[\mathbf{S}_r(\mathbf{q}_r - \mathbf{x}_r) - d_r\dot{\mathbf{x}}_r - k_r(\mathbf{x}_r - \mathbf{q}_l(t - T_l(t)))\right]\end{aligned} \quad (9)$$

The kinetic energy of the local rigid-joint manipulator is given by

$$\mathcal{K}_l(\dot{\mathbf{q}}_l) = \frac{1}{2}\dot{\mathbf{q}}_l^\top \mathbf{M}_l(\mathbf{q}_l)\dot{\mathbf{q}}_l, \quad (10)$$

and the kinetic energy of the remote (transformed) flexible-joint manipulator is

$$\mathcal{K}_r(\dot{\mathbf{q}}_r, \mathbf{x}_r) = \frac{1}{2}\dot{\mathbf{q}}_r^\top \mathbf{M}_r(\mathbf{q}_r)\dot{\mathbf{q}}_r + \frac{1}{2}\dot{\mathbf{x}}_r^\top \mathbf{I}_r\dot{\mathbf{x}}_r. \quad (11)$$

Moreover, the potential energy stored in the $\mathbf{x}_r$-coordinate and the remote link virtual spring, is

$$\mathcal{U}_r(\mathbf{q}_r, \mathbf{x}_r) = \frac{1}{2}(\mathbf{x}_r - \mathbf{q}_r)^\top \mathbf{S}_r(\mathbf{x}_r - \mathbf{q}_r), \quad (12)$$

and the potential energy stored in the local-remote interconnection fulfills

$$\mathcal{U}_{lr}(\mathbf{q}_l, \mathbf{q}_r) = \frac{1}{2}|\mathbf{q}_l - \mathbf{x}_r|^2. \quad (13)$$

Defining the (scaled) total energy as

$$\mathcal{E} = \mathcal{K}_l + \frac{k_l}{k_r}\left(\mathcal{K}_r + \mathcal{U}_r\right) + k_l\mathcal{U}_{lr}$$

and evaluating $\dot{\mathcal{E}}$ along (9) and using Property P1, yields

$$\begin{aligned}\dot{\mathcal{E}} &= \dot{\mathbf{q}}_l^\top\boldsymbol{\tau}_h - \frac{k_l}{k_r}\dot{\mathbf{q}}_r^\top\boldsymbol{\tau}_e - d_l|\dot{\mathbf{q}}_l|^2 - \frac{k_l d_r}{k_r}|\dot{\mathbf{x}}_r|^2 \quad (14)\\ &- k_l\dot{\mathbf{q}}_l^\top(\mathbf{x}_r - \mathbf{x}_r(t - T_r(t))) - k_l\dot{\mathbf{x}}_r^\top(\mathbf{q}_l - \mathbf{q}_l(t - T_l(t)))\\ &= \dot{\mathbf{q}}_l^\top\boldsymbol{\tau}_h - d_l|\dot{\mathbf{q}}_l|^2 - \frac{k_l}{k_r}\dot{\mathbf{q}}_r^\top\boldsymbol{\tau}_e - \frac{k_l d_r}{k_r}|\dot{\mathbf{x}}_r|^2\\ &- k_l\dot{\mathbf{q}}_l^\top\int_{t-T_r(t)}^{t}\dot{\mathbf{x}}_r(\sigma)d\sigma - k_l\dot{\mathbf{x}}_r^\top\int_{t-T_l(t)}^{t}\dot{\mathbf{q}}_l(\sigma)d\sigma\end{aligned}$$

Before going further, let us borrow from [19] a lemma that is instrumental in the stability proof.

Lemma 1: [19] For any vector signals $\mathbf{y}, \mathbf{z} \in \mathbb{R}^n$, any variable time-delay $0 \leq T(t) \leq {}^*T < \infty$ and any constant $\alpha > 0$, the following inequality holds

$$-\int_0^t \mathbf{y}^\top(\theta)\int_{\theta-T(\theta)}^{\theta}\mathbf{z}(\sigma)d\sigma d\theta \leq \frac{\alpha}{2}\|\mathbf{y}\|_2^2 + \frac{{}^*T^2}{2\alpha}\|\mathbf{z}\|_2^2.$$

$\Diamond$

Proposition 1: Consider the teleoperator (1)–(2), controlled by (8), (5) and (7). Suppose that Assumption A1 holds and that $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e$ verify (3). Set the control gains such that

$$4d_l d_r > ({}^*T_l + {}^*T_r)^2 k_l k_r \quad (15)$$

Then:

I. Joint and link velocities and position errors are bounded, i.e., $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_r, |\mathbf{q}_r - \boldsymbol{\theta}_r|, |\mathbf{q}_l - \mathbf{q}_r| \in \mathcal{L}_\infty$. Moreover, $\dot{\mathbf{q}}_l, \dot{\mathbf{x}}_r \in \mathcal{L}_2$ and $|\ddot{\mathbf{x}}_r|, |\dot{\mathbf{x}}_r| \to 0$ as $t \to \infty$.

II. If the human and the environment do not inject any forces on the local and the remote manipulators, respectively, i.e. $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$, the local and remote link position error asymptotically converges to zero, i.e., $\lim_{t\to\infty}|\mathbf{q}_l(t) - \mathbf{q}_r(t)| = 0$ and the remote transformed coordinate $\mathbf{x}_r$ satisfies $\lim_{t\to\infty}|\mathbf{x}_r(t) - \mathbf{q}_r(t)| = 0$. Moreover, all velocities asymptotically converge to zero.

Proof: Let us start by integrating, from 0 to t, (14). This yields

$$\begin{aligned}\mathcal{E}(t) - \mathcal{E}(0) &= \int_0^t\left(\dot{\mathbf{q}}_l^\top(\sigma)\boldsymbol{\tau}_h(\sigma) - \frac{k_l}{k_r}\dot{\mathbf{q}}_r^\top(\sigma)\boldsymbol{\tau}_e(\sigma)\right)d\sigma -\\ &-d_l\int_0^t|\dot{\mathbf{q}}_l(\sigma)|^2 d\sigma - k_l\int_0^t\dot{\mathbf{q}}_l^\top(\theta)\int_{\theta-T_r(\theta)}^{t}\dot{\mathbf{x}}_r(\sigma)d\sigma d\theta -\\ &-\frac{k_l d_r}{k_r}\int_0^t|\dot{\mathbf{x}}_r(\sigma)|^2 d\sigma - k_l\int_0^t\dot{\mathbf{x}}_r^\top(\theta)\int_{\theta-T_l(\theta)}^{t}\dot{\mathbf{q}}_l(\sigma)d\sigma d\theta\end{aligned}$$

Using (3) and invoking Lemma 1, to the double integral terms with α_l and α_r, respectively, yields

$$\mathcal{E}(t) + \lambda_l\|\dot{\mathbf{q}}_l\|_2^2 + \lambda_r\|\dot{\mathbf{x}}_r\|_2^2 \leq \mathcal{E}(0) + \epsilon_l + \frac{k_l}{k_r}\epsilon_l, \quad (16)$$

where $\lambda_l := d_l - \frac{k_l}{2}\left(\alpha_l + \frac{{}^*T_l^2}{\alpha_r}\right)$ and $\lambda_r := \frac{k_l d_r}{k_r} - \frac{k_l}{2}\left(\alpha_r + \frac{{}^*T_r^2}{\alpha_l}\right)$. Solving, simultaneously for $\lambda_i > 0$, it holds that

$$\frac{2d_r\alpha_l - k_r{}^*T_r^2}{k_r\alpha_l} > \alpha_r > \frac{k_l{}^*T_l^2}{2d_l - k_l\alpha_l}$$

which yields the inequality $a\alpha_l^2 + b\alpha_l + c < 0$, where $a := 2d_r k_l$, $b := k_l k_r({}^*T_l^2 + {}^*T_r^2) - 4d_l d_r$ and $c := 2d_l k_r{}^*T_r^2$. Solving for $a\alpha_l^2 + b\alpha_l + c = 0$ ensures that the inequality holds, and if $-b > 0$ then there is, at least, one positive solution for α_l. However, first we need to check if real solutions do exist by establishing positivity of the discriminant, i.e., $b^2 - 4ac > 0$. After some algebraic manipulations we get that if (15) holds then $b^2 - 4ac > 0$. Moreover, since $4d_l d_r > ({}^*T_l + {}^*T_r)^2 k_l k_r > ({}^*T_l^2 + {}^*T_r^2)k_l k_r$ then $-b$ is positive. Hence, setting the controller gains such that (15) holds, λ_l and λ_r are strictly positive.

Further, $\mathcal{E}$ is proper (positive definite and radially unbounded) with respect to $\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_r, |\mathbf{x}_r - \mathbf{q}_r|$ and $|\mathbf{q}_l - \mathbf{x}_l|$. This last, Assumption A2, the fact that $\lambda_i > 0$ and (16) prove that $\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_r, |\mathbf{x}_r - \mathbf{q}_r|, |\mathbf{q}_l - \mathbf{x}_r| \in \mathcal{L}_\infty$ and that $\dot{\mathbf{q}}_l, \dot{\mathbf{x}}_r \in \mathcal{L}_2$.

Using Property P3, the fact that $\dot{\mathbf{x}}_r = \dot{\boldsymbol{\theta}}_r - \mathbf{S}_r^{-1}\frac{\partial \mathbf{g}_r(\mathbf{q}_r)}{\partial \mathbf{q}_r}\dot{\mathbf{q}}_r$, and boundedness of $\dot{\mathbf{x}}_r, \dot{\mathbf{q}}_r$ it is proved that $\dot{\boldsymbol{\theta}}_r \in \mathcal{L}_\infty$. Moreover, Property P3 and $|\mathbf{x}_r - \mathbf{q}_r|, |\mathbf{q}_l - \mathbf{x}_r| \in \mathcal{L}_\infty$ also show that $|\boldsymbol{\theta}_r - \mathbf{q}_r|, |\mathbf{q}_l - \mathbf{q}_r| \in \mathcal{L}_\infty$.

Now, $|\mathbf{q}_l - \mathbf{x}_r| \in \mathcal{L}_\infty$, $\dot{\mathbf{q}}_l \in \mathcal{L}_2$ and

$$\mathbf{x}_r - \mathbf{q}_l(t - T_l(t)) = \mathbf{x}_r - \mathbf{q}_l + \int_{t-T_l(t)}^{t}\dot{\mathbf{q}}_l(\sigma)d\sigma,$$

ensure that $|\mathbf{x}_r - \mathbf{q}_l(t - T_l(t))| \in \mathcal{L}_\infty$. This last, boundedness of $\dot{\mathbf{x}}_r$ and $|\mathbf{x}_r - \mathbf{q}_r|$ support the claim that, from (9), $\ddot{\mathbf{x}}_r \in$

$\mathcal{L}_\infty$. Thus, Barbălat's Lemma ensures that $\lim_{t\to\infty} \dot{\mathbf{x}}_r(t) = \mathbf{0}$. This, in turn, shows that

$$\lim_{t\to\infty} \int_0^t \ddot{\mathbf{x}}_r(\sigma)d\sigma = \lim_{t\to\infty} \dot{\mathbf{x}}_r(t) - \dot{\mathbf{x}}_r(0) = -\dot{\mathbf{x}}_r(0).$$

Hence, if $\frac{d}{dt}\ddot{\mathbf{x}}_r \in \mathcal{L}_\infty$ then, by Barbălat's Lemma, $\lim_{t\to\infty} \ddot{\mathbf{x}}_r(t) = \mathbf{0}$. Note that

$$\begin{aligned} \frac{d}{dt}\ddot{\mathbf{x}}_r = & - \mathbf{I}_r^{-1}\left[\mathbf{S}_r(\dot{\mathbf{x}}_r - \dot{\mathbf{q}}_r) - d_r\ddot{\mathbf{x}}_r\right] - \\ & - k_r\mathbf{I}_r^{-1}\left[\dot{\mathbf{x}}_r - (1-\dot{T}_l(t))\dot{\mathbf{q}}_l(t - T_l(t))\right], \end{aligned} \tag{17}$$

thus, Assumption A1 and boundedness of $\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_r$ and $\ddot{\mathbf{x}}_r$ ensures that $\frac{d}{dt}\ddot{\mathbf{x}}_r \in \mathcal{L}_\infty$ as required. This completes Part I of the proof.

For the proof of Part II, assume $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$. From (9), that $\dot{\mathbf{q}}_l, |\mathbf{q}_l - \mathbf{x}_r| \in \mathcal{L}_\infty$ and $\dot{\mathbf{q}}_l \in \mathcal{L}_2$ imply that $\ddot{\mathbf{q}}_l \in \mathcal{L}_\infty$. Thus, invoking Barbălat's Lemma with $\dot{\mathbf{q}}_l \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $\ddot{\mathbf{q}}_l \in \mathcal{L}_\infty$ it is proved that $\lim_{t\to\infty} \dot{\mathbf{q}}_l(t) = \mathbf{0}$. Hence

$$\lim_{t\to\infty} \int_0^t \ddot{\mathbf{q}}_l(\sigma)d\sigma = \lim_{t\to\infty} \dot{\mathbf{q}}_l(t) - \dot{\mathbf{q}}_l(0) = -\dot{\mathbf{q}}_l(0).$$

Furthermore, $\dot{\mathbf{q}}_r, |\mathbf{x}_r - \mathbf{q}_r| \in \mathcal{L}_\infty$ imply, from (9), that $\ddot{\mathbf{q}}_r \in \mathcal{L}_\infty$ and Assumption A1, Property P4 together with $\ddot{\mathbf{q}}_l, \dot{\mathbf{q}}_l, \dot{\mathbf{x}}_r \in \mathcal{L}_\infty$ support the fact that $\frac{d}{dt}\ddot{\mathbf{q}}_l \in \mathcal{L}_\infty$. Hence $\lim_{t\to\infty} \ddot{\mathbf{q}}_l(t) = \mathbf{0}$.

At this point, note that all the terms in the right hand side of (17) converge to zero, except for $\dot{\mathbf{q}}_r$. Hence, if it is proved that $\frac{d}{dt}\ddot{\mathbf{x}}_r$ converges to zero, then $\lim_{t\to\infty} \dot{\mathbf{q}}_r(t) = \mathbf{0}$. For, since $\lim_{t\to\infty} \ddot{\mathbf{x}}_r(t) = \mathbf{0}$, it suffices to prove that $\frac{d^2}{dt^2}\ddot{\mathbf{x}}_r \in \mathcal{L}_\infty$. Which indeed is bounded due to Assumption A1 and boundedness of $\frac{d}{dt}\ddot{\mathbf{x}}_r, \ddot{\mathbf{x}}_r, \ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$. Thus, $|\dot{\mathbf{q}}_r| \to 0$ as $t \to \infty$. From (4), that convergence to zero of $\dot{\mathbf{q}}_r$ and $\dot{\mathbf{x}}_r$ ensure that $\dot{\boldsymbol{\theta}}_r$ asymptotically converges to zero.

Similar to the convergence proof of $\ddot{\mathbf{q}}_l$ it can be easily established that $\lim_{t\to\infty} \ddot{\mathbf{q}}_r(t) = \mathbf{0}$ and, from the closed–loop system, this implies that $\lim_{t\to\infty} |\mathbf{x}_r(t) - \mathbf{q}_r(t)| = 0$. This last and the fact that the signals $\ddot{\mathbf{q}}_l, \dot{\mathbf{q}}_l, \ddot{\mathbf{x}}_r, \dot{\mathbf{x}}_r$ converge to zero ensure the claim that $\lim_{t\to\infty} |\mathbf{q}_l(t) - \mathbf{q}_r(t)| = 0$, as required. This completes the proof. ∎

Remark 1: If the local manipulator has joint flexibility, the local controller can be easily extended using similar terms as in (5) and (7). In such a case, the local kinetic energy (10) transforms to

$$\mathcal{K}_l(\dot{\mathbf{q}}_l) = \frac{1}{2}\dot{\mathbf{q}}_l^\top \mathbf{M}_l(\mathbf{q}_l)\dot{\mathbf{q}}_l + \frac{1}{2}\dot{\mathbf{x}}_l^\top \mathbf{J}_l\dot{\mathbf{x}}_l,$$

and a new potential energy term arises, i.e.,

$$\mathcal{U}_l(\mathbf{q}_l, \mathbf{x}_l) = \frac{1}{2}(\mathbf{x}_l - \mathbf{q}_l)^\top \mathbf{S}_l(\mathbf{x}_l - \mathbf{q}_l).$$

The boundedness and convergence proofs are established following *verbatim* the proof of Proposition 1.

Remark 2: Interestingly, but not surprisingly, the stability condition (15) is the same as that for bilateral teleoperators with rigid joints, see [19], [2]. This fact may be due to the passive interconnection between the actuated and the non-actuated parts of the flexible joint manipulators.

IV. Simulations

By means of some numerical simulations, this section shows the effectiveness of the gravity compensation scheme for bilateral teleoperators with joint flexibility. The local manipulator contains only rigid joints and the remote manipulators only flexible joints. Both manipulators have two revolute DOF. Their corresponding nonlinear dynamics are modeled by (1) and (2). The inertia matrix, the Coriolis and centrifugal effects matrix and the gravity vector are given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i + 2\beta_i c_{i_2} & \delta_i + \beta_i c_{i_2} \\ \delta_i + \beta_i c_{i_2} & \delta_i \end{bmatrix},$$

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \begin{bmatrix} -2\beta_i s_{i_2}\dot{\mathbf{q}}_{i_2} & -\beta_i s_{i_2}\dot{\mathbf{q}}_{i_2} \\ \beta_i s_{i_1}\dot{\mathbf{q}}_{i_2} & 0 \end{bmatrix}$$

and $\mathbf{g}_i(\mathbf{q}_i) = \mathrm{col}(g_{1i}, g_{2i})$, respectively, where $g_{1i} := l_{i_1}(m_{i_1}+m_{i_2})c_{i_1} + gl_{i_2}m_{i_2}c_{i_{12}}$; $g_{2i} := gl_{i_2}m_{i_2}c_{i_{12}}$); c_{i_k}, s_{i_k} are the short notation for $\cos(\mathbf{q}_{i_k})$ and $\sin(\mathbf{q}_{i_k})$; $c_{i_{12}}$ stands for $\cos(q_{i_1}+q_{i_2})$; $\mathbf{q}_{i_k}$ represents the angular position of link k of manipulator i, with $k \in 1, 2$; $\alpha_i = l_{i_2}^2 m_{i_2} + l_{i_1}^2(m_{i_1}+m_{i_2})$, $\beta_i = l_{i_1}l_{i_2}m_{i_2}$ and $\delta_i = l_{i_2}^2 m_{i_2}$, where l_{i_k} and m_{i_k} are the respective lengths and masses of each link and $g = 9.81$ is the acceleration of gravity constant.

The remote motor inertia is $\mathbf{I}_r = \mathrm{diag}(0.39, 0.39)$ and the stiffness of the remote flexible joints is $\mathbf{S}_r = \mathrm{diag}(200)$. The rest of the physical parameters for both manipulators are: the length of the links, $l_{i_1} = l_{i_2} = 0.38$ and, the masses, $m_{l_1} = 3.5$, $m_{l_2} = 0.5$, $m_{r_1} = 0.5$ and $m_{r_2} = 0.35$ (all units are in the *SI*). The initial conditions have been set to $\dot{\mathbf{q}}_i(0) = \dot{\boldsymbol{\theta}}_r(0) = 0$, $\mathbf{q}_l(0) = \left[\frac{-1}{8}\pi, \frac{1}{8}\pi\right]^\top$ and $\mathbf{q}_r(0) = \boldsymbol{\theta}_r(0) = \left[\frac{1}{5}\pi, \frac{-1}{2}\pi\right]^\top$.

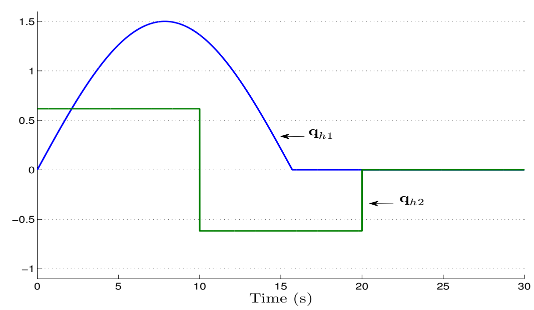

Fig. 1. Desired trajectory of the human operator.

The human operator is modeled as a spring-damper system $\boldsymbol{\tau}_h = K_h(\mathbf{q}_h - \mathbf{q}_l) + B_h\dot{\mathbf{q}}_l$, where $\mathbf{q}_h$ is the desired human link position, shown in Fig. 1. K_h and B_h are the spring and damper gains, respectively, and have been set to $K_h = 10$ and $B_h = 2$.

For simplicity, the variable time-delays are the same in both directions, i.e., $T_l(t) = T_r(t)$. Fig. 2 shows such delays and a sinusoidal signal sent through the communication channel and received with this delay. This delay aims at emulating the behavior of Internet-based communications. Their upper bound is $^*T_i = 0.42$s. The controller gains are set to $d_l = 2$, $k_l = 6$, $d_r = 9$ and $k_r = 11$. Easy calculations show that the controllers gains satisfy (15).

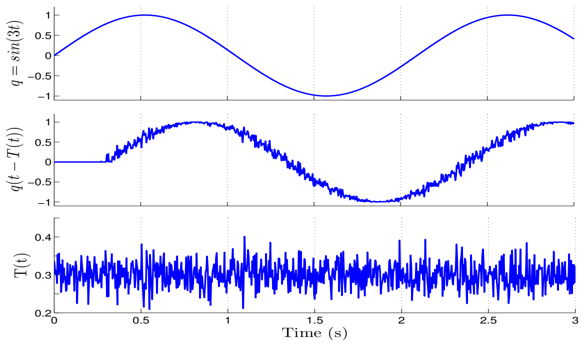

Fig. 2. Variable time-delays.

The first set of simulations presents the results when the human operator injects forces to the local manipulator and the remote manipulator moves freely in space, i.e., $\boldsymbol{\tau}_e = \mathbf{0}$. Fig. 3 shows the local and remote joint positions together with the local and remote position error. Despite the presence of time-delays and the fact that both have different initial positions, when time evolves, position error asymptotically converges to zero. Fig. 4 depicts the tracking results in Cartesian coordinates.

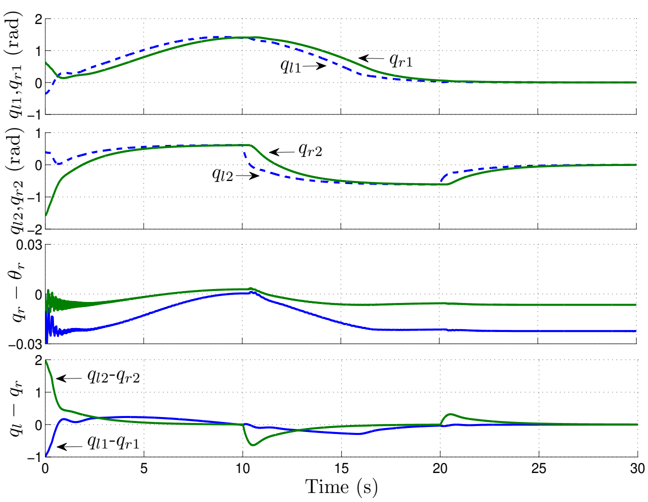

Fig. 3. Articular position and error for the local and remote manipulator.

In the second set of simulations, the human manipulator injects forces to the local manipulator and the remote manipulator comes into contact with a stiff wall. This wall is modeled as a stiff spring-damper system with stiffness equal to $10,000$Nm, and damping equal to 200 Nms, and it is located at 0.5m in the y-coordinate. The local and remote joint positions, together with the position error can be seen in Fig. 5. Fig. 6 shows the position evolution in Cartesian coordinates. From these figures, it can be seen that, around second 4, even when the remote manipulator comes into contact with the wall, position tracking is established and hence the position error asymptotically converges to zero when the human and environment forces become zero. The controller torques are shown in Fig. 7.

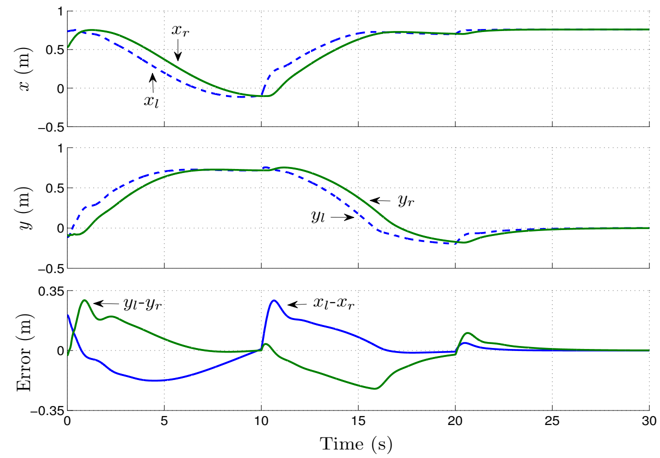

Fig. 4. Cartesian local and remote position and Cartesian position error.

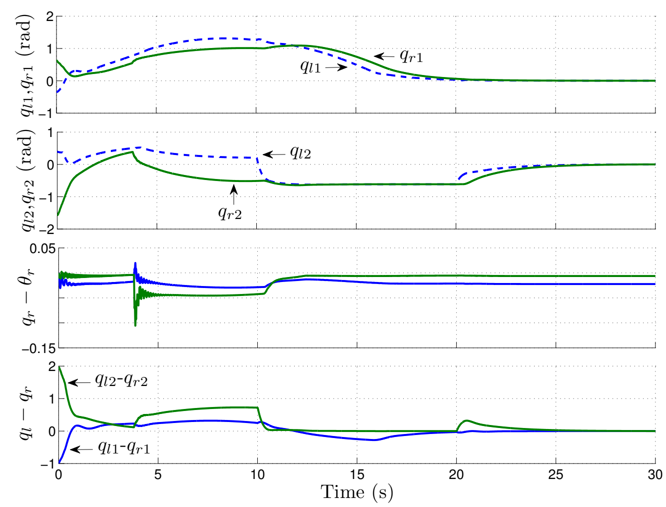

Fig. 5. Local and remote articular position and error when interacting with a stiff wall.

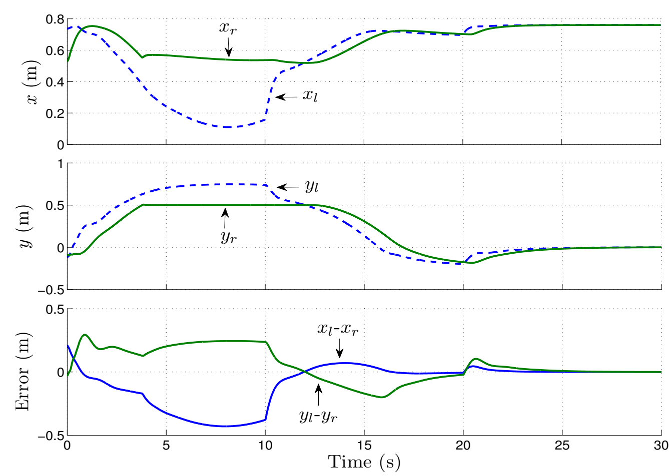

Fig. 6. Cartesian local and remote positions and Cartesian position error when interacting with a stiff wall.

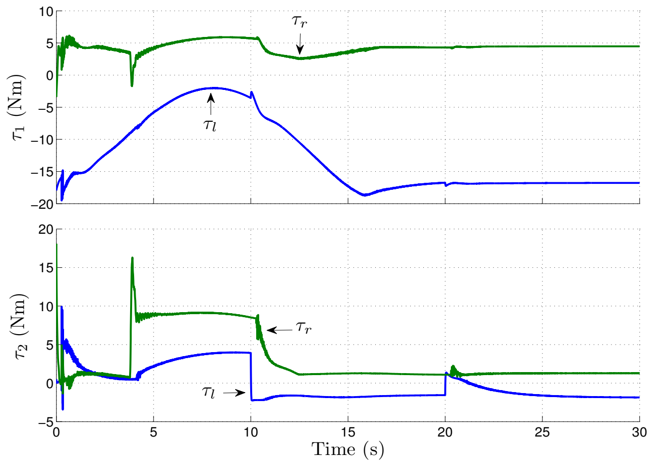

Fig. 7. Local and remote controller torques when interacting with a stiff wall.

V. Conclusions

This paper reports a new controller which dynamically compensates the gravity drift effects in bilateral teleoperators composed of flexible joint robot manipulators with variable time-delays in the communications. The solution to this long-lasting control problem consists of using a proportional plus damping controller together with an exact gravity compensation term inspired in the work of De Lucca and Flacco [26], [27]. Assuming that the human operator and the environment are passive, it is shown that (joint and link) velocities and position errors are bounded. Furthermore, if the human operator and the environment forces are zero, asymptotic convergence to zero of (joint and link) velocities and local and remote position error is ensured provided that sufficiently large damping is injected in the system. The theoretical results are supported by simulations with a rigid local manipulator and a flexible joint remote manipulator which interacts with a stiff wall.

The next working steps are the real implementation of the controllers using two 3-DOF manipulators and the extension to the more general case of synchronization and consensus of networks with multiple flexible joint manipulators.

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